MACM 316 - Computing Assignment 6

- Read the *Guidelines for Assignments* first.
- Submit a one-page PDF report to Canvas and upload your Matlab scripts (as m-files). Do not use any other file formats.
- Keep in mind that Canvas discussions are open forums.
- You must acknowledge any collaborations/assistance from colleagues, TAs, instructors etc.

Cubic splines

In lectures you've learned about cubic splines, piecewise interpolants with continuous second derivatives. The goal of this assignment is to implement clamped cubic spline interpolation for the function

$$f(x) = \sin^2(x).$$

First you'll need a set of interpolation points, `x`. For simplicity, let these be equally spaced points spanning between 0 and π. The number of points, `n`, will change throughout the experiment.

Next pick a set of points on which to evaluate the interpolant. This set of points, `z`, needs to be much larger than the set of interpolation points and remain the same for all experiments.

Finally, calculate the data you need for the interpolation: $y = f(x)$, $a = f'(0)$, $b = f'(\pi)$.

The function `spline()` in Matlab constructs both natural and clamped cubic splines. The function has the following syntax for clamped splines:

```
S = spline(x,[a, y, b],z);
```

Test the interpolation error for several values of `n`. The error should be in the L_∞–norm, which can be calculated in Matlab in two ways:

```
error = max(abs(S - f(z)));
error = norm(S - f(z),inf);
```

What method should be used to solve the cubic spline system? Measure the computation time of the `spline()` function to find evidence of the $\mathcal{O}(n)$ operation count of this method.

What error behaviour do you expect? Confirm this behaviour. Find an estimate for the number M that appears in the error formula.

Tips

- The computation time for the function `spline()` is very small. As such, any amount of noise will affect your computation time calculations. To overcome this, try taking an average of times over several trials.