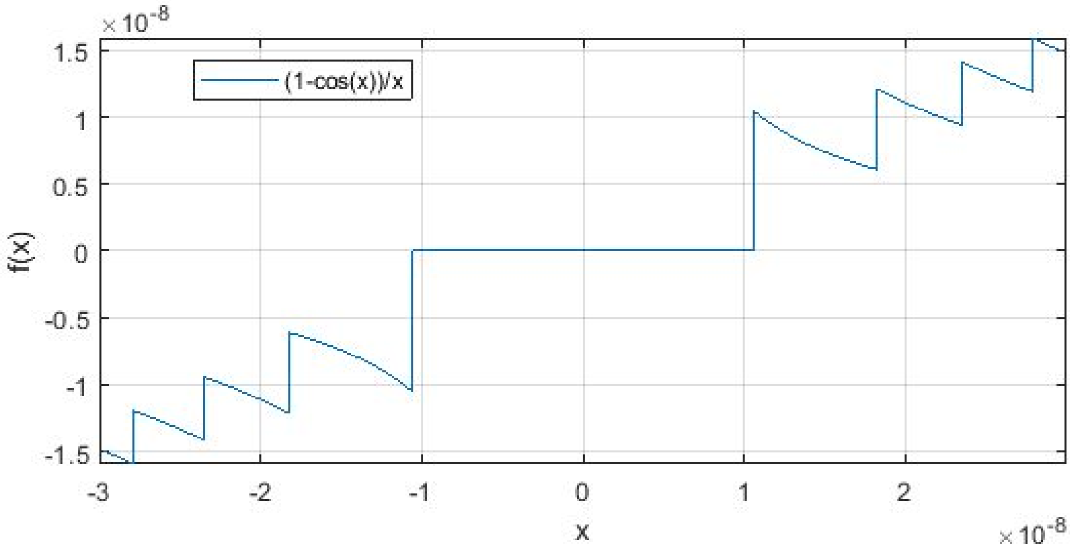
Benjamin Martin 301347720 <u>Computing Assignment 1</u> MACM316 2018-05-25

$$f(x) = \frac{1-cos(x)}{x}\ over\ (-2^{-25}, 2^{-25})$$



<u>Computed Values for f(x) in Matlab</u>

x	-2^{-10}	-2^{-20}	-2^{-26}	$-2^{-26.5}$	$2^{-26.5}$	2^{-26}	2^{-20}	2^{-10}
$\frac{1-cosx}{x}$	$-4.8828e-04$	-2^{-21}	-2^{-27}	0	0	2^{-27}	2^{-21}	$-4.8828e-04$

Finding the order of convergence by Taylor Series expanding around $xo = 0$,

$$f(x) = \frac{1-(1-\frac{x^2}{2}+...)}{x} = \frac{x}{2} + \ldots \text{ and } \frac{1-cosx}{x} = O(x).$$

Comparing this to my numerical data f(x) is indeed converging to zero faster than x.

The smallest absolute value of x for which matlab can compute $\frac{1-cosx}{x}$ is 2^{-26}. Matlab represents 1 using an exponent of 0 and a mantissa of 52 zero bits. The closest representable number uses an exponent of -1 and 52 1 bits. The difference being 2^{-53} (any smaller difference will be rounded to zero). The smallest value of x is found using:

$$acos(1-2^{-53}) = +2^{-26} or -2^{-26}$$

<u>Note</u>: for x, 2^-26<x<2^-26.5, cos(x) is computed by rounding to 2^-26. So the calculation for f(x) is completed, but inaccurate.

The three sources of round-off error are: Cancellation-error of 1-cos(x), the computation of cos(x), and the division by x. Mantissas have limited amount of bits to represent a number, and digits can be lost(rounded of chopped) when two values represented with different exponents are operated on.