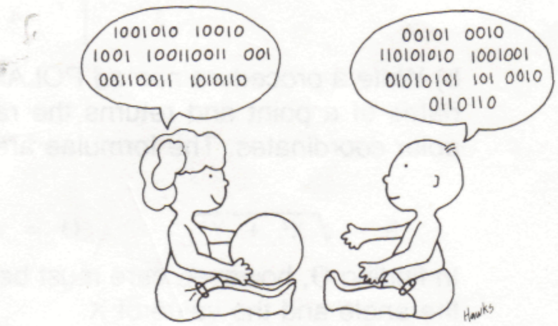
17. a) A simple but often useful function is called the SIGNUM function which returns +1 if its argument is positive, −1 if its argument is negative, and 0 if its argument is exactly zero. Write the SIGNUM function as a Pascal function which takes a REAL parameter and returns an INTEGER.

    b) Show how SIGNUM can be used to simplify the complicated IF..THEN in Program 5.2 by rewriting the ROOT function.

18. Write a recursive procedure SUMUP which returns the sum of all positive integers less than or equal to its argument.



Advanced Exercise

19. Write a program that will solve a non-obtuse triangle (compute the unknown sides and angles) for the following situations. The law of sines and the law of cosines are needed to solve this problem.

    a) Given two sides and the included angle:

    Sides 1 and 2: 3 3
    Included angle: 60

Side	Angle
3.00	60.00
3.00	60.00
3.00	60.00

    b) Given two angles and the included side:

    Angles 1 and 2: 20 70
    Side: 25

Side	Angle
8.55	20.00
23.49	70.00
25.00	90.00

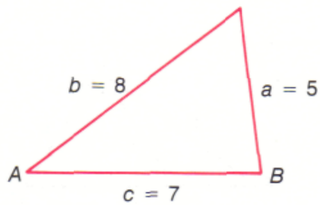
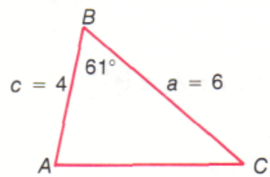
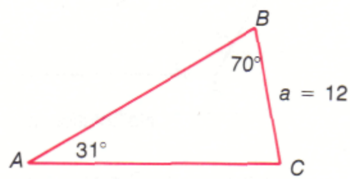
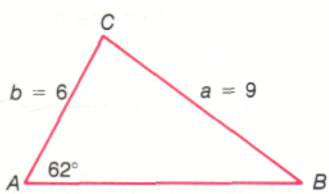
    c) Given three sides:

    Three sides: 3 4 5

Side	Angle
3.00	36.87
4.00	53.13
5.00	90.00

4-3 The Ambiguous Case

To apply either the Law of Sines or the Law of Cosines to solving a triangle, three parts of the triangle must be known.

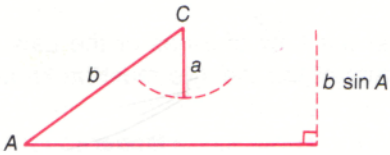
Given	Model	Procedure
1. Three sides		Use the Law of Cosines.
2. Two sides and the included angle		Use the Law of Cosines.
3. Two angles and any side		Use the Law of Sines.
4. Two sides and an angle opposite one side		Use the Law of Sines (Ambiguous Case).

The last case in the table is called the **ambiguous case** because there may be no triangle, one triangle, or two triangles satisfying the given conditions.

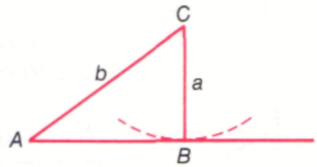
The diagrams below illustrate the various possibilities, given A, b, and a. They are divided into two cases:

$$A < 90^\circ \quad \text{and} \quad A \geq 90^\circ.$$

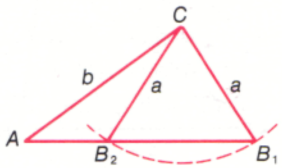
Case I: $A < 90^\circ$



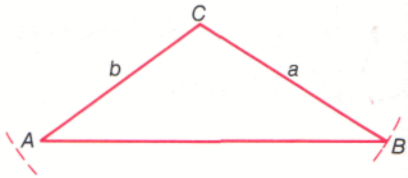
No solution: $a < b \sin A$



One solution: $a = b \sin A$

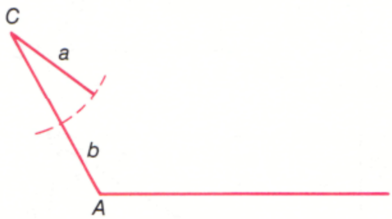


Two solutions: $a < b$
and $a > b \sin A$

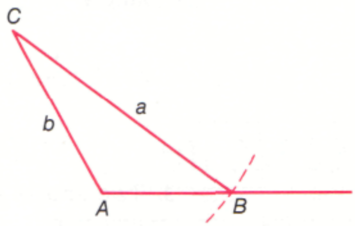


One solution: $a \geq b$

Case II: $A \geq 90^\circ$



No solution: $a \leq b$



One solution: $a > b$

The Law of Sines can be used to solve a triangle for which the data are ambiguous. However, it is important to sketch and label the triangle first. Then determine the number of possible solutions.

Example 1

Find the number of solutions for each triangle.

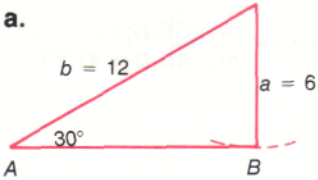
a. $A = 30^\circ$
$a = 6$
$b = 12$

b. $A = 30^\circ$
$a = 8$
$b = 12$

c. $A = 30^\circ$
$a = 4$
$b = 12$

Solutions: Sketch each triangle.

a.

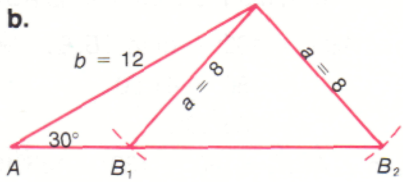


$\sin 30° = \frac{1}{2}$

$\therefore a = b \sin A$

One Solution

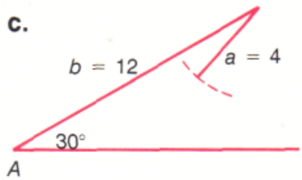
b.



$a < b$ and $a > b \sin A$

Two Solutions

c.



$a < b \sin A$

No Solutions

When there are two solutions, you must solve both triangles.

Example 2

In $\triangle ABC$, $A = 30°$, $a = 15$, and $b = 20$.

a. Find B to the nearest ten minutes.

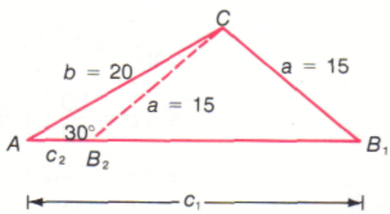
b. Find c to the nearest tenth.

Solutions: **a.** Since $\sin 30° = \frac{1}{2}$ and $b \sin 30° = 10$,

$b \sin 30° < a < b$ ⟵ $10 < 15 < 20$

Thus, there are two solutions, corresponding to two triangles AB_1C and AB_2C.

First, solve $\triangle AB_1C$.



$$\frac{a}{\sin 30°} = \frac{b}{\sin B_1}$$

$$\frac{15}{\frac{1}{2}} = \frac{20}{\sin B_1}$$

$$\sin B_1 = \frac{10}{15} = \frac{2}{3}$$

B_1 is about $41°50'$.

$\therefore m\angle ACB_1$ is about $180° - (30° + 41°50')$, or $108°10'$.

b. Now find c_1, where $c_1 = AB_1$.

$$\frac{c_1}{\sin(108°10')} = \frac{15}{\sin 30°}$$

$$c_1 = \frac{\sin(108°10') \cdot 15}{\sin 30°}$$

$$= \sin(71°50') \cdot 30$$

$$= (.9502) \cdot 30$$

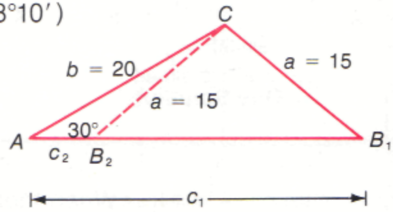
$$= 28.5$$ ⟵ To the nearest tenth

a. $\triangle B_1CB_2$ is isosceles. (Refer to the figure below.) Thus, its base angles have equal measures. In $\triangle AB_2C$,

$$\begin{aligned} m\angle AB_2C &= 180° - m\angle CB_2B_1 \\ &= 180° - 41°50' \\ &= 138°10'. \end{aligned}$$

$m\angle CB_2B_1 = m\angle AB_1C = 41°50'$

Thus, $m\angle ACB_2 = 180° - (30° + 138°10')$
$= 11°50'.$



b. Now find c_2, where $c_2 = AB_2$.

$$\frac{c_2}{\sin ACB_2} = \frac{a}{\sin A}$$

$$\begin{aligned} c_2 &= \frac{\sin(11°50')\cdot 15}{\sin 30°} \\ &= (.2051)(30) \\ &= 6.2 \end{aligned}$$

To the nearest tenth

Example 3

In $\triangle ABC$, $C = 121°$, $c = 40$ and $b = 30$. Find B.

Solution: Since $C = 121°$, $C > 90°$. Also, $c > b$. Thus, there is one solution.

$$\frac{\sin B}{30} = \frac{\sin 121°}{40}$$

$\sin 121° = \sin(180° - 59°) = \sin 59°$

$$\sin B = \frac{30}{40}\sin 59°$$

$$\sin B = .6429$$

$$B = 40°$$

B equals either 40° or 140°. Since $C > 90°$, $B \neq 140°$.