Lecture 19: Advanced Aerospace Vehicle Attitude Control

Textbook Section 11.4, 11.5, 12.2, & 12.3

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Introduction

- Classical attitude control design: SISO principles to design
  - Pitch attitude controller using elevator input
  - Roll attitude controller using aileron control input
  - Sideslip attitude controller using rudder control input
  - Airspeed controller using thrust input

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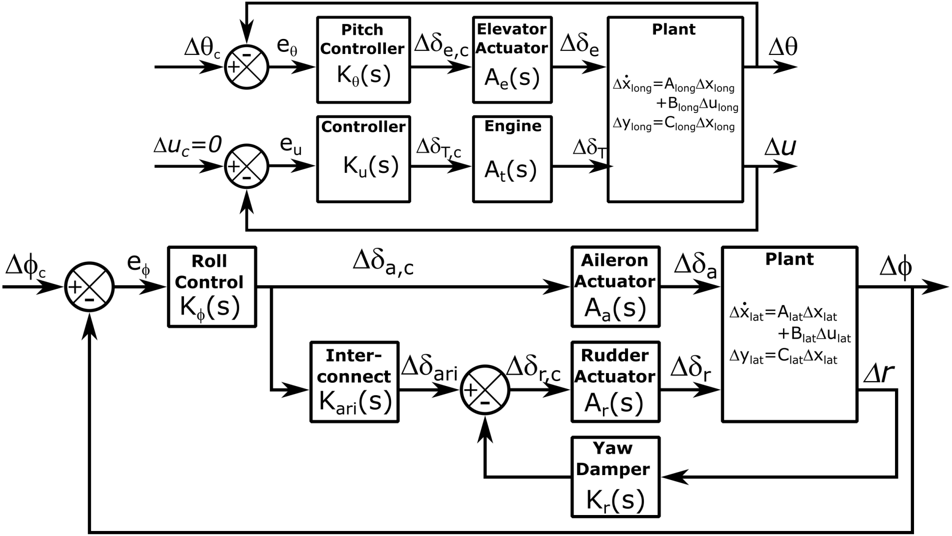
  - Pitch attitude controller using elevator input
  - Roll attitude controller using aileron control input
  - Sideslip attitude controller using rudder control input
  - Airspeed controller using thrust input
- Decoupled approach sub-optimal as SISO systems do not coordinate control actions to manage airplane energy or heading for commanded trajectory
  - Airspeed controller does not incorporate flight-path angle in determining required thrust
  - Pitch attitude controller does not know vertical velocity nor limits
  - Roll and sideslip angles interrelated to yaw and heading angles: required to use yaw damper and turn compensation for coordinated turns in inner-loop
  - Thrust asymmetry compensation in event of engine-out condition

Classical Airplane Attitude Control Systems



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- Use small angle dynamics to provide processed error signals that decouple yaw and sideslip before providing command to ailerons and rudder actuator using fundamental physics of the lateral and directional angles of flight vehicle
- Modern airplanes often use total energy and total heading control systems for MIMO attitude control

Energy Dynamics

- From point-mass perspective with constant-wind assumption in straight flight, total energy of flight vehicle, $\mathcal{E}$: sum of kinetic and potential energy

$$\mathcal{E} = mg\left(\frac{1}{2}\frac{v_\infty^2 + v_w^2}{g} + h\right) \tag{1}$$

  - m: mass
  - v_∞: airspeed
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  - h: altitude

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  - m: mass
  - v_∞: airspeed
  - v_w: constant wind
  - g: assumed constant acceleration due to gravity
  - h: altitude
- Lagrangian, $\mathcal{L}$: difference of kinetic and potential energy

$$\mathcal{L} = mg\left(\frac{1}{2}\frac{v_\infty^2 + v_w^2}{g} - h\right) \tag{2}$$

Energy Dynamics (continued)

- Differentiating w.r.t. time:

$$\dot{\mathcal{E}} = mg\left(\frac{v_\infty \dot{v}_\infty}{g} + \dot{h}\right) \tag{3}$$

$$\dot{\mathcal{L}} = mg\left(\frac{v_\infty \dot{v}_\infty}{g} - \dot{h}\right) \tag{4}$$

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- Rearranging and substituting for $\dot{h} \approx \bar{v}_\infty \gamma$ for small angle approximation:

$$\dot{\tilde{\mathcal{E}}} = \frac{\dot{\mathcal{E}}}{mgv_\infty} = \frac{\dot{v}_\infty}{g} + \gamma \tag{5}$$

$$\dot{\tilde{\mathcal{L}}} = \frac{\Delta\dot{\mathcal{L}}}{mgv_\infty} = \frac{\dot{v}_\infty}{g} - \gamma \tag{6}$$

  - $\dot{\tilde{\mathcal{E}}}$: normalized total energy
  - $\dot{\tilde{\mathcal{L}}}$: normalized Lagrangian

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- Recall x_W-axis force balance:

$$-D + T\cos\alpha\cos\beta - mg\sin\gamma = m\dot{v}_\infty \tag{7}$$

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- For some steady-flight conditions:

$$\bar{T} + \Delta T = mg\left(\frac{\dot{v}_\infty}{g} + \bar{\gamma} + \Delta\gamma\right) + \bar{D} - \Delta D \tag{9}$$

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- Assuming $\bar{T} \approx mg\bar{\gamma} + \bar{D}$ for trim and $\Delta D \approx 0$:

$$\frac{\Delta T}{mg} = \delta_T = \frac{\dot{v}_\infty}{g} + \Delta\gamma = \frac{\dot{\mathcal{E}}}{mgv_\infty} \tag{10}$$

Flight-Path Angle Approximation

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Flight-Path Angle Approximation

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  - Demonstrates pitch, θ, used effectively to change rate of normalized Lagrangian
- TECS: elevator deflection, δ_e, provides additional SISO inner-loop attitude control for pitch

Total Energy Control System

- Total energy control system (TECS) design: planning system provides TECS:
  - Commanded airspeed acceleration in g's, $\frac{\dot{v}_{\infty,c}}{g}$
  - Commanded flight-path angle, γ_c

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  - Commanded airspeed acceleration in g's, $\frac{\dot{v}_{\infty,c}}{g}$
  - Commanded flight-path angle, γ_c
- Commanded total energy rate:

$$\dot{\tilde{\mathcal{E}}}_c = \frac{\dot{v}_{\infty,c}}{g} + \gamma_c \tag{13}$$

- Commanded Lagrangian rate:

$$\dot{\tilde{\mathcal{L}}}_c = \frac{\dot{v}_{\infty,c}}{g} - \gamma_c \tag{14}$$

Total Energy Control System (continued)

- Reference signal to track:

$$\vec{r} = \begin{bmatrix} \dot{\tilde{\mathcal{E}}}_c \\ \dot{\tilde{\mathcal{L}}}_c \end{bmatrix} = \begin{bmatrix} \frac{\dot{v}_{\infty,c}}{g} + \gamma_c \\ \frac{\dot{v}_{\infty,c}}{g} - \gamma_c \end{bmatrix} \tag{15}$$

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- Output

$$\vec{y} = \begin{bmatrix} \dot{\tilde{\mathcal{E}}} \\ \dot{\tilde{\mathcal{L}}} \end{bmatrix} = \begin{bmatrix} \frac{\dot{v}_{\infty}}{g} + \gamma \\ \frac{\dot{v}_{\infty}}{g} - \gamma \end{bmatrix} \tag{16}$$

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- Tracking error

$$\vec{e} = \vec{y} - \vec{r} \tag{17}$$

Total Energy Control System (continued)

- Consider tracking constant step inputs to $\vec{r}$ and reject constant disturbances, $\vec{w}$, e.g. additive drag: use servomechanism state-space augmentation with 1 integration

$$\vec{z} = \begin{bmatrix} \vec{e} \\ \dot{\vec{y}} \end{bmatrix} \tag{18}$$

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$$\vec{u} = \begin{bmatrix} K_e & K_y \end{bmatrix} \begin{bmatrix} \int \vec{e} \\ \vec{y} \end{bmatrix} \tag{21}$$

  - $K_e \in \mathbb{R}^{n_u \times n_y}$ and $K_y \in \mathbb{R}^{n_u \times n_y}$

Total Energy Control System (continued)

- For TECS, following inputs and gains:

$$\begin{bmatrix}\delta_T \\ \theta_c\end{bmatrix} = \begin{bmatrix} K_{e,\dot{\tilde{\mathcal{E}}}} & 0 & K_{y,\dot{\tilde{\mathcal{E}}}} & 0 \\ 0 & K_{e,\dot{\tilde{\mathcal{L}}}} & 0 & K_{y,\dot{\tilde{\mathcal{L}}}} \end{bmatrix} \begin{bmatrix} \int \left(\dot{\tilde{\mathcal{E}}} - \dot{\tilde{\mathcal{E}}}_c\right) \\ \int \left(\dot{\tilde{\mathcal{L}}} - \dot{\tilde{\mathcal{L}}}_c\right) \\ \dot{\tilde{\mathcal{E}}} \\ \dot{\tilde{\mathcal{L}}} \end{bmatrix} = K_{TECS} \begin{bmatrix} \int \left(\dot{\tilde{\mathcal{E}}}_c - \dot{\tilde{\mathcal{E}}}\right) \\ \int \left(\dot{\tilde{\mathcal{L}}}_c - \dot{\tilde{\mathcal{L}}}\right) \\ \dot{\tilde{\mathcal{E}}} \\ \dot{\tilde{\mathcal{L}}} \end{bmatrix} \quad (22)$$

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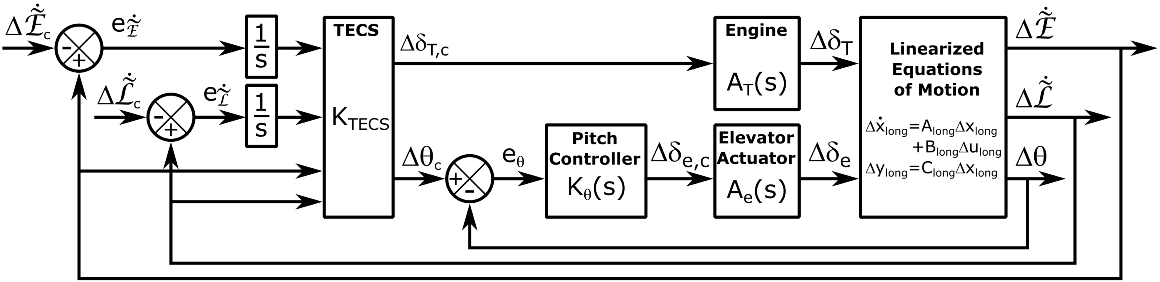
- Gains as matrix, K_z, designed using linear control methods, i.e. eigenvalue placement or optimal control
- For control design, model TECS system as block diagram

TECS Block Diagram



- Option: $\Delta\dot{\tilde{\mathcal{E}}}_c$ and $\Delta\dot{\tilde{\mathcal{L}}}_c$ computed from $\Delta\gamma_c$ and $\Delta v_{\infty,c}$

TECS Dynamics

- θ_c passed to SISO inner-loop controller for $\delta_{e,c}$:

$$\delta_{e,c}(s) = K_\theta(s)\theta_c(s) \tag{23}$$

  - Typically designed first to track θ_c signal faster than outer TECS loop and improve stability characteristics of short-period dynamics

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- For control design of TECS outer-loop, linearized plant will correspond to some LTI state-space model:

$$\begin{aligned} \dot{\vec{x}}_{cl} &= A_{cl}\vec{x}_{cl} + B_{cl}\vec{u} \\ \vec{y} &= C_{cl}\vec{x}_{cl} + D_{cl}\vec{u} \end{aligned} \tag{24}$$

  - $\vec{x}_{cl}$ includes state of vehicle, elevator actuator, engine, controller
  - $\vec{u} = [\Delta\delta_T \;\; \Delta\theta_c]^T$
  - $\vec{y} = [\dot{\tilde{\mathcal{E}}} \;\; \dot{\tilde{\mathcal{L}}}]^T$

TECS Dynamics (continued)

- Define augmented state:

$$\vec{x}_{aug} = \begin{bmatrix} \int e_{\dot{\tilde{\mathcal{E}}}} \\ \int e_{\dot{\tilde{\mathcal{L}}}} \\ \vec{x}_{cl} \end{bmatrix} \tag{25}$$

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- Output feedback:

$$\vec{u} = K_{TECS} C_{aug} \vec{x}_{aug} \tag{26}$$

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- Closed-loop dynamics:

$$\begin{aligned} \dot{\vec{x}}_{aug} &= \left(A_{aug} + B_{aug} K_{TECS} (I_2 + D_{aug} K_{TECS})^{-1} C_{aug} \right) \vec{x}_{aug} + \begin{bmatrix} -I_2 \\ 0 \end{bmatrix} \vec{r} \\ \vec{y}_{aug} &= C_{aug} \vec{x}_{aug} \end{aligned} \tag{27}$$

TECS Dynamics (continued)

$$A_{aug} = \begin{bmatrix} 0 & C_{cl} \\ 0 & A_{cl} \end{bmatrix} \tag{28}$$

$$B_{aug} = \begin{bmatrix} D_{cl} \\ B_{cl} \end{bmatrix} \tag{29}$$

$$C_{aug} = \begin{bmatrix} I_2 & 0 \\ 0 & C_{cl} \end{bmatrix} \tag{30}$$

$$D_{aug} = \begin{bmatrix} 0 \\ D_{cl} \end{bmatrix} \tag{31}$$

Total Heading Control System

- Analogously to TECS coupled commands and error rates of γ and $\frac{\dot{v}_\infty}{g}$ for efficient decoupling of thrust and elevator: **total heading control system (THCS)**
  - Couples yaw angle, ψ, with sideslip angle, β, to efficiently control flight vehicle via error rates of heading, σ, i.e. $\psi + \beta$, & differential $\psi - \beta$
  - Form commanded roll angle and commanded yaw rate to inner-loop controllers

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- Compute commanded yaw rate:

$$\dot{\psi}_c = K_\psi(\psi_c - \psi) \tag{32}$$

- Commanded sideslip:

$$\dot{\beta}_c = K_\beta(\beta_c - \beta) \tag{33}$$

  - Typically also limited using some minimum and maximum values for rate commands

Total Heading Control System

- Defining error rates:

$$e_{\dot{\psi}} = \dot{\psi}_c - \dot{\psi} \tag{34}$$

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- Form commanded roll angle:

$$\phi_c = \frac{K_{\phi,i}}{s}\frac{v_\infty}{g}(e_{\dot{\psi}} + e_{\dot{\beta}}) \tag{36}$$

- Commanded yaw rate angle:

$$r_c = \frac{K_{r,i}}{s}(e_{\dot{\psi}} - e_{\dot{\beta}}) \tag{37}$$

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  - $\frac{V_\infty}{g}$ term used to coordinate r_c with ϕ_c
- Coordinated turns: requires $K_\psi = K_\beta$
- For $\phi_c = \frac{V_\infty r_c}{g}$, requires $K_{\phi,i} = K_{r,i}$

Total Heading Control System

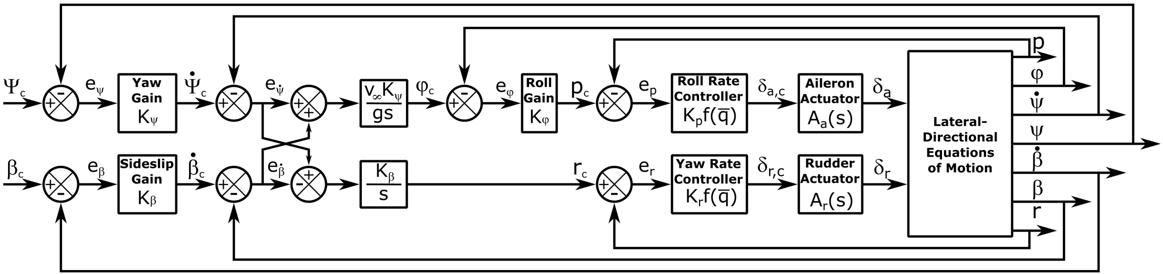
- Further design control gains for decoupled inner-loops:

$$\delta_{a,c} = K_p f(\bar{q})(K_\phi(\phi_c - \phi) - p) \tag{38}$$

$$\delta_{r,c} = K_r f(\bar{q})(r_c - r) \tag{39}$$

- Provide stability augmentation for all control modes and flight conditions for specific flight vehicle
  - $f(\bar{q})$: multiplicative “inversion” function, dependent on trim pitch rate, $\bar{q}$
  - Alternative: implement dynamics inversion block instead of multiplicative function

THCS System Block Diagram



Summary

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  - Determine total and differential energy from flight-path and velocity rate
  - Map to thrust and pitch $\rightarrow$ control pitch via elevator
  - Can formulate control as robust servomechanism framework
  - Provides eigenvalue placement/LQR/$\mathcal{H}_\infty$ design

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- Total Heading Control System
  - Determine commanded roll and yaw rate from yaw and heading
  - Decouples roll and yaw rate controllers