International Geopolitics*

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Abstract

Since the Age of Exploration, the world has become economically integrated while remaining politically disintegrated as a collection of nation-states. The nation-state system remains robust because borders, which divide the global landmass into states, interact with economic integration to absorb shocks. We build a general equilibrium model of international trade and national borders. Over a long time horizon, declining trade costs incentivize the redrawing of borders, leading to the formation, modification, and dissolution of states. Our model has significant implications for the economic and political geography of the world, revealing location-specific geopolitics both within and across states. The border patterns predicted by the model are consistent with the data from digitized historical maps.

Keywords: endogenous borders, trade costs, gravity model

JEL Classification Numbers: F50, P16, N40.

1 Introduction

The Age of Exploration created connections between parts of the world that had previously been isolated. A time traveler from the 18th century might have mixed feelings about the present world. Economically, the world has become remarkably integrated. Thanks to low trade costs, consumers buy goods globally and producers sell worldwide. However, politically, the world remains disintegrated: politics are often local, policies are at most regional, and nation-states still serve as the fundamental units of global affairs. She would quickly grasp the political map of our time, just as we readily understand the one from her era.

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Although the nation-state system persists, individual states have come and gone. The system acts as a stage where states form, change, and dissolve, periodically redrawing the world's political boundaries. In this paper, we present a general equilibrium model of international trade and national borders. Consider a linear world populated by a continuum of locales. These locales form states based on a tradeoff between the gains from trade and the challenges of governance. Domestic trade is less costly than foreign trade, encouraging locales to choose larger states, while larger states face more internal conflicts, leading locales to opt for smaller states. Essentially, the nation-state system functions as a market of statehood where locales select others with whom to form states.

While the terms world, states, and borders originate from the nation-state system, they can also represent other political structures. For example, if the world is a metropolis composed of multiple districts, then the states refer to the districts within the metropolis. As long as cross-district business costs are higher than within-district business costs, our model remains applicable. At the core of the model are differential locational advantages across locales. Locales closer to other locales have stronger locational advantages. Joining the state of neighbors with greater proximity to the rest of the world becomes a dominant strategy for every locale. As shown in Section 2, the nation-state system, as a set of world partitions, has a unique equilibrium.

When the terms world, states, and borders are interpreted literally, our model offers a framework to analyze international economics and politics in relation to each other, as illustrated in Section 3. On the economic front, our model demonstrates that the relationship between trade volume and trade costs is more complex than previously thought. It is well-known that trade volume between two countries increases when trade costs decrease. However, this is just one of three effects involved. When unit trade costs decrease, states become smaller because they no longer need large domestic markets to maintain low consumption prices. Consequently, states tend to trade less with each other due to their reduced economic sizes, but they also tend to trade more because the states between them shrink, bringing them geographically closer. These additional effects arise from border adjustments and are concisely depicted by a long-run gravity equation that allows for changing borders. On the political front, our model illustrates the sensitivity of regions geographically close to the rest of the world in the process of globalization. According to our model, national borders with the highest proximity to the rest of the world face the most pressure to change. To our knowledge, our model is the first to reveal location-specific geopolitics both within and across states.

Despite its highly stylized nature, our model closely mirrors the distribution of national borders worldwide. In Section 4, we identify two data patterns in digitized historical maps that are consistent with our model. Although the earth is spherical, the landmass on its surface is fragmented and irregularly shaped, creating locational advantages for some locales and disadvantages for others. Therefore, our model provides a reasonable approximation of real-world political geography.

We take a bold step to endogenize nations in the context of international trade. The literature has examined the relationship between international trade and various modern institutions. Domestic institutions influenced by international trade include checks and balances (Acemoglu, Johnson, and Robinson, 2005), parliamentary operations (Puga and Trefler, 2014), democratization (Galiani and Torrens, 2014), military operations (Acemoglu and Yared, 2010; Bonfatti and O'Rourke, 2018; Martin, Mayer, and Thoenig, 2008; Skaperdas and Syropoulos, 2001), contract enforcement (Anderson, 2009; Anderson and Marcouiller, 2002; Ranjan and Lee, 2007), and societal organization (Greif, 1994). There are also extensive studies on the relationship between international trade and international institutions, such as Baier and Bergstrand (2002, 2004), Egger, Larch, Staub, and Winkelmann (2011), Galiani and Torrens (2021), Guiso, Herrera, and Morelli (2016), Keller and Shiue (2014), and Krishna (2003). Notice that all modern institutions, both domestic and international, rest on nation-states as their fundamental units. The rise of nation-states ended feudalism and initiated modern international relations. Our study is a theory of nation states *à la* international trade.

Our study also contributes to the literature on the efficient size of states (Alesina and Spolaore, 1997, 2005, 2006; Alesina, Spolaore, and Wacziarg, 2000, 2005; Brennan and Buchanan, 1980; Desmet, Le Breton, Ortuño-Ortín, and Weber, 2011; Friedman, 1977; Gancia, Ponzetto, and Ventura, 2022). Our modeling of the tradeoff between trade and governability builds on a similar setup in Alesina, Spolaore, and Wacziarg (2000, 2005). In this literature, when trade and state sizes are interdependent, analytical solutions are not obtainable without assuming symmetric state sizes. However, assuming symmetric state sizes would hinder the model's ability to analyze real-world economies and polities. In reality, state sizes are heterogeneous, and this is precisely why small and large countries face disparate economic and political situations. To introduce state asymmetry tractably, we convert the state-sizing problem into a border-drawing problem. Similar to modern cartography, which partitions a two-dimensional landmass into states (polygons) using two-dimensional dividers (lines), we use one-dimensional dividers (points) to divide a one-dimensional landmass into states (intervals). This one-degree reduction in dimensionality provides solvable borders and tractable characterizations of every locale's common interests with every other locale, within their own state, and with contiguous states, enabling us to unify economies and polities in one single model.

There is a small but rapidly growing body of literature that simulates world-border formation (Allen, 2023; Fernández-Villaverde, Koyama, Lin, and Sng, 2023; Turchin, Currie, Turner, and Gavrilets, 2013; Weese, 2016). Since borders are interdependent, the analysis of borders often triggers the curse of dimensionality. Efficient algorithms and fast computation can mitigate this challenge, as shown in this literature. Among the extant studies, the most relevant to ours is Allen (2023), which provides conditions under which a simple algorithm can solve borders to minimize trade costs for any given geography. Our model takes an opposite approach to this literature. Economics has a long tradition of using a one-dimensional "world" to highlight key mechanisms in high-dimensional problems, such as Hotelling (1929) on spatial competition, Black (1948) and Downs (1957) on majority-rule voting, Ogawa and Fujita (1980) on urban structures, and Dornbusch, Fischer, and Samuelson (1977) on comparative advantage. We follow this tradition to demonstrate, with a linear world, how locational advantages confer power to economic agents in the global economy and polity. The goal of our model is not to produce accurate borders but to solve for minimalist borders that can produce geopolitics.

Lastly, our model adds to the gravity model literature. Recent gravity models underscore the importance of including remoteness terms, also known as multilateral resistance, to account for general equilibrium effects in bilateral trade analysis (Allen, Arkolakis, and Takahashi, 2020; Anderson and van Wincoop, 2003; Head and Mayer, 2014). Our linear-world gravity model integrates general equilibrium effects without these remoteness terms, maintaining the elegant form of Newton's law of universal gravitation. A pedagogical tool to visualize the gravity model with general equilibrium effects has long been overdue. To our knowledge, we are the first to provide such a tool.

The rest of the paper is structured as follows. In Section 2, we present our theoretical model. In Section 3, we discuss our model's implications on international trade and security. In Section 4, we derive world-border statics and dynamics from our model and link them to data from digitized historical maps. In Section 5, we conclude.

2 Theory

2.1 Environment

Consider a world represented by a continuum of locales, indexed by $t \in [-1, 1]$. The midpoint locale $t = 0$, serving as the World Geometric Center (WGC), divides the world into left and right hemispheres. In each hemisphere, there are two directions: proximal (towards the WGC) and distal (away from the WGC). While our analysis primarily uses the right hemisphere $[0, 1]$ as an example, it applies equally to the left hemisphere.

All locales have identical quantities of land z and initial labor l^0, both inelastically supplied to produce locale-specific differentiated goods. Locales utilize equally efficient technologies, represented by

$$y(t) = z(t)^{\alpha} l(t)^{1-\alpha}, \tag{1}$$

where $0 < \alpha \leq 1$, $z(t)$ represents the land at locale t, and $l(t)$ the labor at locale t. The land $z(t)$ is immobile (i.e., affixed to locale t) and owned by the lord of locale t. Labor within a state (defined later) migrates freely. Firms compete perfectly in production and sales.

At each locale t, the lord and labor have an aggregate consumption of $C(t) = C^z(t) + C^l(t)$. The lord has utility function

$$U(t) = \frac{1}{1-\gamma} C^z(t)^{1-\gamma} - hS(n_t), \tag{2}$$

where $\gamma > 1$, $h > 0$, and h represents a constant marginal disutility h from its state's size $S(n_t)$, where n_t represents the state to which locale t belongs. As in Alesina et al. (2000, 2005), the disutility from state size results from the heightened domestic conflicts in larger states. The labor has utility function

$$V(t) = \frac{\psi}{1-\gamma} C^l(t)^{1-\gamma}, \tag{3}$$

where $\psi > 0$ is a free scalar that allows a potential difference in their marginal utility of consumption. The consumption of both the lord and the labor consists of all goods produced globally, aggregated using a Cobb-Douglas function:

$$C(t) \equiv \exp\left(\int_{-1}^{1} \ln c(t,s)ds\right), \tag{4}$$

where $c(t,s)$ is the quantity of the good made by locale s and consumed at locale t.

Without loss of generality, we assume that consumers bear the trade costs. Specifically, to consume $c(t,s)$ units of the good produced at locale s, consumers at locale t pay for $c(t,s)$ and a trade cost. Following Alesina et al. (2000), we specify the trade cost as

$$d(t,s) = \exp(a(t,s)g(t,s)), \tag{5}$$

where $a(t,s)$ represents a political cost and $g(t,s)$ a physical cost. As in their analysis, we assume $a(t,s) = 0$ for domestic trade and $a(t,s) = 1$ for foreign trade. Unlike their symmetric locales, our locales differ in locations such that $g(t,s) = \tau|s-t|$, where $\tau > 0$ is the physical cost per unit of distance.[1] Thus,

$$d(t,s) = \begin{cases} 1, & \text{if } s \in n_t, \\ \inf_{\tilde{t}\in n_t} \exp(\tau\,|s-\tilde{t}|), & \text{if } s \notin n_t, \end{cases} \tag{6}$$

where n_t refers to the state to which locale t belongs. The limit inferior $\inf_{\tilde{t}\in n_t}$ indicates that trade costs apply only beyond the farthest national border.[2] Also, as in Alesina et al. (2000), $d(t,s)$ is in the iceberg form: only one unit of the good reaches locale t if $d(t,s) \geq 1$ units are shipped from locale s to locale t.

The world is partitioned into states by a set of borders:

$$\{b_n\} \equiv \{b_{-N}, ..., b_{-1}, b_{-0}, b_0, b_1, ..., b_N\}, \tag{7}$$

where $b_{-0} \leq 0 \leq b_0$ and $-1 \leq b_{-N} < b_N \leq 1$. Hereafter, State $n > 0$ ($n < 0$) denotes a state in the right (left) hemisphere with size $S_n > 0$:

$$S_n = b_n - b_{n-1}. \tag{8}$$

State 0, referring to the state formed by b_{-0} and b_0, has size $S_0 = b_0 - b_{-0}$ and is in both hemispheres. Note that in both hemispheres, the left border of state n is b_{n-1}, the right border b_n. As a convention, we let the distal (proximal) border of every state be an open (closed) endpoint. For example, if state n is in the right hemisphere, its left border b_{n-1} is its territory while its right border b_n is not its territory (but state $n+1$'s territory).

[1]To introduce physical trade costs that vary with distance, we follow the trade literature to use the exponential function form (see Anderson and van Wincoop (2004) for a review), micro-founded by the aggregation of incremental iceberg costs as the distance between the increments tends to zero (Allen and Arkolakis, 2014).

[2]The assumption of zero domestic trade costs is not critical in our context, which will be discussed in Section 2.4.

2.2 Definition of equilibrium

The lord of each locale determines to which state the locale belongs. The timing of events is as follows.

Date 1 The lord of locale t makes offers to the lords of his proximal (to the WGC) locales and himself to be his overlord. If any of these offers is accepted by a proximal lord, that lord becomes his overlord. If multiple offers are accepted by proximal lords, the most proximal one among them becomes his overlord. If none of the offers are accepted by proximal lords, the lord becomes his own overlord and decides whether to accept any offer from his distal locales. All lords act simultaneously to make offers, then decide on offers, and settle their roles as overlords or (ordinary) lords.

Date 2 Overlords decide on the state sizes for themselves and distal neighbors whose offers they have taken.

Date 3 Production and consumption occur. Lords (including overlords) and labor produce goods, which are then traded and consumed worldwide.

The three dates occur sequentially, with each providing information necessary for the subsequent one. On each date, information is complete, making the order of activities irrelevant. For example, offers are made and then accepted on date 1, and goods are produced and then consumed on date 3. Such first-then orders within a date do not affect the equilibrium, since all agents know precisely what is happening on the date.

The offer rules we set for Date 1 reflect the importance of locational advantages. On the line segment $[-1, 1]$ representing the world, the midpoint $t = 0$ (i.e., the aforementioned WGC) has the shortest total distance to the rest of the world, while locale t has a total distance of $t^2 + 1$ to the rest of the world. Given that domestic trade is costless, locales prefer to be in the same state as their proximal neighbors to minimize trade costs (recall equation (6)). Additionally, since locales are located on a continuum, we require that locales form states only with their adjacent locales (i.e., enclaves are not allowed).

The limited sizes of states stem from utility function (2) of the lords. A larger state size raises consumption by reducing foreign trade costs, but accommodating more locales causes more domestic conflicts. The term $-hS(\cdot)$ in the utility function, as first introduced by Alesina et al. (2000, 2005), counteracts the overgrowth of state sizes. When τ is sufficiently large or h is sufficiently small, borders disappear, and a single global state would encompass all locales. Conversely, when τ is sufficiently small or h is sufficiently large, each locale becomes its own state. Our research interest lies in the scenario where τ and h are neither too large nor too small. Now we introduce

Definition 1. *$\{b_n^*\}$ is equilibrium world partitions if it satisfies criteria (i) and (ii):*

$$\text{(i) } b^L(t) = \sup\{b_n^* | b_n^* < t\} \text{ and } b^R(t) = \inf\{b_n^* | b_n^* > t\} \text{ for any } t \in [-1, 1], \tag{9}$$

and

$$\text{(ii) For any } \widehat{b}^L(t) \neq b^L(t) \text{ and } \widehat{b}^R(t) \neq b^R(t)\text{, if } U(t|\widehat{b}^L(t), \widehat{b}^R(t)) > U(t|b^L(t), b^R(t)),$$
$$\text{there must be at least one } t' \neq t \text{ such that } U(t'|\widehat{b}^L(t), \widehat{b}^R(t)) < U(t'|b^L(t), b^R(t)). \quad (10)$$

In the definition, criterion (i) technically links locales with their states through geometric coordinates. Locales in the same state share left and right borders, established by their overlord. In equilibrium, lords located at geometric coordinates $\{b_n^*\}$ are the overlords. Criterion (ii) ensures that no overlord can benefit from deviating from the equilibrium partitions without harming other overlords. Since labor does not participate in the drawing of borders, we hereafter use the terms locale and lord interchangeably in border affairs. Additionally, since borders imply states, the terms equilibrium world partitions (i.e., $\{b_n^*\}$) and equilibrium states are equivalent. If excluding a locale from a state can improve the welfare of other locales in that state, then the state with the included locale is not part of the equilibrium partitions. Moreover, even if all locales in a state agree on the state's borders, the state is still not part of the equilibrium partitions if there exists one foreign locale that wants to join the state and allowing it to join does not harm any existing locale.

To close the model, we need the markets of goods to be in equilibrium. Given that all locales have identical factor endowments, the Cobb-Douglas technology, and the Cobb-Douglas consumption structure, we set the factory-gate price of all goods to be p, regardless of their origins. Firms at locale s are indifferent to the destination of their sales. The market clearing condition for the good produced at locale s is given by

$$\int_{-1}^{1} y(t,s)dt = y(s), \tag{11}$$

where $y(s)$ is the total output of locale s. The market-clearing condition (11) is invariant across origins (e.g., between s and s'), ensured by the Cobb-Douglas consumption structure (4) and the fact that consumers pay trade costs. Now we are ready to define an equilibrium of the model:

Definition 2. *An equilibrium of the model takes the form of*

$$\Omega \equiv \left(b^L(t), b^R(t), C^z(t), C^l(t), l(t), y(t), \forall t \in [-1,1]\right), \tag{12}$$

where

$$\{b^L(t), b^R(t), \forall t\} = \{b_n^*\}. \tag{13}$$

Since the model has three dates, the equilibrium is a subgame perfect equilibrium.

2.3 Solving the equilibrium

The model can be solved by backward induction.

Solving Date 3. On this date, production is conducted at every locale, and all lords and labor in the world as consumers purchase goods worldwide. At locale t, the lord maximizes utility function (2) subject to

$$\int_{-1}^{1} p(t,s)c^z(t,s)ds = r(t)z(t), \tag{14}$$

where $r(t)$ is the rental price of land at locale t, while the labor maximizes utility function (3) subject to

$$\int_{-1}^{1} p(t,s)c^l(t,s)ds = w(t)l(t), \tag{15}$$

where $w(t)$ is the wage rate at locale t. Their total expenditure on the good made by locale s equals[3]

$$\kappa(t) \equiv p(t,s)c(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} + \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)}, \tag{16}$$

where $p(t,s)$ is the delivery price of the good produced by locale s and consumed at locale t, while $\lambda^z(t)$ and $\lambda^l(t)$ are the shadow prices. By taking the integral of equation (16) across destination locale t, we obtain the GDP of the good's origin locale s:

$$py(s) = \int_{-1}^{1} p(t,s)c(t,s)dt = \int_{-1}^{1} \kappa(t)dt \equiv \kappa. \tag{17}$$

Since term $\int_{-1}^{1} \kappa(t)dt$ in equation (17) does not vary by s, $y(s)$ is locale-invariant as well. Intuitively, trade costs are all paid by consumers and thus the Cobb-Douglas consumption structure ensures that all locales face the same global demand. Thus, we denote locale GDP uniformly by κ in equation (17). Then, the income of lord and labor follows:

$$r(s)z(s) = \alpha py(s) = \alpha\kappa, \text{ and } w(s)l(s) = (1-\alpha)py(s) = (1-\alpha)\kappa, \tag{18}$$

where $l(s)$ is the labor supply at locale s.

On date 3, every state has a statewide labor market. In this market, the total labor supply is the aggregate of the initial labor l^0 across locales of the state, while the total labor demand is the aggregate of the $l(s)$ in equation (18) across locales of the state. Since land is immobile within a state, the resulting wage rate is equalized across locales within the state. That is, for any state n, its initial labor will be distributed uniformly across locales in equilibrium:

$$l(s) = l(s') \text{ for any } s, s' \in n, \text{ and } \int_{s\in n} l(s)ds = \int_{s\in n} l^0(s)ds. \tag{19}$$

Then we have

$$y(s) = z^{\alpha} {l^0}^{1-\alpha}, \text{ and } py(s) = r(s)z + w(s)l^0, \tag{20}$$

[3]See Appendix A.1.1 for the derivation.

for any locale s in the world.

Three observations are noteworthy. First, the locale-level variables in equations (18)–(20) are equalized across locales of the world, so they can be written as $\{p, y, r, z, w, l\}$ hereafter. Second, $\{p, y, r, z, w, l\}$ holds true regardless of the partition of the world, allowing all agents in the model to have perfect foresight regarding these variables on the two prior dates. Third, what remains unsolved in the equilibrium Ω are $C^z(t)$, $C^l(t)$, and $\{b_n^*\}$. $C^z(t)$ and $C^l(t)$, despite being consumed on date 3, can immediately be determined once the world partitions $\{b_n^*\}$ are established. We now move on to Date 2 to solve for the $\{b_n^*\}$ established by overlords.

Preparation for Solving Date 2. As solved above, there are three locale-level prices, $p(t)$, $r(t)$, and $w(t)$, and they are all locale-invariant: $p(t) = p$, $r(t) = r$, and $w(t) = w$. Thus, we can conduct normalization with one of them. Now we introduce

Lemma 1. *Normalize $p = rz/2$ then*

$$C^z(t) = 1/R(t), \tag{21}$$

where

$$R(t) \equiv \exp\left(\int_{-1}^{1} \ln d(t,s)ds\right). \tag{22}$$

Proof. See Appendix A.1.2. □

The $R(t)$ in Lemma 1, defined as an aggregate of locale t's bilateral distance from the rest of the world, measures locale t's *remoteness* from the rest of the world. It can alternatively be interpreted as the price index faced by locale t's consumers. Since all lords in the world have the same income, their real income can be expressed as $1/R(t)$ after the normalization. The function $R(t)$ greatly facilitates the analysis of the world partitions $\{b_n^*\}$ due to its following properties.

First, $R(t)$ applies to all locales in the same state as locale t. This is because domestic trade cost is zero (recall equation (6)). Denote the state of locale t by n_t, then

$$R(t) = \exp\left(\int_{-1}^{b_{n_{t-1}}} \tau(b_{n_t-1} - s)ds + \int_{b_{n_t}}^{1} \tau(s - b_{n_t})ds\right) \tag{23}$$

$$= \exp\left(\frac{\tau}{2}[(1 + b_{n_{t-1}})^2 + (1 - b_{n_t})^2]\right) \equiv R_{n_t}, \tag{24}$$

where the first (second) term in the exponential function corresponds to the remoteness to the left-side (right-side) rest of the world. The n_{t-1} in equation (24) refers to the left-side neighboring state of state n_t. The right border of state n_{t-1}, namely $b_{n_{t-1}}$, is the left border of state n_t and is part of state n_t. By equation (24), $R(t)$ is increasing with state n_t's minimal distance from the WGC.[4]

[4] If locale t is in the right hemisphere (i.e., $t > 0$), $b_{n_{t-1}}$ is the point where locale t and its fellow locales start paying trade costs for their imported goods from their left-side foreign states. If locale t is in the left hemisphere (i.e., $t < 0$), b_{n_t} is the point where locale t and its fellow locales start paying trade costs for their imported goods from their right-side foreign states.

Now consider the right hemisphere of the world, so state n is the n-th nearest state to the WGC. Its left border is b_{n-1}, its right border b_n, its size $S_n = b_n - b_{n-1}$, and its remoteness R_n. The following properties of R_n follow from equation (24):

$$\begin{aligned}
\frac{\partial R_n}{\partial S_n} &= -\tau(1 - b_{n-1} - S_n)R_n < 0, && (25)\\
\frac{\partial R_n}{\partial b_{n-1}} &= \tau(2b_{n-1} + S_n)R_n > 0, && (26)\\
\frac{\partial R_n}{\partial \tau} &= \frac{1}{2}[(1 + b_{n-1})^2 + (1 - b_{n-1} - S_n)^2]R_n > 0. && (27)
\end{aligned}$$

According to equation (25), R_n decreases if state n increases in size. The state size change may take the form of (a) fixing the left border and pushing the right border rightward, (b) fixing the right border and pushing the left border leftward, or (c) pushing both borders outward. According to equation (26), R_n decreases if state n moves leftward with its size unchanged. According to equation (27), R_n decreases if no border changes but the foreign trade cost per unit of distance (τ) decreases. These properties of $R(t)$ prepare us well for solving for the world partitions $\{b_n^*\}$ on Date 2.

Solving Date 2. On date 2, the overlords (designated on date 1, as discussed later) draw borders for themselves and other lords in their states, with perfect foresight of the events on date 3 (as discussed above). For the lord of locale t, a marginally larger state results in a disutility h (see utility function (2)) and a consumption gain (see Lemma 1 and property (25)).

To solve for the world partitions $\{b_n^*\}$, we start from the center of the world and move outward. This solving process will yield one equilibrium of the model. Consider locale $t = 0$ located at the WGC. It is the most proximal locale in the world, and therefore its lord must be an overlord.[5] By equation (24), it has the lowest possible remoteness. Thus, if locale $t = 0$ sets its borders to include any other locale in the world as its fellow lord, that locale will attain the lowest possible remoteness, and its utility cannot increase further. The last locales in the two hemispheres included by locale $t = 0$ in its state can be determined from this first-order condition:[6]

$$\tau R_0^{\gamma-1}(1 - b_0^* - b_{-0}^*) = h. \tag{28}$$

It is straightforward to verify that the two borders are symmetric. $R(t = 0)$ applies to all locales (b_{-0}^*, b_0^*), namely state 0. Therefore, $R(t = 0)$ can also be written as R_0 as we did in equation (28).

Notice that the locales at b_{-0}^* and b_0^* are excluded from state 0 because, by convention, borders belong to their distal states. For example, consider locale b_0^* in the right hemisphere. Its overlord must govern his own state, forming a new state to the right of state 0. This new state, namely state 1, extends

[5]The lord at the WGC can only take his own offer to be an overlord.

[6]First-order condition (28) stems from equations (2), (21), and (25).

rightward until the following first-order condition is met:

$$\tau R_1^{\gamma-1}(1 - b_0^* - S_1) = h, \tag{29}$$

where the last term S_1 is solvable. Since $S_1 \equiv b_1^* - b_0^*$, b_1^* is determined from the perspective of locale $t = b_0^*$, and the locale at b_1^* is excluded from state 1. All locales in the interval $[b_0^*, b_1^*)$ attain their lowest possible remoteness by joining state 1, and their utility cannot increase further. Similarly, for any state $n \geq 1$, the first-order condition is

$$\tau R_n^{\gamma-1}(1 - b_{n-1}^* - S_n) = h, \tag{30}$$

where the last term S_n is solvable. With S_n, the overlord at locale b_{n-1}^* determines b_n^* and excludes the locale at b_n^*.

The above solving process, starting from the center and moving rightward, applies equally to the left hemisphere. At the end, all borders in the world, namely $\{b^L(t), b^R(t), \forall t\} = \{b_n^*\}$, settle. The number of states equals $2N + 1$ in equilibrium, with $2N$ satisfying:[7]

$$2N = \{2n : \frac{S_0}{2} + \sum_{i=1}^{n} S_i \leq 1 \text{ and } \frac{S_0}{2} + \sum_{i=1}^{n+1} S_i > 1\}. \tag{31}$$

This nation-state system leaves very distal locales out. Specifically, locales in $[-1, b_{-N}]$ and $[b_N, 1]$ are not accepted into their proximal side states. At each pole, the last few locales have incentives to form their own states to eliminate foreign trade costs among themselves. These two polar states, unlike the $2N + 1$ states, are smaller than optimal and thus we call them *polar semi-states*. They correspond to small island states in reality, with further discussion provided in Section 2.4. Thus far, one set of world partitions $\{b_n^*\}$ has been solved for. One might wonder whether different sets of world partitions could emerge if the borders are solved for in a different order, a topic that will be discussed below.

Solving Date 1. The world partitions $\{b_n^*\}$ solved for Date 2 represent a set of critical coordinates—locales at these coordinates become the most proximal locales in their states. The lords of these locales are overlords who draw borders for themselves and distal-side constituent locales. These overlords are designated on Date 1 by lords who have perfect foresight about Date 2. Being inside $\{b_n^*\}$ is both sufficient and necessary to be an overlord. To see the sufficiency, note that none of the lords at these locales can find a more proximal locale to serve as their overlord, as shown above, and therefore must be their own overlords. The necessity refers to the fact that there are no overlords in the model other than those at the locales $\{b_n^*\}$, as we explain and prove below.

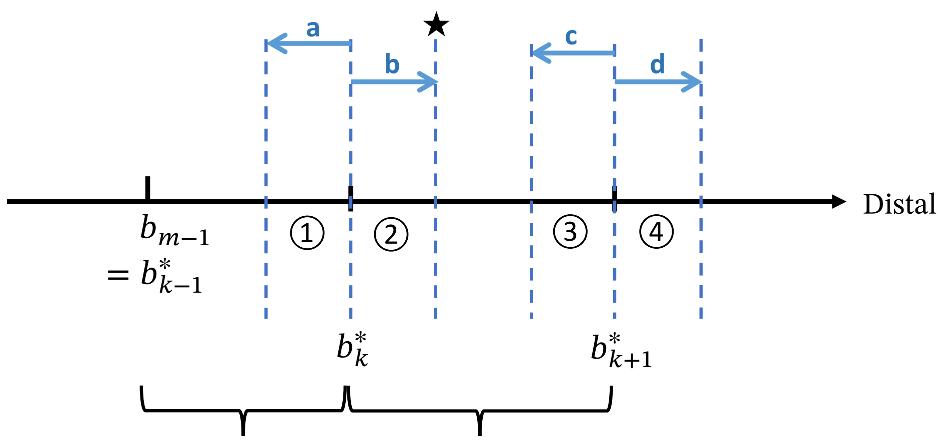
First, the composition of state 0 is determined through utility maximization. Therefore, regardless of the starting point for solving the borders, the overlord at locale $t = 0$ would select the same composition (b_{-0}^*, b_0^*). All locales within this range would find state 0 optimal because being in the same state as locale $t = 0$ minimizes their remoteness, attaining the global minimum of remoteness. Consequently,

[7] A special case where the number of state equals $2N$ is discussed in Section 2.4.

there is no other overlord within (b^*_{-0}, b^*_0).

Other states implied by $\{b^*_n\}$ achieve minimized remoteness, which is not the global minimum, but locales in them cannot attain a lower remoteness. To understand why, pick three arbitrary borders in $\{b^*_n\}$ and denote them as b^*_{k-1}, b^*_k, and b^*_{k+1}. We assume they are in the right hemisphere, though the same reasoning applies if they are in the left hemisphere. These three borders form state k and state $k+1$ in the equilibrium partitions found earlier. Now, consider three borders b_{m-1}, b_m, and b_{m+1} that form states m and $m+1$. Assume $b_{m-1} = b^*_{k-1}$, $b_m \neq b^*_k$, and $b_{m+1} \neq b^*_{k+1}$. That is, overlord $m-1$ is in $\{b^*_n\}$, while overlords at b_m, b_{m+1}, and beyond are not in $\{b^*_n\}$.[8] We aim to show that such b_m and b_{m+1} do not exist.

Figure 1: b_m and b_{m+1} Are Not Equilibrium Partitions



Notes: States k and $k+1$ are two states formed by borders b^*_{k-1}, b^*_k and b^*_{k+1} as previously solved for. Hypothetical borders b_m and b_{m+1} deviate from b^*_k and b^*_{k+1} in the forms of arrows a, b, c, d, or a combination of them. As explained in the text, none of the deviations can make equilibrium partitions.

Figure 1 illustrates why such b_m and b_{m+1} do not exist. If they did exist, forming states m and $m+1$, they would have to take the form of arrows a, b, c, d, or a combination of two such arrows. These represent an exhaustive list of all possible deviations from b^*_k and b^*_{k+1}. Such b_m and b_{m+1}, to be in equilibrium, must not make any locale in the current states k and $k+1$ worse off. Consider arrow a. It would make the locales in interval (1) worse off because their remoteness $R(\cdot)$ depends on their most proximal fellow locale. Thus, arrow a is not part of any equilibrium partitions. Next, consider arrow b. All locales currently in state $k+1$ except those in interval (2) would be worse off because their remoteness would then be calculated based on border ★. Then consider arrow c. Locales in interval

[8]Conceivably, the smallest possible value of m is 1. That is, $b_{m-1} = b^*_{k-1} = b^*_0$ is in $\{b^*_n\}$ (as previously shown), b_1 is different from b^*_1 and not in $\{b^*_n\}$, b_2 is different from b^*_2 and not in $\{b^*_n\}$, and so on.

3 would be worse off because they would be in a more distal state $m+2$ than the current state $k+1$ and thus end up with a higher remoteness. Turning to arrow d, state $m+1$ that includes interval 4, would be larger than the current state $k+1$ and thus too large, violating the first-order condition (30) for locales in the state. For the same reason, combinations of these arrows would also make certain locales worse off.

Since all overlords are in $\{b_n^*\}$ and no other overlords exist, the world partitions we identified are the only possible ones in equilibrium. These overlords take offers from themselves and some distal-side neighbors. In sum, we formally prove the following proposition:

Proposition 1. *There exists a unique set of equilibrium world partitions.*

Proof. The existence of the equilibrium partitions is shown by the process through which we solved the model above, starting from the WGC and moving outward. The uniqueness of the equilibrium is proved in Appendix A.1.3. □

Finally, with $\{b_n^*\}$, $C^z(t)$ and $C^l(t)$ settle to complete the unique equilibrium Ω.

2.4 Remarks

In this subsection, we discuss the rationale behind various settings of the model, their generality, and limitations.

Overlord, lord, and labor. Overlords can personify facilities that require a convenient location relative to surrounding areas and the support of those areas. Government is an example of such facilities. Delegated by the surrounding constituents, governments provide services but have limited capacities, making it difficult to govern oversized territories. The facilities personified by overlords need not be political. For instance, a warehouse also needs a convenient location and support from its surroundings, while large warehouses are risky because they are vulnerable to theft, attacks, and disasters. Communal storage is indeed considered a reason why early human societies evolved into states with limited sizes (e.g., Adams (1981) and Mayshar, Moav, and Pascali (2022)). Customs, as a mixed example that has both political functions (such as duty collection) and non-political functions (such as storage), is another example of the overlords in our model.

International trade is not the only mechanism favoring large states. Alternative mechanisms, such as leviathans collecting taxes from various locales or public goods produced with increasing returns to scale, can also make large states desirable. We focus on the international trade mechanism because a linear world provides an ideal setting to quantify trade costs and their impacts on geopolitics. If used solely as a geometric coordinate system for indexing locations, the world line's potential in geopolitical analysis would be underutilized. The world line integrates geometric coordinates, varying trade costs, and endogenous state sizes into a highly tractable structure. We assume equal land endowments across the world line, in order to demonstrate that trade costs alone, without comparative advantages or increasing returns in production, can endogenously create world partitions.

A common characteristic of the facilities personified by overlords is that they exhibit both economies of scale and capacity constraints. The $-hS(n_t)$ term in equation (2) acts as a penalty function, imposing capacity constraints. Since our model includes lords as landowners, we assign the penalty function to the lords (including overlords). The penalty function ensures limited state sizes, regardless of the penalty's nature and the bearer. The "domestic conflicts," as the source of disutility mentioned earlier, is a general term referring to the penalty function, which can be assigned to all lords, only overlords, or other agents such as monarchs, politicians, and customs officials.

While labor is not strictly necessary in our model, it would be unusual for an economic model to have only one immobile production factor (land) at work. Letting labor freely migrate within states not only makes the model more realistic but also allows labor endowments l_0 to have arbitrary initial distribution across locales within a state. Imagine that labor does not enter the model until employed in production on Date 3. Then, as long as the labor supply is proportional to state sizes, the equilibrium would remain unchanged.

Number of states. The equilibrium number of states in the model excludes the very distal locales at the two ends of the world line. These distal locales form two polar semi-states, providing stability and comparative statics. Every single border change in the model has general equilibrium effects on the rest of the world. While such changes may alter the number of states, if they are not sufficiently large, they will be absorbed by adjustments in the size of the two polar semi-states. Their technical role in cushioning border shocks is crucial for maintaining the stability of the number of states.

The number of states in equilibrium is typically $2N + 1$ (see equation (31)) with one exception. When state 0 collapses to a stateless point, namely $b_0^* = b_{-0}^* = 0$, the number of states is $2N$. A necessary condition for this extreme scenario to occur is $\tau \exp(\tau(\gamma - 1)) = h$ (see first-order equation (28)). This specific combination of parameters is unlikely to hold, so we generally disregard this special case. Nonetheless, even in this special case, the first-order condition (30) would still hold, as would the rest of the equilibrium.

Specific geography. The necessity of using a specific geography in this study arises from the need to model the behaviors of borders (lords). Without a specific geography, defining borders would be challenging.[9] In our case, the linear geography makes the equilibrium partitions determinable by imposing the constraint that locales can only form states with their adjacent locales. This greatly simplifies the analysis, as every state must be an interval, and the total mass of the states (intervals) in the world automatically adds up to the constant 2 (ranging from -1 to 1), representing the total area of landmass in

[9] If borders are shapeless, the set of states in the world has numerous possible configurations. This is because the locales $\{t\}$ can then be partitioned arbitrarily into any $\{S_n\}$, where both the composition and number of elements are endogenous. See Allen (2023) for an algorithm-based solution to tackle such problems.

the world.

A circular geography might seem to be an alternative for our modeling purposes. However, a ring (i.e., a circle without an interior) is inherently symmetric, making it impractical for analyzing differential remoteness across locales. Using a disk (i.e., a circle with an interior) could generate differential remoteness, but defining borders within a disk remains problematic.[10]

Cobb-Douglas consumption & production. The two Cobb-Douglas structures provide us with the elegant sufficient statistic $1/R(t)$, as depicted in equation (21). The two Cobb-Douglas structures, when combined with equal endowments across locales, significantly simplify the equilibrium analysis. Because of them, the focus of the model-solving process shifts from understanding how lords organize themselves into states to understanding how they pick sides to minimize $R(t)$. The implications of relaxing the Cobb-Douglas assumptions hinge on whether the equal endowment assumption is upheld or not.[11]

If we maintain the equal endowment assumption, substituting either Cobb-Douglas structure with a more general functional form would not materially impact the equilibrium. For example, if the production function becomes CES, the model would remain unchanged because production would continue being symmetric across locales, given the symmetry of technology and endowments. If the consumption structure becomes CES, the advantages of proximal locales would be magnified: not only would their consumption costs decrease (as currently in the model), but their income would also rise due to their larger share of total world expenditure (which is currently not accounted for). In such a scenario, the 1 in the numerator of $1/R(t)$ would be replaced by a complex function inversely correlated with $R(t)$. The model will be sophisticated to solve, but the additional sophistication tends to strengthen the location advantages.

If we abandon the equal endowment assumption, replacing either Cobb-Douglas structure would introduce considerable complexities, making it unclear whether an equilibrium exists, and if so, whether it is unique. The root of the complexity lies in the following: for overlords, including a marginal locale with large (small) endowments could decrease (increase) the price of the produced good, impacting all his fellow locales and thus creating a vast array of decision combinations involving large and small endowments across the world. This curse of dimensionality arises when either consumption or production is more elastic than Cobb-Douglas. Further, if the ratio of land/labor endowments varies across locales, a higher elasticity of substitution in production than that of Cobb-Douglas would introduce further complications: lords may or may not choose to include locales with abundant labor, depending on whether the increase in output outweighs their smaller share of sales revenue.

[10] In a unit disk, any two straight lines can intersect in numerous ways, leading to countless possible divisions of the disk.

[11] Although lords and labor need not be interpreted literally, the Cobb-Douglas technology can be viewed as representing the common practice of sharecropping with fixed shares under the feudal system in European history.

Forms of trade costs. Assuming zero domestic trade costs ensures that siding with proximal neighbors is the dominant strategy for all locales in the world. The main mechanism of our model only requires that the trade cost per unit of distance be lower in domestic trade than in foreign trade. Introducing positive domestic trade costs does not alter this main mechanism but introduces additional complexities to the analysis. For example, with positive domestic trade costs, locale $t = 0$ wants to include some moderately distant locales in state 0, even though those locales may prefer to join their distal neighbors.

Using heterogeneous trade costs across locales will further reduce the model's solvability. At the core of the model are locational advantages and the consensus formed within states based on these advantages. When trade costs are heterogeneous enough to offset some of these advantages, the consensus among neighboring locales is disrupted, leading to unpredictable consequences. There are a few specific geographic features that yield heterogeneous trade costs across locales but have foreseeable effects. Firstly, a geographic barrier with prohibitively high trade costs (such as an untamable ocean) will divide the world into two parts, each undergoing partitioning similar to the previous analysis. Secondly, the insertion of an uninhabitable but passable segment with high trade costs per unit of distance (like a river or desert) into the world line will expand the states on its distal side, while the proximal state may either grow or diminish in size. Lastly, if hilly areas are as inhabitable as regular land, they can be treated as such. However, trade costs per unit of distance must be adjusted to account for the presence of ridges.

3 International Trade and Security

Trade volumes and routes are influenced by borders, and they in turn shape those borders. Our model facilitates the analysis of this bidirectional relationship.

3.1 International trade

International trade typically follows a gravity pattern: two states have a larger bilateral trade volume if they are larger in size and closer in distance. A gravity model of bilateral trade can be derived from our model:

Proposition 2. *The exports from state m to a nonadjacent state n follow*

$$X_{m,n} = \zeta S_m S_n \exp(-\tau D_{m,n}), \tag{32}$$

where ζ is a positive scalar, S_m and S_n are the sizes of the two states as defined before, and $D_{m,n}$ is the shortest distance between the two states (e.g., if $n > m + 1 \geq 1$, $D_{m,n} = b_{n-1} - b_m$).

Proof. See Appendix A.1.4. □

Proposition 2 relates to the gravity model literature in international trade.[12] Early versions of the gravity model, represented by equation (32), were analogous to Newton's law of universal gravitation. However, these early models were found lacking because they did not account for the differential remoteness of the two states from the rest of the world. Trade between any two states depends not only on their bilateral relationship but also on their remoteness from the rest of the world through general equilibrium effects. Over the past two decades, gravity models have added remoteness-related terms to account for such general equilibrium effects. Our gravity equation accounts for these general equilibrium effects while maintaining the original Newtonian form. Additionally, remoteness in our model has a clear geometrical interpretation: a state is considered remote if it is far from the WGC.

Moreover, because borders are endogenous in our model, our gravity equation (32) can be used to analyze how parameters influence the interplay between international trade and national borders. In the following analysis, we use $\hat{v} = dv/v$ to denote the percentage change in any variable v.

First, consider an exogenous reduction in τ such that $d\tau < 0$. Its impact on the bilateral trade volume $X_{m,n}$ can be decomposed into three effects:

$$\underbrace{\hat{X}_{m,n}}_{\lesseqgtr 0} = \underbrace{\hat{S}_m + \hat{S}_n}_{\text{size effect}<0} \underbrace{-D_{m,n} d\tau}_{\text{direct effect}>0} \underbrace{-\tau dD_{m,n}}_{\text{location effect}>0} . \tag{33}$$

Among the three effects in equation (33), the *direct effect* is self-explanatory. The *size effect* refers to the fact that both states shrink in size when τ decreases. In fact, all states shrink in size when τ decreases because the resulting consumption boost can now sustain smaller states. The net of these two effects, the direct effect and the size effect, has an ambiguous sign, depending on which is greater in magnitude. There is a third *location effect* that adds to the ambiguity. As reducing τ leads to smaller states worldwide, the size shrinkage of the states located between state m and state n brings the two states closer to each other. The size effect and the location effect are absent in the extant studies, which assume fixed national borders. Thus, equation (32) can be considered a long-run gravity equation. In short runs, borders are fixed, such that the direct effect is the only effect.

Next, consider an exogenous reduction in h such that $dh < 0$. Its impact on the bilateral trade volume $X_{m,n}$ can also be decomposed into three effects:

$$\underbrace{\hat{X}_{m,n}}_{\lesseqgtr 0} = \underbrace{\hat{S}_m + \hat{S}_n}_{\text{size effects}>0} \underbrace{-D_{m,n} d\tau}_{=0} \underbrace{-\tau dD_{m,n}}_{\text{location effect}<0} . \tag{34}$$

Here, the size effect is positive since state sizes grow when h decreases. The negative association between h and state size stems from the greater "tolerance" among locales within all states. The location effect is negative because m and n are farther apart due to the expansion of the states between them. Once again, the net effect of these two influences is ambiguous. In other words, even if states become more integrated politically and thus larger in size, they do not necessarily trade more with each other.

[12]See Anderson (2011), Head and Mayer (2014), and Allen et al. (2020) for reviews.

This is reminiscent of historical periods when empires were widespread but did not trade much with each other because trade routes between empires were blocked by other empires.

3.2 International security

A change in trade cost per unit of distance (i.e., the aforementioned $d\tau$) impacts global security by altering the world partitions, a force that can be counterbalanced by a dh in the same direction. To see that balance, totally differentiate the first-order condition (30):[13]

$$\frac{dh}{d\tau} = R_n^{\gamma-1}(1-b_n)(1+\frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2+(1-b_n)^2]) > 0. \tag{35}$$

Intuitively, when the trade cost per unit of distance decreases, locales need to be more tolerant of each other to maintain the current world partitions. Without this increased tolerance, the existing partitions would collapse, leading to the emergence of smaller states.

By equation (35), the need for a h-compensation in response to a τ change is less for more remote states, because their territories are less valued.[14] In other words, borders are not created equal. To this end, we can show

Proposition 3. *Borders closer to the WGC are more contentious, because changes to them result in greater welfare impacts.*

Proof. Recall that $1/R_n$ is the sufficient statistic for welfare for all lords in state n. A simple manipulation of the previous equation (24) gives:

$$-\frac{\partial(R_n/\partial S_n)}{R_n} = \tau(1-(b_n-b_{n-1})), \tag{36}$$

where $S_n \equiv b_n - b_{n-1}$. Consider three borders in the right hemisphere: b_{k-1}, b_k, and b_{k+1}. Locales $[b_{k-1}, b_k)$ constitute state k, and locales $[b_k, b_{k+1})$ constitute state $k+1$. Holding borders b_{k-1} and b_{k+1} constant, a change in b_k affects state k more than it does state $k+1$. □

The assertion that some borders are more contested than others and the quest to identify such borders are central themes of geopolitical analysis in international relations.[15] Studies in international relations, including geopolitical analysis, are more qualitative and less integrated compared to those in international trade. Proposition 3 formalizes this claim and highlights where to identify these contested borders.

[13] Appendix A.1.5 gives the full derivation.

[14] When state n refers to a farther state from the WGC, the effects through $(1+b_{n-1})^2$ and $(1-b_n)^2$ cancel each other, while the effect through $(1-b_n)$ at the front of equation (35) gives this finding.

[15] Geopolitical analysis, as a branch of international relations established by Mackinder (1904), Huntington (1907), and Fairgrieve (1917), has significantly influenced the work of historians (Braudel, 1949), human geographers (Diamond, 1999), and political scientists (Morgenthau, 1948; Kissinger, 1994, 2014; Brzezinski, 1997). For recent influential works in geopolitical analysis, see Deudney (1983, 2006).

4 National Borders Over the Past Three Centuries

Our theory is based on a linear (one-dimensional) world, while the earth's surface is two- or three-dimensional. We propose that the mechanism described by our model remains applicable to higher-dimensional geography as long as locational advantages exist. Since the inhabitable landmass on the earth is fragmented and irregularly shaped, some locations are inherently closer to the rest of the world than others. As demonstrated in this section, our theory's border-drawing mechanism can capture certain aspects of the world's political geography. Below, we first derive testable predictions from our theory and then use data to test them. Finding empirical evidence in the data supports not the world-line setting of our model, but the role of locational advantages in global border formation as approximated by the linear structure.

4.1 Testable predictions

Since the sizes of nation-states throughout history are observable and quantifiable, we derive the following testable predictions regarding state sizes and their evolution over time:

Proposition 4.

(i) *Within a given time period, states closer to the WGC (except state 0) are smaller, provided that the trade cost parameter τ is not too small.*

(ii) *In time periods when state 0 is larger in size, state n is farther from the WGC, and this effect is more pronounced if n is greater.*

Proof. See Appendix A.1.6. □

The intuition behind Part (i) of Proposition 4 is straightforward. As long as trade costs are not minimal, states closer to the WGC (locale $t = 0$) are nearer to the rest of the world and therefore have less need for a large domestic market to keep consumption costs low. Applying this reasoning to state 0, one might expect it to be the smallest state in the world. However, a competing force specifically affects state 0. Unlike all other states, state 0 sets its borders in two opposite directions. Consequently, the marginal return relative to the disutility from a unit distance of expansion is greater for state 0 than for other states. To illustrate this difference, compare state 0's first-order condition (28) with state 1's first-order condition (29):

$$\tau R_0^{\gamma-1}(1-\frac{S_0}{2}) = h \quad \text{versus} \quad \tau R_1^{\gamma-1}(1-\frac{S_0}{2}-S_1) = h.$$

Recall $R_0 < R_1$ and $\gamma > 1$. The only requirement on the relative sizes of S_0 and S_1 is that $1 - S_0/2$ must be greater than $1-S_0/2-S_1$. That always holds. Thus, S_0 could be greater than, less than, or equal to S_1.

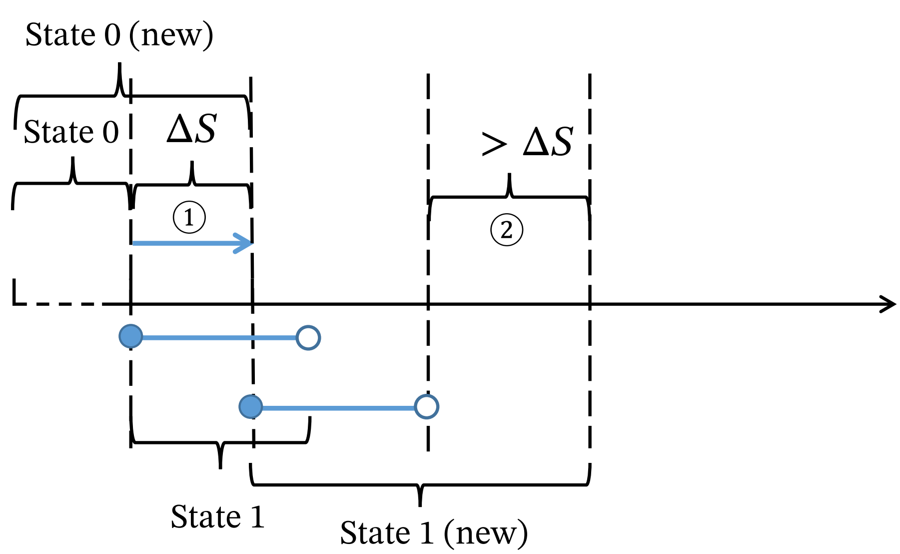
The same comparison applies between state 0 and any state with index $|n| > 1$. The exception of state 0 reminds us of empires in history which usually located in the center of continents or subcontinents.

The size of state 0 varies over time due to various shocks. When state 0 increases in size, every other state n moves farther from the WGC, with this effect becoming more pronounced as the index n increases. Technically, this is because

$$\frac{\partial^2 b_n}{\partial b_{n-1} \partial b_0} > 0, \tag{37}$$

and the cross partial derivative grows with the index n. The intuition behind this result, namely Part (ii) of Proposition 4, is as follows. When state 0 becomes larger, all other states are "pushed away" from the WGC. As these states are pushed away, they need to grow in size because the new locales they acquire are less advantageous than those they lose. Figure 2 illustrates this mechanism using state $n = 1$ as an example. Suppose state 0 expands its right-side border by a distance of ΔS. Locales in region 1 with a measure of ΔS, which previously belonged to state 1, are now part of state 0. Consequently, state 1 must now include region 2 with a measure greater than ΔS. This size increase occurs because the newly acquired territory 2 has worse locations than the lost territory 1.

Figure 2: Part (ii) of Proposition 4



Notes: Solid (hollow) circles represent closed (open) interval endpoints. The locales represented by solid (hollow) circles are in the distal (proximal) state.

4.2 Empirical findings

We collected data from digitized world maps, using the political world map from the year 1994 as our benchmark.[16] To supplement the modern map, we compiled three historical world maps with base years 1750, 1815, and 1914-1938. The rationale behind selecting these base years, along with other data details and descriptive statistics, is discussed in Appendix A.2.1. Hereafter, we refer to the base years 1750, 1815, 1914-1938, and 1994 as the 18th century, 19th century, early 20th century, and modern period, respectively.

A key concept in our theory is the world geometric center (WGC), which refers to the midpoint of all locales on the world line. To estimate the location of the WGC, we define a locale as an administrative division on the world map with a population of at least 15,000.[17] We calculated $Distance(t, t')$, which is the orthodromic distance (great-circle distance) between any two locales in the world ($t, t' \in W$), and then calculated each locale t's total distance from all other locales. The locale with the smallest total distance is designated as the WGC:

$$WGC \equiv \arg\min_{t} \sum_{t' \in W} Distance(t, t'). \tag{38}$$

Table 1 reports the WGCs for the four periods, which have remained generally stable, reflecting the stability of human habitats over the past centuries. The locales where the WGCs are located, referred to as state 0, were part of large states in three out of the four periods in our sample (the Austrian Empire, Prussia, and the Austro-Hungarian Empire). These states played the role of state 0 in their respective periods. The modern-period state 0, the Czech Republic, has the smallest size in comparison with its historical counterparts.

Table 1: Estimated Locations of the WGC

Modern period	*The 18th century*
Hradec Kralove, Czech Republic (50.21,15.83)*	Kisvarda, Austrian Empire (48.22,22.08)*
The 19th century	*Early 20th century*
Weißwasser, Prussia (51.50,14.64)*	Hradec Kralove, Austro-Hungarian Empire (50.21,15.83)*

Notes: * Geographic coordinates in the parentheses are in the form (latitude, longitude).

In Table 2, we regress each local t's total distance from the rest of the world, $\sum_{t' \in W} Distance(t, t')$,

[16] No major border changes have occurred in the world since 1994.

[17] The population threshold is set moderately low to ensure that the landmass is used for permanent residence. A high threshold would limit the sample to industrial clusters, while a very low threshold would cause locales with only temporary public projects, scattering periodic employers, or seasonal school enrollments to be overrepresented. The value of 15,000 is the lowest population requirement used by the US Census to determine central cities of metropolitan statistical areas. Lowering that population threshold to zero is equivalent to treating every state as a polygon. We explore this approach in Appendix A.2.3.

on its distance from the WGC, $D(t)$, at various polynomial orders for the modern period. Columns (1) and (2) show that the coefficients for the constant term, first-order term, and second-order (quadratic) term are all positive and statistically significant. When higher-order terms of $D(t)$ are incrementally added to the regression in column (3), the constant and the second-order terms remain positive and statistically significant. Overall, the total distance increases quadratically with the distance from the WGC, which is consistent with the linear pattern where the distance between locale t and the rest of the line $[-1, 1]$ is $t^2 + 1$. This successful approximation is likely due to the fragmented and irregularly shaped inhabitable landmass on the earth's surface. That is, some locations on the earth's surface are inherently closer to the rest of the world than others. If the landmass were uniformly distributed, we would not observe the quadratic patterns since locational advantages would not exist.

Table 2: Distance from WGC and Distance from the Rest of the World

Dependent variable is ln(total distance from the rest of the world)			
	(1)	(2)	(3)
Constant term	1.187e+08***	1.234e+08***	1.248e+08***
	(1810979.989)	(647,497.479)	(662,080.449)
Distance from WGC	5,206.437***	2,382.739***	-992.376
	(818.453)	(509.458)	(1,845.611)
Distance from WGC^2	0.412***	0.732***	2.452**
	(0.082)	(0.160)	(0.973)
Distance from WGC^3		0.000	-0.000
		(0.000)	(0.000)
Distance from WGC^4		-0.000***	0.000
		(0.000)	(0.000)
Distance from WGC^5			-0.000
			(0.000)
Distance from WGC^6			0.000
			(0.000)
Coast and island dummies	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes
Observations	21,068	21,068	21,068

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05.

To test Part (i) of Proposition 4, we regress the territorial size of state n (in logarithm) on its shortest distance from the contemporary WGC, $\ln D(n)$. The results, reported in Table 3, show a positive and statistically significant correlation between the two variables. Column (1) of Panel A, which is the highlighted column, corresponds to the modern period. Since states have different numbers of locales, the state-level minimum distance from the WGC, as a sample statistic, may cause heteroskedasticity in the regression. To address this, we experiment with weighting regressions by the number of locales at the state level, and the results remain similar. We minimize the use of control variables to maximize sample sizes. In column (1) of Panel B, we control for military expenses, iron and steel production, and primary energy consumption. With these national powers controlled for, our sample size slightly decreases (from 162 to 156). The coefficient of $\ln D(n)$ remains positive and statistically significant, both unweighted and weighted. Columns (2) to (4) report similar results for other periods.[18]

Table 3: State Sizes and Locations within Periods

Dependent variable is ln(Area)	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: Full sample				
ln(Distance from WGC)	0.628***	0.760***	0.651***	0.383***
	(0.196)	(0.204)	(0.122)	(0.130)
Coast dummy	1.745**	-0.116	0.704***	0.456*
	(0.703)	(0.359)	(0.266)	(0.275)
Island dummy	-2.089***	-1.038**	-1.439***	-1.376***
	(0.598)	(0.401)	(0.467)	(0.371)
If weights are used:#				
ln(Distance from WGC)	0.607***	0.701***	0.628***	0.639***
	(0.153)	(0.234)	(0.196)	(0.102)
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	162	121	137	174
Panel B: With national power controls				
ln(Distance from WGC)	0.522***		1.937***	0.850***
	(0.110)		(0.643)	(0.248)
Coast dummy	-0.400*		0.939**	0.012
	(0.223)		(0.406)	(0.452)
Island dummy	-1.025***		-2.474*	-1.006***
	(0.328)		(1.273)	(0.349)
ln(Military expenses)	0.003		-0.068	0.037
	(0.130)		(0.290)	(0.127)
ln(Iron & steel production)	0.027		0.449*	0.001
	(0.056)		(0.254)	(0.099)
ln(Primary energy consumption)	0.487***		-0.116	0.255***
	(0.103)		(0.206)	(0.068)
If weights are used:#				
ln(Distance from WGC)	0.774***		2.239***	1.511***
	(0.129)		(0.768)	(0.254)
Continent fixed effects	Yes		Yes	Yes
Observations	156		51	75

Notes: # In both panels, regressions are rerun under the same specification but with weights (number of locales), with only the coefficient of ln(Distance from WGC) reported as a separate row (other coefficients available upon request). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

To test Part (ii) of Proposition 4, we first assign a state index n to each state. State n represents the

[18]The relevance of $\ln D(n)$ to international trade is discussed in Appendix A.2.2, and robustness checks are conducted and reported in Appendix A.2.3.

Table 4: State Sizes and Locations across Periods

Dependent variable is ln(Distance from the contemporary WGC)				
	(1)	(2)	(3)	(4)
Panel A: 30 Nearest States to WGC				
State index normalized#	4.052***	4.940***	3.573***	5.025***
	(0.852)	(0.886)	(0.554)	(0.887)
Area of State 0	-0.382	1.363**		
	(0.486)	(0.532)		
State index normalized × Area of State 0	12.458***	9.034**	15.163***	9.420***
	(3.413)	(3.449)	(2.722)	(3.516)
Period fixed effect	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island & coast dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	120	78	120	78
Panel B: 50 Nearest States to WGC				
State index normalized#	4.361***	5.263***	4.254***	5.297***
	(0.402)	(0.471)	(0.270)	(0.475)
Area of State 0	0.049	1.552***		
	(0.346)	(0.380)		
State index normalized × Area of State 0	8.295***	5.694***	9.124***	6.078***
	(1.408)	(1.725)	(1.193)	(1.742)
Period fixed effect	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island & coast dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	200	121	200	121
Panel C: 70 Nearest States to WGC				
State index normalized#	5.220***	5.322***	5.082***	5.363***
	(0.203)	(0.418)	(0.215)	(0.419)
Area of State 0	0.706**	1.757***		
	(0.274)	(0.358)		
State index normalized × Area of State 0	3.538***	3.333**	4.006***	3.508**
	(0.826)	(1.501)	(0.803)	(1.516)
Period fixed effect	No	No	Yes	Yes
National power countrols¥	No	Yes	No	Yes
Island & coast dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	280	151	280	151

Notes: # The normalized state index equals 0 (respectively, 1) for the state with the shortest (longest) distance to its contemporary WGC. ¥ National power controls include military expenses, iron & steel production, and primary energy consumption (all in log terms). Robust standard errors in parentheses. *** $p<0.01$, ** $p<0.05$.

n-th nearest state to the WGC, which may correspond to different state identities over time. In our data analysis, we limit the state index n to 1-30, 1-50, and 1-70, respectively. We exclude $n > 70$ because states with higher indexes do not exist in every period. Equation (37) informs the following regression:

$$\ln D_{pr}(n) = \eta_0 I(n) + \eta_1 State0Area_{pr} + \eta_2 \left(I(n) \times State0Area_{pr}\right) + \bar{\xi}' X_{n,pr} + \epsilon_{n,pr}, \tag{39}$$

where $\ln D_{pr}(n)$ is the shortest distance between state $n = 1, 2, ..., 30/50/70$ and the WGC in period pr. $I(n)$ is the state index normalized between 0 and 1, with $I(n) = 0$ if state n is the nearest to the WGC within the sample, and $I(n) = 1$ if state n is the farthest. The coefficient η_0 is expected to be positive. $State0Area_{pr}$ is the area of state 0 in period pr, and $X_{n,pr}$ is a vector of control variables. $\epsilon_{n,pr}$ is the error term. η_1, expected to be positive, captures the effect that a larger state 0 "pushes" all other states further away from the WGC. The primary interest is in η_2, which is expected to be positive. As an alternative to including $State0Area_{pr}$ in the regression, we can use a period fixed effect to absorb its variation, with the interaction term $I(n) \times State0Area_{pr}$ unchanged.

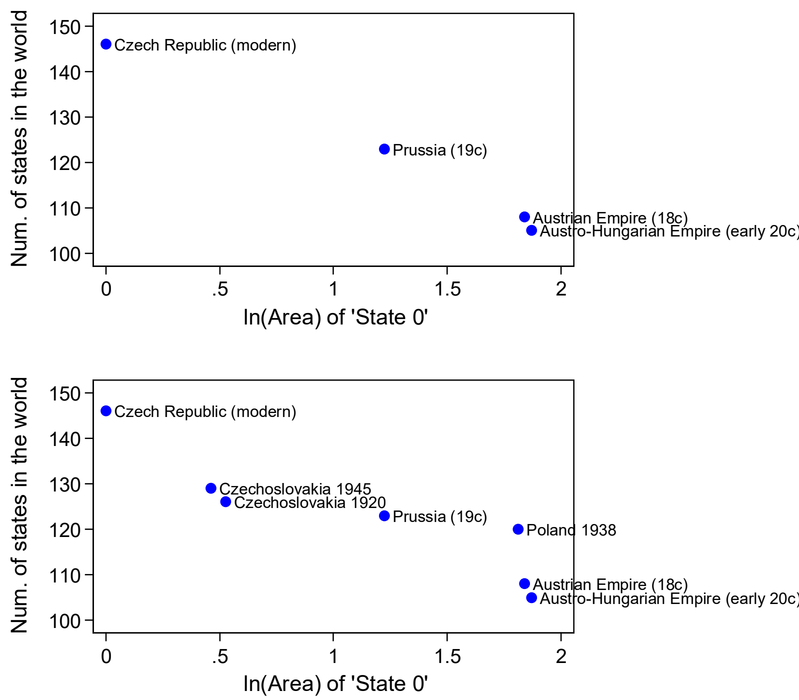
The results are presented in Table 4. Panel A includes data from states 1-30 across four periods, resulting in a total sample size of 120. In columns (1) and (2), we use $State0Area_{pr}$, while in columns (3) and (4), we apply period fixed effects. Columns (1) and (3) exclude national power control variables, thus both have 120 observations. Columns (2) and (4) include national power control variables, which are unavailable for all states in the 18th century and for some states in later periods, reducing the sample size to 78. The coefficient of the interaction term, $\hat{\eta}_2$, is positive and statistically significant in all columns. Panels B and C use the same specifications as Panel A but include states 1-50 and states 1-70, respectively, yielding similar findings.

Part (ii) of Proposition 2 also implies that a larger state 0 results in larger sizes for all states in the world, leading to a smaller number of states globally. We examine the correlation between the area of state 0 and the number of states in the world over different time periods, as shown in the upper panel of Figure 3. A clear negative association between the two variables is evident. In the lower panel, we include additional observations: a post-war observation (Czechoslovakia in 1920), an interwar observation (Poland in 1938), and another post-war observation (Czechoslovakia in 1945). The negative correlation remains and becomes even more pronounced.

5 Concluding Remarks

Linearization is a widely used modeling technique in economics, and we apply it to world geography to understand the interactions between national borders and international trade. It is particularly useful for our purpose as it allows for the modeling of endogenous borders. Building on a linear world framework, our general equilibrium model offers a political geography shaped by international trade. Our model connects local economies to the global economy, links local welfare with foreign welfare, and ties national borders to the nation-state system. Additionally, we find patterns in historical maps that

Figure 3: State 0 and Number of States



Notes: In the upper panel, 18c, 19c, and early 20c refer to the 18th century, 19th century, and early 20th century, respectively. Modern refers to the year 1994. The lower panel includes three additional observations related to the two world wars. Czech Republic (modern) has the smallest area among all state 0's. We normalize it to one (zero in log). For all other periods, the ln(Area) of state 0 refers to the difference between actual ln(Area) and the ln(Area) of Czech Republic (modern). This normalization is in order to keep the horizontal axis short.

are consistent with our model's predictions.

We envision two promising avenues for future research. First, although colonization is not addressed in this study, our model provides a framework for examining this complex process. Colonization involves migration, trade, and borders for both empires and colonies. During the era of colonization, the world map was closer to linearity (Eurocentric, with very few Pacific routes) than in later periods. Thus, our linear world model offers a promising approach to modeling the general equilibrium of colonization. Second, we did not find worldwide bilateral trade data dating back to the 18th century. If such data were available, it would be valuable for evaluating how trade volumes and nation-states have influenced each other over time. Although such data are scarce, they are starting to become available for certain regions, such as Western Europe and East Asia.

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"International Geopolitics" Appendices

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A.1 Proofs and derivations

A.1.1 Equation (16)

At locale t, the lord maximizes $U(t) = \frac{1}{1-\gamma} C^z(t)^{1-\gamma} - hS(t)$, where $C^z(t) \equiv \exp\left(\int_{-1}^{1} \ln c^z(t,s) ds\right)$, subject to the budget constraint $\int_{-1}^{1} p(t,s) c^z(t,s) ds = r(t) z$. His first-order condition is

$$p(t,s) c^z(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} \equiv \kappa^z(t). \tag{A.1}$$

By inserting it back into the budget constraint, we obtain $\kappa^z(t) = r(t) z(t)/2$.

At locale t, the labor maximizes $V(t) = \frac{\psi}{1-\gamma} C^l(t)^{1-\gamma}$, where $C^l(t) \equiv \exp\left(\int_{-1}^{1} \ln c^l(t,s) ds\right)$, subject to budget constraint $\int_{-1}^{1} p(t,s) c^l(t,s) ds = w(t) l(t)$. His first-order condition is

$$p(t,s) c^l(t,s) = \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)} \equiv \kappa^l(t). \tag{A.2}$$

By inserting it back into the budget constraint, we obtain $\kappa^l(t) = w(t) l(t)/2$.

So, the aggregate first-order condition is the sum of equations (A.1) and (A.2):

$$p(t,s) c(t,s) = \frac{C^z(t)^{1-\gamma}}{\lambda^z(t)} + \frac{\psi C^l(t)^{1-\gamma}}{\lambda^l(t)} \equiv \kappa(t).$$

This is equation (16) in the text. The value of $\kappa(t)$ is

$$\kappa(t) = \kappa^z(t) + \kappa^l(t) = (r(t) z + w(t) l(t))/2.$$

Notice that the aggregate first-order condition is used to derive the aggregate expenditure on locale s's product at locale t, namely $p(t,s)c(t,s)$. The lord and labor solve their own utility maximization.

A.1.2 Equation (21)

By equation (A.1), we have $c^z(t,s) = \kappa^z(t)/p(t,s)$. By inserting the $c^z(t,s)$ into $C^z(t)$, we obtain

$$
\begin{aligned}
C^z(t) &= \exp\left(\int_{-1}^{1}(\ln \kappa^z(t) - \ln p(t,s))ds\right) \\
&= \exp\left(\int_{-1}^{1}(\ln \kappa^z(t)/p - \ln d(t,s))ds\right) \\
&= \exp\left(2\ln \kappa^z(t)/p - \int_{-1}^{1}\ln d(t,s)ds\right) \\
&= (\kappa^z(t)/p)^2 \exp\left(-\int_{-1}^{1}\ln d(t,s)ds\right) \\
&= \left(\frac{rz}{2p}\right)^2 \exp\left(-\int_{-1}^{1}\ln d(t,s)ds\right) \\
&= \left(\frac{rz}{2p}\right)^2 /R(t),
\end{aligned}
$$

where $p = rz/2$ is the normalized factory-gate price. Thus, $C^z(t) = 1/R(t)$, which is equation (21) in the text.

A.1.3 Proof of Proposition 1: Uniqueness

Assume that there exists another set of equilibrium world partitions:

$$\{\tilde{b}_n\} \equiv \{\tilde{b}_{-\tilde{N}}, ..., \tilde{b}_{-1}, \tilde{b}_{-0}, \tilde{b}_0, \tilde{b}_1, ..., \tilde{b}_{\tilde{N}}\}. \tag{A.3}$$

We now prove $\tilde{b}_n = b_n^*$ for all $n = 0, 1, \dots, N$. If that is untrue, then either (1) $\tilde{b}_0 \neq b_0^*$, or (2) there exists $m \geq 1$ such that $\tilde{b}_n = b_n^*$ for all $n = 0, 1, \dots, m-1$ and $\tilde{b}_m \neq b_m^*$. Below we show a contradiction in each scenario.

In scenario (1), we consider locale $t = 0$ with two borders $\tilde{b}_{-0}$ and $\tilde{b}_0$. At the previously known equilibrium partitions $\{b_n^*\}$,

$$\frac{\partial U(t|-b_0, b_0)}{\partial b_0}|_{b_0=b_0^*} = 0.$$

Thus locale $t = 0$ would always be worse off and thus deviate from the new equilibrium.

Then move on to scenario (2). When $n \geq 1$, from the previously known equilibrium $\{b_n^*\}$, we have

$$\frac{\partial U(t|b_{n-1}, b_n)}{\partial b_n}|_{b_{n-1}=b_{n-1}^*, b_n=b_n^*} = 0.$$

If $\tilde{b}_m < b_m^*$, we consider an arbitrary locale $t \in (b_{m-1}^*, \tilde{b}_m) \subseteq (b_{m-1}^*, b_m^*)$. Given the two borders $\tilde{b}_{m-1}$

and $\tilde{b}_m$, locale t's utility function gives

$$U(t|\tilde{b}_{m-1}, \tilde{b}_m) = U(t|b^*_{m-1}, \tilde{b}_m) < U(t|b^*_{m-1}, b^*_m).$$

If $\tilde{b}_m > b^*_m$, we consider an arbitrary locale $t \in (b^*_{m-1}, b^*_m)$. Given the two borders $\tilde{b}_{m-1}$ and $\tilde{b}_m$, locale t's utility function gives

$$U(t|\tilde{b}_{m-1}, \tilde{b}_m) = U(t|b^*_{m-1}, \tilde{b}_m) < U(t|b^*_{m-1}, b^*_m).$$

Thus locale t would always be worse off and thus deviate from the new equilibrium. This locale t is from state m. The same reasoning applies to an arbitrary locale t from state $m+1$.

A.1.4 Proof of Proposition 2

We assume, without loss of generality, that $n > m + 1 \geq 1$ (i.e., the two states are both in the right hemisphere and nonadjacent) and that state n is farther from the WGC than state m (i.e., $D_{m,n} = b_{n-1} - b_m$). The export volume from state m to state n is

$$\begin{aligned} X_{m,n} &= S_m \int_{b_{n-1}}^{b_n} p(s)c(s, b_m)ds = \frac{S_m}{2} \int_{b_{n-1}}^{b_n} \kappa d(s, b_m)^{-1} ds \\ &= \frac{\kappa}{2\tau} S_m [\exp(-\tau(b_{n-1} - b_m)) - \exp(-\tau(b_n - b_m))] \\ &= \frac{\kappa}{2\tau} S_m \exp\left(-\tau D_{m,n}\right) \times (1 - \exp(-\tau S_n)). \end{aligned}$$

Here, the second equality stems from equation (17). Since states sizes are small compared with 1 (the size of each hemisphere is 1), $1 - \exp(-\tau S_n) = \tau S_n$. So, equation (32) is obtained:

$$X_{m,n} = \zeta S_m S_n \exp\left(-\tau D_{m,n}\right),$$

where $\zeta = \kappa/2$ applies to all pairs worldwide.

A.1.5 Equation (35)

The first-order condition (30) for state n is equivalent to

$$F \equiv \tau R_n^{\gamma-1}(1 - b_{n-1} - S_n) - h = 0, \tag{A.4}$$

which implies the following partial derivatives:

$$
\begin{aligned}
F_h &= -1 < 0, & \text{(A.5)}\\
F_S &= -(\gamma-1)R_n^{\gamma-1}\tau^2(1-b_n)^2 - \tau R_n^{\gamma-1} < 0, & \text{(A.6)}\\
F_{b_{n-1}} &= (\gamma-1)R_n^{\gamma-1}\tau^2(2b_{n-1}+S_n)(1-b_{n-1}-S_n) - \tau R_n^{\gamma-1}, & \text{(A.7)}\\
F_\tau &= R_n^{\gamma-1}(1-b_n)\left(1+\frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2+(1-b_n)^2]\right) > 0. & \text{(A.8)}
\end{aligned}
$$

So,

$$
\frac{dh}{d\tau} = -\frac{F_\tau}{F_h} = F_\tau = R_n^{\gamma-1}(1-b_n)\left(1+\frac{\tau(\gamma-1)}{2}[(1+b_{n-1})^2+(1-b_n)^2]\right) > 0.
$$

A.1.6 Proof of Proposition 4

Part (i). By equation (A.7), $F_{b_{n-1}} > 0$ if

$$
\tau > \frac{1}{(\gamma-1)(b_0(1-b_0))}. \tag{A.9}
$$

By total differentiation, $\frac{\partial S_n}{\partial b_{n-1}} = -\frac{F_{b_{n-1}}}{F_S}$. Recall $F_S < 0$ in equation (A.6). Thus, $\frac{\partial S_n}{\partial b_{n-1}} > 0$ so long as inequality (A.9) holds.

Part (ii). Since $\frac{\partial^2 b_n}{\partial b_{n-1}\partial b_0} = \frac{\partial^2 S_n}{\partial b_{n-1}\partial b_0} = \frac{\partial^2 S_n}{\partial b_0 \partial b_{n-1}}$, we can show instead that $\frac{\partial^2 S_n}{\partial b_0\partial b_{n-1}} > 0$ and is increasing with n. A simple manipulation of equation (30) shows

$$
\frac{\partial b_n}{\partial b_{n-1}} = \frac{\partial(b_{n-1}+S_n)}{\partial b_{n-1}} = 1 + \frac{\partial S_n}{\partial b_{n-1}} > 0, \tag{A.10}
$$

where $\frac{\partial S_n}{\partial b_{n-1}} > 0$ comes from part (i). By equation (A.10),

$$
\frac{\partial b_n}{\partial b_0} = \prod_{i=0}^{n-1}\frac{\partial b_{n-i}}{\partial b_{n-i-1}} > 0, \tag{A.11}
$$

for any $n \geq 1$, and thus

$$
\frac{\partial S_n}{\partial b_0} = \frac{\partial S_n}{\partial b_{n-1}}\frac{\partial b_{n-1}}{\partial b_0} > 0. \tag{A.12}
$$

The first term in equation (A.12) is positive and increasing with n. Specifically, for a greater n (and thus $n-1$), b_n has to be extended further from b_{n-1}, resulting in a larger S_n.

Now move on to the second term in equation (A.12), which equals

$$
\frac{\partial b_{n-1}}{\partial b_0} = \prod_{i=1}^{n-1}\frac{\partial b_{n-i}}{\partial b_{n-i-1}},
$$

following equation (A.11). Here, every term inside the product is weakly greater than 1. They all equal 1 if all states from 1 to $n-1$ keep their original sizes. For a greater n (and thus $n-1$), the product has one more term in it and thus weakly increases. Notice that this result is independent from the change in b_n (and thus S_n).

To combine the two terms, one can see that $\frac{\partial^2 S_n}{\partial b_0 \partial b_{n-1}} > 0$ and is increasing with n.

A.2 Data work

A.2.1 Sources

Historical maps. We use multiple historical atlases as our data sources, including Barraclough (1994), Rand McNally (1992, 2015), and Overy (2010). Since maps from historical atlases are often available only for specific region-time blocks, combining these different sources allowed us to compile comprehensive world maps for various periods. Each period begins from a base year and extends approximately 20-30 years forward.

Selecting base years involves careful judgment, as it requires balancing historical significance with map availability. In principle, we chose years that (i) followed major wars and (ii) preceded relatively peaceful 20-30 year periods. The world political geography in these base years resulted from resolving the power imbalances that triggered the wars, leading to temporary regional stability. For example, 1750 followed the War of the Austrian Succession, and 1815 marked the signing of the Treaty of Paris. However, finding a suitable base year in the early 20th century is challenging, as the interwar years (1919-1938) were too short to be considered a peaceful period. Selecting a single year in this context could result in using a political map marked by regional tensions and changing borders. Nonetheless, the first half of the 20th century, a significant period in modern history, should not be excluded from this study. As a compromise, we pooled data from three separate base years—1914, 1920, and 1938—into a combined dataset.[19]

Similar judgments were applied when determining which states to exclude from the world maps. In general, territories with ambiguous sovereignty statuses were excluded. For instance, small island states were typically omitted because many were dependent territories. There are two exceptions to this rule. First, although colonies had ambiguous sovereignty statuses, they serve as good examples of border reshuffling and state formation. Therefore, colonies were treated as independent states during their respective periods if they eventually became independent. Second, 18th-century kingdoms were considered independent states if they were free from neighboring states with clear sovereignty statuses. Without these two exceptions, the number of states in historical periods would be too small.

[19] If a state changed its name across these base years, we treated it as a new state. If a state retained its name, we treated it as a "steady state" and averaged its variables across the three base years.

Locales in the world. The information on within-state administrative divisions is sourced from the GeoNames database.[20] The GeoNames database provides geographic coordinates for administrative divisions worldwide and includes current population data. For modern times, this dataset includes 21,068 such divisions, referred to as "locales" in our study. Since GeoNames does not have historical data, we employed two methods to address this gap. First, we used the current GeoNames population as a proxy for historical populations, as it represents the largest possible set of human habitats on the earth. Second, we utilized historical urban population data compiled by Reba, Reitsma, and Seto (2016), which, although based on historical records, covers only a limited number of cities, mainly megacities, throughout history. We compared the locale maps generated by both methods with each other and against historical maps with estimated population densities. The results are consistent. We prefer the first method due to its extensive coverage and compatibility with other country-level control variables.[21]

Other historical data. In addition to historical world maps, we extracted data on population, iron and steel production, military expenditures, and primary energy consumption from the National Material Capabilities Dataset (version 4) compiled by Singer (1987), which is now part of the Correlates of War (COW) project.[22] This dataset is regularly updated and extends beyond 1987, providing control variables that reflect each country's national power and level of industrialization. The dataset begins coverage in 1815, so data are unavailable for our 18th-century sample.

Summary statistics for all the variables described above are provided in Table A1. Figure A1 shows the distribution of $\ln Area(n)$ across different time periods.

A.2.2 Economic Relevance of Proximity to the WGC

In this appendix, we examine the economic relevance of proximity to the WGC, denoted as $D(t)$. We use the following gravity regression model:

$$\ln T(n,n') = \mu \ln Distance(n,n') + \bar{\vartheta} \cdot \begin{bmatrix} \ln Size(n) \\ \ln Size(n') \end{bmatrix} + \bar{\omega} \cdot \begin{bmatrix} \ln D(n) \\ \ln D(n') \end{bmatrix} + \iota' Z_{nn'} + \epsilon_{nn'}, \tag{A.13}$$

where $T(n,n')$ is the trade volume (imports) between states n and n', $Distance(n,n')$ is the distance between the two states, $Size(n)$ and $Size(n')$ are their respective sizes (either population or area), $Z_{nn'}$ are control variables, and $\epsilon_{nn'}$ is the error term. We introduced two novel terms, $D(n) \equiv \min_{t\in n} D(t)$ and $D(n') \equiv \min_{t\in n'} D(t)$, which measure the shortest distance between each state and the WGC. These two terms are our variables of interest as they capture whether proximity to the WGC has any implications for trade. We hypothesize that their coefficients are negative, such that states farther from the WGC have

[20] The database is accessible online at www.geonames.org, offering both free and paid data services.

[21] Note that the location of the WGC changes over time because uncharted areas differ by period. Locales mapped to uncharted areas in a historical period are excluded from the collections for that period, leading to shifts in the estimated WGC over time.

[22] The COW project is accessible online at www.correlatesofwar.org.

Table A1: Summary Statistics

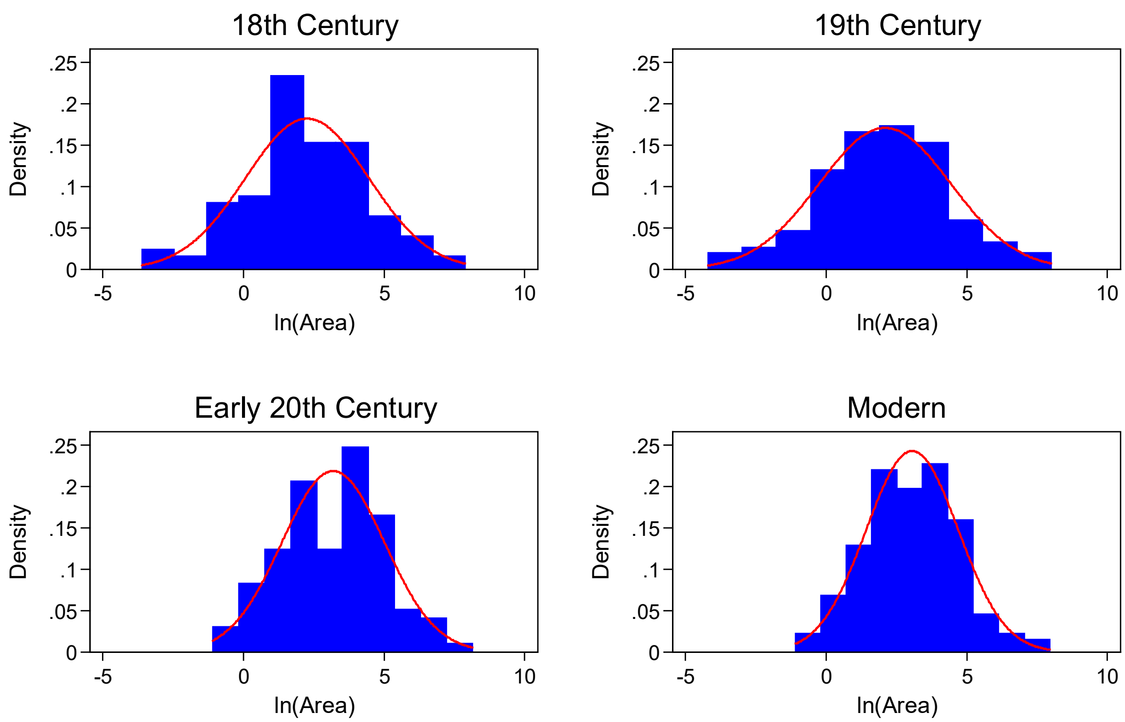
Variable	Obs	Mean	STD	Min	Max
Panel A: Modern period					
Distance from WGC (km)	162	5365	3575	132.2	17968
Area (square km)	162	86.41	274.7	0.338	2806
Coast dummy	162	0.753	0.433	0	1
Island dummy	162	0.123	0.330	0	1
Military expenditure#	156	3.548e+06	9.153e+06	4783	5.700e+07
Iron and steel production (tons)	156	5054	19802	0	205259
Primary energy consumption*	156	118773	308762	25.74	2.461e+06
Panel B: The 18th century					
Distance from WGC (km)	121	4959	3609	364.7	17620
Area (square km)	121	71.00	269.7	0.0269	2664
Coast dummy	121	0.752	0.434	0	1
Island dummy	121	0.182	0.387	0	1
Panel C: The 19th century					
Distance from WGC (km)	137	4945	3867	110.9	17970
Area (square km)	137	84.07	308.4	0.0148	2976
Coast dummy	137	0.679	0.469	0	1
Island dummy	137	0.153	0.362	0	1
Military expenditure#	51	5146	4316	14.73	20687
Iron and steel production (tons)	51	325.5	444.2	0	2806
Primary energy consumption*	51	7100	9968	0	62639
Panel D: The early 20th century					
Distance from WGC (km)	174	5606	3523	194.0	17968
Area (square km)	174	120.1	387.7	0.338	3401
Coast dummy	174	0.828	0.379	0	1
Island dummy	174	0.126	0.333	0	1
Military expenditure#	75	745823	1.919e+06	0	9.970e+06
Iron and steel production (tons)	75	1908	5953	0	45349
Primary energy consumption*	75	30703	100490	0	809321

Notes: # Following the COW database, the unit is 1,000 US dollars (1,000 British Pounds) in Panels A and D (Panel C). * The unit is 1,000 of coal-ton equivalents.

locational disadvantages in their trade with *every* trade partner. To test this hypothesis, we extracted data from the year 1994 from the CEPII gravity dataset to estimate the gravity regression (A.13).[23] As shown in Table A2, a shorter distance from the WGC is associated with higher bilateral trade volumes between states n and n'.

[23]The CEPII data are widely used in international trade studies. It is accessible online at `www.cepii.fr`. For details, see Head, Mayer, and Ries (2010) and Head and Mayer (2014).

Figure A1: Histogram of Territorial Area (Log-scale)



Notes: Red curves represent normal distributions.

A.2.3 Additional Robustness Checks

In our theory, world geography is considered a continuous landmass. However, the earth's landmass is divided by oceans into continents. Among all continents, Eurasia's geography best fits our theoretical construct. We reran the regressions in Table 3 using subsamples of Eurasian and non-Eurasian states for each period. The results, shown in Table A3, reveal similar patterns for both subsamples.

Additionally, we experimented with using the rank value of $D(n)$ instead of $\ln D(n)$ as the main explanatory variable. The state that is the n-th nearest to the contemporary WGC has a rank value of n. We normalized the rank value between 0 (nearest to the WGC) and 1 (farthest from the WGC) within each period to ensure it is unaffected by the varying numbers of states across periods. In Table A4, we use the rank value instead of $\ln D(n)$ while keeping the specifications otherwise the same as in Table 3. The results are similar to those in Table 3. However, the rank value lacks cardinal meaning, as its variation is ordinal and the differences between values are difficult to interpret. Therefore, it serves only as a robustness check.

Furthermore, we computed the centroid of each state (i.e., the arithmetic mean position of all the

Table A2: Economic Relevance of Distance from the WGC

Dep. variable is ln(Trade volume)				
	(1)	(2)	(3)	(4)
	Size=population		*Size=area*	
ln(Size of exporter)	0.518***	0.499***	0.323***	0.313***
	(0.009)	(0.009)	(0.009)	(0.009)
ln(Size of importer)	0.444***	0.426***	0.260***	0.251***
	(0.009)	(0.009)	(0.009)	(0.008)
ln(Bilateral distance)	-0.466***	-0.253***	-0.404***	-0.222***
	(0.021)	(0.023)	(0.022)	(0.025)
ln(Exporter's distance from WGC)	-0.305***	-0.331***	-0.404***	-0.408***
	(0.014)	(0.014)	(0.015)	(0.016)
ln(Importer's distance from WGC)	-0.255***	-0.281***	-0.332***	-0.335***
	(0.014)	(0.014)	(0.016)	(0.016)
Other control variables+	No	Yes	No	Yes
Observations	18,839	18,839	19,019	19,019

Notes: The data are for the year 1994 in both panels. + Control variables include dummies for being in the same regional trade agreements, sharing legal origins, sharing currencies, sharing borders, sharing official languages, dummies for being a GATT member (each side), and dummies for selling to and buying from a colony, respectively. Robust standard errors in parentheses. *** p<0.01.

points in the state as a polygon) and the centroid of the modern (1994) world.[24] Using these coordinates, we recalculated $D(n)$ and reran our study for the modern period. The centroid-based results are presented in Table A5, with both regression specifications and sample states following Table 3. Similar to Table 3, a positive and statistically significant correlation is found between $\ln Area(n)$ and $\ln D(n)$. The centroid approach serves only as a robustness check, as it may overstate the importance of territories with low (including zero) population density.

[24]We found the centroid of the modern world to be at (40.52N, 34.34E), located in Yarımca, Uğurludağ, Çorum, Turkey.

Table A3: Robustness Check: Eurasia vs. Non-Eurasia

Dependent variable is ln(Area)				
	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: The Eurasian subsample				
ln(Distance from WGC)	0.410***	0.868***	0.620***	0.356**
	(0.135)	(0.229)	(0.132)	(0.136)
Island and coast dummies	Yes	Yes	Yes	Yes
Observations	82	67	81	90
Panel B: The Non-Eurasian subsample				
ln(Distance from WGC)	1.033**	1.554***	1.673***	1.027**
	(0.427)	(0.451)	(0.270)	(0.445)
Island and coast dummies	Yes	Yes	Yes	Yes
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	80	54	56	84

Notes: Robust standard errors in parentheses. *** p<0.01, ** p<0.05.

Table A4: Robustness Check: Rank of Distance

Dependent variable is ln(Area)				
	(1)	(2)	(3)	(4)
	Modern	18th century	19th century	Early 20th century
Panel A: Full sample				
Rank(Distance from WGC)	0.007**	0.021***	0.017***	0.007**
	(0.003)	(0.005)	(0.004)	(0.003)
Coast dummy	0.202	-0.205	0.709**	0.528*
	(0.254)	(0.364)	(0.281)	(0.276)
Island dummy	-1.355***	-1.067***	-1.401***	-1.383***
	(0.371)	(0.403)	(0.458)	(0.356)
Continent fixed effects	Yes	Yes	Yes	Yes
Observations	162	121	137	174
Panel B: With national power controls				
Rank(Distance from WGC)	0.008***		0.058***	0.014**
	(0.003)		(0.018)	(0.005)
Coast dummy	-0.342		1.155***	0.192
	(0.230)		(0.427)	(0.483)
Island dummy	-0.965***		-2.403*	-0.885**
	(0.328)		(1.316)	(0.371)
ln(Military expenses)	0.022		-0.025	0.044
	(0.133)		(0.288)	(0.137)
ln(Iron & steel production)	-0.014		0.463	-0.038
	(0.054)		(0.297)	(0.098)
ln(Primary energy consumption)	0.512***		-0.235	0.267***
	(0.105)		(0.253)	(0.074)
Continent fixed effects	Yes		Yes	Yes
Observations	156		51	75

Notes: All specifications here are the same as those in Table 3 except that the main regressor is Rank(Distance from WGC) instead of ln(Distance from WGC). Rank 0 (respectively, 1) means the shortest (longest) distance from WGC. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table A5: Robustness Check: Distance from the World Centroid

Dependent variable is ln(Area)		
	(1)	(2)
ln(Distance from the world centroid)	0.554**	0.411**
	(0.236)	(0.172)
Coast dummy	0.226	-0.365
	(0.284)	(0.257)
Island dummy	-1.674***	-1.127***
	(0.448)	(0.407)
ln(Military expenses)		0.047
		(0.152)
ln(Iron & steel production)		-0.052
		(0.064)
ln(Primary energy consumption)		0.580***
		(0.127)
Continent fixed effects	Yes	Yes
Observations	162	156

Notes: The data is based on the 1994 world map. The set of states is the same as in column (1) of Table 3. Robust standard errors in parentheses. *** p<0.01, ** p<0.05.