MACM 316 - Computing Assignment 7

- Read the *Guidelines for Assignments* first.
- Submit a one-page PDF report to Canvas and upload your Matlab scripts (as m-files). Do not use any other file formats.
- Keep in mind that Canvas discussions are open forums.
- You must acknowledge any collaborations/assistance from colleagues, TAs, instructors etc.

Monte Carlo integration

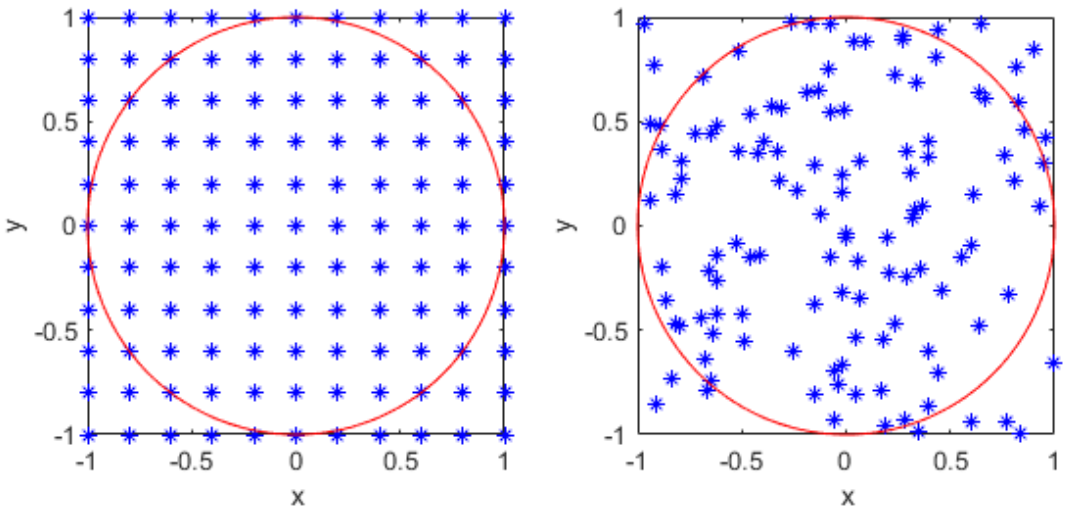
In this assignment we will tackle calculating the area of a circle. This can be done using numerical integration in two dimensions.

Consider the piecewise function

$$f(x,y) = \begin{cases} 1 & \sqrt{x^2+y^2} \leq 1, \\ 0 & \sqrt{x^2+y^2} > 1. \end{cases}$$

This function is equal to 1 inside and on the circle of radius 1 centered at the origin and zero everywhere else. Therefore, the integral over all space of this function is equal to the area of the circle.

We will calculate this integral in two ways: using an equally spaced grid, and using a set of points distributed randomly over the area. The latter option is called Monte Carlo integration.



The figure shows the two distributions of points. We can take the ratio of the number of points lying inside the circle to the total number of points and multiply by the total area to approximate

the area of the circle. This is equivalent to the following quadrature rule:

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)dxdy \approx \frac{4}{N}\sum_{n=1}^{N} f(x,y),$$

where N is the total number of points and $f(x,y)$ has been defined previously.

The evenly distributed points can be defined as:

$$x_i = -1 + \frac{2i}{m}, \quad y_j = -1 + \frac{2j}{m}, \quad 0 \le i,j \le m.$$

For Monte Carlo integration, both x and y should be two random numbers from the interval $[-1,1]$. (Use the Matlab function `rand()`.)

The exact area of the circle is π. Calculate the error of integration for both sets of quadrature points for different N.

We know the error has the form $\mathcal{O}\left(1/N^p\right)$; what is p for evenly distributed points? What restrictions are there on the number N for this distribution of points?

What is p for Monte Carlo integration? Why might Monte Carlo integration fail?

Tips

- Pick large enough N. If N is too small, the value of p may be unclear.