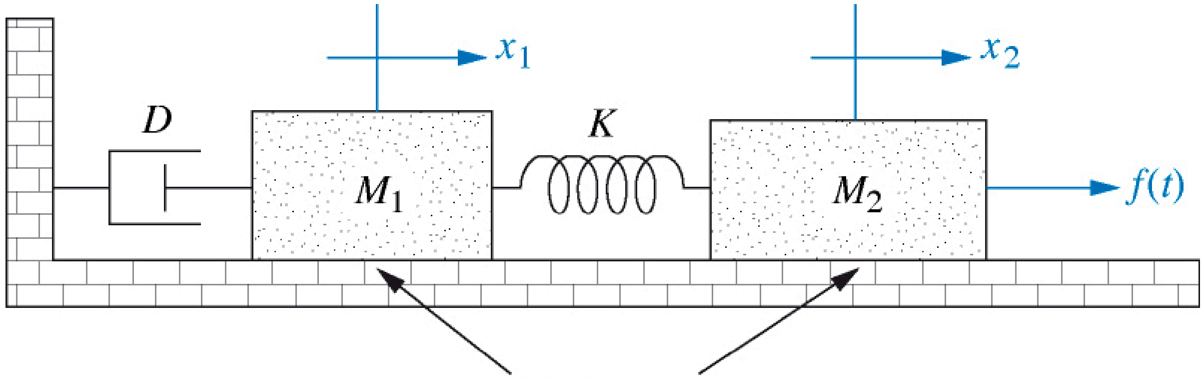
Example: Full-State Feedback Controller Design



1. *Differential equation:* Two second-order differential equations can be obtained using Newton's Law.

$$M_1 \frac{d^2 x_1}{dt^2} + D\frac{dx_1}{dt} + Kx_1 - Kx_2 = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} - Kx_1 + Kx_2 = f(t)$$

2. *State variables:* From each of the second-order differential equations, two linearly independent state variables need to be selected. In this problem, they are $x_1, \dot{x}_1, x_2, \dot{x}_2$.

Mechanical System (Cont'd)

3. *State equations:* Form two simultaneous first-order differential equations from each of the two second-order differential eq's.

$$\frac{dx_1}{dt} = \dot{x}_1$$

$$\frac{d\dot{x}_1}{dt} = -\frac{K}{M_1}x_1 - \frac{D}{M_1}\dot{x}_1 + \frac{K}{M_1}x_2$$

$$\frac{dx_2}{dt} = \dot{x}_2$$

$$\frac{d\dot{x}_2}{dt} = \frac{K}{M_2}x_1 - \frac{K}{M_2}x_2 + \frac{1}{M_2}f(t)$$

5. *In matrix form:*

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/M_1 & -D/M_1 & K/M_1 & 0 \\ 0 & 0 & 0 & 1 \\ K/M_2 & 0 & -K/M_2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \end{bmatrix} f(t)$$

Using Ackermann's Formula

Ackermann's formula

$$K = [0 \quad \cdots \quad 0 \quad 1]\, C^{-1} \alpha_c(F)$$

where C is the controllability matrix:

$$C = [G \quad FG \quad \cdots \quad F^{n-1}G]$$

$\alpha_c(F)$ is a matrix defined as

$$\alpha_c(F) = F^n + \alpha_1 F^{n-1} + \alpha_2 F^{n-2} + \cdots + \alpha_n I$$

where the α_i are the coefficients of the desired characteristic polynomial.

Matlab Code

```
clear;
M1 = 1;       % kg
M2 = 2;       % kg
K = 1000;      % N/m
D = 50;        % N-s/m

Ka = 1*2000;    % N/m, adding a spring between M1 and wall

F = [0 1 0 0;
    (-K/M1-Ka/M1) (-D/M1) (K/M1) 0;
    0 0 0 1;
    (K/M2) 0 (-K/M2) 0];
G = transpose([0 0 0 (1/M2)]);

C1 = G;
C2 = F*G;
C3 = F*F*G;
C4 = F*F*F*G;
C = [C1 C2 C3 C4];
```

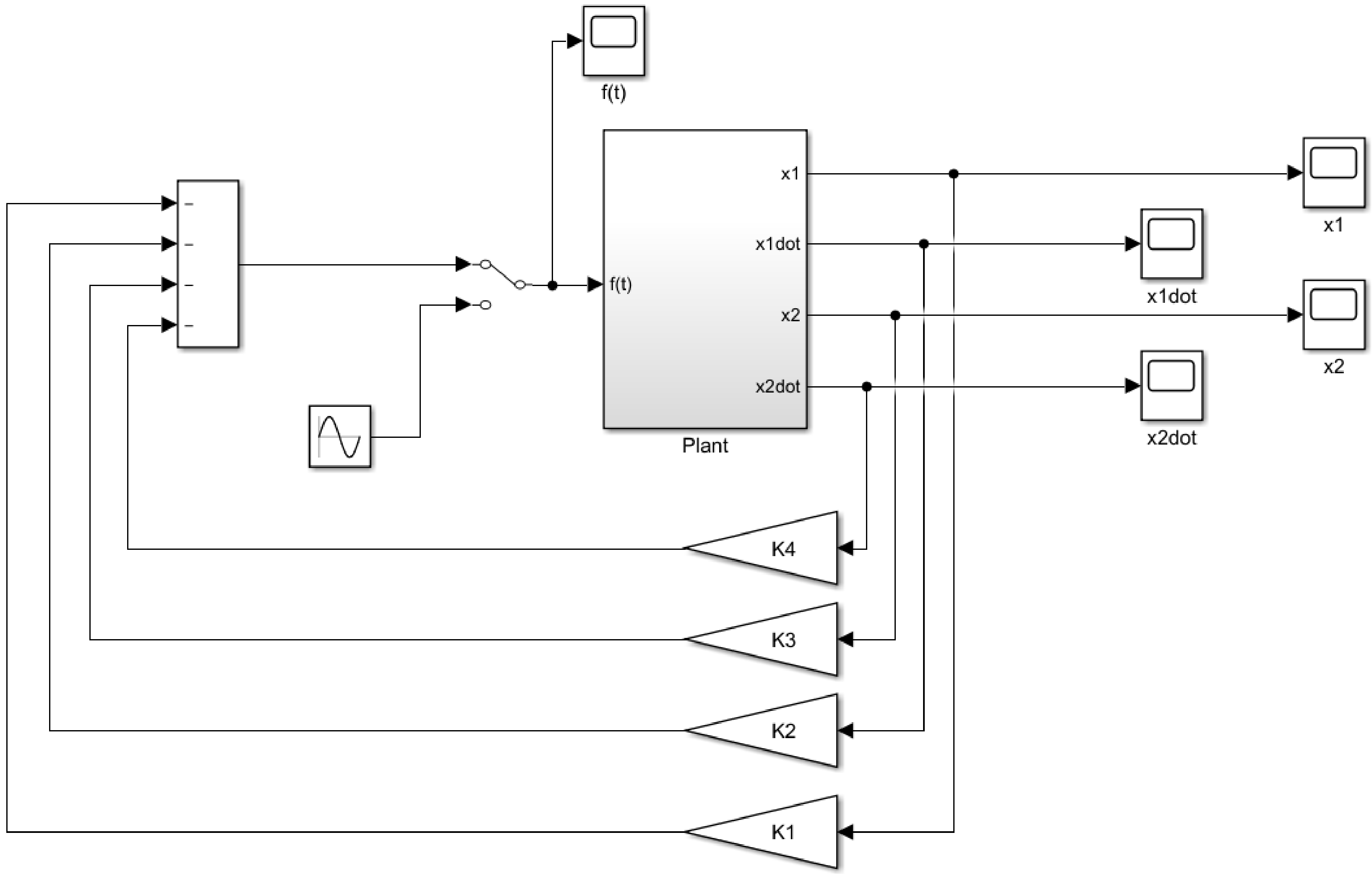
Matlab Code (continued)

```
tn = [0 0 0 1]*inv(C);
T_inv = [tn*F^3;
    tn*F^2;
    tn*F;
    tn];
A = T_inv*F*inv(T_inv);
B = T_inv*G;

% Try repeated poles
p = 50;
alpha1 = 4*p;
alpha2 = 6*p^2;
alpha3 = 4*p^3;
alpha4 = p^4;
alphaC = F^4 + alpha1*F^3 + alpha2*F^2 + alpha3*F + alpha4*eye(4);

KC = [0 0 0 1]*inv(C)*alphaC
K1 = KC(1);
K2 = KC(2);
K3 = KC(3);
K4 = KC(4);
```

Simulink Model - System



Simulink Model - Plant