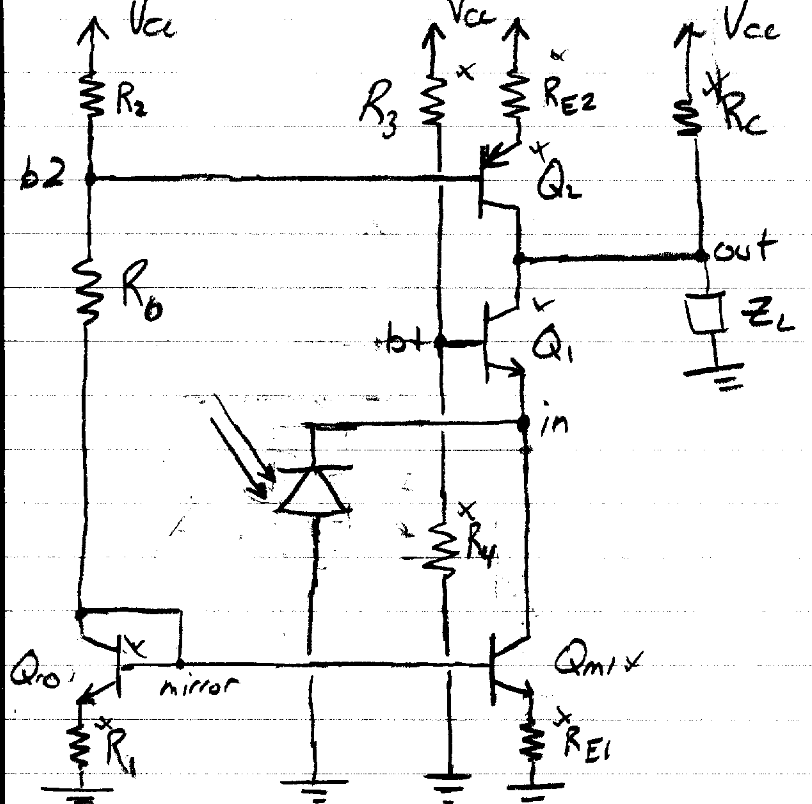
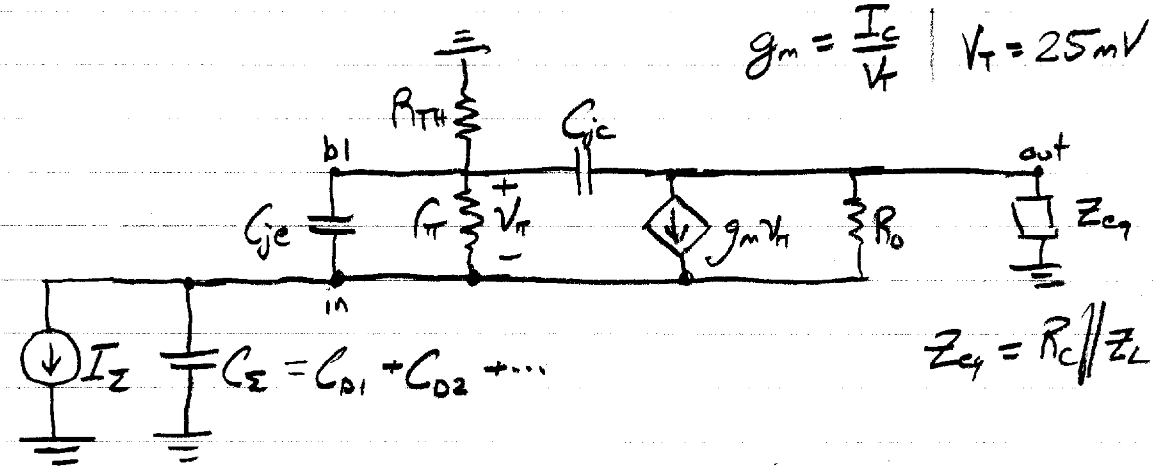
Improved Transimpedance Stage

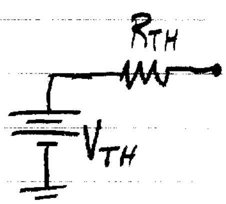


$$r_\pi = \frac{V_T\beta}{I_c} \quad \Big| \quad R_0 = \frac{V_A}{I_c}$$

$$g_m = \frac{I_c}{V_T} \quad \Big| \quad V_T = 25\,mV$$

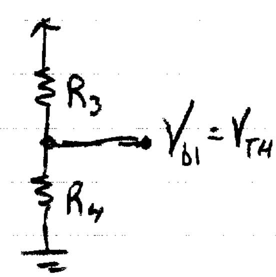


$$Z_{eq} = R_C \,\|\, Z_L$$



$$R_{TH} = \frac{R_3 R_4}{R_3 + R_4} \quad \Big| \quad R_3 = R_{TH}\,\frac{V_{cc}}{V_{TH}}$$

$$V_{TH} = \frac{R_4 V_{cc}}{R_3 + R_4} \quad \Big| \quad R_4 = R_{TH}\,\frac{V_{cc}}{V_{cc} - V_{TH}}$$



KCL at the emitter/input:

$$I_\Sigma + \frac{V_{IN}}{\left(\frac{1}{j\omega C_\Sigma}\right)} + \frac{V_{IN}-V_{B1}}{\underbrace{\left(r_\pi + \frac{1}{j\omega C_{je}}\right)}} + (V_{IN}-V_{B1})(-g_m) + \frac{V_{IN}-V_{OUT}}{R_o} = \emptyset$$

$$\left[r_\pi + \frac{1}{j\omega C_{je}}\right] = r_\pi\left(1 - \frac{j}{\omega r_\pi C_{je}}\right) = r_\pi\sqrt{1+\left(\frac{1}{\omega r_\pi C_{je}}\right)^2}\, e^{-j\angle\tan^{-1}\left(\frac{1}{\omega r_\pi C_j}\right)}$$

$$V_{IN}\cdot j\omega C_\Sigma + \frac{V_{IN}-V_{B1}}{r_\pi\sqrt{1+(\omega_o/\omega)^2}}\, e^{j\angle\tan^{-1}(\omega_o/\omega)} + (V_{IN}-V_{B1})(-g_m) + \frac{V_{IN}-V_{OUT}}{R_o} = -I_\Sigma$$

[4] where $\boxed{\omega_o = \frac{1}{r_\pi C_{je}} = \frac{I_C}{V_T C_{je}\beta}}$

[1] $$V_{IN}\left[j\omega C_\Sigma + \frac{e^{j\angle\tan^{-1}\omega_o/\omega}}{r_\pi\sqrt{1+(\omega_o/\omega)^2}} - g_m + \frac{1}{R_o}\right] + V_{B1}\left[g_m - \frac{e^{j\angle\tan^{-1}(\omega_o/\omega)}}{r_\pi\sqrt{1+(\omega_o/\omega)^2}}\right] - V_{OUT}\left[\frac{1}{R_o}\right] = -I_\Sigma$$

KCL at the base b_1:

$$\frac{V_{B1}-V_{IN}}{\left(\frac{1}{j\omega C_{je}}+r_\pi\right)} + \frac{V_{B1}}{R_{TH}} + \frac{V_{B1}-V_{OUT}}{\left(\frac{1}{j\omega C_{jc}}\right)} = \emptyset$$

[2] $$V_{IN}\left[\frac{-e^{j\angle\tan^{-1}(\omega_o/\omega)}}{r_\pi\sqrt{1+(\omega_o/\omega)^2}}\right] + V_{B1}\left[\underbrace{\frac{1}{R_{TH}} + \frac{e^{j\angle\tan^{-1}(\omega_o/\omega)}}{r_\pi\sqrt{1+(\omega_o/\omega)^2}} + j\omega C_{jc}}\right] + V_{OUT}\left[-j\omega C_{jc}\right] = \emptyset$$

$$\left[\frac{1}{R_{TH}} + j\omega C_{jc}\right] = \frac{1}{R_{TH}}\left(1+j\omega R_{TH}C_{jc}\right) = \frac{1}{R_{TH}}\sqrt{1+(\omega R_{TH}C_{jc})^2}\, e^{j\angle\tan^{-1}\omega R_{TH}C_{jc}}$$

Intentionally design R_{TH} such that the ω_o term is negligible:

$$\omega C_{jc}\sqrt{1+\left(\frac{1}{\omega R_{TH}C_{jc}}\right)^2} = \frac{1}{R_{TH}}\sqrt{1+(\omega R_{TH}C_{jc})^2} \geq 10\cdot\frac{1}{r_\pi\sqrt{1+(\omega_o/\omega)^2}}$$

Let $\boxed{\omega_1 = \frac{1}{R_{TH}C_{jc}}}$. Assume that $\frac{\omega}{\omega_1} << 1$,

or that $\boxed{R_{TH} \leq \frac{1}{10\,\omega C_j}}$.

Then the design equation simplifies to

$$\frac{1}{R_{TH}} \geq 10 \frac{1}{r_\pi \sqrt{1+(\omega_0/\omega)^2}}$$

[5] $$\boxed{R_{TH} \leq \frac{r_\pi}{10} \cdot \sqrt{1+(\omega_0/\omega)^2} = \frac{I_c}{10\beta V_T} \sqrt{1+\left(\frac{I_c}{V_T C_{je} \beta \omega}\right)^2}}$$

But equation [5] is no longer valid if the resulting R_{TH} does not satisfy the relation:

[6] $$\boxed{R_{TH} \leq \frac{1}{10\omega C_{jc}}} \qquad \frac{1}{R_{TH}} \geq 10\omega C_{jc}$$

Going back to the base current analysis...

[2'] $$V_{IN}\left[\frac{-e^{j\angle \tan^{-1}\omega_0/\omega}}{r_\pi \sqrt{1+(\omega_0/\omega)^2}}\right] + V_{B1}\left[\frac{1}{R_{TH}}\right] + V_{OUT}\left[-j\omega C_{jc}\right] = \emptyset$$

For the last independent equation, KCL at the collector/output:

$$\frac{V_{OUT}-V_{B1}}{\left(\frac{1}{j\omega C_j}\right)} + (V_{B1}-V_{IN})(g_m) + \frac{V_{OUT}-V_{IN}}{R_o} + \frac{V_{OUT}}{Z_{eq}} = \emptyset$$

[3] $$V_{IN}\left[-g_m - \frac{1}{R_o}\right] + V_{B1}\left[g_m - j\omega C_{jc}\right] + V_{OUT}\left[\frac{1}{R_o} + \frac{1}{Z_{eq}} + j\omega C_{jc}\right] = \emptyset$$

Next we make the following substitutions into [1], [2], [2'], and [3] to get a system in terms of two design parameters $I_{c(sat)}$ and R_C.

$$r_\pi = \frac{\beta V_T}{I_c}, \quad g_m = \frac{I_c}{V_T}, \quad R_o = \frac{V_A}{I_c}$$

$$V_{IN}\left[j\omega C_{j2} + \frac{I_c e^{j\angle \tan^{-1}\omega_0/\omega}}{V_T \beta \sqrt{1+(\omega_0/\omega)^2}} - \frac{I_c}{V_T} + \frac{I_c}{V_A}\right] + V_{B1}\left[\frac{I_c}{V_T} - \frac{I_c e^{j\angle \tan^{-1}\omega_0/\omega}}{\beta V_T \sqrt{1+(\omega_0/\omega)^2}}\right] - V_{out}\left[\frac{I_c}{V_A}\right] = -I_\Sigma$$

[1'] $$V_{IN}\left[\frac{I_c}{V_T}\left(\frac{e^{j\angle \tan^{-1}(\omega_0/\omega)}}{\beta\sqrt{1+(\omega_0/\omega)^2}}-1\right)+j\omega C_\Sigma\right]+V_{B1}\left[\frac{I_c}{V_T}\left(1-\frac{e^{j\angle \tan^{-1}\omega_0/\omega}}{\beta\sqrt{1+(\omega_0/\omega)^2}}\right)\right]-V_{OUT}\left[\frac{I_c}{V_A}\right]=-I_\Sigma$$

[2''] $$V_{IN}\left[\frac{-I_c\, e^{j\angle \tan^{-1}\omega_0/\omega}}{\beta V_T\sqrt{1+\omega_0/\omega}}\right]+V_{B1}\left[\frac{1}{R_{TH}}+\frac{I_c\, e^{j\angle \tan^{-1}\omega_0/\omega}}{\beta V_T\sqrt{1+(\omega_0/\omega)^2}}+j\omega C_{jc}\right]+V_{OUT}\left[-j\omega C_{jc}\right]=\emptyset$$

[3'] $$V_{IN}\left[\frac{-I_c}{V_T}\right]+V_{B1}\left[\frac{I_c}{V_T}-j\omega C_{jc}\right]+V_{OUT}\left[\frac{I_c}{V_A}+\frac{1}{Z_{eq}}+j\omega C_{jc}\right]=\emptyset$$

Adding 3 to 2'':

$$V_{IN}\left[\frac{-I_c}{V_T}\left(1+\frac{e^{j\angle \tan^{-1}\omega_0/\omega}}{\beta\sqrt{1+(\omega_0/\omega)^2}}\right)\right]+V_{B1}\left[\frac{1}{R_{TH}}+\frac{I_c}{V_T}\left(1+\frac{e^{-j\angle \tan^{-1}\omega_0/\omega}}{\beta\sqrt{1+(\omega_0/\omega)^2}}\right)\right]+V_{OUT}\left[\frac{I_c}{V_A}+\frac{1}{Z_{eq}}\right]=\emptyset$$

[2'''] $$V_{IN}\left[\frac{-I_c}{V_T}\right]+V_{B1}\left[\frac{1}{R_{TH}}+\frac{I_c}{V_T}\right]+V_{OUT}\left[\frac{I_c}{V_A}+\frac{1}{Z_{eq}}\right]=\emptyset$$

Subtracting the simplified, approximated 2'' back from 3:

[3''] $$V_{B1}\left[\frac{-1}{R_{TH}}-j\omega C_{jc}\right]+V_{OUT}\left[j\omega C_{jc}\right]=\emptyset$$

Equation 1' can actually be simplified by itself now that all terms are given in I_c.

$$V_{IN}\left[\frac{-I_c}{V_T}+j\omega C_\Sigma\right]+V_{B1}\left[\frac{I_c}{V_T}\right]+V_{OUT}\left[\frac{-I_c}{V_A}\right]=-I_\Sigma$$

[1''] $$V_{IN}\left[\frac{I_c}{V_T}-j\omega C_\Sigma\right]+V_{B1}\left[\frac{-I_c}{V_T}\right]+V_{OUT}\left[\frac{I_c}{V_A}\right]=I_\Sigma$$

Adding 2''' to 1'' and dropping R_o:

[1'''] $$V_{IN}\left[-j\omega C_\Sigma\right]+V_{B1}\left[\frac{1}{R_{TH}}\right]+V_{OUT}\left[\cancel{\frac{I_c}{V_A}}+\frac{1}{Z_{eq}}\right]=I_\Sigma$$

Scaling 1''' in preparation for addition to 2''':

$$-j\omega C_Z \mapsto \frac{I_c}{V_T} \qquad \times\frac{-I_c}{V_T\, j\omega C_Z}$$

$$V_{IN}\left[\frac{I_c}{V_T}\right] + V_{B1}\left[\frac{+j I_c}{V_T R_{TH}\omega C_Z}\right] + V_{OUT}\left[\frac{+j I_c}{V_T \omega C_Z Z_{eq}}\right] = \frac{+j I_c I_Z}{V_T \omega C_Z}$$

Combining 1''' and 2''' into 2^{IV} and dropping R_o again

$$V_{B1}\left[\frac{1}{R_{TH}} + \frac{I_c}{V_T} + \frac{j I_c}{V_T R_{TH}\omega C_Z}\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\left(1 + j\frac{I_c}{V_T \omega C_Z}\right)\right] = \frac{+j I_c I_Z}{V_T \omega C_Z}$$

$$V_{B1}\left[\frac{-j V_T \omega C_Z}{I_c R_{TH}} + \frac{-j\omega C_Z}{1} + \frac{1}{R_{TH}}\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\left(\frac{-j V_T \omega C_Z}{I_c} + 1\right)\right] = I_Z$$

$$V_{B1}\left[\frac{1}{R_{TH}} - j\omega C_Z\left(1 + \frac{V_T}{I_c R_{TH}}\right)\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\left(1 - j\frac{V_T \omega C_Z}{I_c}\right)\right] = -I_Z$$

Making a fairly conservative assumption that

$$\boxed{R_{TH} \geq 10\frac{V_T}{I_c}}$$

then we have

[2^{IV}] $$V_{B1}\left[\frac{1}{R_{TH}}\left(1 - j\omega R_{TH} C_Z\right)\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\left(1 - j\frac{V_T \omega C_Z}{I_c}\right)\right] = I_Z$$

To repeat, at this point we have

$$V_{IN}\left[-j\omega C_Z\right] + V_{B1}\left[\frac{1}{R_{TH}}\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\right] = I_Z$$

$$V_{B1}\left[\frac{1}{R_{TH}}\left(1 - j\omega R_{TH} C_Z\right)\right] + V_{OUT}\left[\frac{1}{Z_{eq}}\left(1 - j\frac{V_T \omega C_Z}{I_c}\right)\right] = -I_Z$$

$$V_{B1}\left[\frac{-1}{R_{TH}}\left(1 + j\omega R_{TH} C_{jc}\right)\right] + V_{OUT}\left[j\omega C_{jc}\right] = \emptyset$$

Equation 3'' can be added to 2'V for interesting effect

$$V_{B1}\left[\frac{1}{R_{TH}}\left(1 - j\omega R_{TH}C_{\Sigma} - 1 - j\omega R_{TH}C_{jc}\right)\right] + V_{OUT}\left[\frac{1}{Z_{eq}} - j\frac{V_T\omega C_{\Sigma}}{Z_{eq}I_c} + j\omega C_{jc}\right] = -I_{\Sigma}$$

$$V_{B1}\left[-j\omega\left(C_{\Sigma} + C_{jc}\right)\right] + V_{OUT}\left[\frac{1}{Z_{eq}} + j\omega\left(C_{jc} - \frac{V_T}{Z_{eq}I_c}C_{\Sigma}\right)\right] = -I_{\Sigma}$$

2N3904

THERMAL DATA

$R_{thj\text{-}amb}$ •	**Thermal Resistance** Junction-Ambient	Max	200	°C/W
$R_{thj\text{-}case}$ •	**Thermal Resistance** Junction-Case	Max	83.3	°C/W

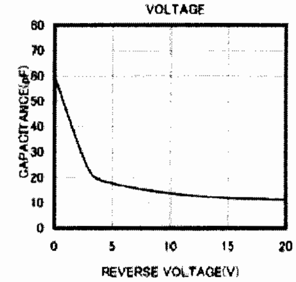
ELECTRICAL CHARACTERISTICS (T_{case} = 25 °C unless otherwise specified)

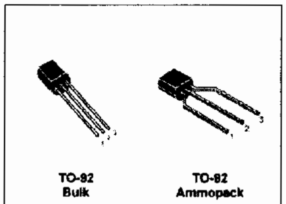
Symbol	Parameter	Test Conditions	Min.	Typ.	Max.	Unit
I_{CEX}	Collector Cut-off Current (V_{BE} = -3 V)	V_{CE} = 30 V			50	nA
I_{BEX}	Base Cut-off Current (V_{BE} = -3 V)	V_{CE} = 30 V			50	nA
$V_{(BR)CEO}$*	Collector-Emitter Breakdown Voltage (I_B = 0)	I_C = 1 mA	40			V
$V_{(BR)CBO}$	Collector-Base Breakdown Voltage (I_E = 0)	I_C = 10 µA	60			V
$V_{(BR)EBO}$	Emitter-Base Breakdown Voltage (I_C = 0)	I_E = 10 µA	6			V
$V_{CE(sat)}$*	Collector-Emitter Saturation Voltage	I_C = 10 mA I_B = 1 mA I_C = 50 mA I_B = 5 mA			0.2 0.2	V V
$V_{BE(sat)}$*	Base-Emitter Saturation Voltage	I_C = 10 mA I_B = 1 mA I_C = 50 mA I_B = 5 mA	0.65		0.85 0.95	V V
h_{FE}*	DC Current Gain	I_C = 0.1 mA V_{CE} = 1 V I_C = 1 mA V_{CE} = 1 V I_C = 10 mA V_{CE} = 1 V I_C = 50 mA V_{CE} = 1 V I_C = 100 mA V_{CE} = 1 V	60 80 100 60 30		300	
f_T	Transition Frequency	I_C = 10 mA V_{CE} = 20 V f = 100 MHz	250	270		MHz
C_{CBO}	Collector-Base Capacitance	I_E = 0 V_{CB} = 10 V f = 1 MHz		4		pF
C_{EBO}	Emitter-Base Capacitance	I_C = 0 V_{EB} = 0.5 V f = 1 MHz		18		pF
NF	Noise Figure	V_{CE} = 5 V I_C = 0.1 mA f = 10 Hz to 15.7 KHz R_G = 1 KΩ		5		dB
t_d t_r	Delay Time Rise Time	I_C = 10 mA I_B = 1 mA V_{CC} = 30 V			35 35	ns ns
t_s t_f	Storage Time Fall Time	I_C = 10 mA I_{B1} = $-I_{B2}$ = 1 mA V_{CC} = 30 V			200 50	ns ns

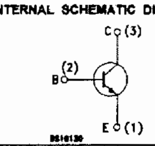
• Pulsed: Pulse duration = 300 µs, duty cycle ≤ 2 %

Photo Diode

Product No: **MTD3010PM**







ABSOLUTE MAXIMUM RATINGS

Symbol	Parameter	Value	Unit
V_{CBO}	Collector-Base Voltage (I_E = 0)	60	V
V_{CEO}	Collector-Emitter Voltage (I_B = 0)	40	V
V_{EBO}	Emitter-Base Voltage (I_C = 0)	6	V
I_C	Collector Current	200	mA
P_{tot}	Total Dissipation at T_C = 25 °C	625	mW
T_{stg}	Storage Temperature	-65 to 150	°C
T_j	Max. Operating Junction Temperature	150	°C

From the datasheets for the photodiode and transistor, it is fairly safe to assume that

$$C_{jc} << C_\Sigma = N\cdot C_{PD}$$

$$@10V, \qquad 4pF << 16\cdot 13\,pF \quad \checkmark$$

So the new equation for 2^v can be approximated by

$[2^v]$ $$V_{B1}\left[-j\omega C_\Sigma\right] + V_{OUT}\left[\frac{1}{Z_{eq}} + j\omega C_{jc} - j\omega C_\Sigma\left(\frac{V_T}{Z_{eq} I_c}\right)\right] = -I_\Sigma$$

Need to scale 2^v for combination with $3''$

$$-j\omega C_\Sigma \longmapsto \frac{1}{R_{TH}} + j\omega C_{jc}$$

$$V_{B1}\underbrace{\left[\frac{1}{R_{TH}}+j\omega C_{jc}\right]} + V_{OUT}\left[\frac{-\left(\frac{1}{R_{TH}}+j\omega C_{jc}\right)}{Z_{eq}\cdot j\omega C_{j\Sigma}} + \left(\frac{1}{R_{TH}}+j\omega C_{jc}\right)\left(\frac{V_T}{Z_{eq}I_c}\right)\right] = -I_\Sigma\cdot\frac{\frac{1}{R_{TH}}+j\omega C_{jc}}{j\omega C_\Sigma}$$

$$\left[\frac{1}{R_{TH}}+j\omega C_{jc}\right] = \frac{1}{R_{TH}}\left(1+j\omega R_{TH}C_{jc}\right) = j\omega C_{jc}\left(1-\frac{j}{\omega R_{TH}C_{jc}}\right)$$

$$= \frac{1}{R_{TH}}\sqrt{1+(\omega R_{TH}C_{jc})^2}\,e^{j\tan^{-1}\omega R_{TH}C_{jc}} = \omega C_{jc}\sqrt{1+\left(\frac{1}{\omega R_{TH}C_{jc}}\right)^2}\,e^{-j\angle\tan^{-1}\frac{1}{\omega R_{TH}C_{jc}}}$$

$$\cancel{\left[\frac{1}{Z_{eq}}+j\omega C_{jc}\right] = j\omega C_{jc}\left(1-\frac{j}{Z_{eq}\omega C_{jc}}\right) = \omega C_{jc}\sqrt{1+\left(\frac{1}{Z_{eq}\omega C_{jc}}\right)^2}\,e^{-j\angle\tan^{-1}(\omega Z_{eq} C_{jc})}}$$

$$V_{B1}\left[\frac{1}{R_{TH}}+j\omega C_{jc}\right] + V_{OUT}\left[\frac{-\omega C_{jc}\sqrt{1+\left(\frac{1}{\omega R_{TH}C_{jc}}\right)^2}\,e^{-j\tan^{-1}\left(\frac{1}{\omega R_{TH}C_{jc}}\right)}}{\omega C_\Sigma\cdot j Z_{eq}} + \left(\frac{V_T}{Z_{eq}I_c}\right)\left(\frac{1}{R_{TH}}+j\omega C_{jc}\right)\right]$$
$$= -I_\Sigma\left[\frac{-j}{\omega R_{TH}C_\Sigma}+\frac{C_{jc}}{C_\Sigma}\right]$$

$$V_{B1}\left[\frac{1}{R_{TH}}+j\omega C_{jc}\right] + V_{OUT}\left[\frac{-C_{jc}}{Z_{eq}C_\Sigma}\sqrt{1+\left(\frac{1}{\omega R_{TH}C_{jc}}\right)^2}\,e^{-j\angle\tan^{-1}\left(\frac{1}{\omega R_{TH}C_{jc}}\right)} + \left(\frac{V_T}{Z_{eq}I_c}\right)\left(\frac{1}{R_{TH}}+j\omega C_{jc}\right)\right]$$
$$= -I_\Sigma\left[\frac{C_{jc}}{C_\Sigma}\left(1-\frac{j}{\omega R_{TH}C_{jc}}\right)\right]$$

Combining 2'' into equation 3''

(3'') $$V_{B1}\left[\frac{-1}{R_{TH}} - j\omega C_{jc}\right] + V_{out}\left[j\omega C_{jc}\right] = \phi$$

(2'') $$V_{B1}\left[\frac{1}{R_{TH}} + j\omega C_{jc}\right] + V_{out}\left[\frac{\frac{1}{R_{TH}} + j\omega C_{jc}}{-Z_{eq}\, j\omega C_\Sigma} + \left(\frac{1}{R_{TH}} + j\omega C_{jc}\right)\frac{V_T}{Z_{eq} I_c}\right] = I_\Sigma \frac{\frac{1}{R_{TH}} + j\omega C_{jc}}{j\omega C_\Sigma}$$

+

$$V_{out}\left[\frac{\frac{1}{R_{TH}} + j\omega C_{jc}}{-Z_{eq}\, j\omega C_\Sigma} + \frac{V_T}{Z_{eq} I_c R_{TH}} + \frac{j\omega C_{jc} V_T}{Z_{eq} I_c} + j\omega C_{jc}\right] = -I_\Sigma \frac{\frac{1}{R_{TH}} + j\omega C_{jc}}{j\omega C_\Sigma}$$

$$\left[\frac{1}{R_{TH}} + j\omega C_{jc}\right] = \omega C_{jc}\left(\frac{1}{\omega R_{TH} C_{jc}} + j\right) = \omega C_{jc}\left(\frac{\omega_1}{\omega} + j\right) = \omega C_j \sqrt{1+\left(\frac{\omega_1}{\omega}\right)^2}\, e^{j\tan^{-1}(\omega/\omega_1)}$$

j $\quad$ ω/ω_1

$$\theta = \tan^{-1}\left(\frac{1}{\omega_1/\omega}\right) = \tan\left(\frac{\omega}{\omega_1}\right)$$

$$= \frac{1}{R_{TH}}\left(1 + j\omega R_{TH} C_{jc}\right) = \frac{1}{R_{TH}}\left(1 + j\frac{\omega}{\omega_1}\right) = \frac{1}{R_{TH}}\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}\, e^{j\tan^{-1}\left(\frac{\omega}{\omega_1}\right)}$$

Dividing through by $\left[\frac{1}{R_{TH}} + j\omega C_{jc}\right]$...

$$V_{out}\left[\frac{j}{\omega Z_{eq} C_\Sigma} + \frac{V_T}{Z_{eq} I_c R_{TH}} \cdot \frac{e^{-j\tan^{-1}(\omega/\omega_1)}}{\frac{1}{R_{TH}}\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}} + \left(\frac{j\omega C_{jc} V_T}{Z_{eq} I_c} + j\omega C_{jc}\right) \cdot \frac{e^{-j\tan^{-1}(\omega/\omega_1)}}{\omega C_{jc}\sqrt{1+(\omega_1/\omega)^2}}\right] = \frac{-I_\Sigma}{j\omega C_\Sigma}$$

$$V_{out}\left[\frac{j}{\omega Z_{eq} C_\Sigma} + \frac{V_T\, e^{-j\tan^{-1}(\omega/\omega_1)}}{Z_{eq} I_c \sqrt{1+(\omega/\omega_1)^2}} + \left(\frac{jV_T}{Z_{eq} I_c} + j\right)\frac{e^{-j\tan^{-1}(\omega/\omega_1)}}{\sqrt{1+(\omega_1/\omega)^2}}\right] = \frac{-I_\Sigma}{j\omega C_\Sigma}$$

$A = \frac{Z_{eq} I_c}{V_T} + 1$

$$V_{out}\left[\frac{j}{\omega Z_{eq} C_\Sigma} + \left(\frac{V_T}{Z_{eq} I_c} e^{-j\tan\left(\frac{\omega}{\omega_1}\right)}\right)\left[\frac{1}{\sqrt{1+(\omega/\omega_1)^2}} + \frac{jA}{\sqrt{1+(\omega_1/\omega)^2}}\right]\right] = \frac{-I_\Sigma}{j\omega C_\Sigma}$$

$$V_{out}\left[\frac{-1}{Z_{eq}} + \frac{j\omega C_\Sigma V_T}{Z_{eq} I_c} \cdot e^{-j\tan^{-1}(\omega/\omega_1)} \cdot \left(\frac{1}{\sqrt{1+(\omega/\omega_1)^2}} + \frac{jA}{\sqrt{1+(\omega_1/\omega)^2}}\right)\right] = -I_\Sigma$$

$$V_{out}\left(\frac{-1}{Z_{eq}}\right)\left[1 - \frac{j\omega C_\Sigma V_T}{I_c} \cdot e^{-j\tan(\omega/\omega_1)} \cdot \left(\frac{1}{\sqrt{1+(\omega/\omega_1)^2}} + \frac{jA}{\sqrt{1+(\omega_1/\omega)^2}}\right)\right] = I_\Sigma$$

[7] $$Z_{gain}^{-1} = \left(\frac{-1}{Z_{eq}}\right)\left[1 - \frac{j\omega C_\Sigma V_T}{I_c} \cdot e^{-j\tan\left(\frac{\omega}{\omega_1}\right)} \cdot \left(\frac{1}{\sqrt{1+(\omega/\omega_1)^2}} + \frac{jA}{\sqrt{1+(\omega_1/\omega)^2}}\right)\right]$$

$$Z_{gain}^{-1} = \frac{1}{Z_{eq}}\left[1+\frac{\omega C_\Sigma V_T}{I_c}\, e^{-j\angle \frac{\pi}{4}+\tan^{-1}\left(\frac{\omega}{\omega_1}\right)}\left(\frac{1}{\sqrt{1+(\omega/\omega_1)^2}}+\frac{jA}{\sqrt{1+(\omega_1/\omega)^2}}\right)\right]$$

So finally we have

[8]
$$Z_{gain}^{-1} = \frac{1}{Z_{eq}}\left[1+\frac{\omega}{\omega_2}\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_1}\right)^2}}+\frac{jA}{\sqrt{1+\left(\frac{\omega_1}{\omega}\right)^2}}\right)e^{-j\angle \frac{\pi}{4}+\tan^{-1}\left(\frac{\omega}{\omega_1}\right)}\right]$$

where

$$A = 1+\frac{Z_{eq} I_c}{V_T}, \quad \omega_1 = \frac{1}{R_{TH} C_{jc}}, \quad \text{and} \quad \omega_2 = \frac{I_c}{C_\Sigma V_T}$$

If the loading from the subsequent stage is insignificant relative to R_c, then the frequency response of the first stage can be adjusted by changing R_c, R_{TH}, and I_c.

[9]
$$R_c = \left|Z_{\text{Low Pass Gain}}\right|$$

$$R_{TH} = \frac{1}{2\pi f_1 C_{jc}}$$

$$I_c = 2\pi f_2 C_\Sigma V_T$$

Lastly the DC operating point design, based on the chosen R_c, R_{TH}, and I_c.

$$R_E = 10\,\Omega = R_{E1},\ R_{E2},\ \text{etc.}$$

$$V_{mirror} = I_c R_E + V_{BE}$$

$$V_{b1} = I_c R_E + 2V_{BE} + V_{margin}, \quad V_{margin} = 0.3\text{V}$$

$$V_{b2} = V_{cc} - I_{c2} R_E - V_{BE}$$

$$V_{out} = \tfrac{1}{2}(V_{b2} - V_{b1}) + V_{b1}$$

$$= \tfrac{1}{2}(V_{cc} - \cancel{2 I_c R_E} - 3V_{BE} - V_{margin}) + \cancel{I_c R_E} + 2V_{BE} + V_{margin}$$

$$= \tfrac{1}{2}V_{cc} - \tfrac{3}{2}V_{BE} + 2V_{BE} - \tfrac{1}{2}V_{margin} + V_{margin}$$

$$= \tfrac{1}{2}(V_{cc} + V_{BE} + V_{margin})$$

For V_{out} ~~assume~~ target, assume $I_{c2} \approx I_c$

$$V_{out} = \tfrac{1}{2}(V_{cc} + V_{BE} + V_{margin})$$

$$I_{c\varnothing} = \tfrac{1}{10} I_c$$

$$I_{c2} = I_c - \frac{V_{out}}{R_c} + I_\ell$$ where I_ℓ is the DC current loading

$$R_1 = \frac{V_{mirror} - V_{BE}}{I_{co}} = \frac{I_c R_E \cancel{+V_{BE}} \cancel{-V_{BE}}}{\frac{1}{10} I_c} = 10\, R_E \qquad \boxed{R_1 = 10\, R_E}$$

$$\cancel{R_2 = \frac{V_{cc} - V_{b2}}{I_{co}} = \frac{10}{I_c}\left(V_{cc} - V_{cc} + I_{c2}R_E + V_{BE}\right) = R_E \frac{10\, I_{c2}}{I_c} + \frac{V_{BE}}{I_c}}$$

$$\cancel{= \frac{R_E(10)}{I_c}\left(I_c\right.}$$

$$R_o + R_1 + R_2 = \frac{V_{cc} - V_{BE}}{I_{co}} = \frac{10(V_{cc} - V_{BE})}{I_c}$$

$$R_o = \frac{10(V_{cc} - V_{BE})}{I_c} - R_1 - R_2 = \frac{10 V_{cc}}{I_c} - \frac{10 V_{BE}}{I_c} - 10R_E - 10R_E - \frac{V_{BE}}{I_c}$$

$$\boxed{R_o = \frac{10(V_{cc} - V_{BE})}{I_c} - (R_1 + R_2)}$$

$$R_3 = R_{TH} \frac{V_{cc}}{V_{TH}} = \frac{R_{TH} V_{cc}}{I_c R_E + 2V_{BE} + V_{margin}} \qquad \boxed{R_3 = \frac{R_{TH} V_{cc}}{I_c R_E + 2V_{BE} + V_{margin}}}$$

$$\boxed{R_4 = \frac{R_{TH} V_{cc}}{V_{cc} - I_c R_E - 2V_{BE} - V_{margin}}}$$

where

$R_{E1} = R_{E2} = R_E$ and V_{margin} should be set to the order of 0.3V

$$R_2 = \frac{V_{cc} - V_{b2}}{I_{co}} = \frac{V_{cc} - V_{cc} + I_{c2} R_E + V_{BE}}{\frac{1}{10} I_c} = \frac{10}{I_c}\left(R_E I_{c2} + V_{BE}\right)$$

$$\cancel{= \frac{10 V_{BE}}{I_c} + \frac{10 R_E}{I_c}\left(I_c - \frac{V_{out}}{R_c} + I_l\right) = \frac{10 V_{BE}}{I_c} + 10 R_E\left(1 - \frac{V_{out}}{I_c R_c} + \frac{I_l}{I_c}\right)}$$

$$= \frac{10}{I_c}\left[R_E\left(I_c - \frac{V_{out}}{R_c} + I_l\right) + V_{BE}\right] = \frac{10}{I_c}\left[R_E\left(I_c + \frac{V_{cc} + V_{BE} + V_{margin}}{2R_c} + I_l\right) + V_{BE}\right]$$

$$= \frac{10}{I_c}\left[V_{BE} + R_E(I_c + I_L) + \frac{R_E}{2R_c}\left(V_{cc} - V_{BE} + V_{margin}\right)\right]$$

$$= 10\left[\frac{V_{BE}}{I_c} + R_E\left[\frac{(I_c + I_L)}{I_c} + \frac{V_{cc} - V_{BE} + V_{margin}}{2R_c I_c}\right]\right]$$

$$\boxed{R_2 = \frac{10}{I_c}\left[V_{BE} + R_E\left(I_c + I_L + \frac{V_{cc} + V_{BE} + V_{margin}}{2R_c}\right)\right]}$$