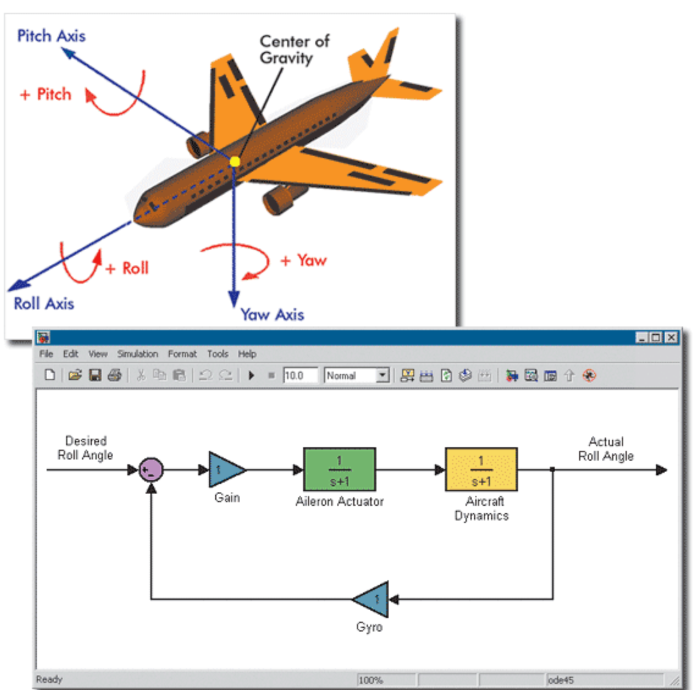
4. Feedback Control and PID Control

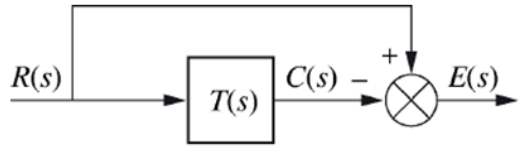
Properties of feedback control systems
PID control
System types

Steady-State Error and System Type



Steady-State Error In Terms of $T(s)$

- General representation of a closed-loop control system, where the closed-loop transfer function is $T(s)$:



- The error, $E(s)$ between the input, $R(s)$ and the output, $C(s)$ is

$$E(s) = R(s) - C(s) = R(s) - R(s)T(s) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- Using the final value theorem, the Steady-State Error (SSE) can be found to be

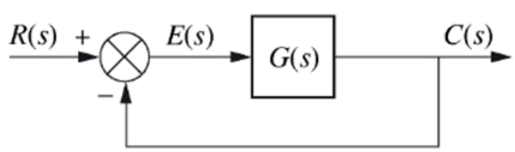
$$e_{ss} = e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sR(s)[1 - T(s)]$$

- For example, if $T(s) = 5/(s^2+7s+10)$ and the input is a unit step,

$$E(s) = \frac{s^2 + 7s + 5}{s(s^2 + 7s + 10)} \Rightarrow e(\infty) = \lim_{s\to 0} sE(s) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Steady-State Error In Terms of $G(s)$

- General representation of a unity feedback system, where the open-loop transfer function is $G(s)$:



- The error, $E(s)$ between the input, $R(s)$ and the output, $C(s)$ is

$$E(s) = R(s) - C(s) = R(s) - E(s)G(s) \quad \Rightarrow \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- Using the final value theorem, the Steady-State Error (SSE) can be found to be

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + G(s)}$$

- Note that the above relationship is **valid only when the closed-loop system is __________**, *i.e.*, all the poles are in the left half-plane and at most one pole at the origin.

Step Input

- If the input is a unit step, then $R(s) = 1/s$.

$$e(\infty) = e_{step}(\infty) = \lim_{s\to 0}\frac{s(1/s)}{1+G(s)} = \frac{1}{1+\lim_{s\to 0} G(s)}$$

- The term $\lim_{s\to 0} G(s)$ is the __________ of the open-loop transfer function, since s, the frequency variable, is approaching zero.
- In order to have zero SSE, $\lim_{s\to 0} G(s) = \infty$.
- Hence, G(s) must take on the following form:

$$G(s) = \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}\ , \text{ where } n \geq 1$$

- Therefore, there must be _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ **at the origin**. A pole at the origin is also interpreted as an integration.
- If there are no integration, then $n = 0$, and

$$\lim_{s\to 0} G(s) = \frac{z_1 z_2\cdots}{p_1 p_2\cdots}$$

Ramp Input

- If the input is a ramp, then $R(s)$ = ________.

$$e(\infty) = e_{ramp}(\infty) = \lim_{s\to 0}\frac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0}\frac{1}{s+sG(s)} = \frac{1}{\lim_{s\to 0} sG(s)}$$

- In order to have zero SSE for a ramp input, we must have

____

- To satisfy the above relationship, $G(s)$ must take on the following form:

$$G(s) = \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}\ , \text{ where } n \geq 2$$

- Therefore, there must be _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ **at the origin**.
- If only one integration exists in the forward path, then

$$\lim_{s\to 0} sG(s) = \frac{z_1 z_2\cdots}{p_1 p_2\cdots}$$

, which is finite rather than infinite and this leads to a constant error for a ramp input.

Parabolic Input

- If the input is a parabola, then $R(s)$ = ________.

$$e(\infty) = e_{parabola}(\infty) = \lim_{s\to 0} \frac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0} \frac{1}{s^2 + s^2G(s)} = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

- In order to have zero SSE for a parabolic input, we must have

$$\lim_{s\to 0} s^2G(s) = \infty$$

- To satisfy the above relationship, $G(s)$ must take on the following form:

$$G(s) = \frac{(s+z_1)(s+z_2)\cdots}{s^n(s+p_1)(s+p_2)\cdots}\ , \text{ where } n \geq 3$$

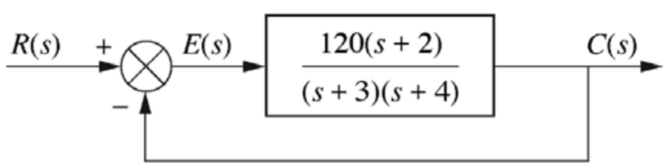
- Therefore, there must be ________________ **at the origin**.
- If only two integrations exist in the forward path, then

$$\lim_{s\to 0} s^2G(s) = \frac{z_1z_2\cdots}{p_1p_2\cdots}$$

, which is finite rather than infinite and this leads to a constant error for a parabolic input.

Example: SSE for systems with no integrators

- Find the steady-state errors for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$ to the system shown below.



- Solutions

The closed-loop system is **stable**.

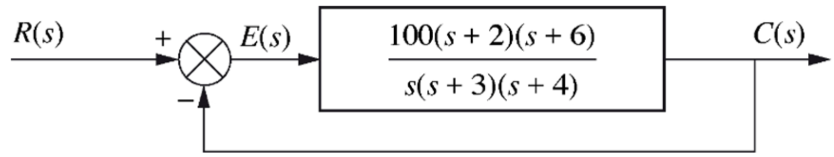
$5u(t)$ ➔ $5/s$: $\quad e_{step}(\infty) = \dfrac{5}{1+\lim_{s\to 0} G(s)} = \dfrac{5}{1+20} = \dfrac{5}{21}$

$5tu(t)$ ➔ $5/s^2$: $\quad e_{ramp}(\infty) = \dfrac{5}{\lim_{s\to 0} sG(s)} = \dfrac{5}{0} = \infty$

$5t^2u(t)$ ➔ $10/s^3$: $\quad e_{parabola}(\infty) = \dfrac{10}{\lim_{s\to 0} s^2G(s)} = \dfrac{10}{0} = \infty$

Example: SSE for systems with one integrator

- Find the steady-state errors for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$ to the system shown below.



- Solutions

  The closed-loop system is **stable**.

$5u(t)$ ➔ $5/s$: $$e_{step}(\infty) = \frac{5}{1+\lim_{s\to 0} G(s)} = \_\_\_\_\_\_\_\_\_\_$$

$5tu(t)$ ➔ $5/s^2$: $$e_{ramp}(\infty) = \frac{5}{\lim_{s\to 0} sG(s)} = \_\_\_\_\_\_\_\_\_\_$$

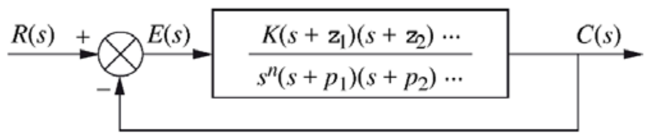
$5t^2u(t)$ ➔ $10/s^3$: $$e_{parabola}(\infty) = \frac{10}{\lim_{s\to 0} s^2G(s)} = \_\_\_\_\_\_\_\_\_\_$$

Static Error Constants

- For unity negative feedback systems, parameters can be defined as **steady-state error** performance specifications, just as we defined damping ratio, natural frequency, settling time, and overshoot as performance specifications for the transient response.
- Steady-state error for different inputs:
  - For a step input, $u(t)$: $$e(\infty) = e_{step}(\infty) = \frac{1}{1+\lim_{s\to 0} G(s)}$$
  - For a ramp input, $tu(t)$: $$e(\infty) = e_{ramp}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)}$$
  - For a parabolic input, $1/2t^2u(t)$: $$e(\infty) = e_{parabola}(\infty) = \frac{1}{\lim_{s\to 0} s^2G(s)}$$
- Static error constants:
  - ________________, K_p: $K_p = \lim_{s\to 0} G(s)$
  - ________________, K_v: $K_v = \lim_{s\to 0} sG(s)$
  - ________________, K_a: $K_a = \lim_{s\to 0} s^2G(s)$

System Type

- Since steady-state errors are dependent upon the number of integrations in the forward path, we define **system type** to be ___ ____________________ ____________________, ***n*** in the forward path.



		Type 0		Type 1		Type 2	
Input	SSE formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, u(t)	$\frac{1}{1+K_p}$	K_p = constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, t	$\frac{1}{K_v}$	$K_v = 0$	∞	K_v = constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $1/2t^2$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	K_a = constant	$\frac{1}{K_a}$

Example: System Type

- A unity feedback system has the following open-loop transfer function:

$$G(s) = \frac{1000(s+8)}{(s+7)(s+9)}$$

Evaluate system type, K_p, K_v, and K_a. Also, find the SSE for the standard step, ramp, and parabolic inputs.

- Solutions

The closed-loop system is **stable**. System type = # of pure integrators = Type 0.

$$K_p = \lim_{s\to 0} G(s) = \underline{\hspace{6cm}}$$

$$K_v = \lim_{s\to 0} sG(s) = \underline{\hspace{6cm}}$$

$$K_a = \lim_{s\to 0} s^2G(s) = \underline{\hspace{6cm}}$$

$$e_{step}(\infty) = \frac{1}{1+K_p} = \frac{1}{1+127} = 7.8\times 10^{-3}, \quad e_{ramp}(\infty) = \infty, \quad e_{parabola}(\infty) = \infty$$