AEM 668 Project 4

MIMO LTI Feedback Control of Tail-Controlled Rocket

Learning Objective

This project is intended to introduce dynamics and control analysis in the presence of uncertainties for a tail-controlled rocket. The equations of motion are taken from R.T. Reichert, "Autopilot Design Using μ-Synthesis," American Control Conference, 1990.

Dynamical System

The nonlinear state dynamics for a tail-controlled rocket can be modeled as

$$\begin{aligned} \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} &= f(\alpha, q, \mathcal{M}) = \begin{bmatrix} f_1(\alpha, q, \mathcal{M}) \\ f_2(\alpha, q, \mathcal{M}) \end{bmatrix} = \begin{bmatrix} \frac{\cos\alpha}{m c_s \mathcal{M}} \left(Q_\infty S_f C_n(\alpha, \mathcal{M}, \delta) \right) + q \\ \frac{1}{I_{yy}} \left(Q_\infty S_f d C_m(\alpha, \mathcal{M}, \delta) \right) \end{bmatrix} \\ \begin{bmatrix} \alpha \\ q \\ a_z \end{bmatrix} &= \begin{bmatrix} \alpha \\ q \\ \frac{Q_\infty S_f C_n(\alpha, \mathcal{M}, \delta)}{m g_0} \end{bmatrix} \end{aligned} \tag{1}$$

where α is the angle of attack in degrees, q is the pitch rate in degrees/second, a_z is the vertical acceleration in g's, g_0 is the standard acceleration due to gravity, $\mathcal{M}$ is the Mach number, c_s is the speed of sound, Q_∞ is the free-stream dynamic pressure and can be modeled as

$$Q_\infty = \frac{1}{2} \gamma_{s-h} P_{s,\infty} \mathcal{M}^2 \tag{2}$$

γ_{s-h} is the specific heat ratio, $P_{s,\infty}$ is the static pressure of the free-stream, S_f is the fin reference area, d is the diameter of the rocket fuselage, m is the mass, I_{yy} is the pitch moment of inertia, g is the acceleration due to gravity, C_n is the nondimensional aerodynamic coefficient for the vertical force modeled as

$$C_n(\alpha, \mathcal{M}, \delta) = sign(\alpha) \left(n_3 |\alpha|^3 + n_2 \alpha^2 + (n_{1,\mathcal{M}} \mathcal{M} + n_{1,0}) |\alpha| \right) + n_0 \delta \tag{3}$$

C_m is the nondimensional aerodynamic coefficient for the pitching moment modeled as

$$C_m(\alpha, \mathcal{M}, \delta) = sign(\alpha) \left(m_3 |\alpha|^3 + m_2 \alpha^2 + (m_{1,\mathcal{M}} \mathcal{M} + m_{1,0}) |\alpha| \right) + m_0 \delta \tag{4}$$

and δ is the tail fin deflection in degrees which is actuated. The acutator can be modeled by a second-order transfer function

$$A_{fin}(s) = \frac{\delta(s)}{\delta_c(s)} = \frac{\omega_a^2}{s^2 + 2\zeta_a \omega_a s + \omega_a^2} \tag{5}$$

where $\omega_a = 150$ rad/s is the actuator undamped natural frequency and $\zeta_a = 0.7$ is the actuator damping ratio. In addition, this actuator is mechanically limited to ±20° deflection and a deflection rate of ±600° per second.

These equations can be used to form a linearized, time-invariant state-space model for the plant, $G(s)$, about trim values $\bar{\alpha}$, $\bar{q}$, $\bar{\delta}$, $\bar{a}_z$, for some Mach number, $\mathcal{M}$, as

$$\begin{bmatrix}\Delta\dot{\alpha}\\ \Delta\dot{q}\end{bmatrix} = \begin{bmatrix}\frac{\partial f_1(\alpha,q,\mathcal{M})}{\partial\alpha} & \frac{\partial f_1(\alpha,q,\mathcal{M})}{\partial q}\\ \frac{\partial f_2(\alpha,q,\mathcal{M})}{\partial\alpha} & \frac{\partial f_2(\alpha,q,\mathcal{M})}{\partial q}\end{bmatrix}_{\bar{\alpha},\bar{q},\bar{\delta},\bar{a}_z} \begin{bmatrix}\Delta\alpha\\ \Delta q\end{bmatrix} + \begin{bmatrix}\frac{\partial f_1(\alpha,q,\mathcal{M})}{\partial\delta}\\ \frac{\partial f_2(\alpha,q,\mathcal{M})}{\partial\delta}\end{bmatrix}_{\bar{\alpha},\bar{q},\bar{\delta},\bar{a}_z} \Delta\delta$$

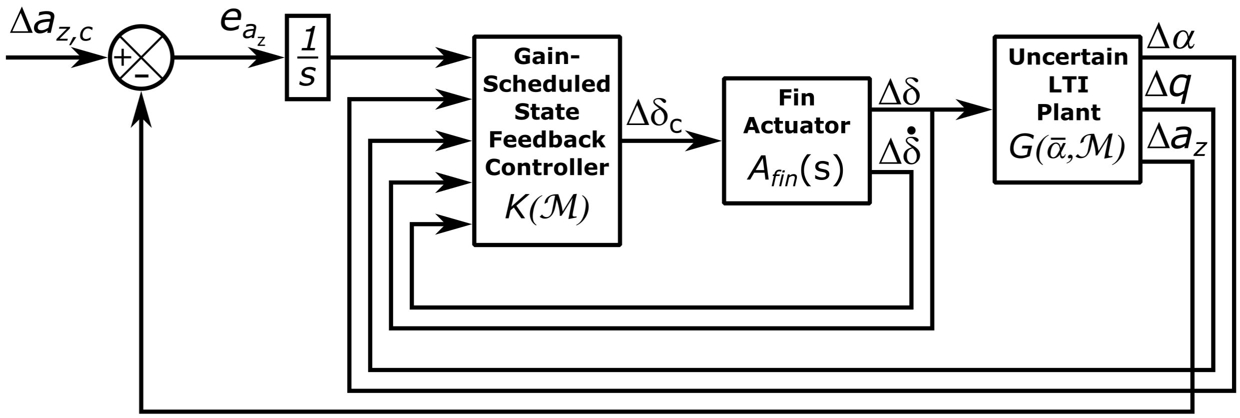
$$\begin{bmatrix}\Delta\alpha\\ \Delta q\\ \Delta a_z\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\\ \frac{\pi}{180}\left(\frac{Q_\infty S_f}{m g_0}\right)\frac{\partial C_n(\alpha,\mathcal{M},\delta)}{\partial\alpha} & \left(\frac{Q_\infty S_f}{m g_0}\right)\frac{\partial C_n(\alpha,\mathcal{M},\delta)}{\partial q}\end{bmatrix}_{\bar{\alpha},\bar{q},\bar{\delta},\bar{a}_z} \begin{bmatrix}\Delta\alpha\\ \Delta q\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{\pi}{180}\left(\frac{Q_\infty S_f}{m g_0}\right)\frac{\partial C_n(\alpha,\mathcal{M},\delta)}{\partial\delta}\end{bmatrix}_{\bar{\alpha},\bar{q},\bar{\delta},\bar{a}_z} \Delta\delta \tag{6}$$

where $\Delta\bullet = \bullet - \bar{\bullet}$ for $\bullet$ as α, q, δ, and n_z.

For this project, the values of the model parameters for this rocket-powered flight vehicle are:

Model Parameter	**Symbol**	**Value**
Fin Reference Area	S_{ref}	0.44 ft^2
Diameter	d	0.75 ft
Mass	m	13.98 slugs
y-moment of inertia	I_{yy}	182.5 slug-ft^2
Standard acceleration due to gravity at 20,000 ft	g	32.2 ft/s^2
Specific heat ratio of air	γ_{s-h}	1.40
Static pressure at 20,000 ft	$P_{s,\infty}$	973.3 lbs/ft^2
First-order C_n term w.r.t δ	n_0	-0.034 /deg
First-order C_n term w.r.t α	$n_{1,0}$	-0.3392 /deg
First-order C_n w.r.t $\mathcal{M}$ & α	$n_{1,\mathcal{M}}$	0.0565333 /deg
Second-order C_n term w.r.t α	n_2	-0.0094457 /deg^2
Third-order C_n term w.r.t α	n_3	0.000103 /deg^3
First-order C_m term w.r.t δ	m_0	-0.206 /deg
First-order C_m term w.r.t α	$m_{1,0}$	-0.357 /deg
First-order C_m w.r.t $\mathcal{M}$ & α	$m_{1,\mathcal{M}}$	0.136 /deg
Second-order C_m term w.r.t α	m_2	-0.019546 /deg^2
Third-order C_m term w.r.t α	m_3	0.000215 /deg^3

A state feedback robust servomechanism control system is to be used to track acceleration commands to the rocket as



where

$$\Delta\delta_c = K \begin{bmatrix} \int a_{z,c} - a_z \\ \Delta\alpha \\ \Delta q \\ \Delta\delta \\ \Delta\dot{\delta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \int a_{z,c} - a_z \\ \Delta\alpha \\ \Delta q \\ \Delta\delta \\ \Delta\dot{\delta} \end{bmatrix} \tag{7}$$

For this project, the control design objectives for this rocket-powered flight vehicle are:

1. Track commanded step inputs for acceleration maneuvers $a_{z,c}$ up to $\pm 1\ g$ with:
   (a) steady-state error of $< 1\%$;
   (b) 5%-settling time < 20 seconds;
   (c) overshoot $< 50\%$; and
   (d) no position or rate saturation of the fin deflection, δ.
2. Robust stability to gain and phase variations in the actuator system and unmodeled high-frequency vibration modes across the range of flight conditions

Project Assignment and Deliverables

For this project, determine in MATLAB/Simulink:

a) Construct a function (at the bottom of the script or as a separate file) to return the linearized, time-invariant state-space model for the plant at a given trim angle of attack, $\bar{\alpha}$, and Mach number, $\bar{\mathcal{M}}$.
   - As a check, the linearization at $\bar{\alpha} = 0°$ and $\mathcal{M} = 3$ should be

$$A = \begin{bmatrix} -0.6 & 1.0 \\ 32.4 & 0 \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} -0.12 \\ -130.9 \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.02 & 0 \end{bmatrix} \tag{10}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ -0.2038 \end{bmatrix} \tag{11}$$

   - and at $\bar{\alpha} = 20°$ and $\mathcal{M} = 3$ should be

$$A = \begin{bmatrix} -1.18 & 1.0 \\ -300.2 & 0 \end{bmatrix} \tag{12}$$

$$B = \begin{bmatrix} -0.11 \\ -130.9 \end{bmatrix} \tag{13}$$

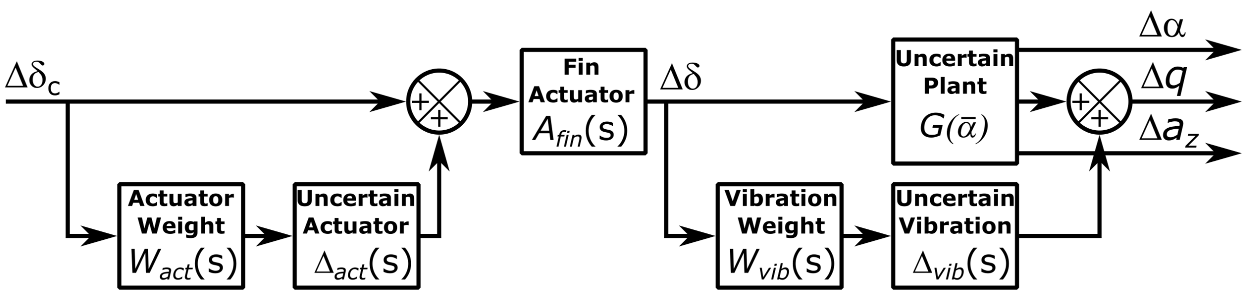
$$C = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -2.54 & 0 \end{bmatrix} \tag{14}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ -0.204 \end{bmatrix} \tag{15}$$

b) Design a robust servomechanism, LTI, state feedback controller at $\bar{\alpha} = 0°$ and $\mathcal{M} = 3$
   - use the `ss` function for A_{fin} as a state-space system to output both δ and $\dot{\delta}$;
   - use the `tf` function for the tracking error integrator block, $\frac{1}{s}$;
   - use the `connect` function to form the generalized plant;
   - determine the controllability of the generalized plant;
   - determine the gain and phase margins, use the 1×1 open-loop transfer function matrix at the plant input, i.e., $L_{in}(s) = [K(s)][T_{A\&G}(s)]$; and
   - (optional) use the `place` function to set the poles of the feedback control system.

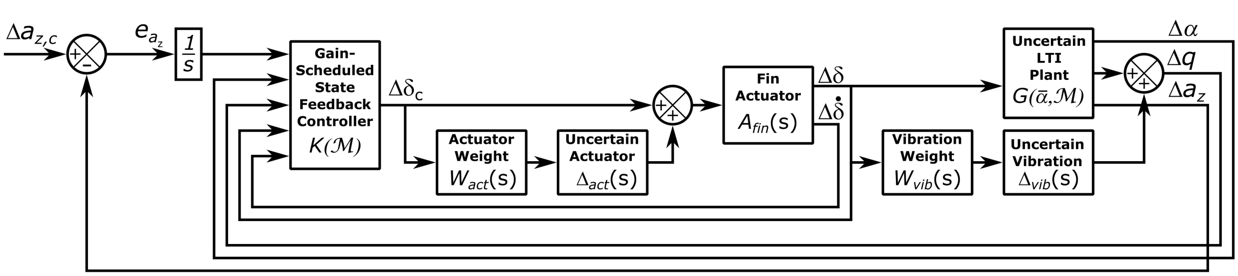
c) Simulate the MIMO LTI system with your robust servomechanism state feedback controller at $\bar{\alpha} = 0°$ and $\mathcal{M} = 3$, actuator, and linearized and nonlinear plants with different step inputs for $a_{z,c}$
   - Start with the provided Simulink model, "rocket.mdl;"
   - output the step response with a suitably small $a_{z,c}$ such that the linearized and nonlinear plants agree; and
   - output the step response with five increasingly larger $a_{z,c}$ until the linearized and nonlinear plants disagree significantly, in your own opinion.

d) Construct an uncertain LTI system for the linearized dynamics model as



   - $\bar{\alpha} \in [0°, 10°]$ as real parametric uncertainty;
   - Δ_{vib} models high-frequency vibration modes;
   - $W_{act} = 0.6$: roughly corresponds to gain variations of 0.6 to 2.5 and phase variations of 35°;
   - $W_{vib} = \frac{150s^3+13{,}000s^2+70{,}000s+48{,}000}{s^3+2{,}000s^2+2{,}000{,}000s+62{,}000{,}000}$;
   - use the `ureal` function on $\bar{\alpha}$ with the `ss` function to form the uncertain plant $G(\bar{\alpha})$ for $\mathcal{M} = 3$ with the *quadratic approximation* for $\cos\alpha$ and $\sin\alpha$ (in radians);
   - use the `simplify` function with the `'full'` option to eliminate redundant copies of the α uncertainty for this uncertain plant;
   - use the `ultidyn` function to form the LTI uncertainties, Δ_{act} and Δ_{vib};
   - use the `connect` function to form the system interconnections; and
   - output the Bode plot of the open-loop transfer functions, $\Delta\alpha/\Delta\delta_c$, $\Delta q/\Delta\delta_c$ and $\Delta a_z/\Delta\delta_c$, using the `bode` function of the uncertain LTI model to generate several samples of the open-loop response.

e) Output to the command window the closed-loop robust stability margin, robust performance, and worst-case gain, for your robust servomechanism controller in part b) shown below for the flight condition $\mathcal{M} = 3$



- use the `connect` function to form the uncertain LTI feedback control system with generalized disturbance, $a_{z,c}$, and generalized error, e_{a_z};
- use the `robstab` function to determine the stability margin of your uncertain system;
- use the `wcgain` function to determine the worst-case uncertainty of your uncertain system; and
- interpret the meaning of the outputs of these functions.