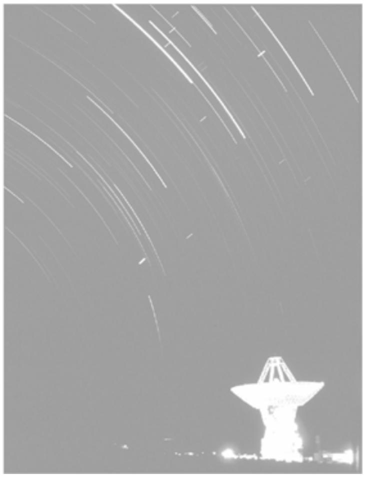
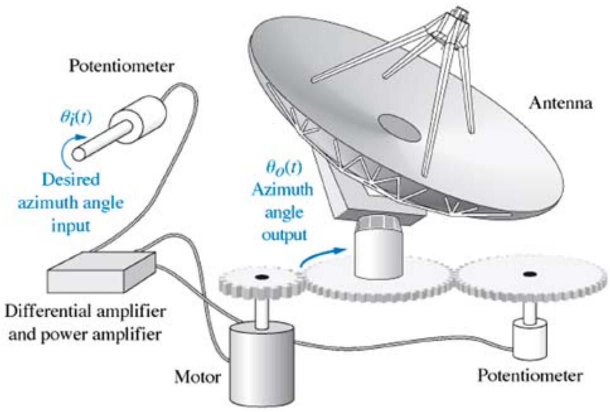
7. Case Study: Antenna Azimuth Position Control System



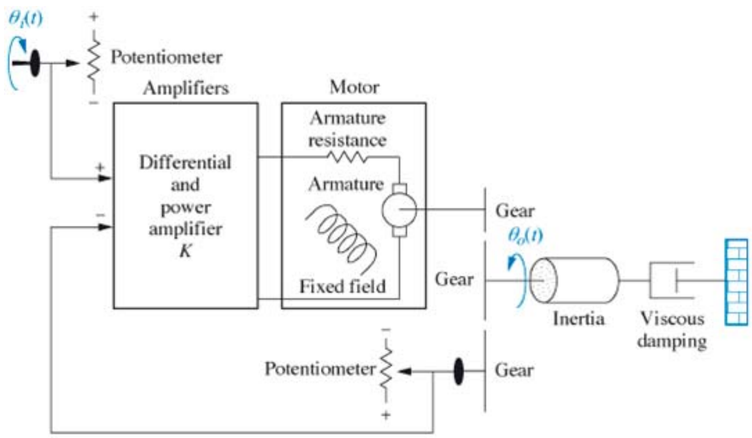
Introduction

- *Position control system*
  1) A position control system converts a position input command to a position output response.
  2) Examples: antennas, robot arms, and computer disk drives.
- *Antenna azimuth position control system*
  1) The goal is to have the azimuth angle output of the antenna, $\theta_o(t)$ follow the input angle of the potentiometer, $\theta_i(t)$.



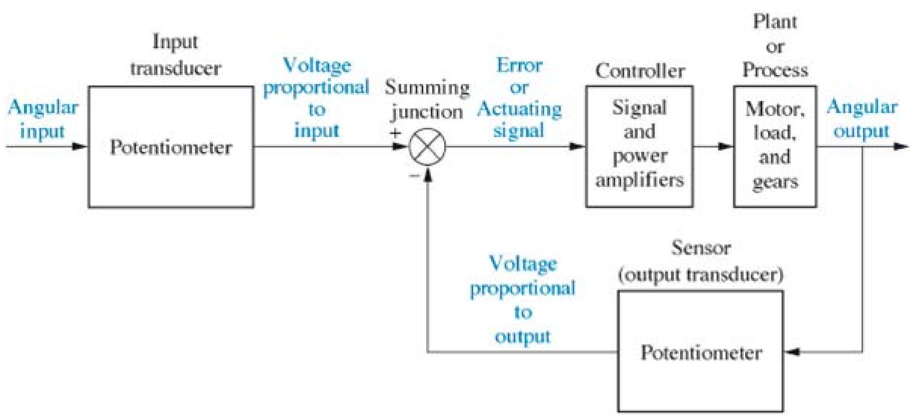
System Schematic

1. Input: angular displacement converted into a voltage by the _______ potentiometer, indicating the _______ antenna angle.
2. Output: ________ angular displacement of the antenna converted into a voltage by the potentiometer in the ________ path.
3. The signal and power amplifiers boost the difference between the input and output voltages. The amplified actuating signal drives the plant.



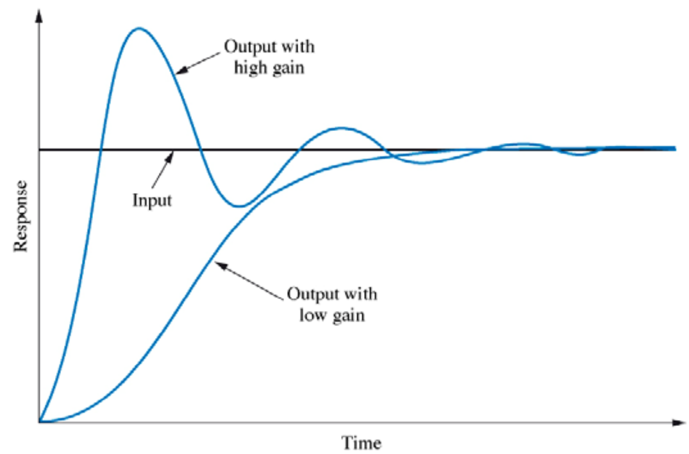
Functional Block Diagram

1. The system's desired function is to drive the error to zero.
2. When the input and output match, the error will be zero and the motor will not turn. Thus, the motor is driven only when the output and the input do not match. (no steady-state position error)
3. The greater the difference between the input and the output, the larger the motor input voltage and the faster the motor will turn. (transient responses will change for different gain values)



System Response

1. Design objectives and the system's performance revolve around the ________________, ________________, and ________.
2. Gain adjustments can affect performance and sometimes lead to trade-offs between the performance criteria.
3. **3.** _____________ can often be designed to achieve performance specifications without the need for trade-offs.



Subsystems

- We need to find the transfer function for each subsystem of the antenna azimuth position control system. The subsystems are shown below with the corresponding input and output variables.

Parameter	Input	Output
Input potentiometer	Angular rotation from user, $\theta_i(t)$	Voltage to preamp, $v_i(t)$
Preamp	Voltage from potentiometers, $v_e(t)=v_i(t)-v_o(t)$	Voltage to power amp, $v_p(t)$
Power amp	Voltage from preamp, $v_p(t)$	Voltage to motor, $e_a(t)$
Motor	Voltage from power amp, $e_a(t)$	Angular rotation to load, $\theta_o(t)$
Output potentiometer	Angular rotation from load, $\theta_o(t)$	Voltage to preamp, $v_o(t)$

Transfer Function: Potentiometers

- Since the input and output potentiometers are configured in the same way, their transfer functions will be the same.
- We neglect the dynamics for the potentiometers and simply find the relationship between the output voltage and the input angular displacement.
- In the center position, the output voltage is zero. Five turns toward either the positive 10 volts or the negative 10 voltages yields a voltage change of 10 volts.
- Thus, the transfer function is found by dividing the voltage change by the angular displacement:

$$\frac{V_i(s)}{\theta_i(s)} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Transfer Function: Amplifiers

- First, we assume that ____________ is never reached.
- Second, the dynamics of the preamplifier are neglected, since its response is typically much ________ than that of the power amplifier.
- Transfer function of the preamplifier:

$$\frac{V_p(s)}{\theta_e(s)} = K$$

- Transfer function of the power amplifier:

$$\frac{E_a(s)}{V_p(s)} = \_\_\_\_\_\_\_\_\_\_\_\_$$

Transfer Function: Motor and Load (1/2)

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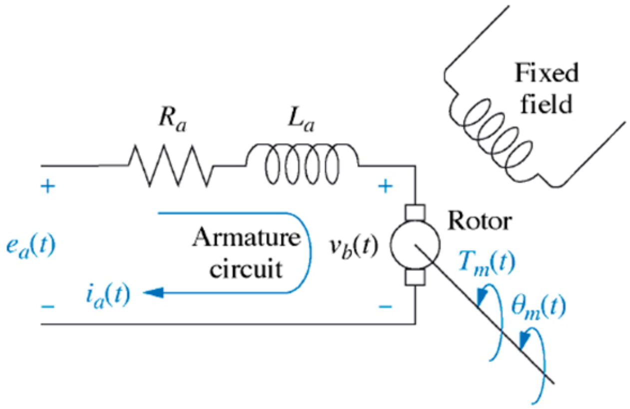

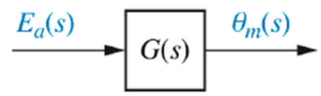

- By assuming that the armature inductance, L_a is small, the transfer function of the DC motor, $\theta_m(s)/E_a(s)$, is found to be

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{\qquad\qquad\qquad\qquad}$$

Transfer Function: Motor and Load (2/2)

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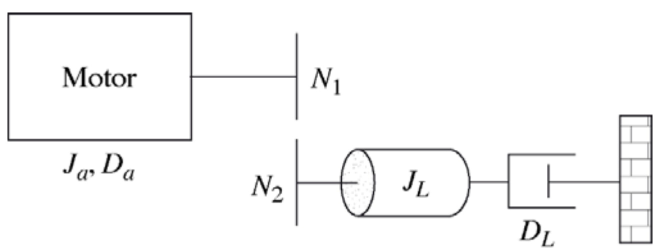
- The DC motor drives the antenna through gears.



- The load inertia of the antenna J_L and load damping D_L are reflected back to the armature as some equivalent inertia and damping to be added to the motor inertia J_a, and damping D_a, respectively.
- Thus, the equivalent inertia and equivalent damping at the armature are

$$J_m = J_a + \underline{\qquad\qquad} \; ; \quad D_m = D_a + \underline{\qquad\qquad}$$

Transfer Function: Final

- System parameters

V	n	K	K_1	a	R_a	J_a	D_a	K_b	K_t	N_1	N_2	N_3	J_L	D_L
10	10		100	100	8	0.02	0.01	0.5	0.5	25	250	250	1	1

- Using the above system parameters, the transfer functions of the system can be determined as

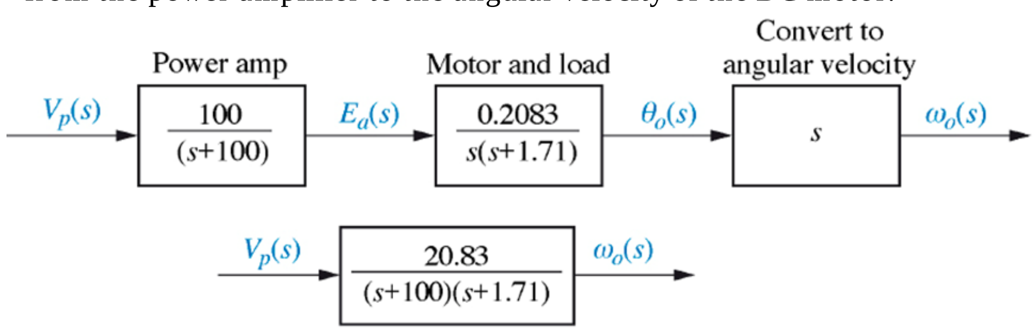
$$\frac{V_i(s)}{\theta_i(s)} = \underline{\qquad\qquad\qquad} \qquad \frac{E_a(s)}{V_p(s)} = \underline{\qquad\qquad\qquad}$$

$$J_m = 0.02 + \underline{\qquad\qquad\qquad} \quad ; \quad D_m = 0.01 + \underline{\qquad\qquad\qquad}$$

$$\frac{\theta_m(s)}{E_a(s)} = \underline{\qquad\qquad\qquad}$$

Antenna Control: Open-Loop Response (1/2)

- Consider the following _________ system (feedback path disconnected) from the power amplifier to the angular velocity of the DC motor.



- The open-loop transfer function is

$$G(s) = \underline{\qquad\qquad\qquad}$$

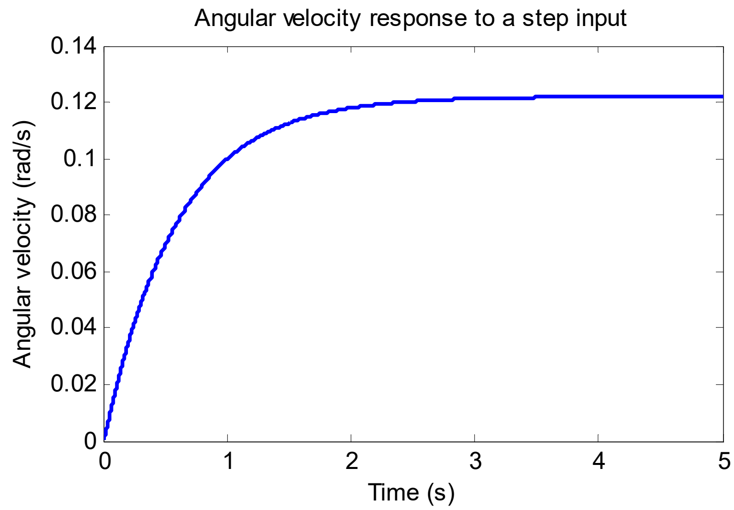
- The natural frequency and damping ratio of the open-loop system are

$$\omega_n = \underline{\qquad\qquad\qquad} \text{ and } \zeta = \underline{\qquad\qquad\qquad}$$

Antenna Control: Open-Loop Response (2/2)

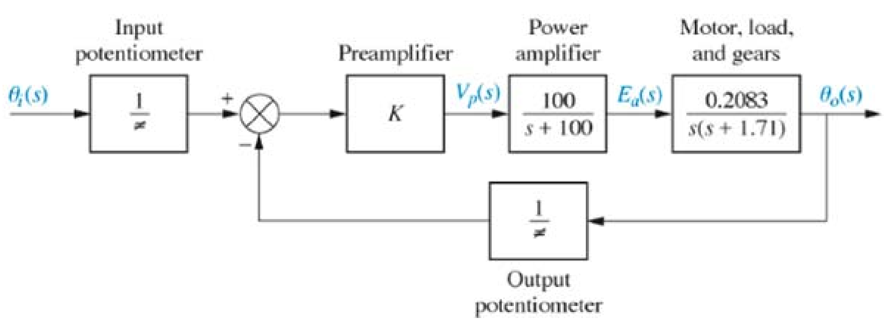
- The open-loop angular velocity response of the load to a step-voltage input to the power amplifier:

$$\omega_o(s) = \frac{20.83}{s(s+100)(s+1.71)}$$



Antenna Control: Closed-Loop System

- Consider the following closed-loop configuration for the antenna system.

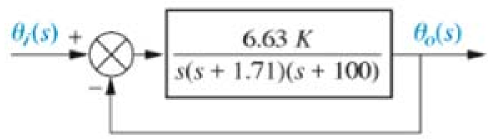


- The closed-loop transfer function is

$$T(s) = \frac{6.63K}{s^3 + 101.71s^2 + 171s + 6.63K}$$
$$= \frac{6.63K/(s^3 + 101.71s^2 + 171s)}{1 + 6.63K/(s^3 + 101.71s^2 + 171s)}$$

Root Locus Techniques (1/2)

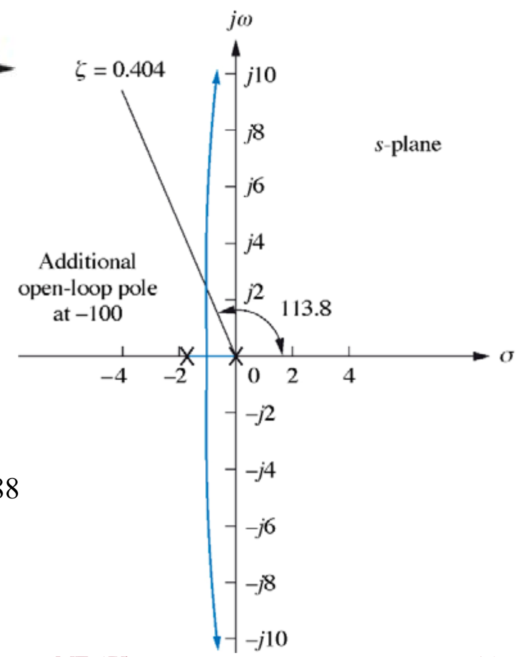
- Problem: Given the antenna azimuth position control system, find the preamplifier gain, K for 25% overshoot.



- Transfer function in the feedforward path:

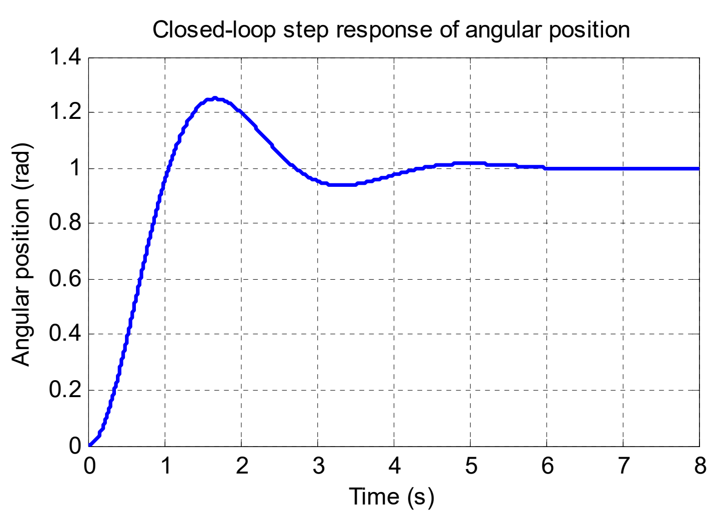
$$G(s) = \frac{6.63K}{s(s+1.71)(s+100)}$$

- 25 % overshoot corresponds to a damping ratio of _______.
- The dominant poles are -0.833 $\pm j1.888$ and the gain is $K = 64.21$.



Root Locus Techniques (2/2)

- Checking the second-order assumption, the third pole must be to the left of the open-loop pole at -100 and is thus greater than five times the real part of the dominant pole pair, which is -0.833. The second-order approximation is thus valid.
- In simulation, 25% overshoot is met.



Antenna Control: Lag-Lead Compensation

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- Problem: Given the antenna azimuth position control system, design cascade compensation to meet the following requirements: (1) 25% overshoot, (2) 2-second settling time, and (3) $K_v = 20$.
- Analysis: With only a gain adjustment, a preamplifier gain of 64.21 yielded 25% overshoot, with the dominant, second-order poles at -0.833 $\pm j1.888$. The settling time is thus $4/\zeta\omega_n$ = ___________ seconds. The open-loop function for the system is

$$G(s) = \frac{6.63K}{s(s+1.71)(s+100)}$$

- Hence K_v = _________________________ . Comparing these values to the problem statement, we want to **improve the settling time by _____ _______** and we want approximately **an _________ improvement in K_v**.

Lead compensator design (1/2)

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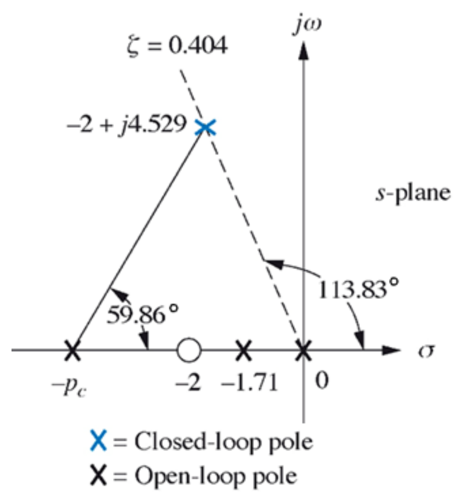
- First locate the dominant second-order pole. To obtain a settling time, T_s of 2 seconds and a percent overshoot of 25%, the real part of the dominant second-order pole should be at ________ . Locating the pole on the 113.83° line (ζ=0.404, corresponding to 25% overshoot) yields an imaginary part of 4.529.
- Second, assume a lead compensator zero and find the compensator pole. Assuming a compensator zero at -2, apply the angle criterion to find that there is an angular contribution of -120.14° at the design point of -2 + j4.529. Therefore, the compensator's pole must contribute 120.14° - 180° = -59.86° for the design point to be on the compensated system's root locus. To calculate the compensator pole, use ___________________________________.

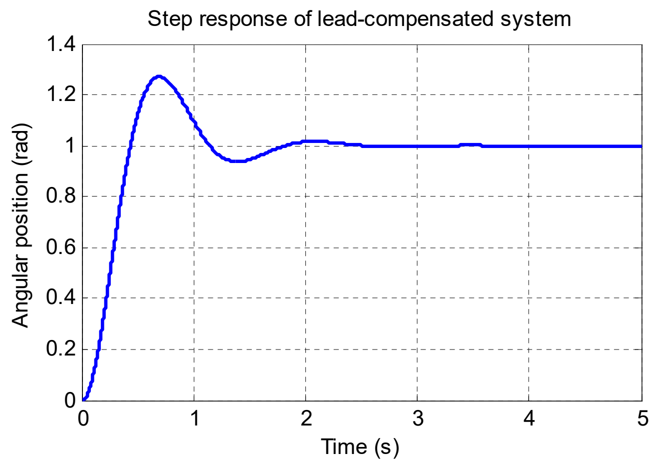


Lead compensator design (2/2)

- Now determine the gain. Using the lead-compensated system's open-loop function,

$$G_{LC}(s) = \frac{6.63K(s+2)}{s(s+1.71)(s+100)(s+4.63)}$$

and the design point -2 + j4.529 as the test point, an application of the magnitude criterion leads to 6.63K = 2549.



Lag compensator design

- K_v for the lead-compensated system is found to be

$$K_v = \lim_{s\to 0} s\frac{2549(s+2)}{s(s+1.71)(s+100)(s+4.63)} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

- Since we want K_v =20, the amount of improvement required over the lead-compensated system is 20/6.44 = 3.1. Choose p_c =-0.01 and calculate z_c = 0.031, which is 3.1 times larger.
- The complete lag-lead-compensated open-loop function, $G_{LLC}(s)$ is

$$G_{LLC}(s) = \frac{6.63K(s+2)(s+\qquad)}{s(s+\qquad)(s+1.71)(s+4.63)(s+100)}$$

- By applying the magnitude criterion, the gain value for the design point on the ζ=0.404 damping line corresponding to 25% overshoot, is found to be K = 382.1.

$$G_{LLC}(s) = \frac{2533(s+2)(s+0.031)}{s(s+0.01)(s+1.71)(s+4.63)(s+100)}$$

Step response of Lag-Lead system