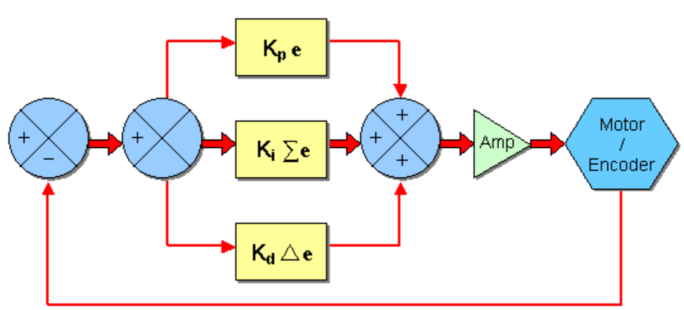
4. Feedback Control and PID Control

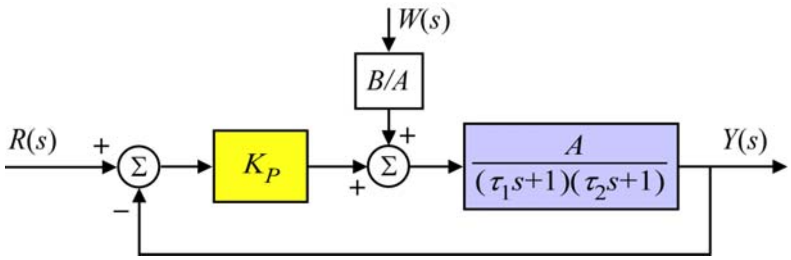
Properties of feedback control systems
PID control
System types

Classical Three Term Control (PID-Control)



Proportional Control (P-Control)

1. *P-Control: output of the controller is* ______________ ***to the error*** *between the reference input, r and the measured output, y.*
2. *The block diagram for the system with P-controller is*



3. *The controller output is* $u=\boldsymbol{K_P}\,e$, *where* $\boldsymbol{K_P}$ *is the proportional* _____.
4. *The transfer function from the reference input, r to the system output, y is*

$$\frac{Y}{R}(s)=\frac{\dfrac{K_P A}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\dfrac{K_P A}{(\tau_1 s+1)(\tau_2 s+1)}}=\frac{K_P A}{(\tau_1 s+1)(\tau_2 s+1)+K_P A}$$

Proportional Control (cont.)

5. *The transfer function from the disturbance, w to the system output, y is*

$$\frac{Y}{W}(s)=\frac{\dfrac{A}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\dfrac{K_P A}{(\tau_1 s+1)(\tau_2 s+1)}}\cdot\frac{B}{A}=\frac{B}{(\tau_1 s+1)(\tau_2 s+1)+K_P A}$$

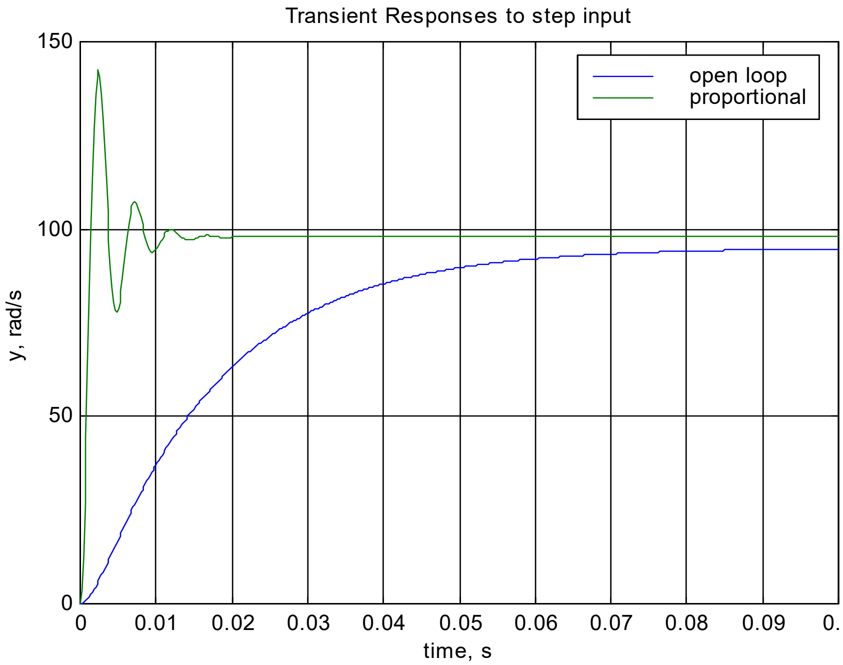
6. *Example problem:* Let K_P=5 with $w=-.1$ ($W(s)=-.1/s$), $r=100$ ($R(s)=100/s$), $\tau_1=1/60$, $\tau_2=1/600$, and $A=10$.

$$Y(s)=\frac{5000}{s\big((\tau_1 s+1)(\tau_2 s+1)+50\big)}+\frac{-5}{s\big((\tau_1 s+1)(\tau_2 s+1)+50\big)}$$

$$=\frac{4995}{s\big((\tau_1 s+1)(\tau_2 s+1)+50\big)}$$

$$y_{ss}=\lim_{s\to 0} sY(s)= \underline{\qquad\qquad\qquad}$$

Proportional Control (cont.)



Comments on Proportional Control

1. *Proportional control __________ the rise time of the system (As the gain K_P is increased).*
2. *As a result, the transient response is ___________. ➔ overshoot is increased.*
3. *Notice that there is still some _____________________ in the response.*
4. *The steady state error with respect to a step input can be eliminated with the addition of ____________________to a proportional controlled system.*

Integral Control

1. *Integral control: output of the controller is equal to the* ________ ***of the error*** *between the reference input, r and the measured output, y.*
2. *The controller output is* $u=K_I\int e\,dt$*, where* K_I *is the integral gain.* K_I *can also be written as* $K_I = \frac{K_P}{T_I}$*, where* T_I *is the integral or reset time. The Laplace transform of the integral controller is*

$$U(s) = \rule{4cm}{0.4pt}$$

3. *Integral control by itself tends to make systems* __________*, thus we use it with a proportional controller* ➔ ***Proportional Integral (PI) Controller***.

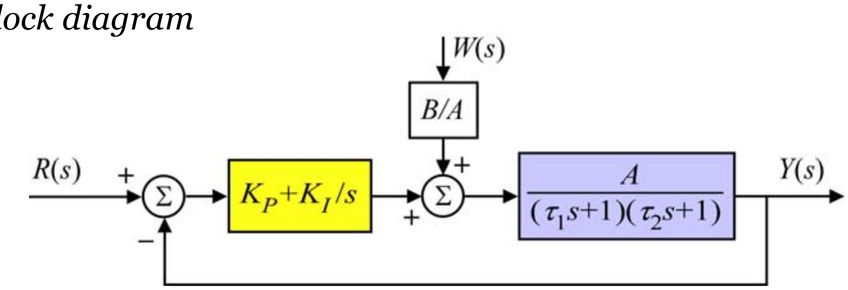
$$u = K_P e + K_I \int e\,dt = K_P e + \frac{K_P}{T_I}\int e\,dt$$

4. *The Laplace transform of the PI controller is*

$$U(s) = \left(K_P + \frac{K_I}{s}\right)E(s) = K_P\left(1 + \frac{1}{T_I s}\right)E(s)$$

Proportional Integral (PI) Control

1. *Block diagram*



2. *Transfer function between the system output, y and the reference input, r is*

$$\frac{Y}{R}(s) = \frac{\dfrac{\left(K_P + K_I/s\right)A}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\dfrac{\left(K_P + K_I/s\right)A}{(\tau_1 s+1)(\tau_2 s+1)}} = \frac{\left(K_P + K_I/s\right)A}{(\tau_1 s+1)(\tau_2 s+1) + \left(K_P + K_I/s\right)A}\cdot\frac{s}{s}$$

PI Control (cont.)

3. *Transfer function (cont'd)*

$$\frac{Y}{R}(s)=\frac{(K_P s+K_I)A}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}$$

4. *The transfer function between the system output, y and the disturbance, w is*

$$\frac{Y}{W}(s)=\frac{\dfrac{A}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\dfrac{\left(K_P+{K_I}/{s}\right)A}{(\tau_1 s+1)(\tau_2 s+1)}}\cdot\frac{B}{A}=\frac{B}{(\tau_1 s+1)(\tau_2 s+1)+\left(K_P+{K_I}/{s}\right)A}\cdot\frac{s}{s}$$

$$\frac{Y}{W}(s)=\frac{Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A}$$

PI Control (cont.)

5. *Example problem:* $K_P = 5$, $K_I = 5/.1 = 500$, $w = -.1$, $r = 100$.

$$Y(s)=\frac{(AK_P r+Bw)s+K_I Ar}{s\left(s(\tau_1 s+1)(\tau_2 s+1)+(K_P s+K_I)A\right)}$$

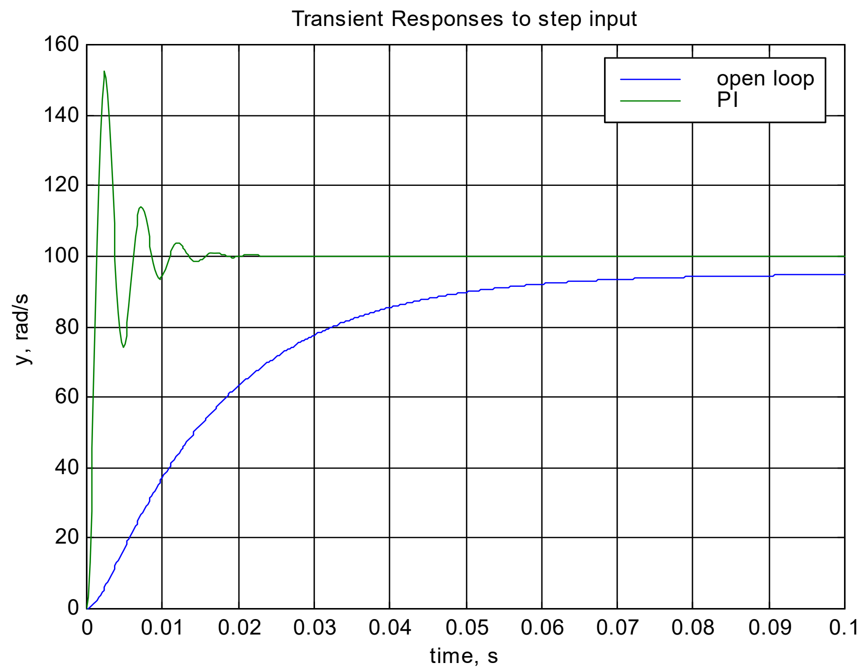
$$y_{ss}=\lim_{s\to 0} sY(s)=\frac{K_I Ar}{K_I A}=r$$

- Therefore, the additional integral control removes the effect of the disturbance.

6. *Comments on PI control*

1) The PI controller __________ the steady state error (for step inputs).
2) However, it adds more instability to system.
3) Adding ______________ to a PI controller fixes the instability problem. → PID Control

PI Control (cont.)



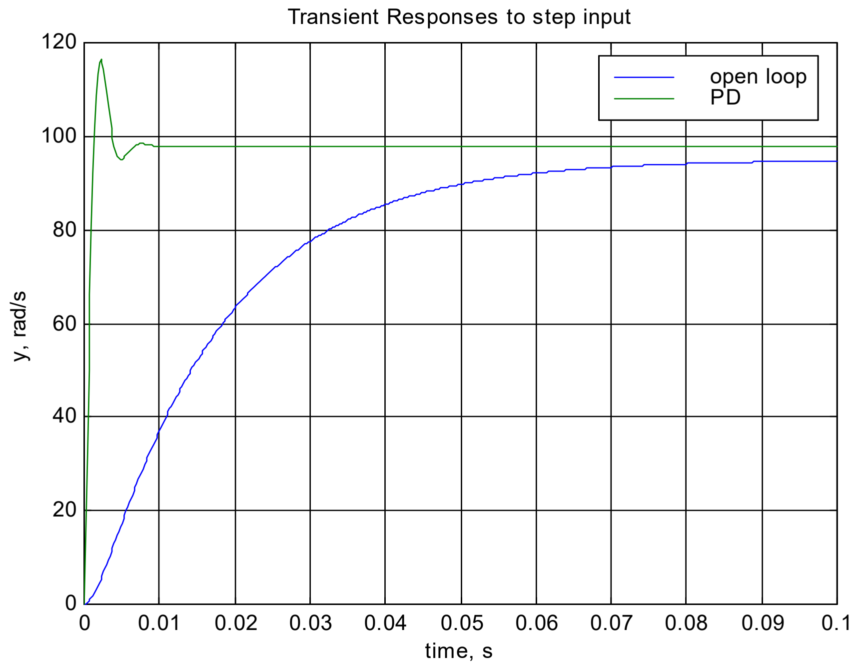
Proportional Derivative (PD) Control

1. *Derivative control: output of the controller is equal to the derivative of the error between the reference input and the system output.*
2. *The controller output is* $u = K_D \frac{de}{dt}$, *where* $\boldsymbol{K_D}$ *is the derivative gain.* $\boldsymbol{K_D}$ *can also be written as* $K_D = K_P T_D$, *where* T_D *is the derivative time. The Laplace transform of the derivative controller is* $U(s) =$ ________________________

- We use derivative control to add stability to P/PI controllers.

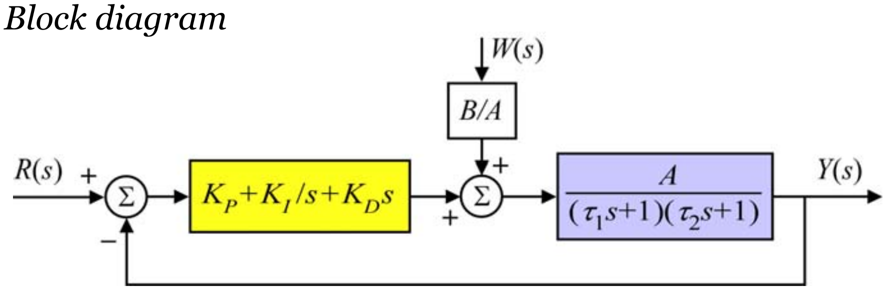
3. *Proportional Derivative (PD) control:*
   1) It adds stability by _______________________.
   2) However, it does not remove ______________________.
   3) Since its output is dependent on the current rate of change of the error, PD control shows ____________________.

PD Control (cont.)



PID Control

1. *Block diagram*



2. *Transfer function between the system output, y and the reference input, r is*

$$\frac{Y}{R}(s)=\frac{\left(K_D s+K_P+K_I/s\right)A}{(\tau_1 s+1)(\tau_1 s+1)+\left(K_D s+K_P+K_I/s\right)A}$$

$$=\frac{(K_D s^2+K_P s+K_I)A}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}$$

PID Control (cont.)

3. *The transfer function between the system output, y and the disturbance, w is*

$$\frac{Y}{W}(s)=\frac{\dfrac{A}{(\tau_1 s+1)(\tau_2 s+1)}}{1+\dfrac{\left(K_D s+K_P+K_I/s\right)A}{(\tau_1 s+1)(\tau_2 s+1)}}\cdot\frac{B}{A}=\frac{B}{(\tau_1 s+1)(\tau_2 s+1)+\left(K_D s+K_P+K_I/s\right)A}$$

$$=\frac{Bs}{s(\tau_1 s+1)(\tau_2 s+1)+(K_D s^2+K_P s+K_I)A}$$

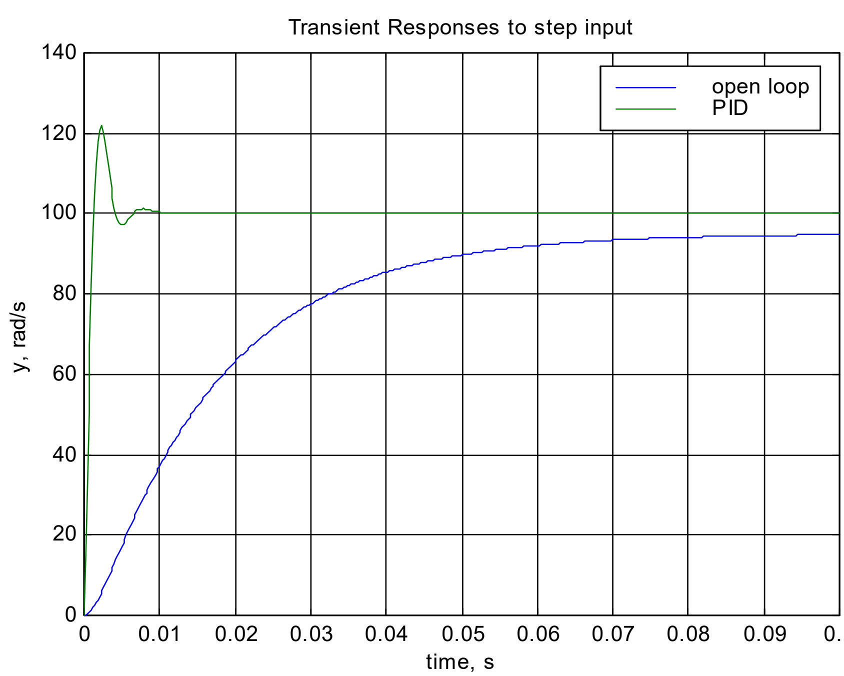
4. *The steady-state value is*

$$y_{ss}=\lim_{s\to 0} sY(s)=\underline{\qquad\qquad\qquad}$$

- Therefore, the steady-state response becomes exactly the same as the reference input. ➔ SSE is completely eliminated.

PID Control (cont.)

$K_I = 500$, $K_P = 5$, $K_D = 5(.0004)=.002$, $w = -.1$, $r = 100$



Comments on PID Control

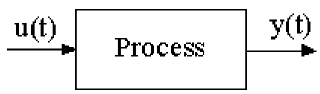
1. *The steady-state error can be* ***completely eliminated*** *due to the integral control.*
2. *The transient response is improved due to the derivative control ➔ Overshoot is reduced.*
3. *Combined, the three controls (**P**roportional, **I**ntegral, and **D**erivative) form the classical **PID controller**, which is widely used in the process industries.*
4. *Now, it remains to find the appropriate values for the three gains, K_P, K_I, and K_D. ➔ **PID tuning**.*
5. *High derivative gain can ______________ to the system.*

Matlab Control Code

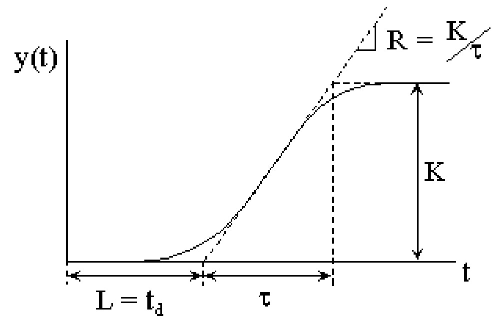
```
clear;

% System responses to controllers
tau1=1/60;  tau2=1/600;  t=[0:.0002:.1]';
W=.1;  A=10;  B=50;  R=100;

% open loop case
num1=R-W*B;
den1=[tau1*tau2 tau1+tau2 1];
yol=step(num1,den1,t);

% proportional control
Kp=5;
num2=R*Kp*A-W*B;
den2=[tau1*tau2 tau1+tau2 1+Kp*A];
yp=step(num2,den2,t);

% proportional/integral control
Ti=.01;
Ki=Kp/Ti;
num3=[R*A*Kp-W*B R*A*Ki];
den3=[tau1*tau2 tau1+tau2 1+Kp*A Ki*A];
ypi=step(num3,den3,t);

% PID control
Td=.0004;
Kd=Kp*Td;
num4=[R*A*Kd R*A*Kp-W*B R*A*Ki];
den4=[tau1*tau2 tau1+tau2+Kd*A 1+Kp*A
Ki*A];
ypid=step(num4,den4,t);

% PD control
num5=[R*A*Kd R*A*Kp-W*B];
den5=[tau1*tau2 tau1+tau2+Kd*A 1+Kp*A];
ypd=step(num5,den5,t);

% plotting data
plot(t,yol,t,ypid)
%plot(t,yol,t,yp,t,ypi,t,ypid,t,ypd)
grid
title('Transient Responses to step input')
xlabel('time, s')
ylabel('y, rad/s')
legend('open loop','PID')
```

PID Tuning (Ziegler-Nichols)

1. *A method for selecting the PID control parameters based on experiments on the process and thus bypass the need for a complete dynamic model. There are two widely-used methods*
2. *Transient-response method*



u(t) is a step input



1) Step response of many process control systems exhibits a process reaction curve that can be approximated by $\frac{Y}{U}(s) = \frac{Ke^{-t_d s}}{\tau s + 1}$.

2) This process is characterized by two parameters:

a. ____________________ , *R* (*slope of the tangent line*)

b. ________________ , *L* (*intersection with time axis*)

PID Tuning (cont.)

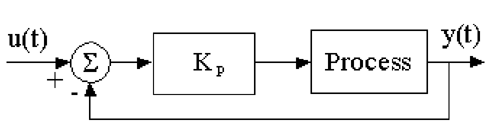
3) In order to achieve a damping of about $\zeta = 0.21$, the parameters are selected according to this table:

	K	T_I	T_d
P	1/RL		
PI	.9/RL	L/.3	
PID	1.2/RL	2L	.5L

2. *Stability-limit method*

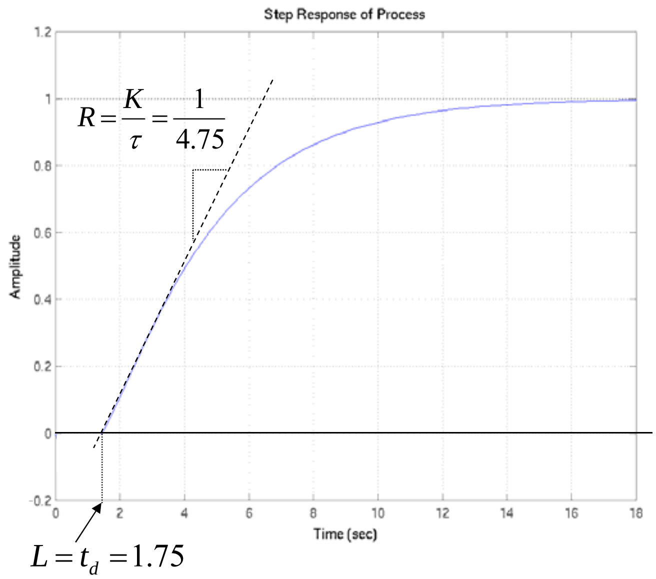
1) Using a proportional controller, the gain is slowly increased until the system becomes ______________ . At this point the gain, Ku, and the period of the system at its marginally stable point, Pu are recorded.

2) Using these two parameters, the PID parameters are selected according to the following table.



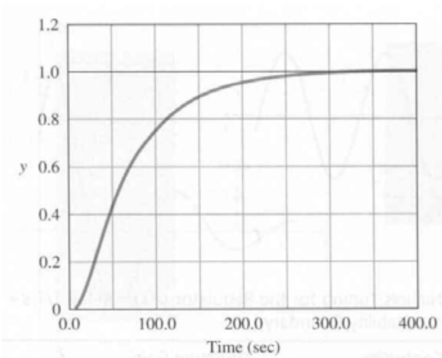
	K	T_I	T_d
P	.5 K_u		
PI	.45 K_u	P_u/1.2	
PID	.6 K_u	P_u/2	P_u/8

Example : Calculating key values



Can be read directly from the step response of a process with a time delay.

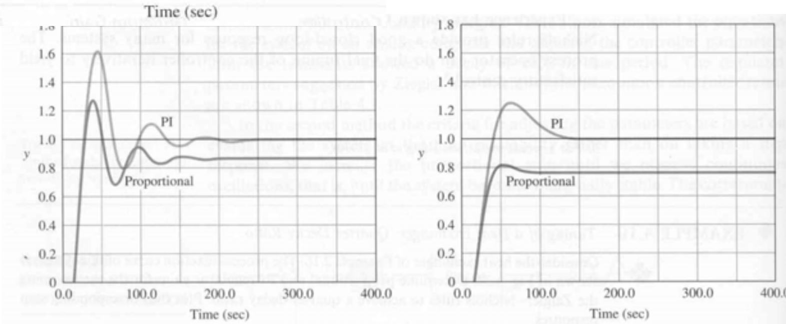
Example - Heat Exchanger (Transient)



From the process reaction curve, R=1/90 and L=13 sec. Then, using

	K	T_I
P	1/RL	
PI	.9/RL	L/.3

$K = 6.92$ for P-control and $K = 6.22$ and $T_I = 43.3$ for PI-control.

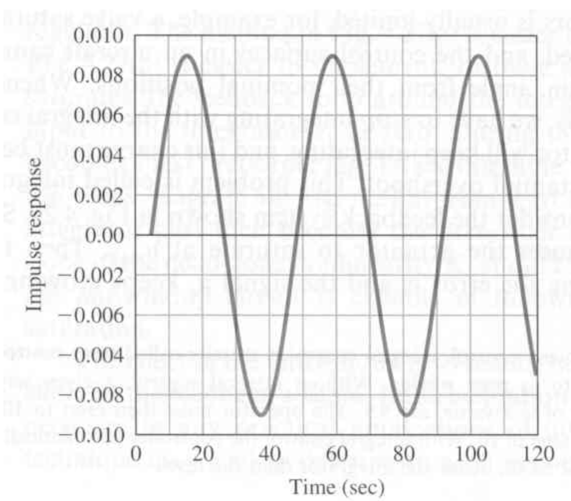


Too much oscillation

Improved responses by reducing K by a factor of 2

Example - Heat Exchanger (Stability Limit)

Proportional feedback is applied to the system. When the gain, Ku = 15.3, the system showed non-decaying oscillation in response to a short pulse.



The gain was Ku =15.3 and the period was measured at Pu = 42 sec. Then, using

	K	T_I
P	$.5K_u$	
PI	$.45K_u$	(Pu/1.2)

K = 7.65 for P-control and K = 6.885 and T_I = 35 for PI-control.