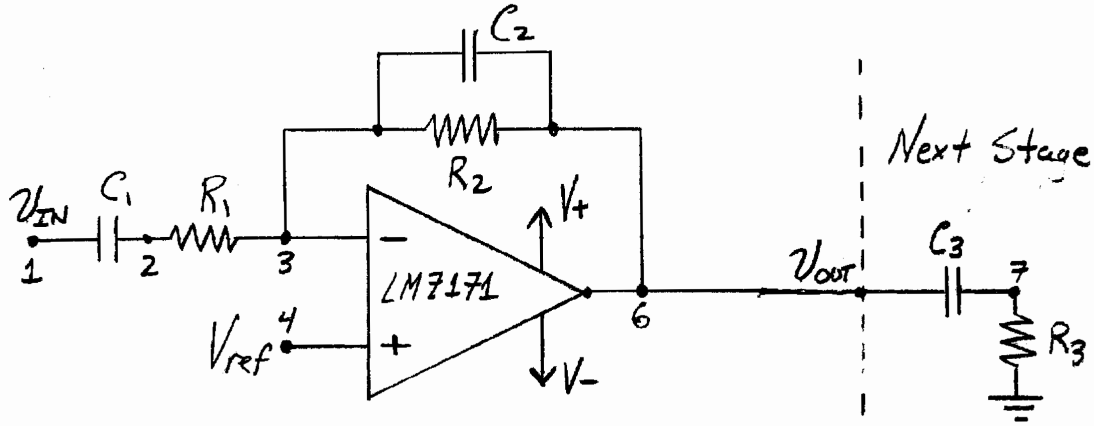
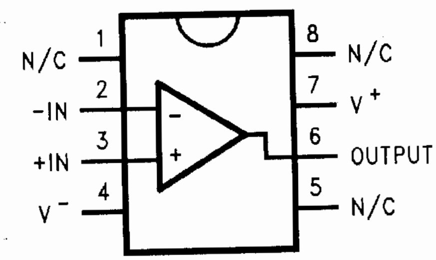
Design of Op-Amp Stage for High Frequency and Stability



First we should lump the 6 passive components into 3 complex impedances, Z_{1-3}.

Texas Instruments LM7171



[1a] $$Z_1 = R_1 - \frac{j}{\omega C_1} = R_1\left(1 - j\frac{\omega_1}{\omega}\right),$$

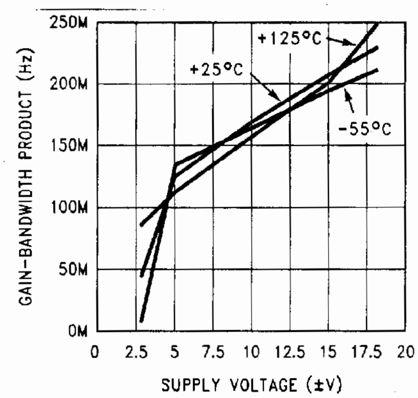
[2a] $$Z_2 = \frac{R_2}{j\omega C_2\left(R_2 + \frac{1}{j\omega C_2}\right)} = R_2\left(\frac{1}{1 + j\frac{\omega}{\omega_2}}\right),$$

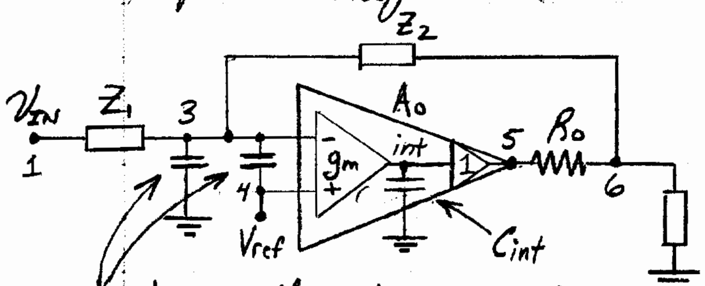
[3a] $$Z_3 = R_3 - \frac{j}{\omega C_3} = R_3\left(1 - j\frac{\omega_3}{\omega}\right), \text{ where}$$

[1b, 2b, 3b] $$\omega_1 = \frac{1}{R_1 C_1}, \quad \omega_2 = \frac{1}{R_2 C_2}, \quad \omega_3 = \frac{1}{R_3 C_3}$$

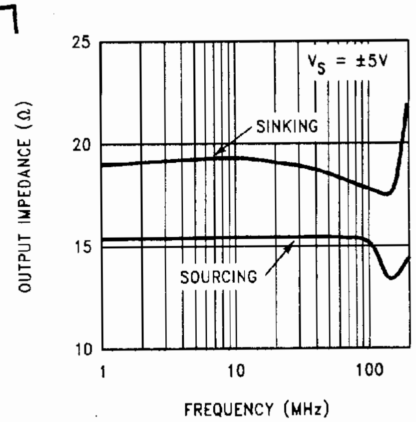
Next we add parasitics to the ideal op-amp, which affect the high frequency response and reflect the LM7171.

|GB|





R_o

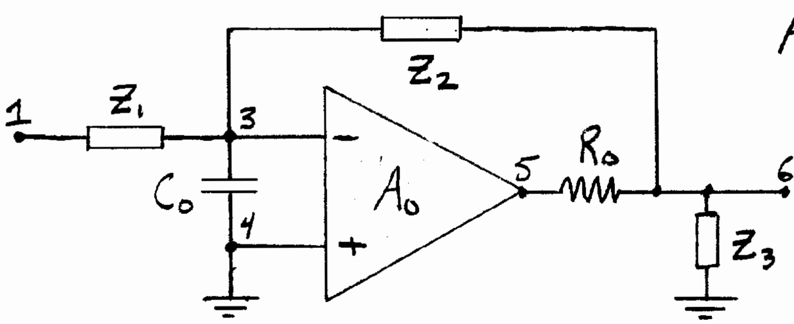


C_o Internally, the op-amp has a differential transconductance amplifier coupled to some parasitic capacitance C_{int} and an output voltage buffer; and that a high frequency pole is created.

$$A_o = \frac{V_5}{V_4 - V_3} = \frac{-j g_m}{2\pi C_{int} f} = \frac{-j}{|GB| f}$$

Typical input capacitance of IC and leads: $\boxed{C_o = 20pF}$

AC Circuit Analysis



$$A_0 = \frac{-j}{|GB|f} = \frac{V_5}{V_4 - V_3}$$

$$V_5 = \frac{-j}{|GB|f}(V_4 - V_3)$$

KCL at node 3:

$$\frac{V_3 - V_1}{Z_1} + \frac{V_3}{\frac{1}{j\omega C_0}} + \frac{V_3 - V_6}{Z_2} = \emptyset$$

[4.1] $$V_3\left[\frac{1}{Z_1} + \frac{1}{Z_2} + j\omega C_0\right] + V_6\left[\frac{-1}{Z_2}\right] = V_1\left[\frac{1}{Z_1}\right]$$

KCL at node 6:

$$\frac{V_6 - V_3}{Z_2} + \frac{V_6}{Z_3} + \frac{V_6 - V_5}{R_0} = \emptyset$$

[5.0] $$V_3\left[\frac{-1}{Z_2}\right] + V_5\left[\frac{-1}{R_0}\right] + V_6\left[\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{R_0}\right] = \emptyset$$

But V_5 is a function of V_3 and the open-loop gain,

$$V_5 = V_3\left[\frac{j|GB|}{f}\right]$$

so now we have

[5.1] $$V_3\left[\frac{-1}{Z_2} - j\frac{|GB|}{R_0 f}\right] + V_6\left[\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{R_0}\right] = \emptyset$$

Collect equations 4.1 and 5.1 into a matrix for solving.

$$\begin{array}{c} (4.1) \\ (5.1) \end{array} \begin{bmatrix} \frac{1}{Z_1} + \frac{1}{Z_2} + j\omega C_0 & \frac{-1}{Z_2} \\ \frac{-1}{Z_2} - j\frac{|GB|}{R_0 f} & \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{R_0} \end{bmatrix} \begin{bmatrix} V_3 \\ V_6 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z_1}V_1 \\ \emptyset \end{bmatrix}$$

For a general 2×2 system, the matrix can be reduced to triangular form by the series of row operations

$$\left[\begin{array}{cc|c} A & B & E \\ C & D & F \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & B/A & E/A \\ -1 & -D/C & -F/C \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & B/A & E/A \\ 0 & \frac{B}{A}-\frac{D}{C} & \frac{E}{A}-\frac{F}{C} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & B/A & E/A \\ 0 & 1 & \left(\dfrac{\frac{E}{A}-\frac{F}{C}}{\frac{B}{A}-\frac{D}{C}}\right) \end{array}\right]$$

So for the AC circuit system, the output voltage is given by

[6a] $$V_o = \frac{\frac{E}{A}-\frac{F}{C}}{\frac{B}{A}-\frac{D}{C}} = \frac{E - \frac{FA}{C}}{B - \frac{DA}{C}}$$

where

[6.b]
$$A = \frac{1}{Z_1} + \frac{1}{Z_2} + j\omega C_o$$

$$B = \frac{-1}{Z_2}$$

$$C = \frac{-1}{Z_2} - j\frac{|GB|}{R_o f} = \frac{-R_o f - jZ_2|GB|}{Z_2 R_o f}$$

$$D = \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{R_o} \approx \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$E = \frac{1}{Z_1}V_1$$

$$F = \emptyset.$$

Substituting in E and F gives us a gain equation

$$V_o = \frac{\frac{1}{Z_1}V_1 - \emptyset}{B - \frac{DA}{C}}$$

[7.0] $$A_v = \frac{1}{Z_1\left(B - \frac{DA}{C}\right)}$$

Getting the expression $B - \frac{DA}{C}$ in an appropriate form takes some algebra.

$$\frac{-DA}{C} = DA(-C^{-1}) = \left[\frac{1}{Z_2} + \frac{1}{Z_3}\right]\left[\frac{1}{Z_1} + \frac{1}{Z_2} + j\omega C_o\right]\left[\frac{Z_2 R_o f}{R_o f + jZ_2|GB|}\right]$$

$$= \left[\frac{R_o f}{jZ_2|GB|}\left(\frac{Z_2}{1 - j\frac{R_o f}{Z_2|GB|}}\right)\right]\left[\frac{Z_2 + Z_3}{Z_2 Z_3}\right]\left[\frac{1}{Z_1 \| Z_2} + j\omega C_o\right]$$

$$\frac{-DA}{C} = \frac{-jR_o f}{Z_2|GB|}\left(\frac{1}{1-j\frac{R_o f}{Z_2|GB|}}\right)\left[\frac{Z_2+Z_3}{-Z_3}\right]\left[\frac{Z_1+Z_2}{Z_1 Z_2}\right]\left(1+j\omega C_o(Z_1\|Z_2)\right)$$

Next $B-\frac{DA}{C}$

$$B-\frac{DA}{C} = \frac{-1}{Z_2} - j\frac{R_o f}{Z_2|GB|}\left(\frac{Z_2+Z_3}{Z_3}\right)\left(\frac{Z_1+Z_2}{Z_1 Z_2}\right)\left(1+j\omega C_o Z_1\|Z_2\right)\left(\frac{1}{1-j\frac{R_o f}{Z_2|GB|}}\right)$$

$$= \frac{-1}{Z_2}\left[1+j\frac{R_o f}{Z_2|GB|}\left(\frac{Z_2+Z_3}{Z_3}\right)\left(\frac{Z_1+Z_2}{Z_1}\right)\left(1+j\omega C_o Z_1\|Z_2\right)\left(\frac{1}{1-j\frac{R_o f}{Z_2|GB|}}\right)\right]$$

and the whole transfer function is

[7.1] $$A_v = \frac{-Z_2}{Z_1}\left[\frac{1}{1+j\dfrac{fR_o(Z_1+Z_2)(Z_2+Z_3)}{|GB|\,Z_1 Z_2 Z_3}\left[\dfrac{1+j\omega C_o Z_1\|Z_2}{1-j\frac{R_o f}{Z_2|GB|}}\right]}\right]$$

where

(1a, 2a, 3a) $$Z_1 = R_1\left[1-j\frac{\omega_1}{\omega}\right],\quad Z_2 = R_2\left[\frac{1}{1+j\frac{\omega}{\omega_2}}\right],\quad Z_3 = R_3\left[1-j\frac{\omega_3}{\omega}\right]$$

and

(1b, 2b, 3b) $$\omega_1 = \frac{1}{R_1 C_1},\quad \omega_2 = \frac{1}{R_2 C_2},\quad \omega_3 = \frac{1}{R_3 C_3}$$

---

op_amp.c

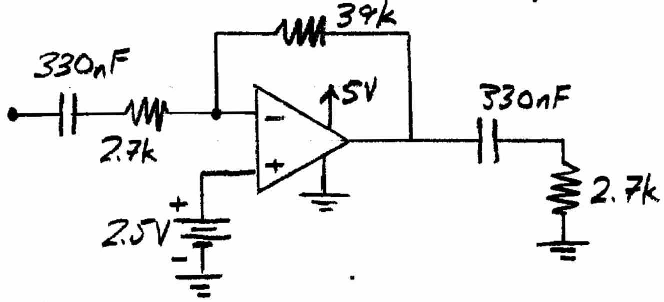
The frequency response of the circuit with a set of values for $\{C_o\ R_1\ R_2\ R_3\ C_1\ C_2\ C_3\}$ and the LM7171 opamp can be quickly evaluated using the C script op_amp.c. You also need the lm7171.cir Ngspice model, gnuplot C interface header/implementation files, and optionally experimental data with row format:

Freq dB Phase(radians)

Dependencies are NgSpice, GNU Scientific Library, and GNU Plot.

```
$ make
$ ./op_amp 20 1 10 1 330 2 330
ngspice 2 -> quit <enter>
```

Experimental, Theoretical, and Spice Results



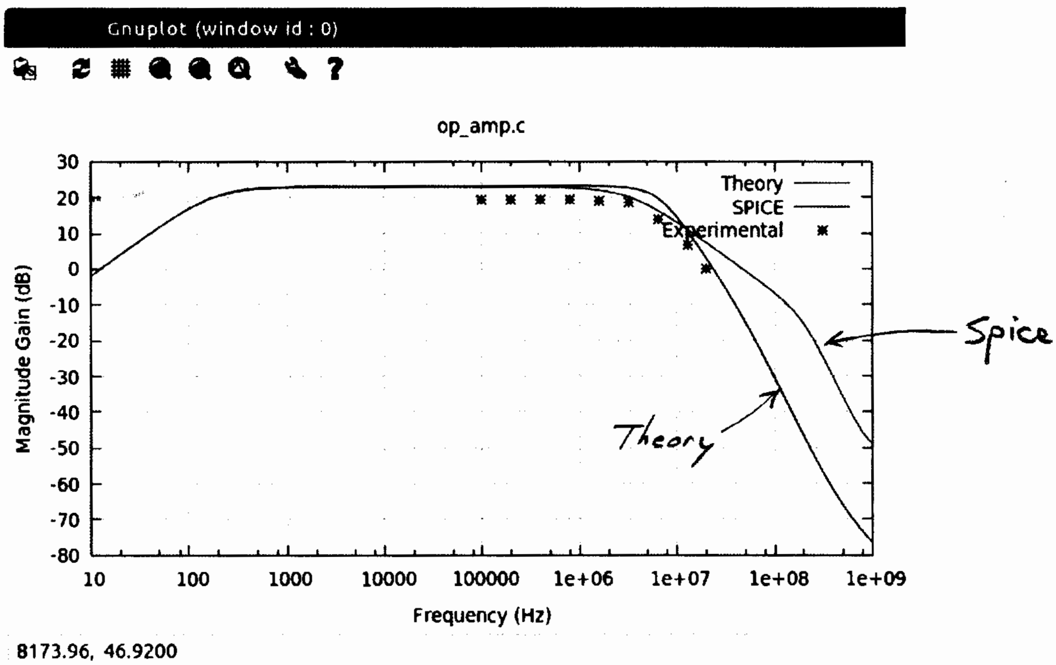
Guessed Values for op_amp.c:

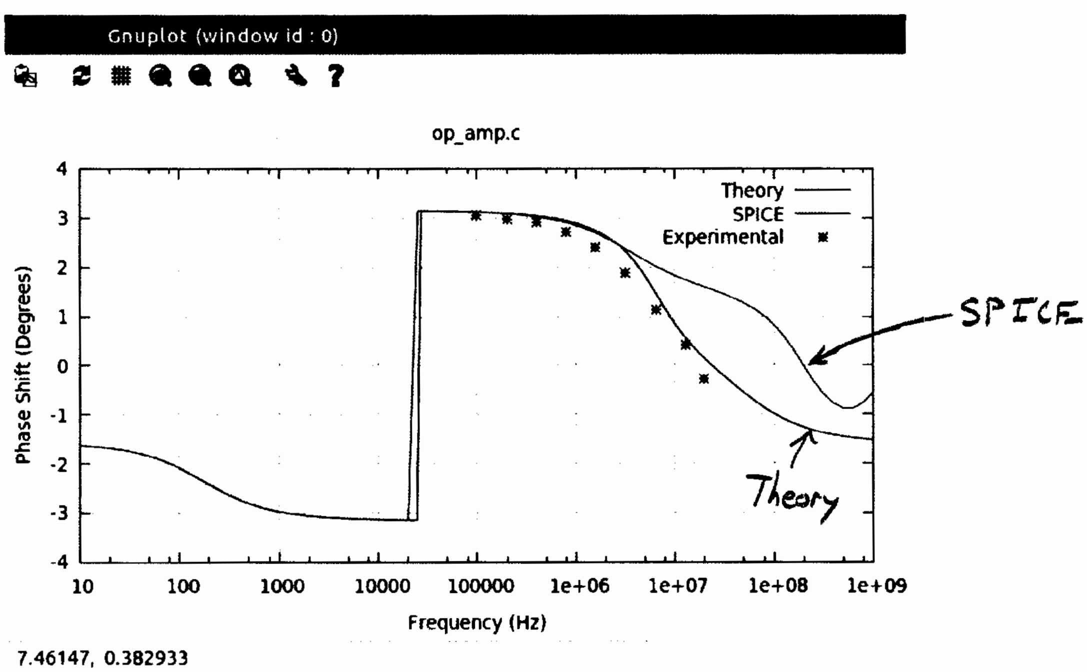
$|GB| = 35\ \text{MHz}$

$R_o = 20\ \Omega$

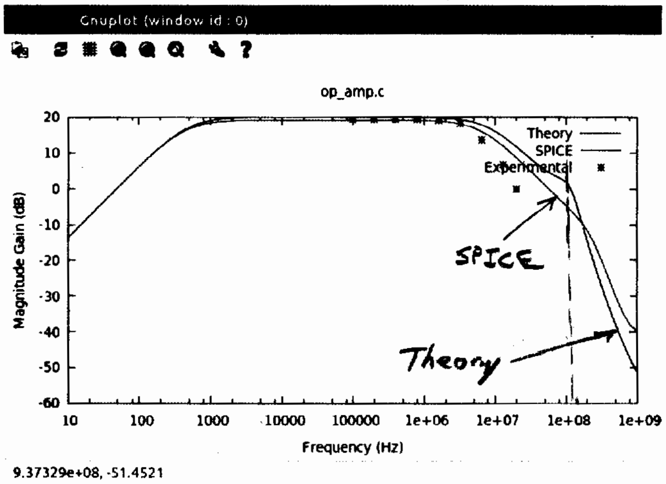
$C_o = 500\ \text{pF}$

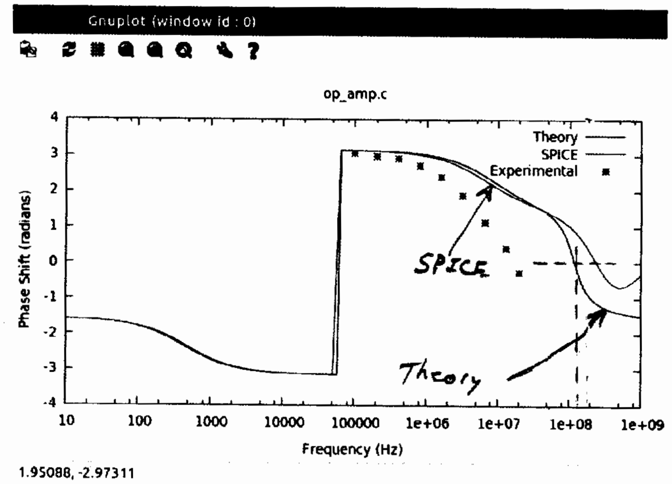
Circuit Constructed on Breadboard





Oscillations and Stability





C_0	100 pF
R_1	1 kΩ
R_2	10 kΩ
R_3	1 kΩ
C_1	330 nF
C_2	2 pF
C_3	330 nF

Look at the simulation with the values in the table. The phase shift crosses zero at about 100 MHz. On the magnitude plot, there is a resonance peak/shoulder at this positive-feedback frequency (100 MHz) whose peak is above zero. i.e. greater than unity, positive feedback = oscillation at 100 MHz.

To analyze/design for stability, it would be helpful to develop a script for generating Nyquist plots for different component values.

For Nyquist plots, we usually want the closed loop transfer function in the form

$$A_v(s) = \frac{A(s)}{1 + L(s)} \qquad [8.0]$$

where $A(s)$ is a complex polynomial and $L(s)$ is a rational function $L(s) = \frac{P(s)}{Q(s)}$, or a fraction of two polynomials.

So that we can analyze when the characteristic equation has a root (is zero) in the right half plane (or along the sinusoidal $j\omega$-axis).

$$F(s) = 1 + L(s) = \emptyset$$

But were getting ahead of ourselves; first we need A_v in the form of eq. 8.0. Starting with equation 7.1, we can assume that the poles ω_1 and ω_3 are negligible for high frequency and stability analysis.

[8.1] $$A_v = \frac{-Z_2}{R_1}\left[\frac{1}{1 + j\frac{f R_0 (R_1 + Z_2)(Z_2 + R_3)}{|GB|\ R_1\ Z_2\ R_3}\cdot\left[\frac{1 + j\omega C_o (R_1 \| Z_2)}{1 - j\frac{R_o f}{Z_2 |GB|}}\right]}\right]$$

We can expand $R_1 \| Z_2$ and $\frac{Z_2 + R_3}{R_3}$ and Z_2 using equation 2a

$$R_1 \| Z_2 = \frac{R_1 Z_2}{R_1 + Z_2} = \frac{R_1 R_2}{\left(1 + j\frac{\omega}{\omega_2}\right)\left(R_1 + \frac{R_2}{1 + j\omega/\omega_2}\right)} = \frac{R_1 R_2}{R_1\left(1 + j\frac{\omega}{\omega_2}\right) + R_2}$$

$$\frac{Z_2 + R_3}{R_3} = \frac{R_2}{R_3\left(1 + j\frac{\omega}{\omega_2}\right)} + 1 = \frac{R_2 + R_3\left(1 + j\frac{\omega}{\omega_2}\right)}{R_3\left(1 + j\frac{\omega}{\omega_2}\right)}$$

[8.2] $$A_v = \frac{-Z_2}{R_1}\left[\frac{1}{1 + j\frac{\omega R_0 \left[R_1\left(1 + j\frac{\omega}{\omega_2}\right) + R_2\right]}{2\pi|GB| \cdot R_2\, R_1} \cdot \frac{\left[R_2 + R_3\left(1 + j\frac{\omega}{\omega_2}\right)\right]}{R_3\left(1 + j\frac{\omega}{\omega_2}\right)}\left[\frac{1 + j\omega C_o \left[\frac{R_2 R_1}{R_1\left(1 + j\frac{\omega}{\omega_2}\right) + R_2}\right]}{1 - j\frac{R_o \omega \left(1 + j\omega/\omega_2\right)}{R_2 \cdot 2\pi \cdot |GB|}}\right]}\right]$$

Next we define ω_0 and ω_4 as

[9] $$\omega_0 = \frac{2\pi |GB| R_2}{R_o}$$

[10] $$\omega_4 = \frac{1}{R_2 C_o}$$

so that

[8.3] $$A_v = \frac{-R_2}{R_1\left(1 + j\frac{\omega}{\omega_2}\right)}\left[\frac{1}{1 + j\frac{\omega}{\omega_0} \cdot \frac{\left[R_1\left(1 + j\frac{\omega}{\omega_2}\right) + R_2\right]}{R_1} \cdot \frac{\left[R_2 + R_3\left(1 + j\frac{\omega}{\omega_2}\right)\right]}{R_3\left(1 + j\frac{\omega}{\omega_2}\right)}\left[\frac{1 + j\frac{\omega}{\omega_4}\left(\frac{R_1}{R_1\left(1 + j\frac{\omega}{\omega_2}\right) + R_2}\right)}{1 - j\frac{\omega}{\omega_0}\left(1 + j\frac{\omega}{\omega_2}\right)}\right]}\right]$$

Next we distribute the term $\frac{1}{(1+j\omega/\omega_2)}$ into the large square brackets and the term $\left[\frac{R_1(1+j\omega/\omega_2)+R_2}{R_1}\right]$ into the second bracket.

[8.4] $$A_v = \frac{-R_2}{R_1}\left[\frac{1}{1+j\frac{\omega}{\omega_2} + j\frac{\omega\left[R_2+R_3\left(1+j\frac{\omega}{\omega_2}\right)\right]}{\omega_0 R_3}\left[\frac{\frac{R_1\left(1+j\frac{\omega}{\omega_2}\right)+R_2}{R_1}+j\frac{\omega}{\omega_4}}{1-j\frac{\omega}{\omega_0}\left(1+j\frac{\omega}{\omega_2}\right)}\right]}\right]$$

Now we are ready to define L as

[8.5] $$L = j\frac{\omega}{\omega_2} + j\frac{\omega\left[R_2+R_3\left(1+j\frac{\omega}{\omega_2}\right)\right]}{\omega_0 R_3}\left[\frac{\frac{R_2}{R_1}+1+j\frac{\omega}{\omega_2}+j\frac{\omega}{\omega_4}}{1-j\frac{\omega}{\omega_0}+\frac{\omega^2}{\omega_0\omega_2}}\right]$$

[8.6] $$L = j\frac{\omega}{\omega_2} + j\frac{\omega}{\omega_0}\frac{\left[\left(\frac{R_2}{R_3}+1\right)+j\frac{\omega}{\omega_2}\right]\left[\left(\frac{R_2}{R_1}+1\right)+j\omega\left(\frac{1}{\omega_2}+\frac{1}{\omega_4}\right)\right]}{\left[\left(1+\frac{\omega^2}{\omega_0\omega_2}\right)-j\frac{\omega}{\omega_0}\right]}$$

[8.7] $$L = j\omega\left[\frac{1}{\omega_2} + \frac{\left[\left(\frac{R_2}{R_3}+1\right)+j\frac{\omega}{\omega_2}\right]\left[\left(\frac{R_2}{R_1}+1\right)+j\omega\left(\frac{1}{\omega_2}+\frac{1}{\omega_4}\right)\right]}{\left[\omega_0+\frac{\omega^2}{\omega_2}-j\omega\right]}\right]$$

[8.8] $$L = j\frac{\omega}{\omega_2}\left[1 + \frac{\left[\left(\frac{R_2}{R_3}+1\right)+j\frac{\omega}{\omega_2}\right]\left[\left(\frac{R_2}{R_1}+1\right)+j\omega\left(\frac{\omega_2+\omega_4}{\omega_2\omega_4}\right)\right]}{\frac{\omega_0}{\omega_2}+\frac{\omega^2}{\omega_2^2}-j\frac{\omega}{\omega_2}}\right]$$

[8.9] $$L = j\frac{\omega}{\omega_2}\left[\frac{\left[\frac{\omega^2}{\omega_2^2}-j\frac{\omega}{\omega_2}+\frac{\omega_0}{\omega_2}\right]+\left[\frac{R_2+R_3}{R_3}+j\frac{\omega}{\omega_2}\right]\left[\frac{R_2+R_1}{R_1}+j\omega\left(\frac{\omega_2+\omega_4}{\omega_2\omega_4}\right)\right]}{\frac{\omega^2}{\omega_2^2}-j\frac{\omega}{\omega_2}+\frac{\omega_0}{\omega_2}}\right]$$

~~Considering just the numerator P(s) after distributing the outer jω/ω2,~~

$$\cancel{P(j\omega) = \frac{j\omega^3}{\omega_0\omega_2^2}+\frac{\omega^2}{\omega_2^2}+\frac{j\omega\omega_0}{\omega_2}+\left[\frac{j\omega(R_2+R_3)}{\omega_2 R_3}-\right.}$$

[8.10] $$\mathcal{L} = \frac{\dfrac{j\omega^3}{\omega_2^2} + \dfrac{\omega^2}{\omega_2} + \dfrac{j\omega\omega_0}{\omega_2} + \left[\dfrac{j\omega(R_2+R_3)}{R_3} - \dfrac{\omega^2}{\omega_2}\right]\left[\dfrac{R_2+R_1}{R_1} + j\omega\left(\dfrac{\omega_2+\omega_4}{\omega_2\omega_4}\right)\right]}{\dfrac{\omega^2}{\omega_2} - j\omega + \omega_0}$$

Breaking $\mathcal{L}$ into two components and converting to s-domain,
$s = j\omega$; $s^2 = -\omega^2$; $s^3 = -j\omega^3$.

$$Q(s) = \frac{-1}{\omega_2}s^2 - s + \omega_0$$

$$P(s) = \frac{-s^3}{\omega_2^2} - \frac{s^2}{\omega_2} + \frac{s\omega_0}{\omega_2} + \left[\frac{s(R_2+R_3)}{R_3} + \frac{s^2}{\omega_2}\right]\left[\frac{R_1+R_2}{R_1} + s\left(\frac{\omega_2+\omega_4}{\omega_2\omega_4}\right)\right]$$

$$P(s) = \frac{-s^3}{\omega_2^2} - \frac{s^2}{\omega_2} + \frac{s\omega_0}{\omega_2} + \frac{s(R_2+R_3)(R_1+R_2)}{R_1R_3} + \frac{s^2(R_2+R_3)(\omega_2+\omega_4)}{R_3\,\omega_2\omega_4} + \frac{s^2(R_1+R_2)}{\omega_2 R_1} + \frac{s^3(\omega_2+\omega_4)}{\omega_2^2\omega_4}$$

$$P(s) = s^3\left[\frac{\omega_2+\omega_4}{\omega_2^2\omega_4} - \frac{1}{\omega_2^2}\right] + s^2\left[\frac{(R_2+R_3)(\omega_2+\omega_4)}{R_3\omega_2\omega_4} + \frac{(R_1+R_2)}{\omega_2 R_1} - \frac{1}{\omega_2}\right] + s\left[\frac{\omega_0}{\omega_2} + \frac{(R_2+R_3)(R_1+R_2)}{R_1R_3}\right]$$

$$P(s) = s^3\left[\frac{1}{\omega_2^2}\left(\frac{\omega_2+\omega_4}{\omega_4} - \frac{\omega_4}{\omega_4}\right)\right] + s^2\left[\frac{1}{\omega_2}\left(\frac{R_1+R_2}{R_1} - \frac{R_1}{R_1} + \frac{(R_2+R_3)(\omega_2+\omega_4)}{R_3\omega_4}\right)\right] + s\left[\frac{\omega_0}{\omega_2} + \frac{(R_2+R_3)(R_1+R_2)}{R_1R_3}\right]$$

So finally we have

[11] $$\mathcal{L}(s) = \frac{As^3 + Bs^2 + Ds}{Es^2 + Gs + \omega_0}$$ where

[11a] $$A = \frac{1}{\omega_2\omega_4},$$

[11b] $$B = \frac{1}{\omega_2}\left[\frac{R_2}{R_1} + \frac{(R_2+R_3)(\omega_2+\omega_4)}{R_3\omega_4}\right],$$

[11d] $$D = \frac{\omega_0}{\omega_2} + \frac{(R_1+R_2)(R_2+R_3)}{R_1R_3},$$

[11e, 11g] $$E = \frac{-1}{\omega_2}, \qquad G = -1$$ such that $$A_v = \frac{-R_2}{R_1}\left[\frac{1}{1+\mathcal{L}(s)}\right]$$

Re-expanding the gain equation for scripting

[8.11] $$A_v = \frac{-R_2}{R_1}\left[\frac{1}{1+L(s)}\right] = \frac{-R_2}{R_1}\left[\frac{1}{1+\frac{-j\omega^3/\omega_2 - \omega^2 B + j\omega D}{\frac{\omega^2}{\omega_2}+j\omega+\omega_0}}\right]$$

[11.2] $$L(\omega) = \frac{-j\frac{\omega^3}{\omega_2} - B\omega^2 + j\omega D}{\frac{\omega^2}{\omega_2}+j\omega+\omega_0} = \frac{-B\omega^2 - j\left(\frac{\omega^3}{\omega_2}-\omega D\right)}{\left(\frac{\omega^2}{\omega_2}+\omega_0\right)+j\omega}$$

Equations 8.11 and 11.2 should work for evaluating Bode and Nyquist plots on the computer, but we might be able to glean more insite by hand.

The system becomes unstable when the characteristic equation is $\emptyset$,

[12] $$\therefore F(s) = 1 + L(s) = \emptyset$$

ie. if there are any roots of $F(s)$ on the right-half s-plane.

We can expand $F(s)$

$$F(s) = 1 + L(s) = 1 + \frac{P(s)}{Q(s)} = \frac{Q(s)+P(s)}{Q(s)}$$

$$F(s) = \frac{As^3 + (B+E)s^2 + (D+G)s + \omega_0}{Es^2 + Gs + \omega_0}$$

$$F(s) = \frac{\left[\frac{1}{\omega_2}\right]s^3 + \left[\frac{R_2}{R_1\omega_2} + \frac{(R_2+R_3)(\omega_2+\omega_4)}{R_3\omega_4\omega_2} - \frac{1}{\omega_2}\right]s^2 + \left[\frac{\omega_0}{\omega_2} + \frac{(R_1+R_2)(R_2+R_3)}{R_1R_3} - 1\right]s + \omega_0}{\frac{-1}{\omega_2}s^2 - s + \omega_0}$$

Multiply through by $\frac{\omega_2}{\omega_2}$ and going back to $j\omega$-domain

$$F(j\omega) = \frac{-j\omega^3 - \omega^2\left[\frac{R_2}{R_1} - 1 + \frac{(R_2+R_3)(\omega_2+\omega_4)}{R_3\omega_4}\right] + j\omega\left[\omega_0 - \omega_2\left(1 - \frac{(R_1+R_2)(R_2+R_3)}{R_1R_3}\right)\right] + \omega_0}{\omega^2 - j\omega + \omega_0}$$