AEM 668 Project 3

Structural-Mode Analysis and Control of Airplane

Learning Objective

This project is intended to introduce structural-mode analysis and control for elastic aerospace vehicle dynamics in the context of dampening the modal displacement.

Dynamical System

Consider an elastic airplane with the following geometric, mass, and structural-mode data

Vehicle Length	l_f	143 ft	**Weight**	W	288,000 lb at MSL
C.g. location from nose	l_{cg}	88.42 ft	**Cockpit location from c.g.**	x_{cp}	63.4 ft
Wing Geometry	S_w $\bar{c}_w$ b_w Λ_w	1,950 ft^2 15.3 ft 70 ft 65°	**Inertias**	I_{xx} I_{yy} I_{zz} I_{xz}	1.5 $\times 10^5$ sl-ft^2 7.0 $\times 10^6$ sl-ft^2 7.1 $\times 10^6$ sl-ft^2 -5.27 $\times 10^4$ sl-ft^2
Mode 3 Generalized Mass	$\mathcal{M}$	184 sl-ft^3	**Mode 3 Frequency**	ω	12.6 rad/sec
Mode 3 Displacement at x_{cl}	$\nu_Z(x_{cp})$	0.32 ft	**Mode 3 Shape**	$\nu'_Z(x_{cp})$	-0.027 rad

where modes 1 and 2 are the implicit rigid-body modes, the phugoid and the short-period and mode 3 can be understood as the elastic-body *first fuselage bending mode*.

Consider the following trimmed coordinated, straight-and-level flight condition for cruise, i.e., $\bar{\gamma} = \bar{\beta} = \bar{\phi} = 0$

Altitude	$\bar{h}$	5000 ft	**Mach number**	M_∞	0.6
Airspeed	$\bar{v}_\infty = \bar{u}$	659 ft/s	**Angle of Attack**	$\bar{\alpha}$	0°
Lift Coefficient	$\bar{C}_L$	0.340	**Drag Coefficient**	$\bar{C}_D$	0.028

with longitudinal stability and control derivatives

State/Input	$X_\bullet$	$Z_\bullet$	$M_\bullet$	Ξ
u	-0.013	-0.1001	0.003	0
α	19.45	-283.3	-3.445	-1075
$\dot{\alpha}$	0	0	-0.1035	0
q	-1.913	16.55	-3.136	-79.44
δ_e	14.83	-42.22	-5.346	-923.0

δ_f	0.742	2.11	3.376	89.53
η	0	-2.812	-0.0663	4.219
$\dot{\eta}$	0	-0.0968	-0.00372	-0.3502

where one can assume a linear aerodynamic and propulsive force and moment model as

$$\Delta X = X_u \Delta u + X_\alpha \Delta\alpha + X_q \Delta q + X_{\delta_e}\Delta\delta_e + X_{\delta_f}\Delta\delta_f \tag{1}$$

$$\Delta Z = Z_u \Delta u + Z_\alpha \Delta\alpha + Z_q \Delta q + Z_{\delta_e}\Delta\delta_e + Z_{\delta_f}\Delta\delta_f + Z_\eta \eta + Z_{\dot\eta}\dot\eta \tag{2}$$

$$\Delta M = M_u \Delta u + M_\alpha \Delta\alpha + M_{\dot\alpha}\Delta\dot\alpha + M_q \Delta q + M_{\delta_e}\Delta\delta_e + M_{\delta_f}\Delta\delta_f + M_\eta \eta + M_{\dot\eta}\dot\eta \tag{3}$$

and

$$\Xi = \frac{Q}{\mathcal{M}} = \Xi_\alpha \Delta\alpha + \Xi_q \Delta q + \Xi_{\delta_e}\Delta\delta_e + \Xi_{\delta_f}\Delta\delta_f + \Xi_\eta \eta + \Xi_{\dot\eta}\dot\eta = \ddot\eta + 2\zeta\omega\dot\eta + \omega^2\eta \tag{4}$$

with $\zeta = 0.01$.

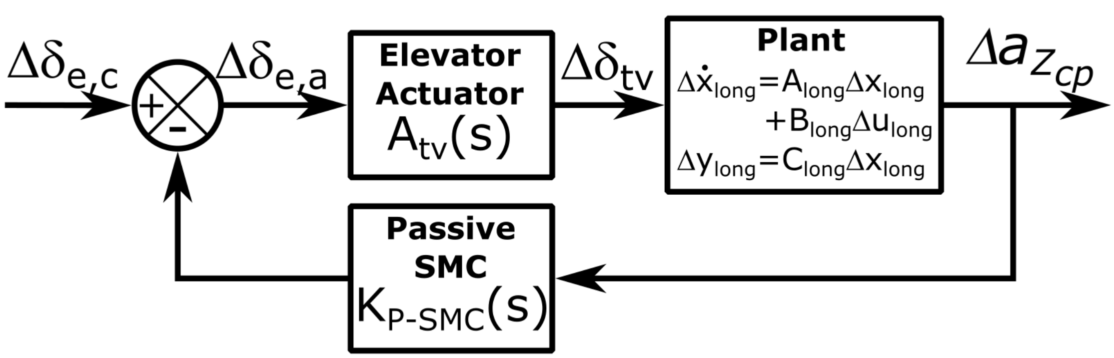
Here δ_e is the *elevator input* for steering the airplane and δ_f is the *fin input* for dampening the structural-mode experienced at the cockpit. For this project, the elevator and fin actuators can be modeled as first-order systems with bandwidths as 50 rad/s. For this airplane, the cockpit, x_{cp}, experiences an acceleration modeled as

$$\Delta a_{Z_{cp}} = \bar{u}\Delta\dot\alpha - \bar{u}\Delta q - x_{cp}\Delta\dot{q} + \nu_Z(x_{cp})\ddot\eta \tag{5}$$

First, for designing a passive *structural-mode controller (SMC)* with a feedback controller, $K_{P-SMC}(s)$, as

$$\delta_{e,a} = -K_{P-SMC}(s)\Delta a_{Z_{cp}} = -K_{P-SMC,p}\left(\frac{s^2 + 2\zeta_n\omega_n s + \omega_n^2}{s^2 + 2\zeta_{id}\omega_n s + \omega_n^2}\right)\Delta a_{Z_{cp}} \tag{6}$$

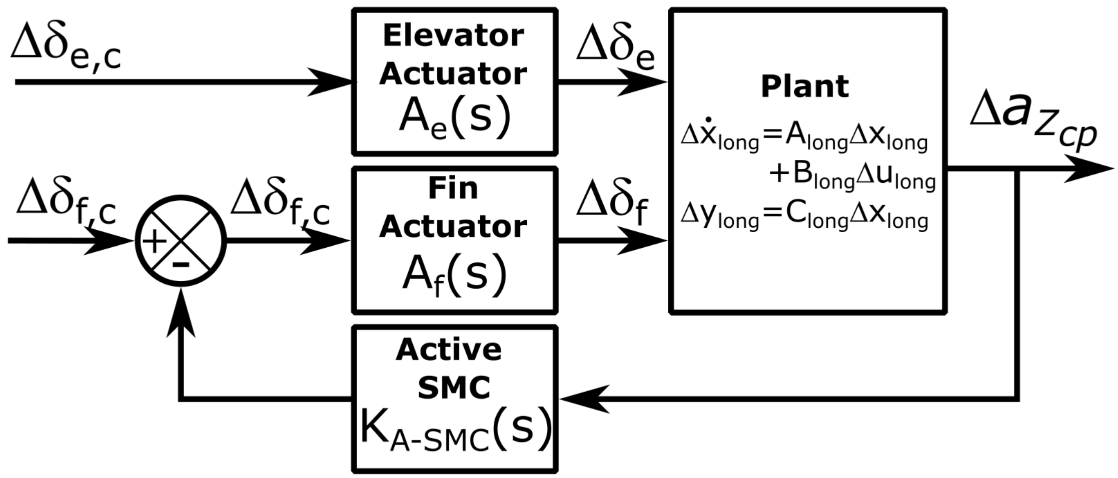
where $K_{P-SMC,p} > 0$ is the proportional gain, ω_n is the *notch filter*'s center frequency, ζ_n is the notch filter's zero damping, and $\zeta_{id} \gg \zeta_n$ increases the damping ratio about the notch filter's center frequency. This passive SMC can be modeled as the following block diagram.



Second, for designing a active *structural-mode controller (SMC)* with a feedback controller, $K_{A-SMC}(s)$, as

$$\delta_{f,c} = -K_{A-SMC}(s)\Delta a_{Z_{cp}} = -K_{A-SMC,p}\left(\frac{s}{s+\omega_{h-p}}\right)\left(\frac{\omega_{l-p}}{s+\omega_{l-p}}\right)\Delta a_{Z_{cp}} \tag{7}$$

where $K_{A-SMC,p} > 0$ is the proportional gain, ω_{h-p} is the high-pass filter corner frequency, ω_{l-p} is the low-pass filter corner frequency and $\omega_{h-p} < \omega_{l-p}$ implies the *band-pass filter* design for this SMC. Here the high-pass filter approximates the integration of the acceleration at the cockpit, $a_{Z_{cp}}$, to obtain the velocity at the payload and the low-pass filter causes the δ_f to ignore low-frequency vertical accelerations. This active SMC can be modeled as the following block diagram.



Project Assignment and Deliverables

For this project, determine in MATLAB/Simulink:

a) Determine, label, and output to the command window the open-loop longitudinal modal characteristics for the phugoid, short-period, and structural modes of the vehicle including:
   (a) eigenvalues;
   (b) natural frequencies;
   (c) damping ratios; and
   (d) comments on the meaning of the values.
b) Output the frequency responses of the cockpit vertical acceleration versus the two inputs, i.e., $\frac{\Delta a_{Z_{cp}}(j\omega)}{\Delta \delta_{e,c}(j\omega)}$, and $\frac{\Delta a_{Z_{cp}}(j\omega)}{\Delta \delta_{f,c}(j\omega)}$.
   - Comment on the ability of the control surfaces to excite the different modes.
   - Compare their magnitudes for the different modes.
c) Output the root locus plot of $K_{P-SMC,p}$ for the passive SMC, $K_{P-SMC}(s)$.
   - Choose ω_n and ζ_n to match the structural-mode natural frequency and damping ratio and $\zeta_{id} = 30\zeta_n$ to notch out the vibration effects of the elevator control input.
   - Use the `rlocus` MATLAB function.
   - Comment on the ability of the passive SMC and the effects of choosing a proportional gain too high.
d) Output the root locus plot of $K_{A-SMC,p}$ for the active SMC, $K_{A-SMC}(s)$.
   - Choose $\omega_{A-SMC,l-p}$ above the structural-mode frequency and $\omega_{A-SMC,h-p}$ below the structural-mode frequency to actuate the fin as additional dampening for the structural-mode.
   - Use the `rlocus` MATLAB function.
   - Comment on the ability of the active SMC and the effects of choosing a proportional gain too high.
e) Design an passive and active SMC, and
   - $K_{P-SMC,p}$;
   - ω_n;
   - ζ_n;
   - ζ_{id};
   - $K_{A-SMC,p}$;
   - $\omega_{A-SMC,l-p}$;
   - $\omega_{A-SMC,h-p}$;
   - output the frequency response of the closed-loop systems
   - determine, label, and output to the command window the closed-loop modal characteristics for the phugoid, short-period, elastic mode, actuators, and both choices of SMC with your choice of proportional gains that stabilize the system, include:
     - eigenvalues;
     - natural frequencies;
     - damping ratios; and
     - comments on the changes in these values from the open-loop system.