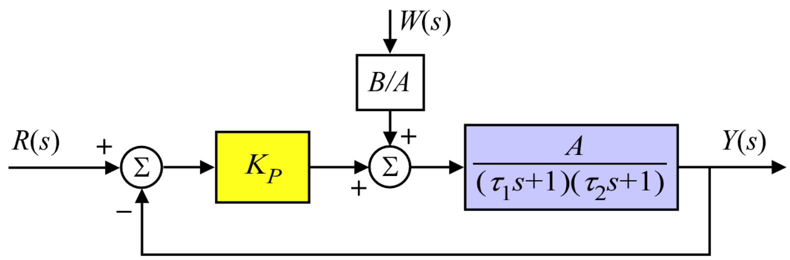
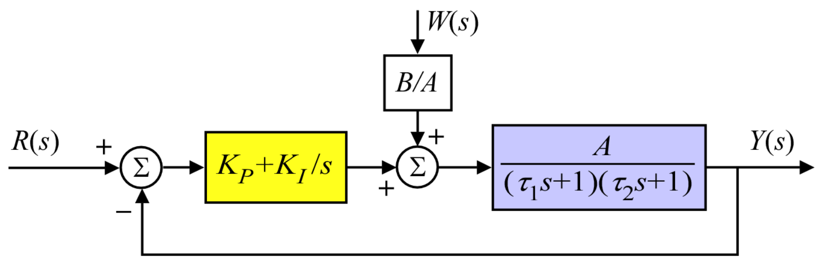
ME 475/575: Control Systems Analysis — Practice #3 Solutions

1. The following block diagrams represent three different controls (P, PI, and PID) for a DC motor.

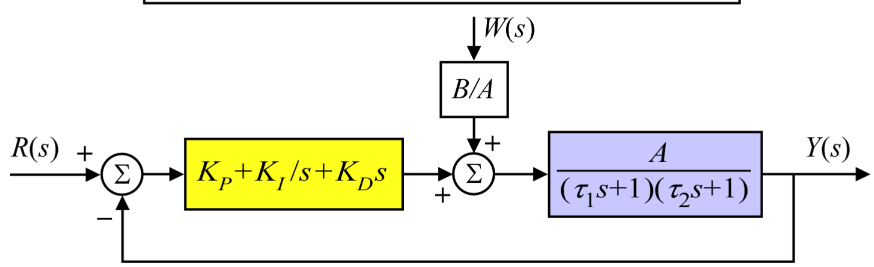
P control:



PI control:



PID control:



(a) Using $K_P = 5$, $K_I = 5/0.01 = 500$ and $K_D = 5(.0004) = 0.002$ with $w = -0.1$ ($W(s) = -0.1/s$), $r = 100$ ($R(s) = 100/s$), $\tau_1 = 1/60$, $\tau_2 = 1/600$, and $A = 10$, $B = 50$, calculate the steady-state response of the three different cases using the final value theorem.

<u>P control:</u>

$$Y(s) = \frac{50}{(\tau_1 s+1)(\tau_2 s+1)+50}\left(\frac{100}{s}\right) + \frac{50}{(\tau_1 s+1)(\tau_2 s+1)+50}\left(\frac{-0.1}{s}\right) = \frac{4995}{s[(\tau_1 s+1)(\tau_2 s+1)+50]}$$

$$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\frac{4995}{(\tau_1 s+1)(\tau_2 s+1)+50} = \frac{4995}{51} = 97.94 \;\Rightarrow\; y_{ss} = 97.94$$

PI control:

$$Y(s)=\frac{(K_P s+K_I)A}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}\left(\frac{100}{s}\right)+\frac{Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}\left(\frac{-0.1}{s}\right)$$

$$y_{ss}=\lim_{s\to 0} sY(s)=\lim_{s\to 0}\frac{100[(K_P s+K_I)A]}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}+\frac{-0.1Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}$$

$$=\frac{100K_I A}{K_I A}=100$$

➔ $y_{ss}=100$

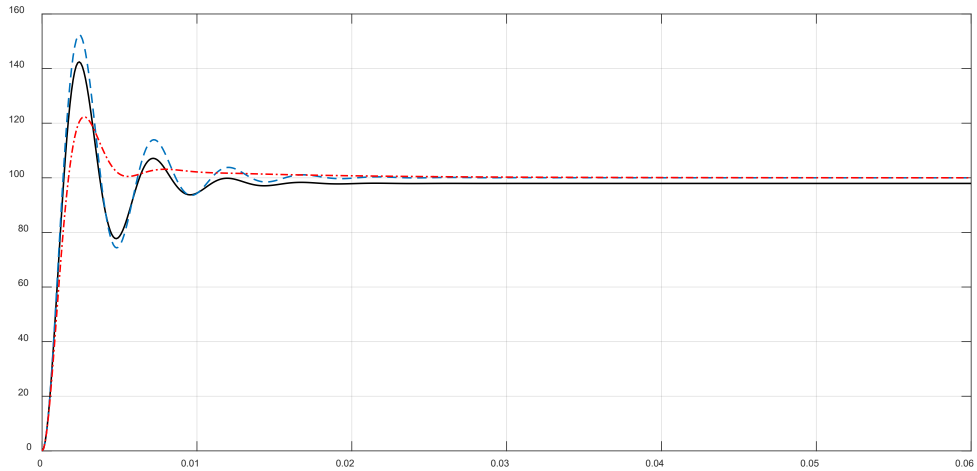
PID control:

$$Y(s)=\frac{(K_D s^2+K_P s+K_I)A}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}\left(\frac{100}{s}\right)$$
$$+\frac{Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}\left(\frac{-0.1}{s}\right)$$

$$y_{ss}=\lim_{s\to 0} sY(s)=\lim_{s\to 0}\frac{100[(K_D s^2+K_P s+K_I)A]}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}$$
$$+\frac{-0.1Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}$$
$$=\frac{100K_I A}{K_I A}=100$$

➔ $y_{ss}=100$

(b) Obtain the system responses up to 0.06 seconds using Simulink and plot them together on the same graph. Attach the Simulink block diagrams as well as the MATLAB script files.



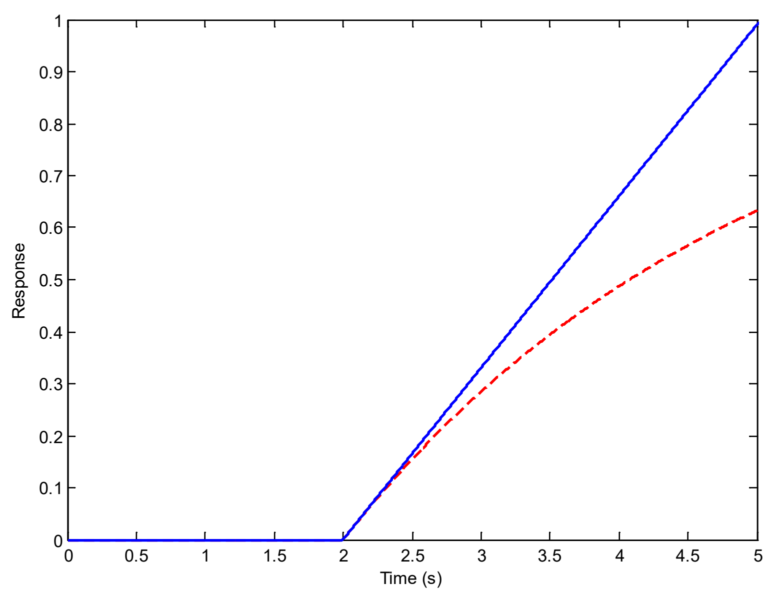
Black line: proportional control; blue line: PI control; red line: PID control.

2. A paper machine is to be controlled by a PID controller. The input is stock flow onto the wire and the output is basis weight or thickness.

   (1) From experimental step response, the machine was found to be approximated by

$$G(s) = \frac{e^{-2s}}{3s+1}$$

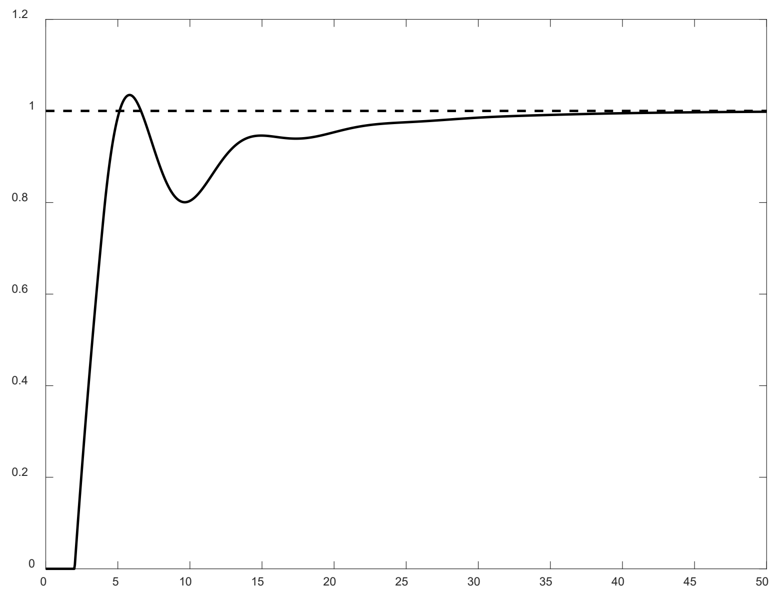
   Find the PI-controller parameters using the transient response method. Simulate the system with the selected parameters for a unit-step input.



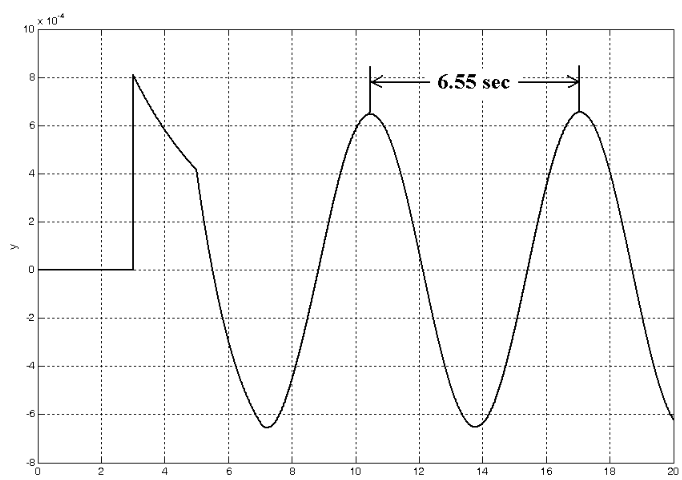
Reaction rate, $R = K/\tau = 0.33$, Time delay, $L = 2$ s.

$K_p = \frac{0.9}{RL} = \frac{0.9}{0.33\times 2} = 1.35$

$T_I = L/0.3 = 6.67$



(2) The system becomes marginally stable for a proportional gain of $K_u = 3.044$, as shown by the unit-impulse response below. Find the optimal PID-controller parameters according to the Zeigler-Nichols tuning rules. Simulate the system with the selected parameters for a unit-step input.



$K_p = 0.6K_u = 0.6\times 3.044 = 1.83$, $T_I = P_u/2 = 6.55/2 = 3.28$,

$$T_D = P_u / 8 = 6.55 / 8 = 0.82$$

$$K_I = \frac{K_P}{T_I} = \frac{1.83}{3.28} = 0.56$$

$$K_D = K_P T_D = 1.83 \times 0.82 = 1.50$$