In[◦]:=
```
Integrate[1/(y^2*R^2*s0 - y*R*s0*Sqrt[y^2 R^2 + 4] + 2*y + 2*s0 - 2), {y, 0, r}]
```

In[1]:=
```
Integrate[1/(y^2*R^2*s0 - y*R*s0*Sqrt[y^2 R^2 + 4] + 2*y + 2*s0 - 2), {y, 0, r},
 Assumptions → 0 < s0 < 1 && 1 < R < 2 && 0 < r < s0]
```

Out[1]=
$$\text{Integrate}\left[\frac{1}{-2+2\,\text{s0}+2\,y+R^2\,\text{s0}\,y^2-R\,\text{s0}\,y\,\sqrt{4+R^2\,y^2}},\ \{y,0,r\},\ \text{Assumptions}\to 0<\text{s0}<1\,\&\&\,1<R<2\,\&\&\,0<r<\text{s0}\right]$$

In[2]:=
```
Integrate[1/(y^2*R^2*s0 - y*R*s0*(2 + y^2 R^2/4) + 2*y + 2*s0 - 2),
 {y, 0, r}, Assumptions → 0 < s0 < 1 && 1 < R < 2 && 0 < r < s0]
```

Out[2]=
$$-4\left(\text{RootSum}\left[8-8\,\text{s0}-8\,\#1+8\,R\,\text{s0}\,\#1-4\,R^2\,\text{s0}\,\#1^2+R^3\,\text{s0}\,\#1^3\,\&,\ \frac{\text{Log}[r-\#1]}{-8+8\,R\,\text{s0}-8\,R^2\,\text{s0}\,\#1+3\,R^3\,\text{s0}\,\#1^2}\,\&\right]-\text{RootSum}\left[8-8\,\text{s0}-8\,\#1+8\,R\,\text{s0}\,\#1-4\,R^2\,\text{s0}\,\#1^2+R^3\,\text{s0}\,\#1^3\,\&,\ \frac{\text{Log}[-\#1]}{-8+8\,R\,\text{s0}-8\,R^2\,\text{s0}\,\#1+3\,R^3\,\text{s0}\,\#1^2}\,\&\right]\right)\ \text{if}\ \textit{condition}$$

In[3]:=
```
Integrate[1/(y^2*R^2*s0 - y*R*s0*(2) + 2*y + 2*s0 - 2),
 {y, 0, r}, Assumptions → 0 < s0 < 1 && 1 < R < 2 && 0 < r < s0]
```

Out[3]=
$$\frac{\text{ArcTan}\left[\frac{-1+R\,\text{s0}}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}\right]+\text{ArcTan}\left[\frac{1+R\,(-1+r\,R)\,\text{s0}}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}\right]}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}$$
$$\text{if}\ \ 1+R\,(-1+r\,R)\,\text{s0}<\sqrt{1-R\,(2+R\,(-2+\text{s0}))\,\text{s0}}$$

In[6]:=
```
Solve[(ArcTan[(-1 + R s0)/Sqrt[-1 + R (2 + R (-2 + s0)) s0]] + ArcTan[
   (1 + R (-1 + r R) s0)/Sqrt[-1 + R (2 + R (-2 + s0)) s0]])/Sqrt[-1 + R (2 + R (-2 + s0)) s0] == -t/2, r]
```

Out[6]=
$$\left\{\left\{r\to\frac{1}{R^2\,\text{s0}}\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}\left(-\frac{1}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}+\frac{R\,\text{s0}}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}+\text{Tan}\left[\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}\left(-\frac{t}{2}-\frac{\text{ArcTan}\left[\frac{-1+R\,\text{s0}}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}\right]}{\sqrt{-1+R\,(2+R\,(-2+\text{s0}))\,\text{s0}}}\right)\right]\right)\ \text{if}\ \textit{condition}\right\}\right\}$$

In[7]:=
```
Simplify[(1 / (R^2 s0)) Sqrt[-1 + R (2 + R (-2 + s0)) s0] (-(1 / Sqrt[-1 + R (2 + R (-2 + s0)) s0]) +
    (R s0) / Sqrt[-1 + R (2 + R (-2 + s0)) s0] + Tan[Sqrt[-1 + R (2 + R (-2 + s0)) s0]
      (-(t / 2) - ArcTan[(-1 + R s0) / Sqrt[-1 + R (2 + R (-2 + s0)) s0]] / Sqrt[-1 + R (2 + R (-2 + s0)) s0])])]
```

Out[7]=

$$\frac{1}{R^2\, s0}\left(-1 + R\, s0 - \sqrt{-1 + R\,(2 + R\,(-2 + s0))\, s0}\ \mathrm{Tan}\left[\frac{1}{2}\sqrt{-1 + R\,(2 + R\,(-2 + s0))\, s0}\ t + \mathrm{ArcTan}\left[\frac{-1 + R\, s0}{\sqrt{-1 + R\,(2 + R\,(-2 + s0))\, s0}}\right]\right]\right)$$

In[13]:=
```
Manipulate[Plot[(1 / (R^2 s0)) (-1 + R s0 - Sqrt[-1 + R (2 + R (-2 + s0)) s0]
      Tan[1 / 2 Sqrt[-1 + R (2 + R (-2 + s0)) s0] t + ArcTan[(-1 + R s0) / Sqrt[-1 + R (2 + R (-2 + s0)) s0]]]),
  {t, 0, 100}], {R, 1.1, 1.8}, {s0, 0.95, 0.999}]
```

Out[13]=