AEM 566 Project 4

Fuel Monitoring System Design

Learning Objective

This project is intended to introduce the use of the Kalman filter and smoother and two variants for a simple sensor fusion problem.

Sensor System

A **fuel monitoring system** estimates the fuel level of a vehicle. A typical monitoring system uses two sensors, a **fuel flow meter** and a **fuel tank level** as a simple system modeled as

$$\begin{aligned} \begin{bmatrix} f_k \\ b_k \end{bmatrix} &= \begin{bmatrix} 1 & A_{line}\Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_{k-1} \\ b_{k-1} \end{bmatrix} + \begin{bmatrix} -A_{line}\Delta t \\ 0 \end{bmatrix} u_{k-1} + \vec{w} \\ y_k &= \begin{bmatrix} A_{tank}^{-1} & 0 \end{bmatrix} \begin{bmatrix} f_k \\ b_k \end{bmatrix} + v \end{aligned} \tag{1}$$

where

- f_k is the fuel remaining in cm^3,
- b_k is the flow meter bias in cm,
- A_{line} is the cross-sectional area of the fuel line in cm^2,
- Δt is the sampling rate in s,
- u_{k-1} is the fuel flow meter measurement at time step $k-1$ in cm/s,
- $\vec{w}$ is the zero-mean fuel flow meter measurement error in [cm^3,cm]^T with covariance Q,
- y_k is the fuel tank level measurement in cm,
- A_{tank} is the cross-sectional area of the fuel tank in cm^2, and
- v is the fuel tank level measurement error in cm with variance R.

Note that in this model, the bias drift is modeled as a Gaussian random walk, but could be modeled as an $AR(1)$ process or a jump process in some similar sensors.

These two sensors can be fused in an univariate filter design where the fuel flow rate integration of the fuel consumption is corrected by the fuel gauge measurement of the current fuel level. This method of fusing a single predictive sensor with a bias drift and a single corrective sensor is known as **complementary sensor fusion**. When a constant gain is used, one obtains a **complementary filter** typically designed with respect to the frequency domain with a high-pass stage on the prediction and a low-pass stage on the correction. However, this project will look at the Kalman filter perspective on this complementary fusion problem.

Sensor Data

The .csv file provided contains by column:

1. the time $t = k\Delta t$,
2. the fuel flow meter sensor data, u_k,
3. the fuel tank level sensor data, y_k,
4. the true fuel remaining, f_k, and
5. the true flow meter bias, b_k.

Note that NaN's appear where data is not used.

For this project assume:

- $\Delta t = 0.5$ s,
- $A_{line} = 1$ cm^2,
- $A_{tank} = 150$ cm^2,
- $Q = \text{diag}(A_{line}^2 \Delta t^2 (0.1)^2,\ 0.1^2)$ [cm^6,cm^2/s^2]^T,
- $R = 1^2$ cm,
- $\vec{x}_0 = [3000\ 0]^T$ [cm^3,cm/s]^T, and
- $P_0 = \text{diag}(10^2,\ 0.1^2)$ [cm^6,cm^2/s^2].

Project Assignment and Deliverables

<u>**Do**</u>: the following tasks in MATLAB or Python.

a) Implement a discrete-time steady-state Kalman filter (SS-KF).
   - Plot the posterior state estimates versus the true states for the fuel remaining.
   - Plot the posterior state estimates versus the true states for the flow meter bias.
   - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the steady-state posterior covariance for the fuel remaining.
   - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the steady-state

posterior covariance for the flow meter bias.

b) Implement a standard discrete-time Kalman filter (KF).
    - Plot the posterior state estimates versus the true states for the fuel remaining.
    - Plot the posterior state estimates versus the true states for the flow meter bias.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the posterior state covariance for the fuel remaining.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the posterior state covariance for the flow meter bias.
    - Plot the Kalman gains versus the steady-state Kalman gains.

c) Implement a discrete-time covariance intersection Kalman filter (CI-KF).
    - Plot the posterior state estimates versus the true states for the fuel remaining.
    - Plot the posterior state estimates versus the true states for the flow meter bias.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the bounding posterior covariance for the fuel remaining.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the bounding posterior covariance for the flow meter bias.

d) Implement a standard discrete-time Kalman smoother (KS).
    - Plot the posterior state estimates versus the true states for the fuel remaining.
    - Plot the posterior state estimates versus the true states for the flow meter bias.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the posterior state covariance for the fuel remaining.
    - Plot the posterior state estimate errors and the $\pm 2\sigma$ from the diagonals of the posterior state covariance for the flow meter bias.

e) Comment on the similarities and differences between the different state estimators and variances.

Deliver: in the Blackboard assignment, all files to run your MATLAB or Python script(s). There is no need to zip your files.