AEM 668 Project 1

Lateral-Directional Stability and Control of Airplane

Learning Objective

This project is intended to introduce in MATLAB the introductory stability analysis and classical control design for flight vehicles using the aerodynamic, geometric, and mass properties of a conventional airplane.

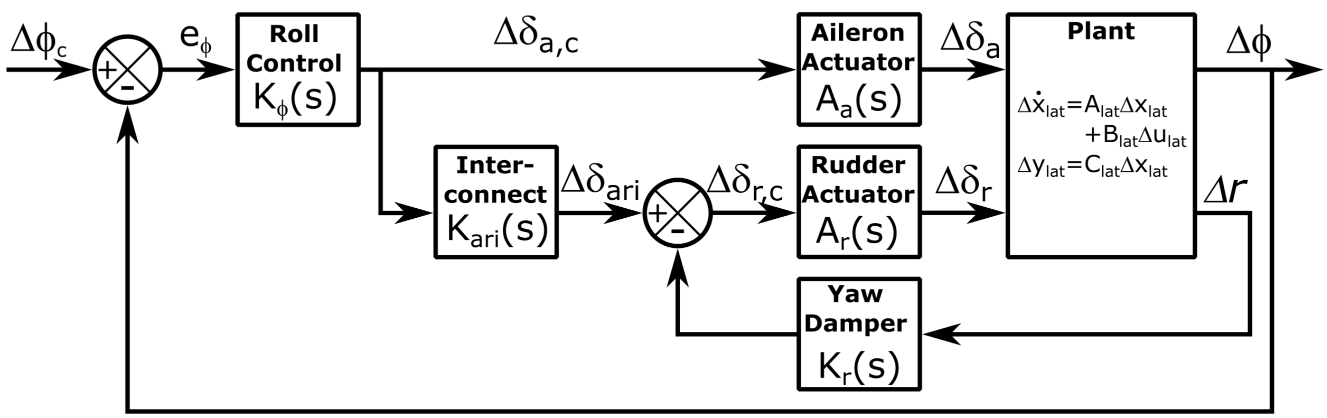
Dynamical System

Given: an airplane with a piston engine operating at an airspeed of 120 m/s and a cruise altitude of 6000 m which corresponds to an air density of 0.66011 kg/m^3, an acceleration due to gravity of 9.788 m/s^2, and a speed of sound of 316.43 m/s. A MATLAB script is provided with the following geometric, aerodynamic, and mass properties as known.

Property Name	**Symbol**	**Value**
Airplane lift coefficient at $\alpha = 0°$ at cruise altitude (relative to fuselage centerline)	C_{L_0}	0.01
Airplane parasitic drag coefficient at cruise altitude	C_{D_0}	0.036
Airplane lift coefficient slope at cruise altitude	C_{L_α}	5.05 /rad
Airplane mass	m	5800 kg
Airplane moment of inertia about x_B axis	I_{xx}	112,000 kg-m^2
Airplane moment of inertia about y_B axis	I_{yy}	93,000 kg-m^2
Airplane moment of inertia about z_B axis	I_{zz}	194,000 kg-m^2
Airplane cross-moment of inertia about x_B-z_B axes	I_{xz}	≈ 0 kg-m^2
Center of gravity location measured from nose	l_{cg}	7.5 m
Quarter chord of wing measured from nose (i.e. assumed aerodynamic center of wing)	l_w	6.8 m
Wing area	S_w	40 m^2
Wing span	b_w	20 m
Wing root chord	$c_{r,w}$	3.0 m
Wing tip chord	$c_{t,w}$	1.0 m
Wing mean aerodynamic chord	$\bar{c}_w$	2.0 m
Wing lift coefficient slope	$C_{L_{\alpha,w}}$	4.95 /rad
Wing efficiency factor	e_w	0.8
Wing dihedral angle	Γ	0.04 rad
Wing sweep angle	Λ	0.1 rad
Rolling moment due to wing dihedral	$\frac{\partial C_{l_\beta}}{\partial \Gamma_w}$	-0.7 /rad
Yawing moment due to change in sideslip angle for wing-fuselage	$C_{n_{\beta,w-f}}$	0 /rad
Inboard aileron y_B position	$y_{a,i}$	7.5 m

Outboard aileron y_B position	$y_{a,o}$	9.5 m
Wing aileron empirical factor	K	-0.1
Aileron area	S_a	0.35 m^2
Vertical tail area	S_v	5.0 m^2
Vertical tail span	b_v	2.5 m
Vertical tail mean aerodynamic chord	$\bar{c}_v$	2.0 m
Vertical tail aerodynamic center x_B coordinate	x_v	-8.5 m
Vertical tail aerodynamic center z_B coordinate	z_v	-0.8 m
Vertical tail lift coefficient slope	$C_{L_{\alpha_v}}$	3.0 /rad
Vertical tail efficiency factor	η_v	0.95
Change in sidewash due to a change in sideslip angle	$\frac{d\sigma}{d\beta}$	0.1
Rudder area	S_r	1.2 m^2

For this airplane assume a roll angle attitude control system is to be designed using the following block diagram for the linearized model-based design.



To form the transfer function, $G_\phi(s) = \frac{\Delta\phi(s)}{\Delta\delta_{a,c}(s)}$, for loop-shaping design with $K_\phi(s)$, one should use multiplication with the matrix of transfer functions for the actuators and plant for $\Delta\delta_{a,c}$ and $\Delta\delta_{r,c}$ to $\Delta\phi$ and Δr, i.e.

$$\begin{bmatrix}\Delta\phi(s)\\ \Delta r(s)\end{bmatrix} = \begin{bmatrix}\frac{\Delta\phi(s)}{\Delta\delta_a(s)} & \frac{\Delta\phi(s)}{\Delta\delta_r(s)}\\ \frac{\Delta r(s)}{\Delta\delta_a(s)} & \frac{\Delta r(s)}{\Delta\delta_r(s)}\end{bmatrix}\begin{bmatrix}\Delta\delta_a(s)\\ \Delta\delta_r(s)\end{bmatrix} \tag{1}$$

$$\begin{bmatrix}\Delta\phi(s)\\ \Delta r(s)\end{bmatrix} = \begin{bmatrix}\frac{\Delta\phi(s)}{\Delta\delta_a(s)} & \frac{\Delta\phi(s)}{\Delta\delta_r(s)}\\ \frac{\Delta r(s)}{\Delta\delta_a(s)} & \frac{\Delta r(s)}{\Delta\delta_r(s)}\end{bmatrix}\left(\begin{bmatrix}A_a(s) & 0\\ 0 & A_r(s)\end{bmatrix}\begin{bmatrix}\Delta\delta_{a,c}(s)\\ \Delta\delta_{r,c}(s)\end{bmatrix}\right) \tag{2}$$

where the MATLAB function `ss(Alat, Blat, Clat, Dlat)` will provide an object that can be used for the transfer function matrix

$$[G(s)] = \begin{bmatrix}\frac{\Delta\phi(s)}{\Delta\delta_a(s)} & \frac{\Delta\phi(s)}{\Delta\delta_r(s)}\\ \frac{\Delta r(s)}{\Delta\delta_a(s)} & \frac{\Delta r(s)}{\Delta\delta_r(s)}\end{bmatrix} \tag{3}$$

in the equation above. MATLAB transfer function matrices can be formed using matrix syntax for

$$\begin{bmatrix} A_a(s) & 0 \\ 0 & A_r(s) \end{bmatrix} \tag{4}$$

Use the MATLAB function `feedback()` to connect Δr with $\Delta\delta_{r,c}$ in negative feedback, i.e.

$$\delta_{r,c}(s) = \delta_{ari}(s) - K_r(s)\Delta r(s) \tag{5}$$

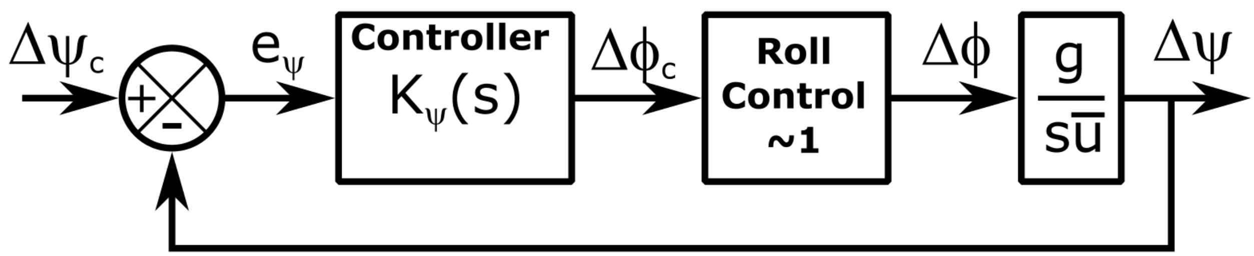
and multiplication with a matrix of transfer functions for splitting $\Delta\delta_{a,c}$ into $\Delta\delta_{a,c}$ and $\Delta\delta_{ari}$, i.e.

$$\begin{bmatrix} \Delta\delta_{a,c} \\ \Delta\delta_{ari} \end{bmatrix} = \begin{bmatrix} 1 \\ K_{ari} \end{bmatrix} \Delta\delta_{a,c} \tag{6}$$

where

$$K_{ari} = -\frac{N^*_{\delta_a}}{N^*_{\delta_r}} \tag{7}$$

Around this roll controller, this airplane is to have a heading hold guidance loop as shown.



Project Assignment and Deliverables

For this project, determine in MATLAB/Simulink:

a) Compute and output to the command window: the linearized 4x4 lateral-directional state and the 4x2 input matrices (for aileron and rudder deflections) about the coordinated straight-and-level flight condition. Use the stability frame, i.e. the airspeed is aligned with the x_B axis (i.e. $\bar{\theta} = \bar{\alpha} = 0°$). Estimate the airplane stability derivatives using the analytical models in Appendix A of the course textbook and assume
   - The lift and drag coefficients are approximately the same in the stability frame as the fuselage frame and do not need to be rotated.
   - The moments of inertia are defined about the stability frame axes.
   - The drag coefficient can be approximated as

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A_w e_w} \tag{8}$$

   - Small angle approximations for β, μ, and ϕ
   - Low-speed flight condition thus derivative terms related to the Mach number can be ignored (e.g. $C_{m_{\mathcal{M}}}$ and $C_{D_{\mathcal{M}}}$).
   - No-wind condition, i.e. the airspeed vector is equal to the ground speed vector
   - Tapered wing geometry

b) Compute and output to the command window:
   - The lateral-directional modes of motion of the airplane, labeling them by name with the following modal characteristics
     - eigenvalues
     - natural frequencies (only for underdamped modes)
     - damping ratios (only for underdamped modes)
       use the MATLAB function: `damp()`
   - the lateral-directional stability of the airplane.

c) Design a lateral-directional outer-loop heading hold guidance system using the simplified plant.
   - Heading hold requirements:
     (a) Closed-loop stability
     (b) Gain margins $\geq \pm 6$ dB
     (c) Phase margins $\geq \pm 45°$
     (d) Loop bandwidth between 0.2 and 1.2 rad/s
     (e) Zero steady-state tracking error for unit step inputs
     (f) $\leq 5\%$ steady-state tracking error for frequencies below 0.05 rad/s
     (g) $\leq 5\%$ gain at frequencies above 10 rad/s
   - Output the heading hold transfer function to the command window.
   - Output the closed-loop poles for the heading hold outer-loop to the command window proving the stability requirement is satisfied.
   - Output the gain and phase margins.
   - Output the gains at the critical frequencies to the command window proving the performance requirements are satisfied.

- If you cannot satisfy the requirements, explain why and what trade-off(s) you made instead.

d) Design a lateral-directional inner-loop roll controller.

- Roll controller requirements:
  (a) Closed-loop stability
  (b) Gain margins $\geq \pm 6$ dB
  (c) Phase margins $\geq \pm 45°$
  (d) Loop bandwidth between 9 and 11 rad/s
  (e) $\leq 1\%$ steady-state tracking error for unit step inputs
  (f) $\leq 5\%$ steady-state tracking error for frequencies below 1 rad/s
  (g) $\leq 5\%$ gain at frequencies above 100 rad/s
- Use first-order control surface actuators for the rudder and ailerons with a time constant of 0.01 seconds.
- Output the roll controller transfer functions to the command window.
- Output the closed-loop poles inner-loop to the command window proving the stability requirement is satisfied.
- Output the gain and phase margins.
- Output the gains at the critical frequencies to the command window proving the performance requirements are satisfied.
- If you cannot satisfy the requirements, explain why and what trade-off(s) you made instead.

e) Use Simulink to simulate (for at least 15 seconds) a 5° step input to the commanded heading

- Include the full roll controller inner-loop in the simulation
- Add to the block diagram the following additive disturbance and noise signals

$$w_{\delta_a}(t) = 0.01 \sin(1t) \text{ rad} \tag{9}$$

$$w_{\delta_r}(t) = 0.01 \sin(1t) \text{ rad} \tag{10}$$

$$v_\phi(t) = 0.02 \sin(100t) \text{ rad} \tag{11}$$

$$v_\psi(t) = 0.02 \sin(100t) \text{ rad} \tag{12}$$

$$v_r(t) = 0.05 \sin(100t) \text{ rad/s} \tag{13}$$

- Plot the simulated heading angle responses with the commanded heading versus time.
- Output the simulated settling time and overshoot for the heading step input to the command window.
- *Use the "Mux" and "Demux" blocks to create and separate signals for vectorized signals.*

Deliver: in the Blackboard assignment, all files to run your MATLAB script(s) and Simulink model(s). There is no need to zip your files.