6. Design via Root Locus

Design of cascade compensators using root locus

PI/Lag Compensator

PD/Lead Compensator

PID/Lag-Lead Compensator

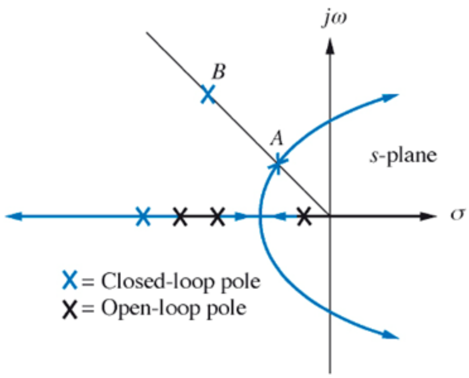
Implementation of Controllers

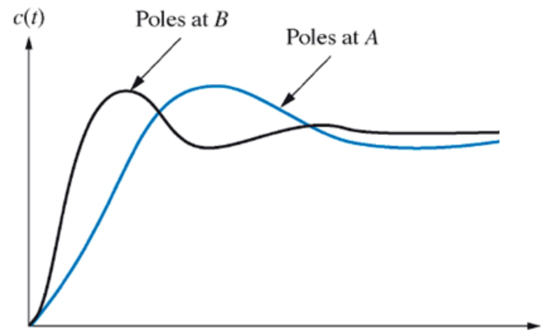
Notch Filter

Introduction

1. *Problem with proportional control*
   1) The root locus typically allows us to choose the proper loop gain to meet a transient response specification. As the gain is varied, we move through different regions of response. Setting the gain to a particular value yields the transient response dictated by the pole locations on the root locus. Thus, we are limited to those responses that ________ ________________________ .
   2) We will get flexibility if we can design for ___________________ that are not on the root locus
2. ***Compensators***
   1) By compensating a system with _____________ poles and zeros, the compensated system has a root locus that goes through the desired pole location for some value of gain. This way, the transient response can be improved.
   2) Compensators are also used _____________ to improve the ______ ____________ characteristics. Thus, by using dynamic compensators, we can meet transient and steady-state error specifications simultaneously.

Improving Transient Response

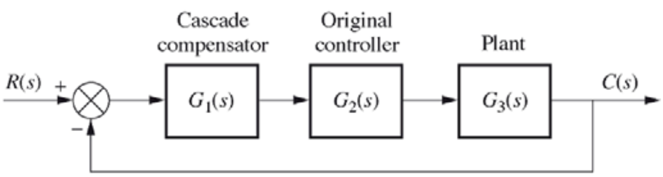




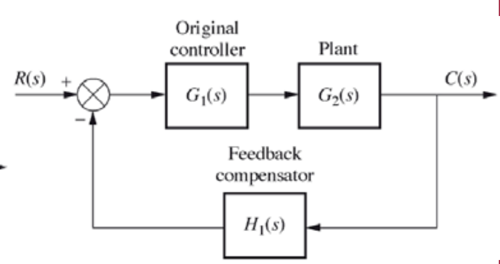
We want to be at *B* for faster response. The overshoot is the same because we are on the ____________ for ζ. If we want a response at *B*, we need to either build a new system with a root locus that goes through *B* or augment or ____________________ with additional poles and zeros so that the compensated system has a root locus that goes through the desired pole location for some value of gain.

Controller Design via Root Locus

- Root locus design techniques can be employed to design various controllers or compensators.
- Cascade compensator design: We can add poles and zeros and vary them to get the desired response.
- This is because anytime you add extra poles and zeros to a system you change the characteristic equation and thus change the system response!
- Two configurations of compensation



________ **compensation**



_______ **compensation**

Improving Steady State Error via Cascade Compensation

- *Two ways to improve the steady-state error*
  1) The objective is to improve the steady-state error without appreciably affecting the transient response.
  2) The first technique is ______________________ **(PI control)**, which uses a pure integrator to place an open-loop, forward-path pole at the origin.
  3) The second technique uses a ____________ that does not employ pure integration. The name of this compensator comes from its frequency response characteristics, where the typical phase angle response is always negative, or ________ in phase angle.
  4) The ideal integral compensator must be implemented with active networks such as op amps.
  5) The lag compensator can be implemented with a less expensive passive network that does not require additional power sources. However, it does not reduce the error to zero perfectly.

Ideal Integral Compensation (PI)

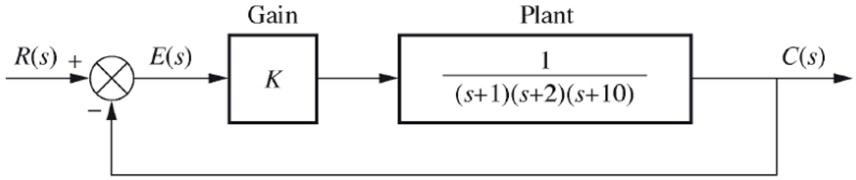
- Steady-state error can be improved by placing an open-loop pole at the origin because this ____________________________ .
- Active circuits can be used to place poles at the origin.
- In order to avoid the situation where the added pole at the origin changes the overall root locus dramatically, __________________ _______ so that the transient response remains the same.
- The general form of the PI compensator will look like

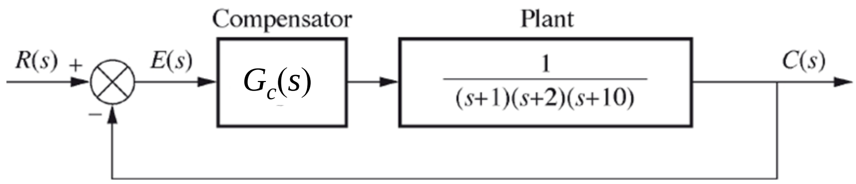
$$G_c(s) = K + \frac{K_I}{s} = \frac{K(s + K_I/K)}{s}$$

- Now the angular contribution of the compensator zero and compensator pole cancel out, and their effect on the root locus is minimized.
- Also, the required gain at the dominant pole is about the same as before compensation since the ratio of lengths from the compensator pole and the compensator zero is approximately unity.

Example: Ideal Integral Compensation

- Add PI control to the following system to improve the steady state error without affecting the transient response. We want a $\zeta = 0.174$.



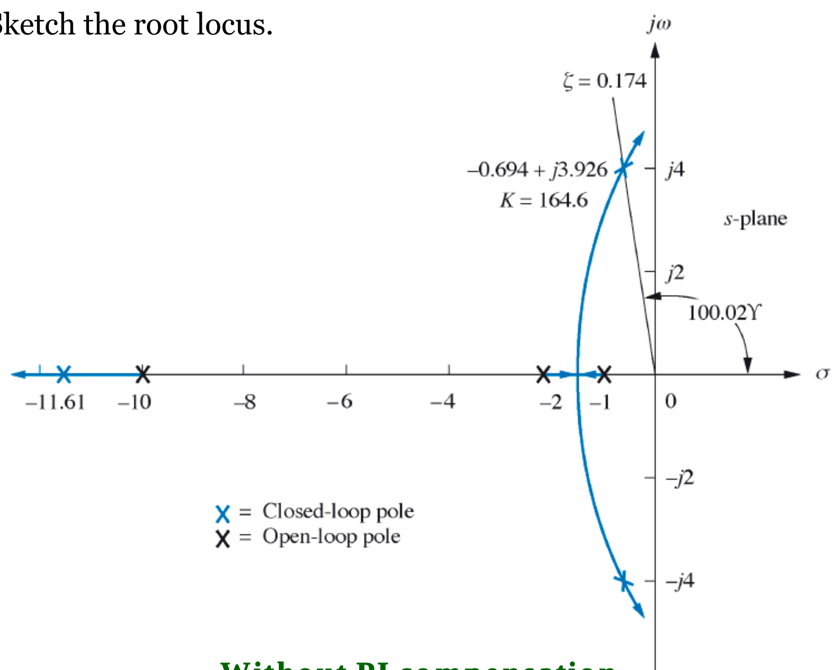


1. Choose values for your poles and zeros. Let the pole be at the origin and set the zero close to it (0.1 is chosen as a good value).

$$G_c(s) = \frac{K(s + K_I/K)}{s}, \quad \text{where} \quad \underline{\qquad\qquad\qquad}$$

Example: Ideal Integral Compensation

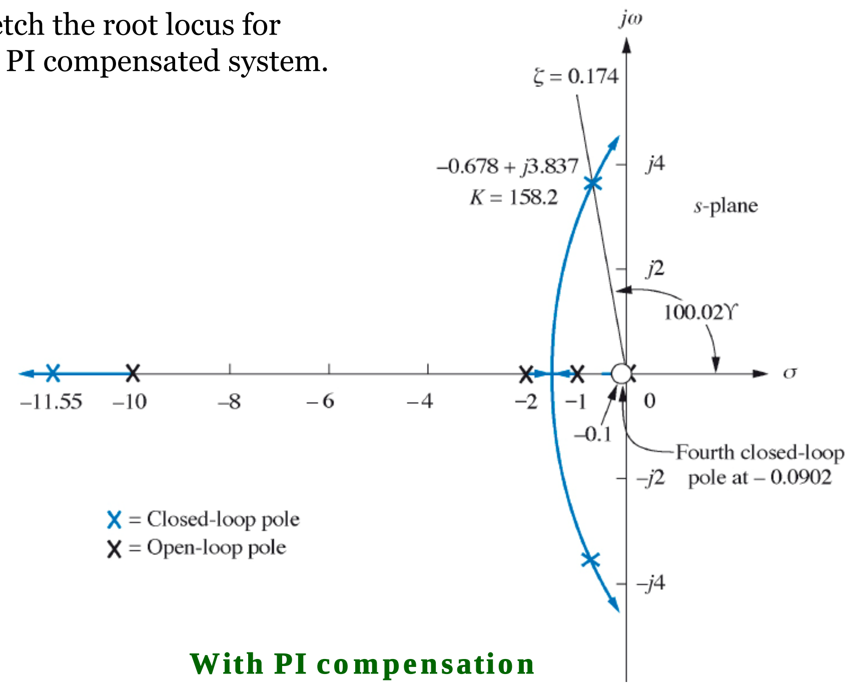
2. Sketch the root locus.



Without PI compensation

Example: Ideal Integral Compensation

2. Sketch the root locus for the PI compensated system.



With PI compensation

Example: Ideal Integral Compensation

3. Assume that the system will behave like a second order system. So one can calculate the values of rise time, settling time, overshoot, etc. For the steady state error to a step input we have:

1) Before PI

$$G(s) = \frac{K}{(s+1)(s+2)(s+10)} \text{, where } K = 164.6$$

$$K_p = \lim_{s\to 0} G(s) = \_\_\_\_\_\_\_\_\_\_ \Rightarrow e_{ss} = \_\_\_\_\_\_\_\_$$

2) After PI

$$G(s) = \frac{K(s+.1)}{s(s+1)(s+2)(s+10)}$$

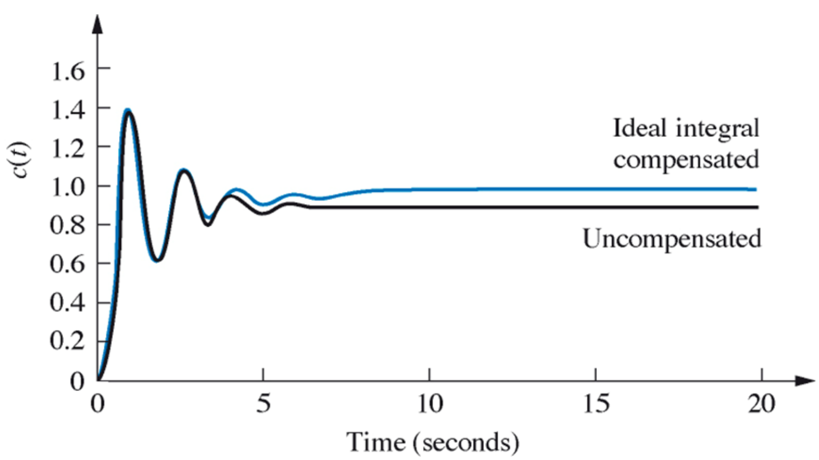
$$K_p = \lim_{s\to 0} G(s) = \_\_\_\_ \Rightarrow e_{ss} = \_\_\_\_\_\_\_\_$$

Example: Ideal Integral Compensation

4. Are the second order assumptions valid?
   1) Since the zero and pole cancel, it won't affect the system greatly
   2) The fourth pole is far away from the system since it starts at 10. So it won't affect the system

5. Simulate the system in Matlab!
   1) Although it looks that the second order assumption is valid, it is still needed to simulate the system and make sure that the design specifications are all met with the compensator.
   2) PI control cannot be implemented without active circuits. This means that op amps must be used. Op amps call for extra power supplies.

Example: Ideal Integral Compensation

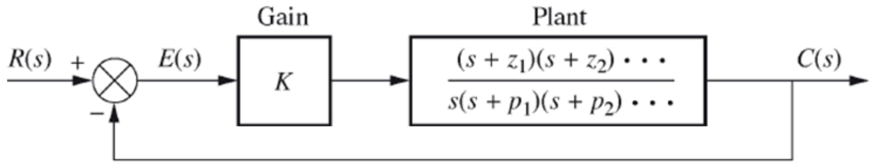


- The step response of the ideal integral compensated system approaches unity in the steady state, while the uncompensated system approaches 0.892.
- The transient response of both the uncompensated and the integral compensated system is the same up to approximately 3 seconds.

Lag Compensation (1/3)

- Lag control approximates PI. It is a cheaper method that can be configured with only ______ components (without op amps).
- Since no pole is added to the origin, this method does not increase the system type. However, it does yield an improvement in steady state error.
- The static error constant, K_{vo} for the uncompensated system shown below is

$$K_{v_O} = \lim_{s \to 0} sG(s) = \underline{\hspace{3cm}}$$

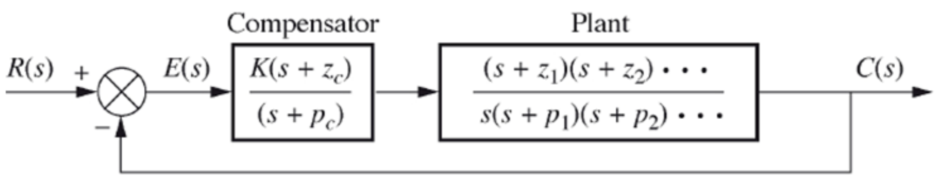


- The pole and zero of a lag compensator are placed **close to the origin** such that $G_c(s) = \dfrac{K(s+z_c)}{s+p_c}$, where $z_c > p_c$.

Lag Compensation (2/3)

- Assuming the lag compensator, the new static error constant is

$$K_{v_N} = \frac{(Kz_1z_2 \ldots)(z_c)}{(p_1p_1 \ldots)(p_c)}$$



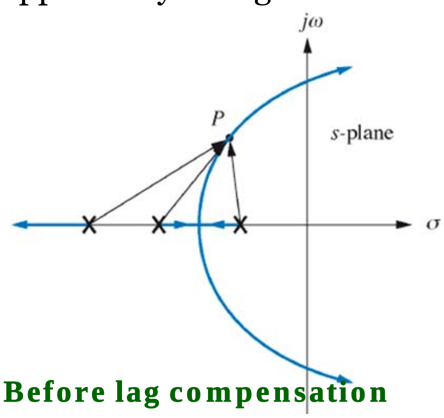
- By comparing the static error constants before and after the lag compensator, we get

$$K_{v_N} = K_{v_O}(\quad) \Rightarrow \frac{K_{v_N}}{K_{v_O}} = \qquad, \quad \text{where} \quad K_{v_N} > K_{v_O}$$

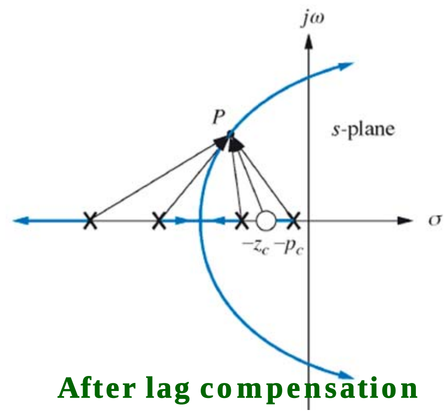
- Thus, the improvement in the compensated system is equal to the ratio of the magnitude of the compensator zero to the compensator pole.

Lag Compensation (3/3)

- If the lag compensator pole and zero are close together, the angular contribution of the compensator to point P is approximately zero degrees. Thus, point P is still at approximately ____________________ on the compensated root locus.
- Since the lengths of the vectors drawn from the lag compensators are approximately equal, the gain K is also not appreciably changed.



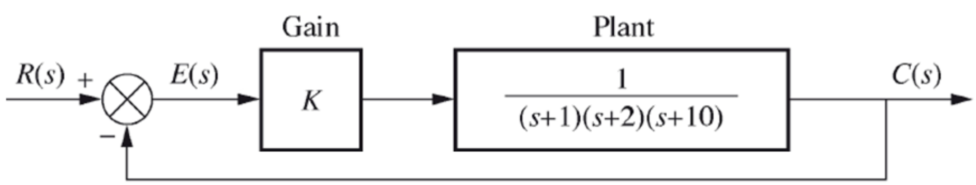
Before lag compensation



After lag compensation

Example: Lag Compensator (1/5)

- Improve the steady state error of the following system by a factor of 10 if the system is operating with a damping ratio of 0.174.



- The SSE of the proportional (uncompensated) system was calculated to be

$$e(\infty) = \frac{1}{1+K_{p_O}} = \frac{1}{1+8.23} = 0.108$$

1. Calculate the new steady-state error (if it is not given) then use that info to choose the values for the pole and zero. To get this we must get the ratio of the static error constant.

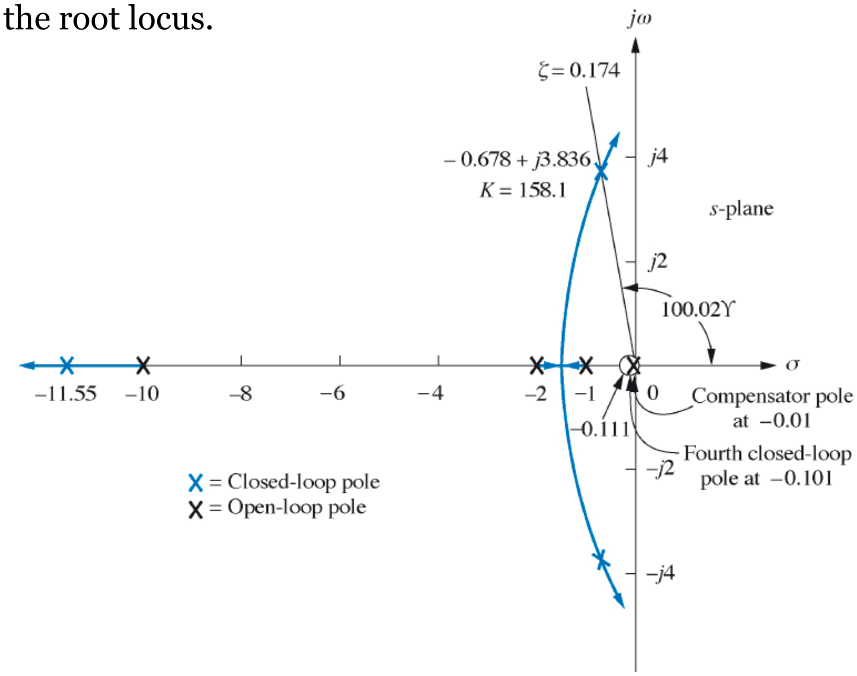
$$e_{lag}(\infty) = \frac{0.108}{10} = 0.0108 = \_\_\_\_\_\_ \Rightarrow \frac{K_{p_N}}{K_{p_O}} = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\Rightarrow K_{p_N} = \_\_\_\_\_\_$$

Example: Lag Compensator (2/5)

Arbitrarily selecting $p_c = 0.01$ results in $z_c = p_c\,(11.13) = 0.111$.

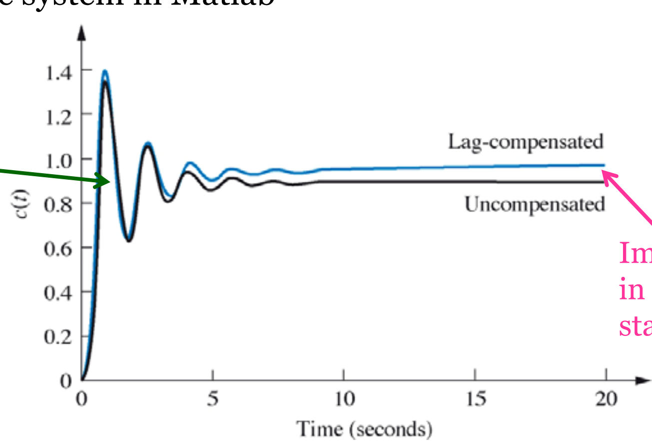
2. Sketch the root locus.



Example: Lag Compensator (3/5)

3. Assume the system will behave like a second order system so one can calculate the values of rise time, settling time, overshoot, etc.
4. Are our assumptions valid?
   - The third pole is far from the dominant system poles, so the 2nd order approximations are valid
   - The fourth pole from the compensator cancels its zero.
5. Simulate the system in Matlab

Transient responses are the same



Improvement in the steady-state response

Example: Lag Compensator (4/5)

6. Comparison of the two systems

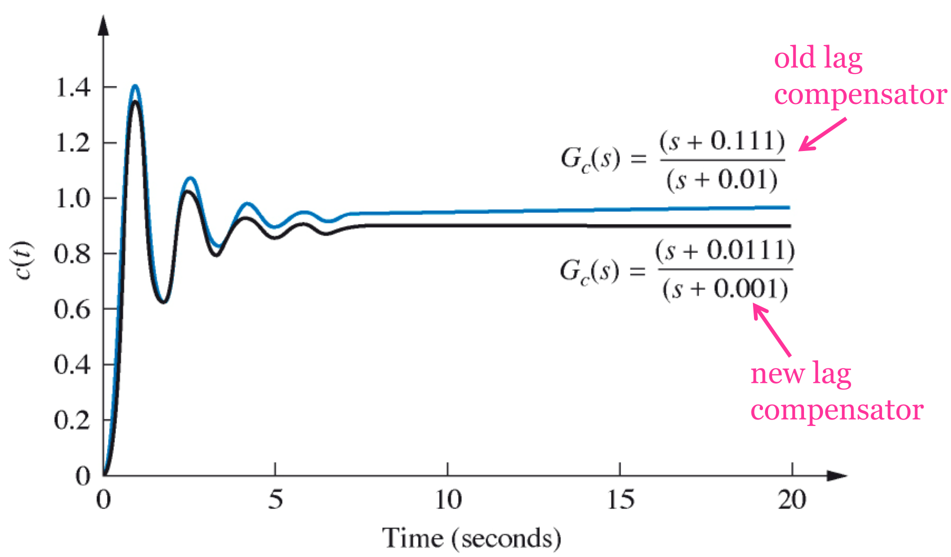
Parameter	Uncompensated	Lag-compensated
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+10)}$	$\frac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
Kp	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant 2nd order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111

7. Now, what if we choose the pole, p_c 10 times closer to the origin? (p_c = 0.001)

$$\frac{z_c}{p_c} = \underline{\qquad\qquad} \Rightarrow z_c = \underline{\qquad\qquad\quad}$$

Example: Lag Compensator (5/5)

6. The new lag compensator has a CL-pole closer to the imaginary axis than the original lag compensator. This pole at -0.01 will produce a longer transient response than the original pole at -0.101, and the steady-state value will not be reached as quickly.



Final Comments on PI/Lag Control

1. In lag and PI control we want to decrease the steady state error at low frequencies (raise the K_v or other error const) while causing very little change to the transient response.
2. In lag control it is important to place the pole-zero combination at as **high** of a frequency possible without influencing the dominant root locations.
3. This is accomplished by guessing the initial value of the pole and then using the needed increase in _____________ to calculate the zero.
4. There may be occasions when we want to use PI or lag control to change the transient response and reduce steady state error. **In that case one can relax the mandate to place the poles and zeroes close to the origin.**