6. Design via Root Locus

Design of cascade compensators using root locus

PI/Lag Compensator

PD/Lead Compensator

PID/Lag-Lead Compensator

Implementation of Controllers

Notch Filter

Improving Transient Response

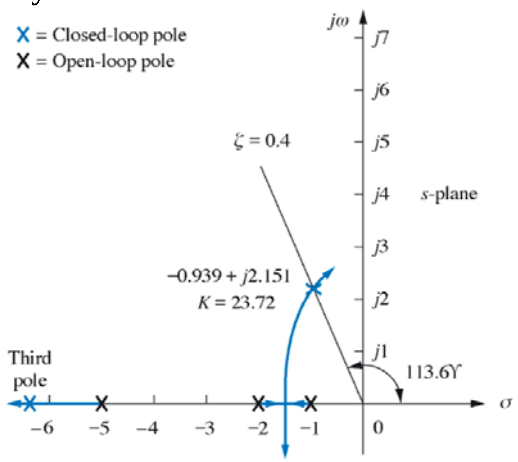
- One method used to improve the transient response is **PD** control/compensation. It is also called _______________ **compensation**.
- The result of adding differentiation is the addition of a zero to the forward-path transfer function.
- Typically, the objective is to design a response that has a desirable overshoot and a shorter settling time than the uncompensated system.
- One major problem with PD compensation is that PD control amplifies high frequency noise.
- **Note:** The transient response of a system can be selected by choosing an appropriate closed-loop pole location (design point) on the s-plane. If this **point is on the root locus**, then all that is required is a simple gain adjustment. If the **pole is not on the root locus**, then __________________ are added to produce a new open loop function whose root locus goes through the design point.

Ideal Derivative Compensation (PD)

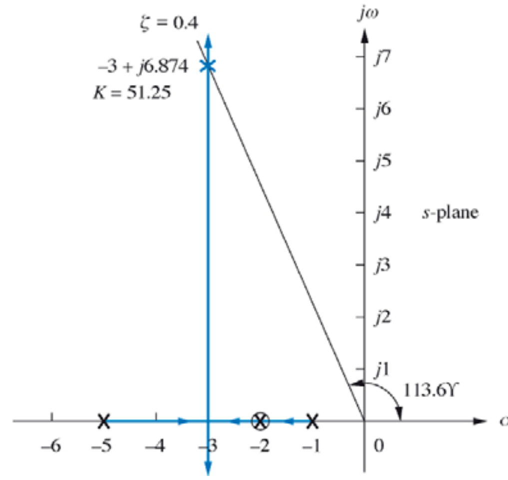
- In PD control, ___________ is added to the forward path such that the transfer function of the compensator becomes

$$G_c(s) = s + z_c$$

- Example: Ideal derivative compensation speeds up the response of a system.

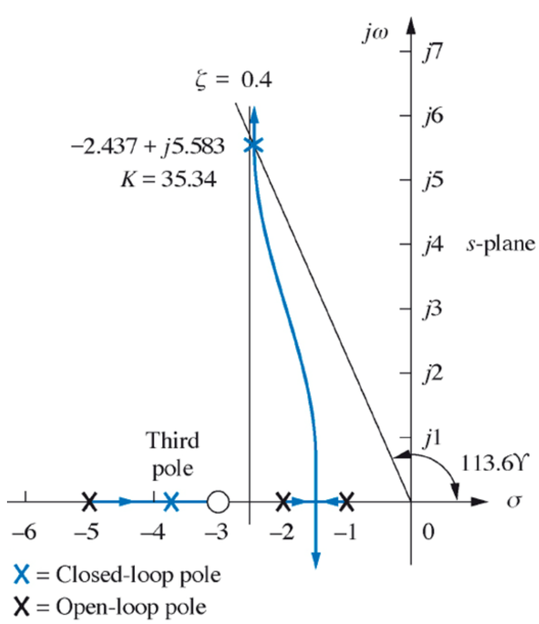


Uncompensated

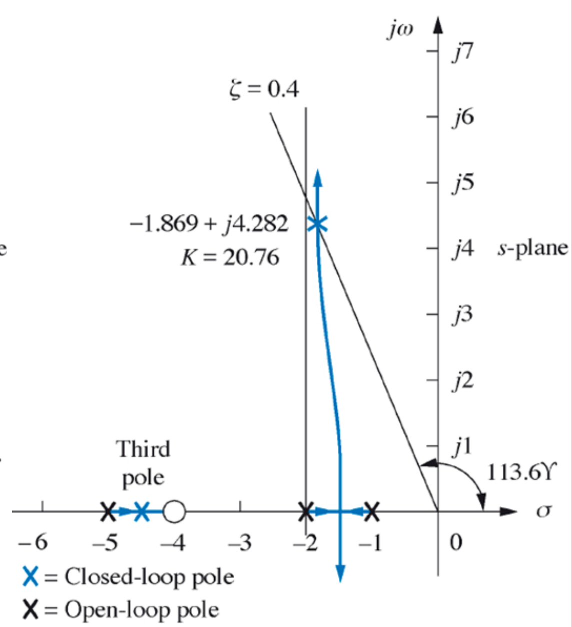


Zero at -2

Ideal Derivative Compensation (PD)



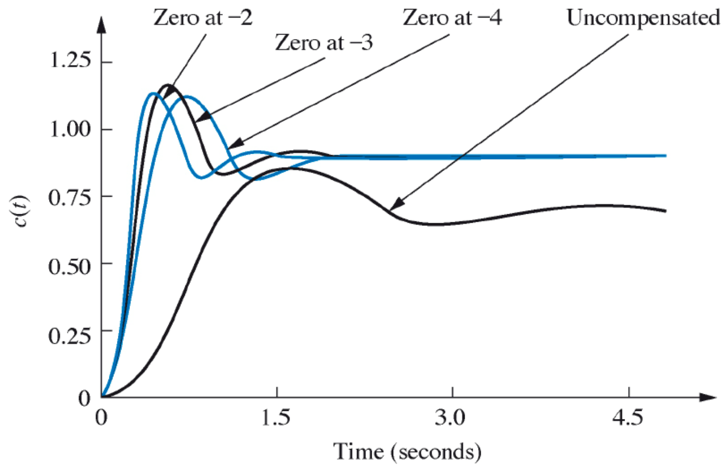
Zero at _____



Zero at _____

Ideal Derivative Compensation (PD)

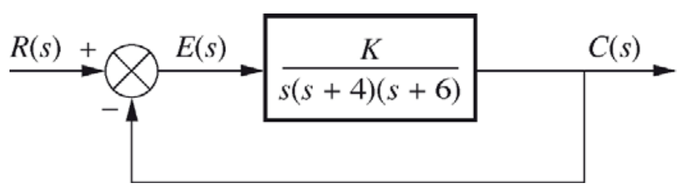
- The time response of the four cases:



- All compensated cases have dominant poles with the ______ _____________ , so the overshoots are almost the same.
- Dominant closed-loop poles with the more negative real parts have the shorter settling times.

Example: PD Compensation (1/7)

- Given the system, design an ideal derivative compensator to yield a 16% overshoot, with a threefold reduction in settling time.



1. Transients
   1) Plot the root locus of the uncompensated system

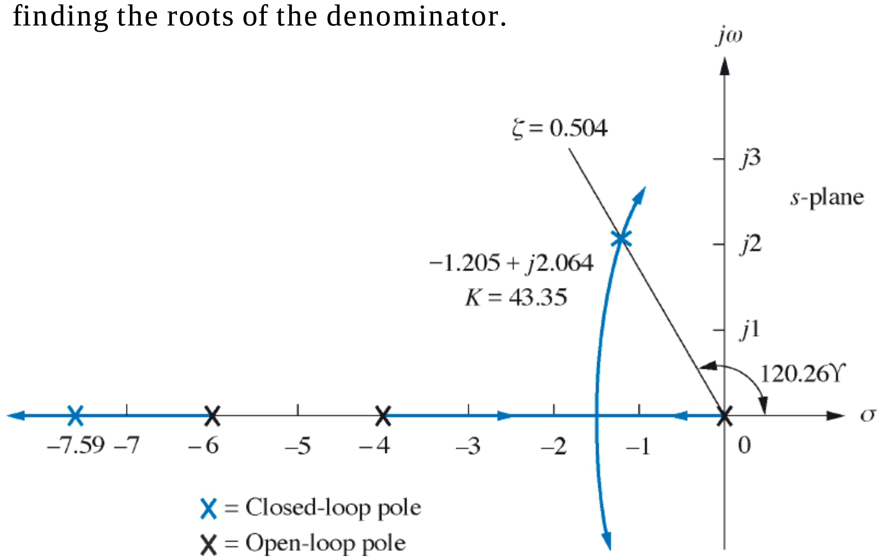
      $M_p = 0.16 \rightarrow \zeta = 0.504$

   2) Assume 2nd order response

      $$t_s = \frac{4}{\zeta\omega_n} = \underline{\hspace{6cm}}$$

Example: PD Compensation (2/7)

3) Assumptions is justified because at $K = 43.35$, the third pole is at 7.59 (6 times farther from the $j\omega$ axis than the dominant poles). This can be found by Matlab or getting the closed loop TF and plugging in K, then finding the roots of the denominator.



Example: PD Compensation (3/7)

2. Calculate the value of the design point (or desired pole).
   - This is the point that we want our new root locus to go through.
   - The real part of the pole can be found from the settling time given in the problem statement.

$$t_s = 1.107 \quad \Rightarrow \quad \sigma = \rule{8cm}{0.4pt}$$

   - In general, $\sigma \pm j\omega_d$ is the pole. Since we know σ and we know the angle θ from the $\zeta = 0.504$ line,

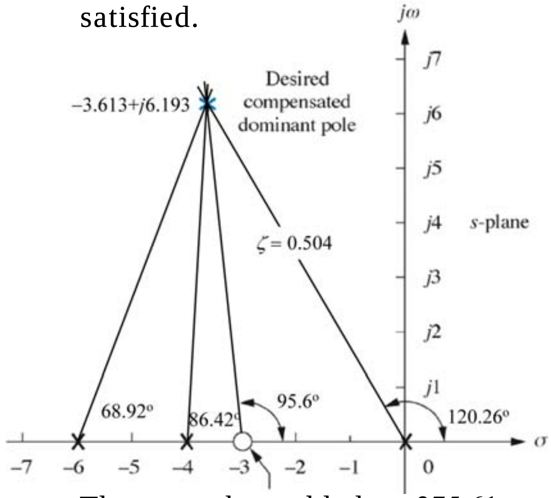
$$\tan\theta = \frac{\sigma}{\omega_d} \quad \Rightarrow \quad \omega_d = \rule{8cm}{0.4pt}$$

   - So the NEW desired pole is ________________

3. Calculate the value of K_P/K_D, the zero in the $K_D\left(s + {K_P}/{K_D}\right)$.
   - We want to add a zero to our system to force the Root Locus through this NEW design point.

Example: PD Compensation (4/7)

- To get the actual value of the zero or (K_P/K_D) we use the fact that for the root locus to go through a point, the angle criterion must be satisfied.



$$\theta_1 = 120.265°$$

$$\theta_2 = \tan^{-1}\left(\frac{6.192}{4-3.613}\right) = 86.42°$$

$$\theta_3 = \tan^{-1}\left(\frac{6.192}{6-3.613}\right) = 68.92°$$

$$\angle \text{Poles} = 120.26 + 68.917 + 86.423 = 275.61°$$

$$\angle KG(s) = \angle \text{Zeros} - \angle \text{Poles} = (2k+1)180°$$

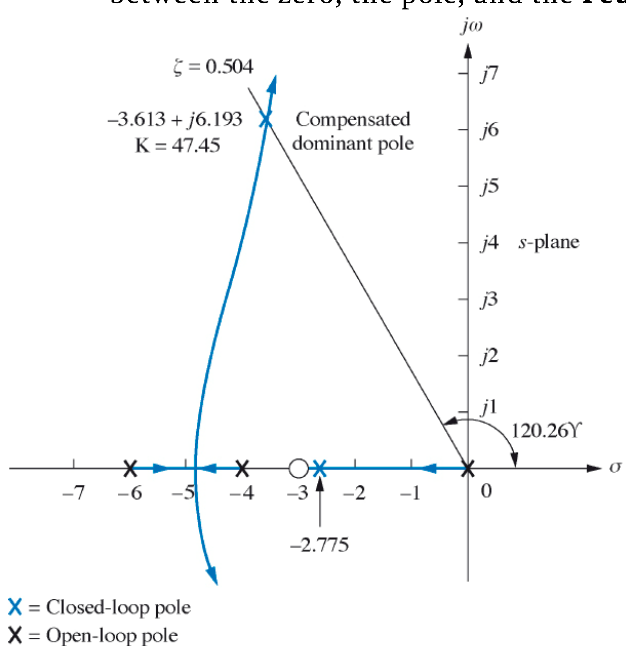
- The zero when added to -275.61° must give some odd multiple of ± 180°

$$\angle \text{Zeros} - 275.61° = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\angle \text{Zeros} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Example: PD Compensation (5/7)

- To find the exact value of the zero, use the small right triangle formed between the zero, the pole, and the **real part (s)** of desired point.

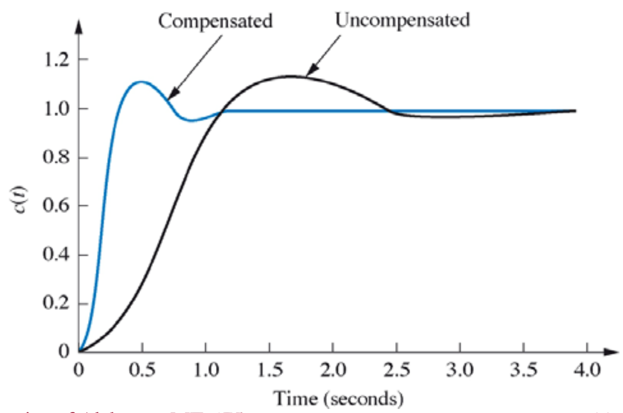


$$\tan(180° - 95.6°) = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\sigma = \_\_\_\_\_\_\_\_\_\_\_\_$$

Example: PD Compensation (6/7)

4. Assuming a second-order system behavior, get transient quantities ζ, rise time, settling time, and the location of the third pole.
   - For uncompensated system: the second-order approximation is accurate since the third pole is at least five times the real part of the dominant, second-order pair.
   - For compensated system: the second-order approximation may be invalid because the CL third pole is at -2.775 ➔ Simulation required!
5. Simulation - the system meets the basic requirements!
   - The gain K is calculated by $K = \frac{1}{|G(s_o)|}$
   - G(s) includes plant and controller



Example: PD Compensation (7/7)

	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$		$\frac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j\,2.064$		$-3.613 \pm j\,6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
% overshoot	16	14.8	16	11.8
t_s	3.320	3.6	1.107	1.2
t_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	-7.591		-2.775	
Zero	None		-3.006	
Comments	2nd order approx OK		Pole-zero not canceling	

Lead Compensation

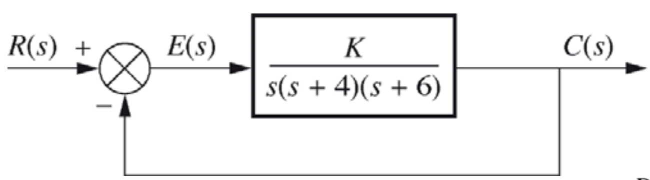
- A passive lead compensator approximates an active ideal derivative compensator (PD controller).

  $G_c(s) =$ __________ , where __________

- Advantages: (1) Passive network → no additional power supplies required. (2) noise due to differentiation is reduced.
- Disadvantage: the additional pole does not reduce the number of branches of the root locus that cross the imaginary axis into the RHP.
- To design a lead compensator, select either a compensator pole or zero arbitrarily (selection of zero first is recommended). Then use the angle criterion to find the remaining zero or pole.
- There exist more than one possible solution. The differences are in the values of static error constants, the gain required to reach the design point, the difficulty in justifying a second-order approximation and the transient response.

Example: Lead Compensation (1/6)

- Given the system, design three lead compensators that will reduce the settling time by a factor of 2 while maintaining 30% overshoot.

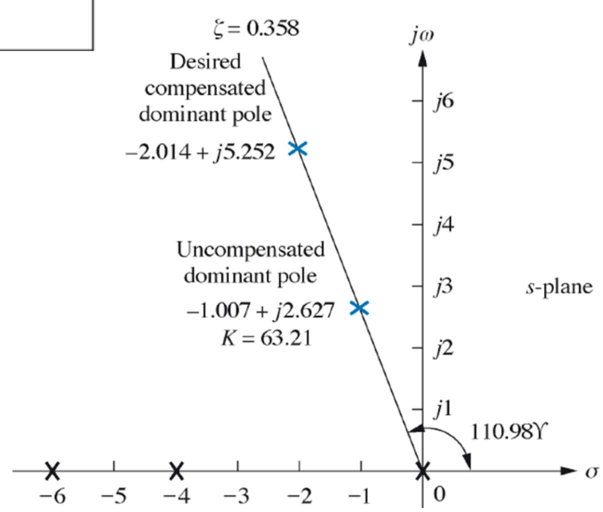


1. Transient response
   1) Plot the root locus of the uncompensated system

      $M_p = 0.30$ → __________

   2) Assume 2nd order response

      $t_s =$ ____________________



Example: Lead Compensation (2/6)

2. Find the design point

- For a two fold reduction in settling time:

$$t_s = \frac{3.972}{2} = 1.986 \quad \Rightarrow \quad \sigma = \frac{4}{t_s} = \frac{4}{1.986} = 2.014$$

- The imaginary part:

$$\theta = \sin^{-1}(.358) = 20.98° \quad \Rightarrow \quad \tan\theta = \frac{\sigma}{\omega_d} = \underline{\hspace{4cm}}$$

$$\Rightarrow \quad \omega_d = \underline{\hspace{2.5cm}}$$

- The design point:

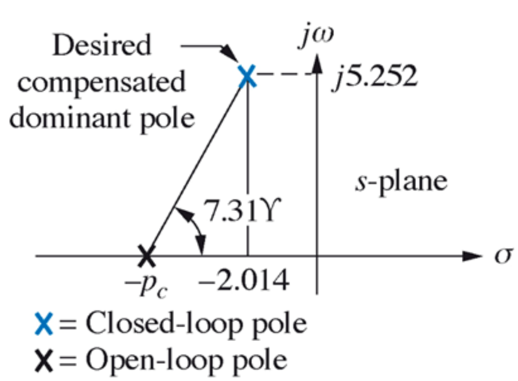
$$s = \sigma + j\omega_d = \underline{\hspace{2.5cm}}$$

3. Arbitrarily choose the value of the zero

- Arbitrarily choose: $z_c = -5$

Example: Lead Compensation (3/6)

4. Use Angle criterion to calculate the pole



$$\psi_c - \theta_1 - \theta_2 - \theta_3 - \theta_c = (2k+1)180°$$

$$\psi_c = \tan^{-1}\left(\frac{5.253}{5-2.014}\right) = 60.38°$$

$$\theta_1 = \tan^{-1}\left(\frac{5.253}{4-2.014}\right) = 69.29°$$

$$\theta_2 = \tan^{-1}\left(\frac{5.253}{6-2.014}\right) = 52.81°$$

$$\theta_3 = 110.98°$$

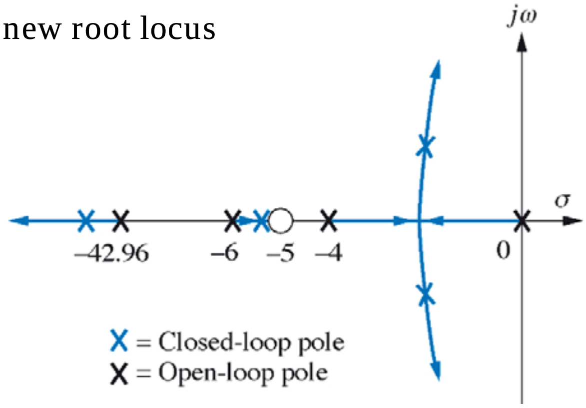
$$(2k+1)180° = \underline{\hspace{2.5cm}} \quad \Rightarrow \quad \theta_c = \underline{\hspace{5cm}}$$

$$\tan(7.31°) = \underline{\hspace{2.5cm}} \quad \Rightarrow \quad p_c = \underline{\hspace{2.5cm}} \qquad G_c(s) = \underline{\hspace{2.5cm}}$$

Lead compensator

Example: Lead Compensation (4/6)

5. Sketch the new root locus



6. Calculate the gain K

$$K = \frac{1}{|G(s_o)|}\text{, where } s_o \text{ is the design point.}$$

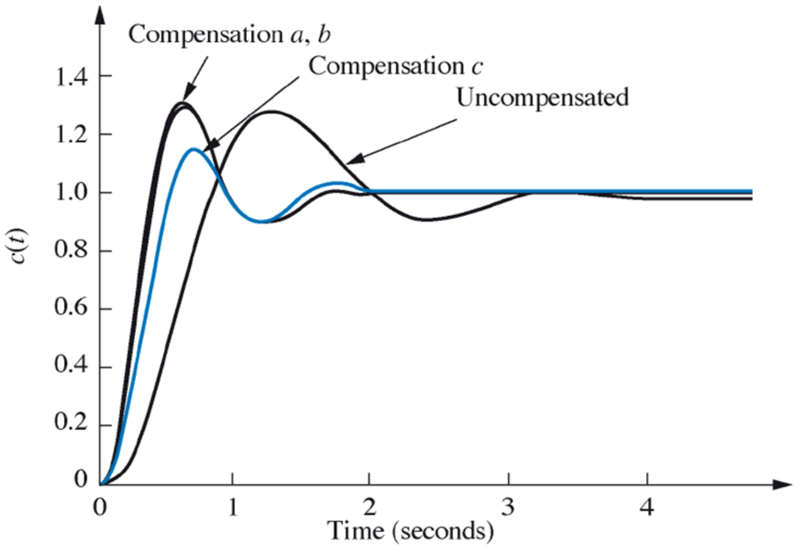
7. Verify the second-order approximation
   - Higher order poles are much farther to the left of the dominant poles.
   - Closed loop poles and zeros cancel, so second order approximation is valid.

Example: Lead Compensation (5/6)

	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
% overshoot	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
t_s	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
t_p	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.90	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.3112
Third pole	–7.986	–43.8, –5.134	–22.06	–13.3, –1.642
Zero	None	–5	None	–2
Comments	2nd order approx OK	2nd order approx OK	2nd order approx OK	No pole-zero cancellation

Example: Lead Compensation (6/6)

8. Notice differences in the following:
   1) The position of the arbitrarily selected zero
   2) The amount of required gain, K.
   3) The position of the third and fourth poles and their relative effect upon the second-order approximation.



Lead Compensation – Less Analytical Method

1. Place the zero in the vicinity of the closed loop natural frequency (ω_n). This is determined by settling time or rise time requirements.
2. Place the pole at a distance between 5 (____________________) and 20 times (______________________) the value of the zero location.
3. Then use steps 5, 6 and 7 for the analytical method.
4. Note: this method will NOT force the root locus to go through a specific point, but it can place the dominant poles in a general region where the response is acceptable.
5. Use trial and error to improve results if necessary