AEM 566 Project 2

Wind Simulator Design

Learning Objective

This project is intended to introduce the use of the stochastic state-space modeling for random phenonema simulation as wind turbulence.

Dynamical System

For assessing the effects of **gusts**, i.e., the **unsteady wind component**, on the velocity EOMs (as opposed to the constant steady wind component), one typically assumes that the unsteady wind component is a zero-mean **random gust** which take on some stochastic, i.e., random, properties of varying magnitude denoted as $\vec{v}_g$. For simplicity, assume one has no steady-wind, i.e. one can redefine the body-fixed frame velocity as

$$\vec{v}_{B/N} = \vec{v}_\infty + \vec{v}_g \tag{1}$$

which uses both $\vec{v}_\infty = [u \; v \; w]^T$ and $\vec{v}_g = [u_g \; v_g \; w_g]^T$ in the state vector for the EOMs. Thus, by component one has

$$\begin{bmatrix} u_{tot} \\ v_{tot} \\ w_{tot} \end{bmatrix} = \begin{bmatrix} u + u_g \\ v + v_g \\ w + w_g \end{bmatrix} \tag{2}$$

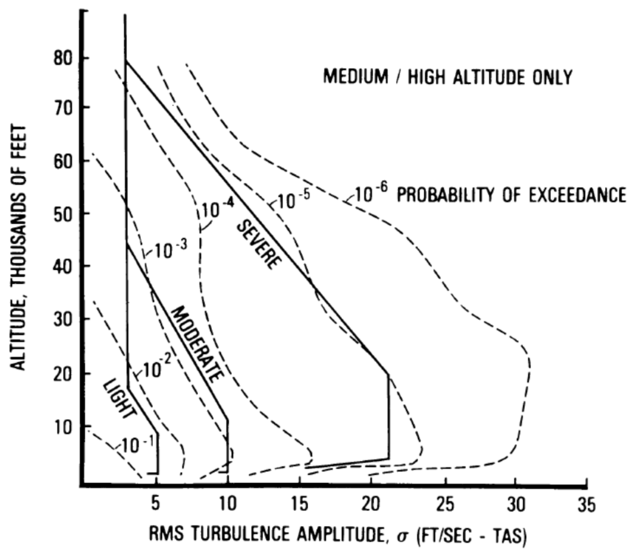
where u, v, and w implicitly model the velocity relative to a hypothetically *still* air mass.

One of the most widely used random gust models for $\vec{v}_g$ are the **Dryden gust model** which is modeled by the continuous-time stochastic state-space model

$$\begin{bmatrix} \dot{u}_g(t) \\ \dot{v}_g(t) \\ \dot{v}_{g_1}(t) \\ \dot{w}_g(t) \\ \dot{w}_{g_1}(t) \end{bmatrix} = \begin{bmatrix} -\frac{\|\bar{v}_\infty\|_2}{L_u} & 0 & 0 & 0 & 0 \\ 0 & -\frac{\|\bar{v}_\infty\|_2}{L_v} & \sigma_v(1-\sqrt{3})\left(\frac{\|\bar{v}_\infty\|_2}{L_v}\right)^{3/2} & 0 & 0 \\ 0 & 0 & -\frac{\|\bar{v}_\infty\|_2}{L_v} & 0 & 0 \\ 0 & 0 & 0 & -\frac{\|\bar{v}_\infty\|_2}{L_w} & \sigma_w(1-\sqrt{3})\left(\frac{\|\bar{v}_\infty\|_2}{L_w}\right)^{3/2} \\ 0 & 0 & 0 & 0 & -\frac{\|\bar{v}_\infty\|_2}{L_w} \end{bmatrix} \begin{bmatrix} u_g(t) \\ v_g(t) \\ v_{g_1}(t) \\ w_g(t) \\ w_{g_1}(t) \end{bmatrix} + \begin{bmatrix} \sigma_u \left(\frac{2\|\bar{v}_\infty\|_2}{\pi L_u}\right)^{1/2} \\ \sigma_v \left(\frac{3\|\bar{v}_\infty\|_2}{L_v}\right)^{1/2} \\ 1 \\ \sigma_w \left(\frac{3\|\bar{v}_\infty\|_2}{L_w}\right)^{1/2} \\ 1 \end{bmatrix} n(t) \tag{3}$$

where the driving function, $dn(t)$, is additive white Gaussian noise (AWGN) of unit intensity, $[L_u \;\; L_v \;\; L_w]^T$ are $[h \;\; 145h^{1/3} \;\; 145h^{1/3}]^T$ for $h < 1750$ ft and $[1750 \;\; 1750 \;\; 1750]^T$ for $h \geq 1750$ ft, and σ_u, σ_v, and σ_w are the standard deviations of the gusts, or **RMS gust intensities**. Since this model is driven by Wiener processes, by inspection, u_g is a first-order Markov process while v_g and w_g are second-order Markov processes.

Realistic RMS gust intensity values can be obtained from data such as "MIL-F-8785C Military Specification: Flying Qualities of Piloted Airplanes" which provides the plot of three levels of RMS gust intensities for different altitudes: *light*, *moderate*, and *severe*.



Note that here the **probability of exceedance** is the probability that the RMS gust intensity would exceed the value shown on the curves at that altitude. Also note that for numerical simulations, $n(t)$ can be approximated as continuous over a short time step of size Δt, thus approximating $n(t)$ by a random sequence n_i which are zero-mean Gaussian with variance $1/\Delta t$.

For this project, consider an aircraft flying at $||\bar{v}_\infty||_2 = 824$ ft/sec and altitude, $20,000$ ft.

For more information, please refer to the following

- Schmidt, D. K., "Appendix C Models of Atmopsheric Turbulence," *Modern Flight Dynamics*, 1st ed., Vol. 1, McGraw-Hill, New York, 2012, pp. 839-851

Project Assignment and Deliverables

Do: the following tasks in MATLAB or Python.

a) Implement a discretized version of the continuous-time Dryden gust model at $\Delta t = 0.01$ s;
   Note: $n[k] \sim \mathcal{N}(0, 1/\Delta t)$.
b) Simulate the Dryden gust model for 600 seconds for three cases:
   1) light: $\sigma_u = \sigma_v = \sigma_w = 5$ ft/sec;
   2) moderate: $\sigma_u = \sigma_v = \sigma_w = 10$ ft/sec; and
   3) severe: $\sigma_u = \sigma_v = \sigma_w = 20$ ft/sec.
c) Generate three plots of the gust components, one for each gust component, u_g, v_g, and w_g, with all three simulation data sets for a gust component on the same plot.
d) Comment on the differences between the light, moderate, and severe gust simulations for the gusts over the 10 minutes of simulation.

Deliver: in the Blackboard assignment, all files to run your MATLAB or Python script(s). There is no need to zip your files.