Full State Feedback Controller for a Dynamic Capacitive Wireless Charging System

Abstract – This digest presents a novel time-domain state-space controller for the operation of capacitive wireless charging systems in dynamic scenarios, allowing the system to transfer power efficiently as the coupling capacitance between the transmitter and receiver is varied. An Active Variable Reactance (AVR) rectifier is used to adjust the inductive compensation of the receiver dynamically in response to the change in coupling capacitance. The control of this AVR rectifier is not well-suited for frequency-domain based control methods due to the high degree of coupling between its inputs. This work presents a model of the system in state-space and the design of a full state feedback (FSF) controller. The controller is validated in both simulation and hardware.

I. Introduction

Wireless power transfer is becoming increasingly important and viable to electric vehicle charging and has the potential to accelerate the widespread adoption of electric vehicles (EVs), especially vehicles that routinely travel along set routes such as buses and warehouse forklifts. Capacitive wireless power transfer (C-WPT) systems can be lighter, cheaper, and smaller than a comparable inductive wireless charging system due to the absence of heavy ferrite cores. The ability of C-WPT systems to transfer high power at high efficiency has been demonstrated. However, one main drawback of C-WPT is that it is extremely sensitive to the value of the coupling capacitance between the receiver and the transmitter. The efficiency and power transfer of the system drop dramatically with the slightest deviation of the coupling capacitance from its nominal value due to the high quality factor of the resonant components in the converter.

To address this shortcoming, we can use an AVR rectifier, which replaces the passive rectifier in standard capacitive wireless charging system. This circuit has the following important properties:

1) Able to provide a continuously variable reactance by varying the power processed in each of its two branches
2) Operates at a fixed frequency (does not violate stringent FCC radio-frequency regulations on EMI)

The use of the AVR rectifier not only allow the converter to transfer power more efficiently, but also allows the vehicle driving over the transmitter to receive, in theory, up to 80% more power in a single pass compared to a baseline C-WPT system without the AVR rectifier. This is because the active compensation expands the range of possible receiver positions for which the converter is able to deliver maximum power from a single point to a relatively large three-dimensional space. Thus, as the vehicle drives over the charging pad, it is receiving the maximum power from the pad for a measurable duration of time, instead of only a single instant.

A control scheme designed in the frequency domain for the AVR rectifier which fully compensates for changes in coupling capacitance is presented in **paper reference**. Though this control scheme was theoretically sound, in practice, the controller was difficult to tune and was extremely sensitive to slight changes in component values. Therefore, it was unable to operate the system in a dynamic scenario. Due to the high degree of cross-coupling between the inputs and outputs of the AVR rectifier as well as the significant nonlinearities in the system, the AVR rectifier control problem is not well-suited for frequency-domain control.

This work proposes a new approach to the AVR rectifier control problem. We first build a model for the AVR rectifier in state-space. We then augment the model with desired control objectives in order to drive certain circuit parameters to their desired dc values, regardless of the coupling capacitance. Finally, we design an optimal FSF controller using a linear quadratic regulator (LQR). This controller is validated on an experimental prototype, and dynamic wireless charging is demonstrated. The proposed state-space model, optimal controller design technique, and hardware validation together comprise a significant contribution to literature.

II. Dynamic Capacitive Wireless Charging System with an AVR Rectifier

III. State Space System Modeling

We begin by deriving the small-signal linearized model of the AVR rectifier. This small signal model must include all elements which contribute to the dynamics of the system at timescales comparable to the change in the coupling capacitance C_s, but no additional elements which introduce unnecessary system complexity. The portion of the AVR rectifier that was modeled and its corresponding small-signal circuit model is shown in Figure **insert figure reference**. It is essentially one boost and one buck converter that have their outputs tied together; both converters also contain an input capacitance.

insert figure

For ease of notation, the **boost** converter elements are referred to with a subscript of **1**, and the **buck** converter elements are referred to with a subscript of **2**.

Next, we derive the pure, unaugmented state-space model for the AVR rectifier. In general, a continuous-time state-space model can be written as:

$$\frac{d}{dt}\vec{x} = A\vec{x} + B\vec{u} \tag{1}$$

where $\vec{x}$ contains the system states, $\vec{u}$ contains the system inputs, A is the system dynamics matrix, and B is the input dynamics matrix.

We first define the states of the circuit which comprise $\vec{x}$. For a linear RLC circuit, it is usually most convenient to define the states as the capacitor voltages and inductor currents, of which we have five: v_1, v_2, i_{L1}, i_{L2}, and v_{out}. We then also define the inputs of the circuit which comprise $\vec{u}$. We have two control handles: d_1' and d_2, the inverse duty cycle of the boost converter and the duty cycle of the buck converter. It is important to remember that these variables all represent small-signal disturbances to the large-signal states, which should have some dc value in the nominal case $\Delta C_s = 0$.

Next, we derive the scalar equations for the derivatives of the circuit states. For example, to find this equation for $\frac{d}{dt}v_{out}$, we write KCL for the output node of the small-signal model of Figure **insert figure reference**:

$$i_{Cout} + \frac{v_{out}}{R} = i_{L2} - I_{L1}d_1 + D_1' i_{L1} \tag{2}$$

Substituting the I-V relation for the output capacitor current and solving for the desired derivative yields:

$$\frac{dv_{out}}{dt} = -\frac{1}{RC_{out}}v_{out} + \frac{1}{C_{out}}i_{L2} - \frac{I_{L1}}{C_{out}}d_1 + \frac{D_1'}{C_{out}}i_{L1} \tag{3}$$

Performing similar analysis for the other four states yields:

$$\begin{cases} \frac{dv_1}{dt} &= -\frac{1}{C_1}i_{L1} \\ \frac{dv_2}{dt} &= -\frac{I_{L2}}{C_2}d_2 - \frac{D_2}{C_2}i_{L2} \\ \frac{di_{L1}}{dt} &= \frac{1}{L1}v_1 + \frac{V_{out}}{L_1}d_1 - \frac{D_1'}{L_1}v_{out} \\ \frac{di_{L2}}{dt} &= \frac{V_2}{L_2}d_2 + \frac{D_2}{L_2}v_2 - \frac{1}{L_2}v_{out} \end{cases} \tag{4}$$

Combining these equations into a single matrix equation, we obtain the small-signal state-space representation of the AVR rectifier:

$$\frac{d}{dt}\begin{bmatrix} v_{out} \\ v_1 \\ v_2 \\ i_{L1} \\ i_{L2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_{out}} & 0 & 0 & \frac{D_1'}{C_{out}} & \frac{1}{C_{out}} \\ 0 & 0 & 0 & -\frac{1}{C_1} & 0 \\ 0 & 0 & 0 & 0 & -\frac{D_2}{C_2} \\ -\frac{D_1'}{L_1} & \frac{1}{L_1} & 0 & 0 & 0 \\ -\frac{1}{L_2} & 0 & \frac{D_2}{L_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{out} \\ v_1 \\ v_2 \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} -\frac{I_{L1}}{C_{out}} & 0 \\ 0 & 0 \\ 0 & -\frac{I_{L2}}{C_2} \\ \frac{V_{out}}{L_1} & 0 \\ 0 & \frac{V_2}{L_2} \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{5}$$

IV. Full State Feedback Controller Design and Simulation

As found in **insert reference to AVR paper**, the AVR rectifier can be controlled without measuring high-frequency voltages or currents, or measuring the change in C_s. This is done by driving the converter's output power p_{out} and real part of the input impedance $Re(z_r)$ to the their values when $\Delta C_s = 0$ ($\equiv P_{out}, Re(Z_r)$, respectively). In order to include these control targets in our model, we define two new augmented states as follows:

$$\begin{cases} z_1 = \int P_{out} - f_1(\vec{x})dt \\ z_2 = \int Re(Z_r) - f_2(\vec{x})dt \end{cases} \tag{6}$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are nonlinear functions of the five existing states in our state-space model; f_1 computes p_{out} and f_2 computes $Re(z_r)$. Thus, the augmented states z_1 and z_2 are the integrals of the errors between the measured value and desired value of p_{out} and $Re(z_r)$ when $\Delta C_s = 0$. Taking the time derivative, we obtain:

$$\begin{cases} \frac{d}{dt}z_1 = P_{out} - f_1(\vec{x}) \\ \frac{d}{dt}z_2 = Re(Z_r) - f_2(\vec{x}) \end{cases} \tag{7}$$

In order to augment our system with this information, we must determine f_1 and f_2, then further linearize them such that a differential change in p_{out} or $Re(z_r)$ can be expressed as a linear combination of the differential changes of the existing states. The following analysis assumes the converter is lossless ($P_{in} = P_{out}$).

The output power can be written as $p_{out} = f_1(\vec{x}) = i_{L1}v_1 + i_{L2}v_{out}$. This is a nonlinear function of our states, so we linearize the system about the operating point by computing the total derivative of p_{out} with respect to i_{L1}, v_1, i_{L2}, and v_{out}. We obtain:

$$dp_{out} = V_{out}d_{iL2} + I_{L2}d_{vout} + V_1 d_{iL1} + I_{L1}d_{v1} \tag{8}$$

The real part of the input impedance $Re(Z_r)$ can be written as the following:

$$Re(Z_r) = f_2(\vec{x}) = \frac{k_{rec}^2 v_1^2 v_2^2 + p_1 p_2 X^2}{k_{rec}(v_1 v_2^2 i_{L1} + v_{out} v_1^2 i_{L2})} \tag{9}$$

where $k_{rec} = 2/\pi^2$ is the gain associated with the half-bridge rectifier under fundamental harmonic analysis, and X is the differential reactance of the two branches of the AVR rectifier ($+jX$ and $-jX$). Plugging in $p_1 = i_{L1}v_1$ and $p_2 = i_{L2}v_{out}$ gives:

$$Re(Z_r) = \frac{k_{rec}^2 v_1^2 v_2^2 + v_1 v_{out} i_{L1} i_{L2} X^2}{k_{rec}(v_1 v_2^2 i_{L1} + v_{out} v_1^2 i_{L2})} \tag{10}$$

Again, this is a nonlinear function of our states, but we perform the same analysis as with the output power to obtain:

$$d[Re(z_r)] = \frac{\partial Re(z_r)}{\partial v_{out}} d_{vout} + \frac{\partial Re(z_r)}{\partial v_1} d_{v1} + \frac{\partial Re(z_r)}{\partial v_2} d_{v2} + \frac{\partial Re(z_r)}{\partial i_{L1}} d_{iL1} + \frac{\partial Re(z_r)}{\partial i_{L2}} d_{iL2} \tag{11}$$

The derivations of these partial derivatives is not shown in this digest. Using the above results, we can augment our state-space model from Sec. III to obtain the following augmented state-space model:

$$\frac{d}{dt}\begin{bmatrix} v_{out} \\ v_1 \\ v_2 \\ i_{L1} \\ i_{L2} \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC_{out}} & 0 & 0 & \frac{D_1'}{C_{out}} & \frac{1}{C_{out}} & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{D_2}{C_2} & 0 & 0 \\ -\frac{D_1'}{L_1} & \frac{1}{L_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{L_2} & 0 & \frac{D_2}{L_2} & 0 & 0 & 0 & 0 \\ -I_{L2} & -I_{L1} & 0 & -V_1 & -V_{out} & 0 & 0 \\ -\frac{\partial Re(z_r)}{\partial v_{out}} & -\frac{\partial Re(z_r)}{\partial v_1} & -\frac{\partial Re(z_r)}{\partial v_2} & -\frac{\partial Re(z_r)}{\partial i_{L1}} & -\frac{\partial Re(z_r)}{\partial i_{L2}} & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{out} \\ v_1 \\ v_2 \\ i_{L1} \\ i_{L2} \\ z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} -\frac{I_{L1}}{C_{out}} & 0 \\ 0 & 0 \\ 0 & -\frac{I_{L2}}{C_2} \\ \frac{V_{out}}{L_1} & 0 \\ 0 & \frac{V_2}{L_2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{12}$$

To close the loop around the plant, we need to find a gains matrix K such that when we set $\vec{u} = r - K\vec{x}$, the system satisfies the two control objectives through a range of disturbances. To do this, we use a linear quadratic regulator (LQR) to design the matrix K. We must choose reasonable values for the diagonal elements of the Q and R matrices. For R, we penalize large control input deviations for two reasons:

1) The range of sensible values for the control values range only from 0 to 1, as the control inputs to the system are duty cycles for power converters.
2) In the nominal operating condition, the buck and boost converters are in pass-through mode (operating with duty cycles of 1), thus any large control input in the wrong direction will simply be rejected as the input saturates.

For Q, we set the penalty for deviations in the values of the original five states from their nominal values to 0, as their values (with the exception with V_{out}) will change from their nominal values in response to a disturbance; we do not want the controller to fight its own control action frivolously.

Once we have a reasonable guess for Q and R, we use the `lqr()` function in MATLAB to solve the corresponding Algebraic Riccati Equation for the optimal gains matrix K.

Figure **insert figure reference** shows the complete block diagram of the system with the loop closed.

V. Hardware and Experimental Results

VI. Conclusion