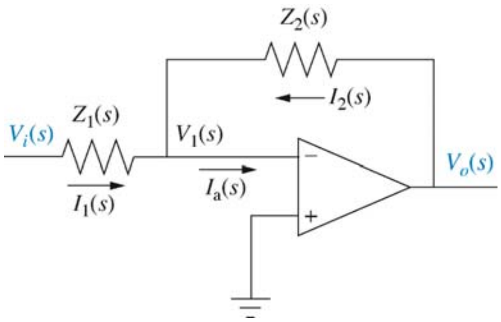
6. Design via Root Locus

Design of cascade compensators using root locus
PI/Lag Compensator
PD/Lead Compensator
PID/Lag-Lead Compensator
Implementation of Controllers
Notch Filter

Physical Realization of Compensator

- Once a controller is designed theoretically, it can be actually constructed using analog circuits.
- For PI, PID, PD, P, Lead and Lag controllers, active circuits (with op-amps) can be used. For ___________________ passive circuits can be used.
- Passive circuits don't require external power and thus they are cheaper. They can be designed from just ___________________ __________ .

Active-Circuit Realization



$$\frac{V_o(s)}{V_i(s)} = \underline{\qquad\qquad}$$

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain	R_1	R_2	$-\frac{R_2}{R_1}$
Integration	R	C	$-\frac{\frac{1}{RC}}{s}$
Differentiation	C	R	$-RCs$

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
PI controller	R_1	R_2, C	$-\frac{R_2}{R_1}\frac{\left(s+\frac{1}{R_2C}\right)}{s}$
PD controller	C, R_1	R_2	$-R_2C\left(s+\frac{1}{R_1C}\right)$
PID controller	C_1, R_1	R_2, C_2	$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_2}}{s}\right]$
Lag compensation	C_1, R_1	C_2, R_2	$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_2C_2 > R_1C_1$
Lead compensation	C_1, R_1	C_2, R_2	$-\frac{C_1}{C_2}\frac{\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_2C_2}\right)}$ where $R_1C_1 > R_2C_2$

Example: Implementing PID Controller (1/2)

- Implement the following PID controller.

$$G_c(s) = \frac{(s+55.92)(s+0.5)}{s}$$

1. From the given system:

$$G_{PID}(s) = \frac{(s^2 + 56.42s + 27.96)}{s} = s + 56.42 + \frac{27.96}{s}$$

2. From the table (for PID controller):

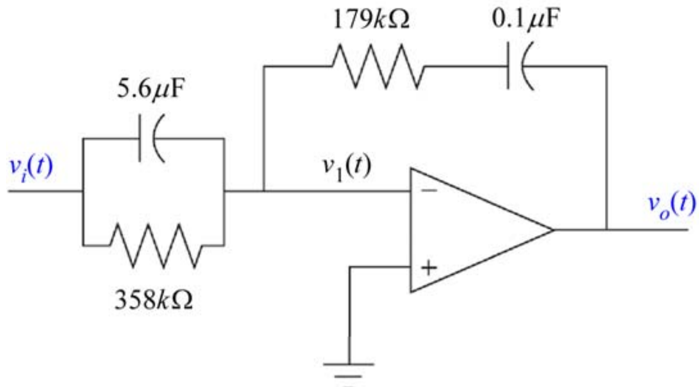
$$G(s) = -\left[\left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right) + R_2C_1 s + \frac{\frac{1}{R_1C_2}}{s}\right]$$

3. By matching terms, we obtain three equations:

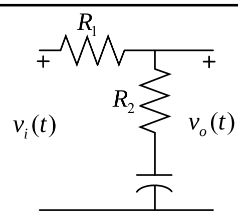
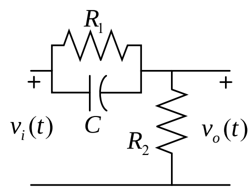
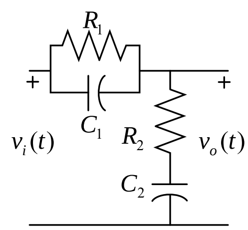
Example: Implementing PID Controller (2/2)

4. Since there are _____ unknowns and ______ equations, we can arbitrarily select a practical value for one of the elements. Selecting $C_2 = 0.1\ \mu F$, the remaining values are found to be $R_1 = 357.65\ k\Omega$, $R_2 = 178.89\ k\Omega$, and $C_1 = 5.59\ \mu F$.

5. Finally, the complete circuit becomes



Passive-Circuit Realization

Function	Network	Transfer function, $\frac{V_o(s)}{V_i(s)}$
Lag compensation		$\frac{R_2}{R_1+R_2}\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$
Lead compensation		$\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$
Lag-lead compensation		$\frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$

Example: Implementing Lead Compensator

- Realize the following lead compensator.

$$G_c(s)=\frac{s+4}{s+20.09}$$

1. From the table (for Lead compensator):

$$G_{Lead}(s)=\underline{\qquad\qquad\qquad}$$

2. By matching terms, we obtain the following two equations.

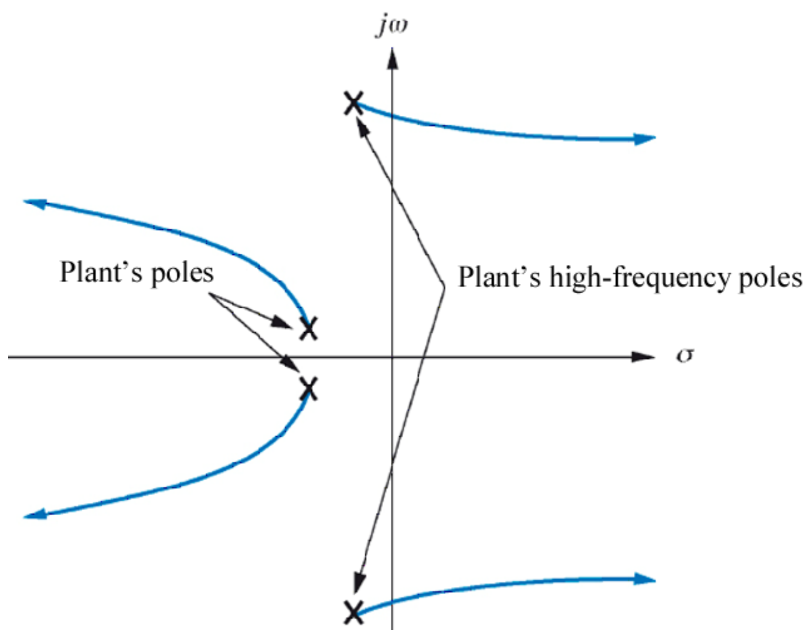
4. Selecting $C = 1\mu F$, the remaining values are found to be $R_1 = 250\ k\Omega$ and $R_2 = 62.2\ k\Omega$.

Other Control System: Notch Filter

- If a plant, such as a mechanical system, has high-frequency vibration modes, then a desired closed-loop response may be difficult to obtain. These high-frequency vibration modes can be modeled as part of the plant's transfer function by _____________________ ____________________.
- In a closed-loop configuration, these poles can move closer to the imaginary axis or even cross into the right half-plane. Instability of _______________________ superimposed over the desired response can result.
- In many cases the system starts off only lightly damped but with a small increase in gain it goes unstable very quickly.
- One way of eliminating the high-frequency oscillation is to cascade a notch filter with the plant. Note that the notch filter suffers from a lack of robustness and should not be used if the uncertainty in the system parameters is large

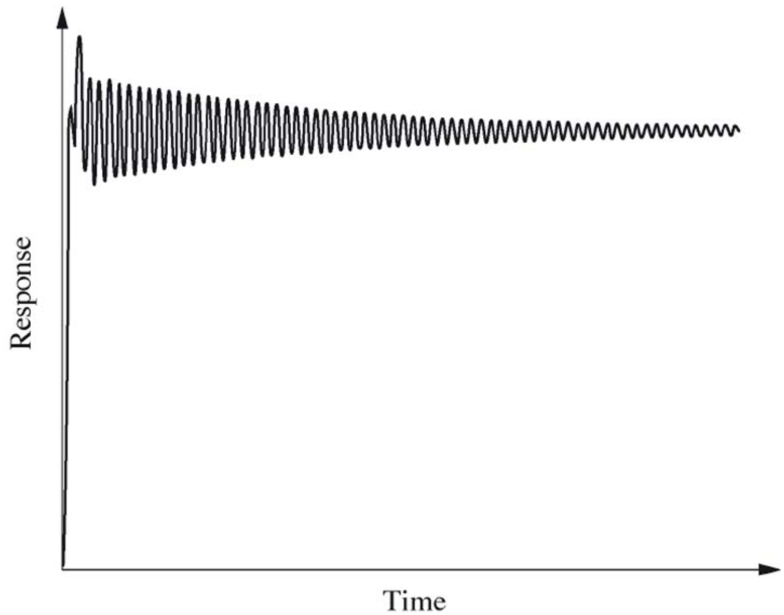
Root Locus of Lightly Damped System

- Root locus before cascading notch filter:



Step Response of Lightly Damped System

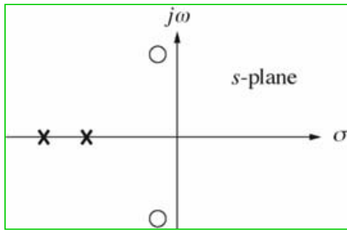
- Typical closed-loop step response before cascading notch filter:



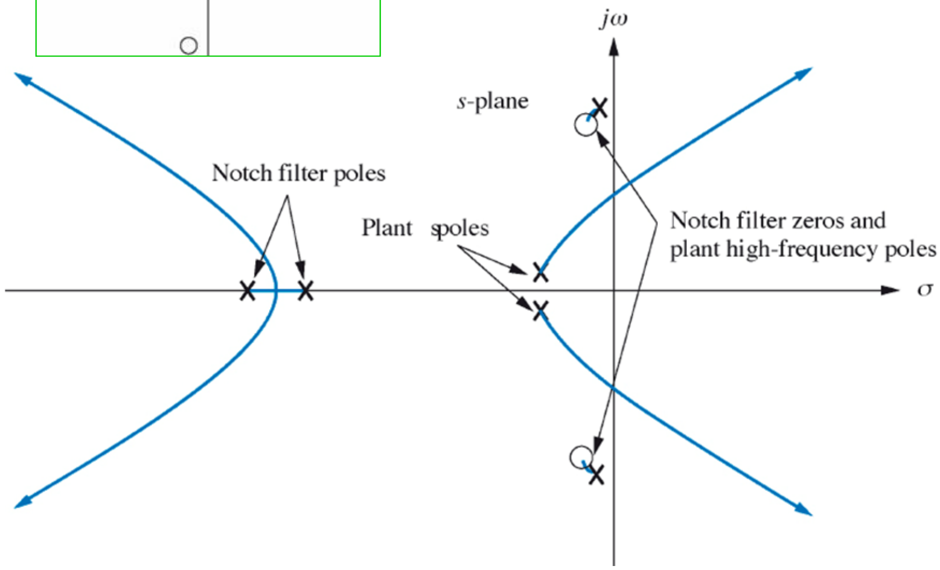
Notch Filter Design Steps

- A notch filter has **two ____________________ and two real poles**. The name of this filter comes from the shape of its magnitude frequency response characteristics, which shows a dip near the damped frequency of the high-frequency poles.
- When applied to lightly damped oscillatory poles, a notch filter places compensator zeros near the poles but at a _____________ _________ and slightly to the left or right of the lightly damped poles. Because of the pole-zero cancellation, the effect of the lightly damped poles is reduced.
- A new set of poles are then placed on the real axis. They are placed fast enough (far enough to the left) to avoid stability issues and slow enough to avoid high frequency noise amplification. This noise amplification comes from having added two zeros earlier.

Root Locus with Notch Filter

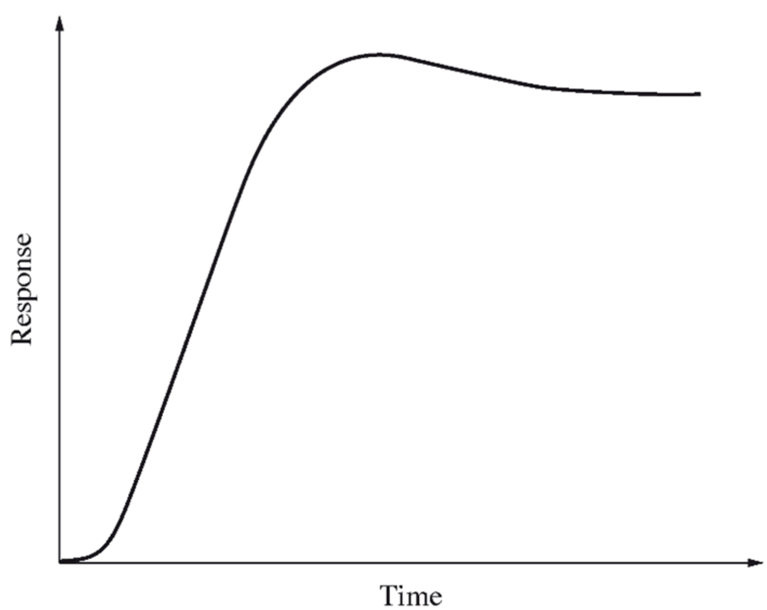


Pole-zero plot of a notch filter



Step Response with Notch Filter

- Closed-loop step response after cascading notch filter:

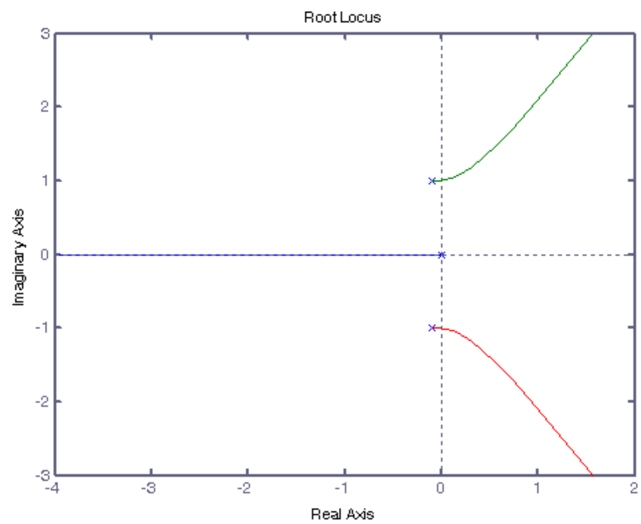


Example: Notch Filter (1/4)

- Design a notch filter for the following unity feedback system.

$$G(s) = \frac{K}{s(s^2 + 0.2s + 1)}$$

```
num  = 1;
den = [1  0.2  1 0];
Gs = tf(num,den);
rlocus(Gs);
```



Example: Notch Filter (2/4)

1. The transfer function for a notch filter is the following:

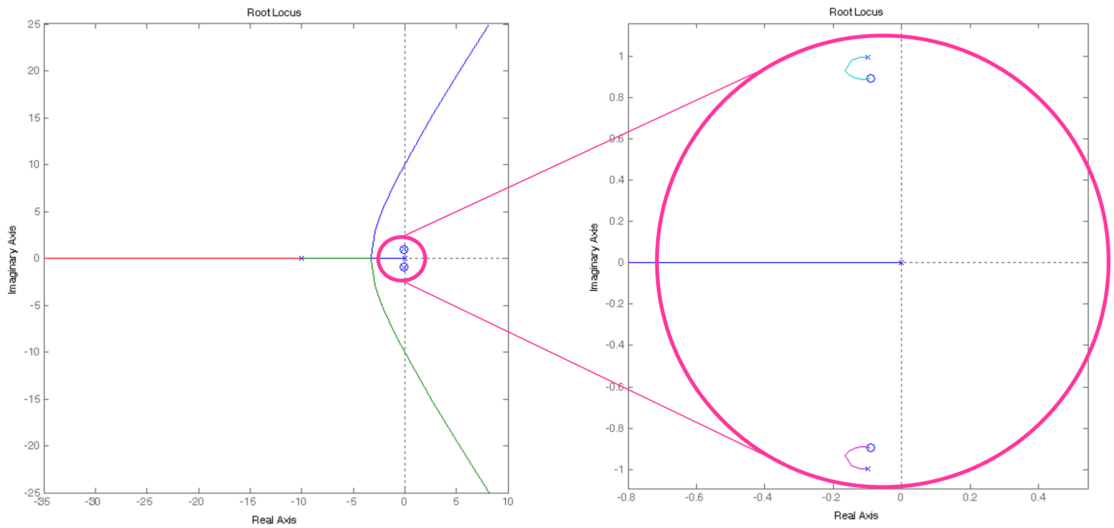
$$D(s) = \underline{\hspace{6cm}}$$

2. Typically, we let $\omega_1 = \omega_2$
3. The lightly-damped oscillatory poles of the uncompensated system are:
$$p_{1,2} = -0.1000 \pm j0.9950$$
4. Using the notch filter design procedure choose the zeros to be
$$z_{1,2} = - \qquad \pm j$$
5. We can now set the two poles to be at $s = -10.0$.
6. With the selected zeros and poles, the notch filter becomes

$$D(s) = \underline{\hspace{6cm}}$$

Example: Notch Filter (3/4)

- The root locus of the system with the designed notch filter



- The pole-zero cancellation is zoomed in on the right plot.

Example: Notch Filter (4/4)

- Step response shows dramatic improvement.
- Good for lightly damped systems. Easy to implement.
- Multiple controllers can be targeted to multiple modes.

$$G(s) = \frac{K}{s(s^2 + 0.2s + 1)}$$

$$D(s) = \frac{s^2 + 0.18s + 0.81}{(s+10)^2}$$

```
G = tf([1], [1 0.2 1 0]);
D = tf(conv([1 .09+j*0.8955],[1 0.09-j*0.8955]),
[1 20 100]);
sysOL = feedback(0.15*G, 1, -1);
K = 200; % proportional gain
sysCL = feedback(K*G*D, 1, -1);
figure (1)
rlocus(G*D);
figure (2)
t = 0:0.01:100;
y_OL = step(sysOL,t);
y_CL = step(sysCL,t);
plot(t,y_OL,'r-',t,y_CL,'b-');
legend('Open-Loop','Closed-Loop');
```