



The cubic spline system can be reduced to  $Ax = b$ , where  $x$  is a column vector containing the  $S''(X_n)/2$ . The system is tridiagonal, and can be solved by the Thomas algorithm, which requires  $O(n)$  steps.

The graph of average time to compute a spline vs.  $n$  interpolating points is linearly proportional to the number of interpolating points (Disregarding  $n = 2$ , which is interpolated as a straight line).

I expect the error to be bound by the given formula  $(5/384) * M * (h^4)$ . Since our nodes are equally spaced from 0 to  $\pi$ ,  $h = \pi/(n-1)$ .

Analytically,  $M$  is the absolute value of the maximum of the fourth derivative of  $\sin^2(x)$  over the interval 0 to  $\pi$ . The fourth derivative is  $-8\cos(2x)$  and the maximum is 8.

Comparing the computed error and the error bound formula, the error is confirmed to be less than the bound at each number of interpolating points.