6. Design via Root Locus

Design of cascade compensators using root locus
PI/Lag Compensator
PD/Lead Compensator
PID/Lag-Lead Compensator
Implementation of Controllers
Notch Filter

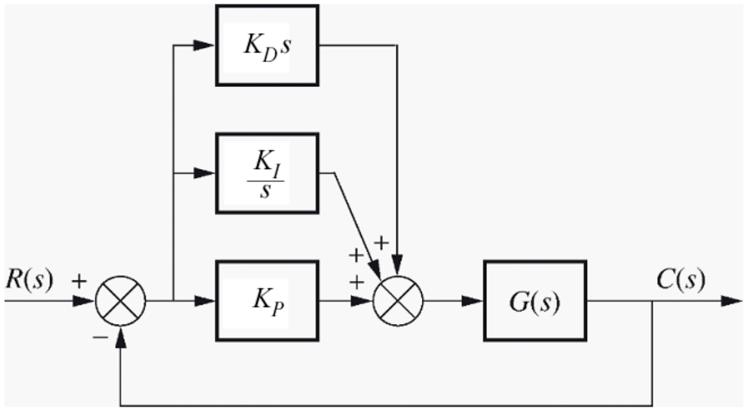
Improving SSE & Transient Response

- We can now combine the various controller types listed earlier to improve both the _______________ and the ________________.
- Basically, we can improve the steady state error by employing PI or lag control. Then we can employ PD or lead control to improve the transient response.
  - Disadvantage: the improvement in transient response yields deterioration in the improvement in steady state error which was designed first.
- We can also improve the transient response by PD or lead control and then improve the steady state error by PI or lag compensation.
  - Disadvantage: slight decrease in the speed of the response when the steady state error is improved.
- In this course, we will design for transient response first, then we will design for steady state error.
- If we design a PD controller followed by a PI controller, the resulting compensator is called a **PID controller**.
- If we design a lead compensator followed by a lag compensator, the resulting compensator is called a **lag-lead compensator**.

PID Controller Design

- It has two zeros and one pole (pole is at the origin). One zero and the pole represents the PI controller. The other zero represents the PD controller.

$$G_c(s) = K_P + K_D s + \frac{K_I}{s} = \frac{K_D\left(\frac{\qquad}{}\right)}{s}$$



PID Controller Design Steps

1. Evaluate the performance of the uncompensated (proportional) system to determine how much improvement in transient response is required. Use the root locus to do this.
2. Design the **PD controller** to meet the transient response specs. It includes the zero location and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Design **PI controller** to yield the required steady-state error.
6. Determine the gains K_D, K_P, and K_I.
7. Simulate system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met.

Example: PID Controller Design (1/7)

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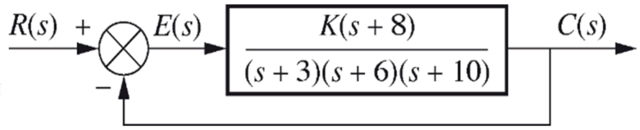
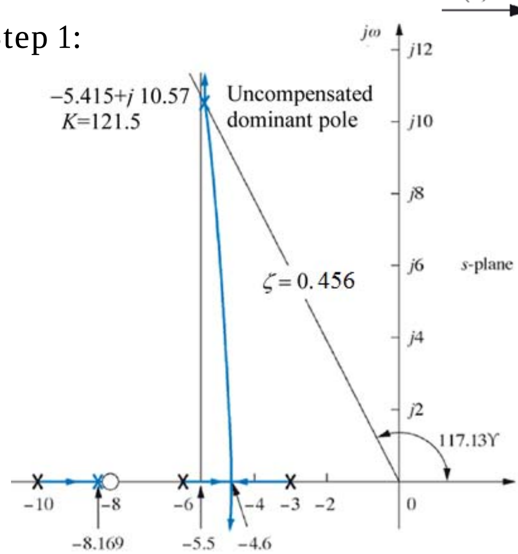
- Given the system, design a PID controller that yields a peak time of two-thirds that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input.



Step 1:



$$M_P = 0.20 \quad \Rightarrow \quad \zeta = 0.456$$

$t_p =$ ____________________

Example: PID Controller Design (2/7)

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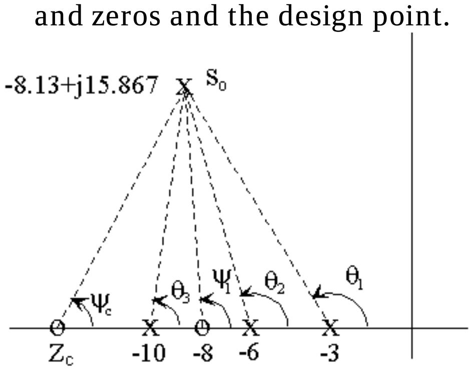
Step 2: Design the PD controller

- Peak time of the new system is 2/3 of the uncompensated system.

$$\omega_d = \frac{\pi}{t_p} = \underline{\qquad\qquad\qquad}$$

- Knowing the imaginary value of $\sigma + j\omega_d$ and the angle $\theta = \sin^{-1}\zeta$, we can calculate σ ________________________.
- Therefore, the design point is $s_o = \sigma + j\omega_d =$ _________________ .
- We next find the angles between the uncompensated system's poles and zeros and the design point.



$$\angle \text{Zeros} - \angle \text{Poles} = (2k+1)180^\circ$$

$$\psi_1 + \psi_c - \theta_1 - \theta_2 - \theta_3 = (2k+1)180^\circ$$

$$\psi_c = -\psi_1 + \theta_1 + \theta_2 + \theta_3 + (2k+1)180^\circ$$

Example: PID Controller Design (3/7)

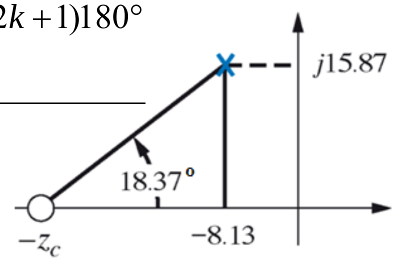
$$\theta_1 = -\tan^{-1}\left(\frac{15.87}{8.13-3}\right) + 180° = -72.09° + 180° = 107.91°$$

$$\theta_2 = -\tan^{-1}\left(\frac{15.87}{8.13-6}\right) + 180° = -82.36° + 180° = 97.64°$$

$$\theta_3 = \tan^{-1}\left(\frac{15.87}{10-8.13}\right) = 83.28°$$

$$\psi_1 = \tan^{-1}\left(\frac{15.87}{-8.13+8}\right) = -89.53° + 180° = 90.47°$$

$$\begin{aligned}\psi_c &= -\psi_1 + \theta_1 + \theta_2 + \theta_3 + (2k+1)180° \\ &= -90.47° + 107.91° + 97.64° + 83.28° + (2k+1)180° \\ &= \underline{\hspace{10cm}}\end{aligned}$$

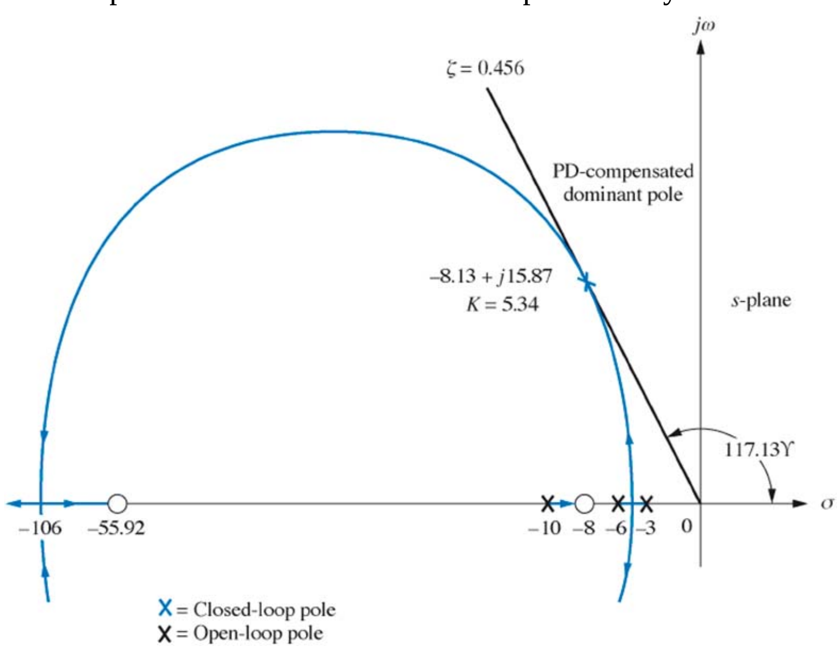


- We can find z_c to be:

____

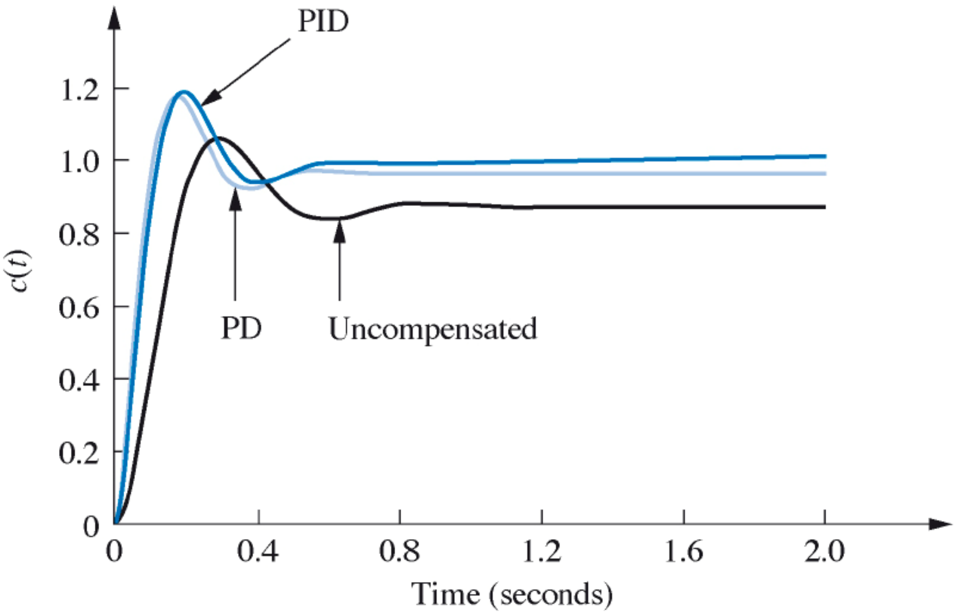
Example: PID Controller Design (4/7)

- Thus the PD controller is: $G_{PD}(s) =$ ___________
- The complete root locus for the PD compensated system:



Example: PID Controller Design (5/7)

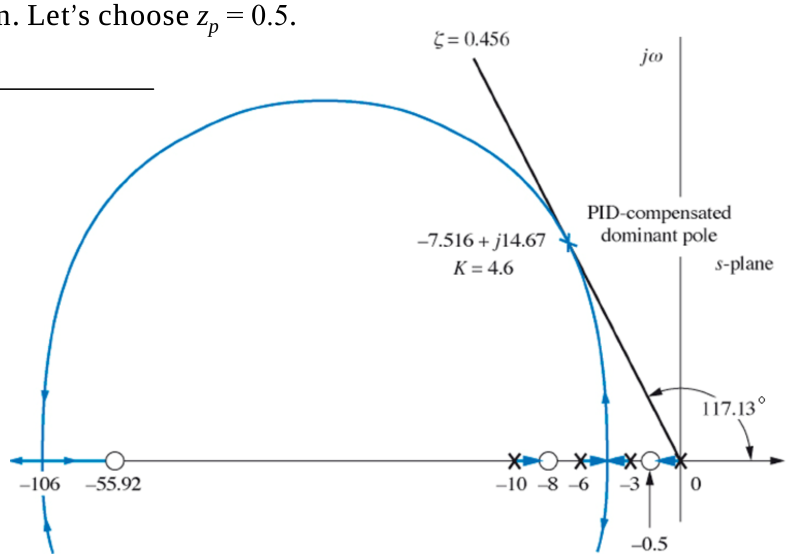
- Steps 3 and 4: Simulate and check the PD system



Example: PID Controller Design (6/7)

- Steps 5: Design the PI controller
  - Any compensator zero will work as long as the zero is placed close to the origin. Let's choose $z_p = 0.5$.

$$G_{PI}(s) = \underline{\hspace{4cm}}$$



  - From the root locus, we calculate K_d to be 4.6 (use the rlocfind(sys) in Matlab)

Example: PID Controller Design (7/7)

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- Steps 6: Determine the gains, K_P, K_I, and K_D.
  - The designed PID controller is

$$G_{PID}(s) = \frac{K_d(s+55.92)(s+0.5)}{s} = \frac{4.6(s+55.92)(s+0.5)}{s}$$

  - By comparing it with a standard PID controller

$$G_{PID}(s) = \frac{K_d(s^2+56.42s+27.96)}{s} = \frac{K_d\left(s^2+\frac{K_p}{K_d}s+\frac{K_I}{K_d}\right)}{s}$$

  - The following controller gains are obtained:

$$K_P = \_\_\_\_\_\_\_\_ \quad , \quad K_I = \_\_\_\_\_\_\_\_ \quad , \quad K_D = 4.6$$

- Step 7 and 8: Simulate and redesign if necessary.

Lag-Lead Compensator Design Steps

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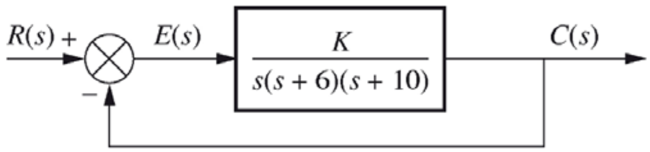
1. Evaluate the performance of the uncompensated (proportional) system to determine how much improvement in transient response is required. Use the root locus to do this.
2. Design the ________________ to meet the transient response specs. It includes the zero location, pole location and the loop gain.
3. Simulate the system to be sure all requirements have been met.
4. Redesign if the simulation shows that requirements have not been met.
5. Evaluate the steady-state error performance for the lead-compensated system to determine how much more improvement in steady-state error is required.
6. Design the ______________ to yield the required steady-state error.
7. Simulate system to be sure all requirements have been met.
8. Redesign if the simulation shows that requirements have not been met.

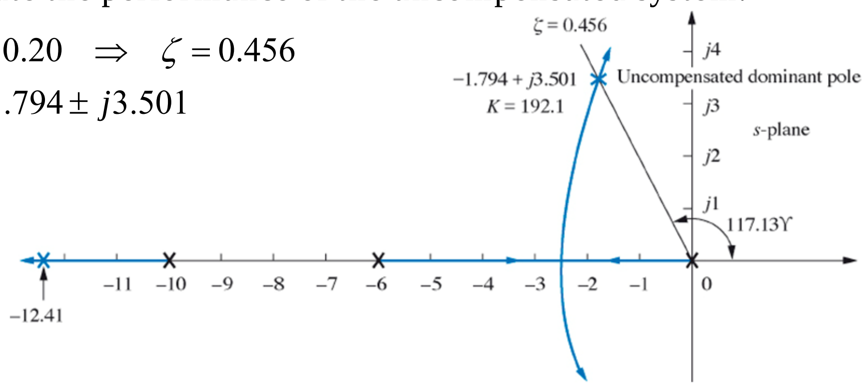
Example: Lag-Lead Compensator (1/6)

- Design a lag-lead compensator for the following system so that the system will operate with 20% overshoot and a twofold reduction in settling time. Further, the compensated system will exhibit a tenfold improvement in steady-state error for a ramp input.



Step 1: Evaluate the performance of the uncompensated system.

$$M_P = 0.20 \quad \Rightarrow \quad \zeta = 0.456$$
$$s = -1.794 \pm j3.501$$



Example: Lag-Lead Compensator (2/6)

Step 2: Design the lead compensator

- Reduction in the settling time by a factor of 2 yields

  $-\zeta\omega_n = \underline{\qquad\qquad\qquad}$

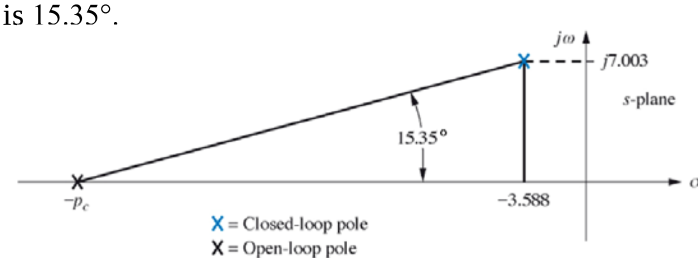
- The imaginary part of the design point is

  $$\omega_d = \zeta\omega_n \tan(117.13°) = 3.588 \tan(117.13°) = 7.003$$

- Arbitrarily select a location for the lead compensator zero. If we choose –6, it will eliminate one of the OL poles and leave the lead-compensated system with three poles.
- Find the location of the compensator pole. By calculation, the sum of all the angular contributions from the OL system poles and zeros and the compensator zero becomes –164.65°. The required angle from the compensator pole is 15.35°.

$\underline{\qquad\qquad\qquad} =$

$\Rightarrow \quad p_c =$



Example: Lag-Lead Compensator (3/6)

- Steps 3 and 4: Check the design with a simulation
- Step 5: Evaluate the steady-state error performance
  - The uncompensated system's OL transfer function

$$G(s) = \frac{192.1}{s(s+6)(s+10)} \quad \Rightarrow \quad K_v = \_\_\_\_\_\_\_\_$$

  - The OL transfer function of the lead-compensated system

$$G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)} \quad \Rightarrow \quad K_v = \_\_\_\_\_\_\_\_$$

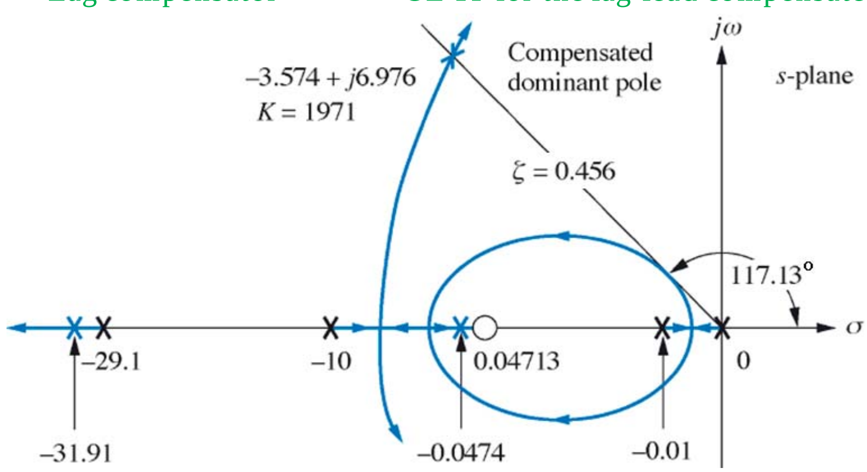
  - Thus, the addition of lead compensation has improved the steady-state error by a factor of 2.122. Since the requirements of the problem specified a tenfold improvement, a lag compensator must be designed to improve the steady-state error by a factor of 4.713 (10/2.122 = 4.713) over the lead-compensated system.

Example: Lag-Lead Compensator (4/6)

- Steps 6: Design the lag compensator
  - Arbitrarily choose the lag compensator pole at 0.01, which then places the lag compensator zero at 0.04713.

$$G_{lag}(s) = \frac{(\qquad\qquad)}{(\qquad\qquad)} \qquad G_{LLC}(s) = \frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$$

Lag compensator — OL TF for the lag-lead compensated system



Example: Lag-Lead Compensator (5/6)

	Uncompensated	Lead-compensated	Lag-lead-compensated
Plant and compensator	$\frac{K}{s(s+6)(s+10)}$	$\frac{K}{s(s+10)(s+29.1)}$	$\frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
% overshoot	20	20	20
t_s	2.230	1.115	1.119
t_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	–12.41	–31.92	–31.91, –0.0474,
Zero	None	None	–0.04713
Comments	2nd order approx OK	2nd order approx OK	2nd order approx OK

Example: Lag-Lead Compensator (6/6)

- Steps 7: Simulate the system to check all requirements