Counterexample-Guided Interval Weakening

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Abstract. Systems deployed for long periods in dynamic environments may experience performance degradation that affects timing guarantees, even when their functional behaviour remains unchanged. In the design and verification of critical systems, such timing guarantees are often expressed using Metric Temporal Logic (MTL). Under degradation, these specifications may no longer hold as stated, although weaker variants that relax timing bounds may still be satisfied and remain meaningful. For example, while an elevator may initially be required to arrive within 30 seconds of a request, degradation of its motor may only allow us to guarantee arrival within 60 seconds. Although weaker, this guarantee is still useful and allows the system to maintain a reasonable level of operation. In this paper we present CEGIW, an iterative, counterexample-guided algorithm for automatically weakening timing intervals in MTL specifications so that they hold for a given system model. The algorithm preserves the logical structure of the original specification and weakens only interval bounds. We prove the correctness and optimality of CEGIW, and conduct an empirical evaluation to demonstrate the practicality of interval weakening using formalised requirements from a number of real-world case-studies. Using a model checker to produce counterexamples, CEGIW either identifies the strongest interval weakening under which the specification holds, or determines that no such weakening exists.

Keywords: System degradation · Specification weakening · Formal methods · Metric temporal logic

1 Introduction

Temporal properties of systems are often specified using logics such as Metric Temporal Logic [24] (MTL), and these properties can be verified to hold using model-checking [11]. However, in the real world, system failures or degradation can invalidate these proofs by breaking their assumptions, in which case the desired properties may no longer hold. Yet, under degradation the system may still have some useful capabilities for reduced operation, and so *logically weaker* versions of these properties may hold.

Given a degraded system $\mathcal{M}$, and an ideal MTL property ϕ that doesn't hold in $\mathcal{M}$, we aim to derive ϕ', the strongest possible *weakening* of ϕ, such that $\phi \Rightarrow \phi'$. To constrain the search space, we focus on modifying the intervals of MTL formulae while preserving the structural form of the specification. For

example, we may want an elevator to always arrive at least 30 seconds after calling it, represented by

$$\Box(\texttt{callElevator} \rightarrow \Diamond_{[0,30]}\texttt{elevatorArrives}) \tag{1}$$

(where $\Box$ is the *always* operator and $\Diamond_{[0,30]}$ is the *eventually* operator bounded between zero and thirty time units). However, if the main motor breaks, a weaker backup motor may start, slowing the system down. In this case the ideal property may not hold, and we may only be able to guarantee that the elevator will arrive within 60 seconds, represented by

$$\Box(\texttt{callElevator} \rightarrow \Diamond_{[0,60]}\texttt{elevatorArrives}). \tag{2}$$

This property is logically weaker than the original, but still guarantees a useful level of functionality. We would like to be able to derive this new property automatically from the system model and the original property.

Related work. Many works consider *unrealisable* set of requirements — where conflicts mean that no satisfying implementation exists — solving the problem by weakening specifications. Some use counterstrategies to strengthen assumptions [5] in the LTL fragment $GR(1)$, while others use heuristic-guided genetic algorithms to mutate assumptions and guarantees towards realisability [10]. However, this is different from the problem of weakening specifications relative to an existing implementation, which we are concerned with. We use a counterexample-guided approach, which has been applied to a large variety of problems including abstraction refinement [13], program synthesis [3], and learning assumptions for compositional verification [15], but not yet to the problem of specification repair in the presence of an existing implementation. This has been explored using techniques from the field of program repair [19], typically heuristically-guided *generate-and-validate* approaches like mutation-based repairs [12] and dynamic invariant detection [1]. We, however, are concerned with correct-by-construction, *semantics-driven* approaches, which have only been explored in the case of propositional logic specifications [6].

Contribution. We present our Counterexample-Guided Interval Weakening (CEGIW) algorithm that, given a degraded system and a desired MTL property ϕ that does not hold on the system, produces a new optimal MTL property ϕ' that both is weakening of ϕ and holds in the degraded system. The weakening is optimal with respect to a formally defined interval order, ensuring that no strictly stronger interval weakening satisfies the degraded system. We use a counterexample-guided approach, generating counterexamples with the nuXmv model checker [11], weakening the property to hold on the counterexamples, and iteratively weakening in this way until the property holds in the system. This approach is aimed at engineers in the design phase of safety-critical systems, who are trying to understand how resilient the timing properties of their system are to various proposed degradations, and how the system's formal guarantees are thus impacted.

The paper is organised as follows: Section 2 sets up the weakening of MTL formulae within contexts, Section 3 describes CEGIW and proves its correctness and optimality, Section 4 demonstrates CEGIW on an example and considers

its usefulness in real-world case-studies, and Section 5 concludes and outlines future work.

2 Weakening Within Contexts

We briefly state the syntax and semantics of Metric Temporal Logic [24] (MTL). Let $\mathcal{P}$ be a set of propositional variables. Well-formed MTL formulae are formed according to the rule:

$$\phi := p \mid \top \mid \neg\phi \mid \phi \wedge \phi \mid \phi \,\mathcal{U}_I\, \phi \mid \phi \,\mathcal{R}_I\, \phi \tag{3}$$

where $p \in \mathcal{P}$ and I is an interval, $[a, b]$, for $a \in \mathbb{N}$ and $b \in \mathbb{N} \cup \{\infty\}$ and $a \leq b$. Other constructs can be defined as usual, e.g. $\Diamond_I \phi = \top \,\mathcal{U}_I\, \phi$. While the release operator $\mathcal{R}$ can be defined in terms of the until operator $\mathcal{U}$, this would leave a negation as the outermost operator, complicating our weakening algorithm. We consider MTL formulae with a pointwise semantics over the natural numbers [4], defined according to a trace π which is an infinite sequence of states in which atomic propositions can hold, and an index of the trace $t \in \mathbb{N}$. The set of atomic propositions that hold in the t-th state is denoted by $\pi(t)$. A trace π satisfies an MTL formula ϕ, denoted by $\pi \vDash \phi$, if and only if $\pi, 0 \vDash \phi$.

$$\begin{array}{lll}
\pi, t \vDash p & \text{iff} & p \in \pi(t) \\
\pi, t \vDash \neg\phi & \text{iff} & \pi, t \nvDash \phi \\
\pi, t \vDash \phi_1 \wedge \phi_2 & \text{iff} & \pi, t \vDash \phi_1 \text{ and } \pi, t \vDash \phi_2 \\
\pi, t \vDash \phi_1 \,\mathcal{U}_I\, \phi_2 & \text{iff} & \exists i \in I.\, ((\pi, t + i \vDash \phi_2) \wedge \forall j \in [0, i) \cap I.\, (\pi, t + j \vDash \phi_1)) \\
\pi, t \vDash \phi_1 \,\mathcal{R}_I\, \phi_2 & \text{iff} & \forall i \in I.\, (\pi, t + i \vDash \phi_1) \\
 & & \quad \vee \exists i \in I.\, (\pi, t + i \vDash \phi_2 \wedge \forall j \in [0, i] \cap I.\, (\pi, t + j \vDash \phi_1))
\end{array}$$

CEGIW weakens a constituent subformula of a larger formula. We show, using the notion of *contexts*, that a weakening of a subformula implies a weakening of the larger formula.

Definition 1 (Contexts). *MTL Contexts are like MTL formulae with a single hole* $[-]$*, and are formed according to the rule:*

$$C ::= [-] \mid C \wedge \phi \mid \phi \wedge C \mid C \vee \phi \mid \phi \vee C \mid C \,\mathcal{U}_I\, \phi \mid \phi \,\mathcal{U}_I\, C \mid C \,\mathcal{R}_I\, \phi \mid \phi \,\mathcal{R}_I\, C \tag{4}$$

where ϕ is an MTL formula and I is an interval. Our definition of contexts does not allow negations on the path to the hole $[-]$, similarly to the restriction imposed by negation normal form (NNF). However, adjacent MTL subformulae ϕ are not required to be in NNF and can contain negations.

We define the notion of *context substitution*, where an MTL formula ψ is substituted into the hole of a context C to produce an MTL formula $C[\psi]$.

$$
\begin{aligned}
[-][\psi] &= \psi \\
(C \wedge \phi)[\psi] &= C[\psi] \wedge \phi & \qquad (C\, \mathcal{U}_I\, \phi)[\psi] &= C[\psi]\, \mathcal{U}_I\, \phi \\
(\phi \wedge C)[\psi] &= \phi \wedge C[\psi] & \qquad (\phi\, \mathcal{U}_I\, C)[\psi] &= \phi\, \mathcal{U}_I\, C[\psi] \\
(C \vee \phi)[\psi] &= C[\psi] \vee \phi & \qquad (C\, \mathcal{R}_I\, \phi)[\psi] &= C[\psi]\, \mathcal{R}_I\, \phi \\
(\phi \vee C)[\psi] &= \phi \vee C[\psi] & \qquad (\phi\, \mathcal{R}_I\, C)[\psi] &= \phi\, \mathcal{R}_I\, C[\psi]
\end{aligned}
\tag{5}
$$

By pushing negations inwards, an MTL formula ϕ can always be transformed into a context C and subformula ψ where ϕ is logically equivalent to $C[\psi]$.

Definition 2 (Weakening and strengthening of MTL formulae). *Let ϕ and ϕ' be MTL formulae. ϕ' is a weakening of ϕ, denoted*

$$\phi \sqsubseteq \phi' \tag{6}$$

if and only if, for all traces π and time-points t, if $\pi, t \vDash \phi$, then $\pi, t \vDash \phi'$. In this case, symmetrically, ϕ is a strengthening of ϕ'. Note that an MTL formula ϕ is always both a strengthening and a weakening of itself, i.e. $\phi \sqsubseteq \phi$.

Theorem 3 (Weakening of contexts). *Let C be a context and ψ and ψ' be MTL formulae. If $\psi \sqsubseteq \psi'$, then $C[\psi] \sqsubseteq C[\psi']$.*

Proof. We do an induction proof over the grammar of contexts using the induction hypothesis $P(C)$, that $C[\psi] \sqsubseteq C[\psi']$. The non-temporal inductive cases are omitted for brevity.

- Base case $[-]$: We assume that $\psi \sqsubseteq \psi'$, and by the definition of context substitution we have that $[-][\psi] \sqsubseteq [-][\psi']$ and thus $P([-])$.
- Inductive case $C\, \mathcal{U}_I\, \phi$: Assuming $P(C)$, we take an arbitrary trace π and time-point t, assume $\pi, t \vDash C[\psi]\, \mathcal{U}_I\, \phi$, and want to prove $\pi, t \vDash C[\psi']\, \mathcal{U}_I\, \phi$. We know that there exists an $i \in I$ such that $\pi, t + i \vDash \phi$, and that for all $j \in [0, i) \cap I$ we have $\pi, t + j \vDash C[\psi]$. Taking arbitrary i and j, by the induction hypothesis we have that $\pi, t + j \vDash C[\psi']$, and so by the semantics $\pi, t \vDash C[\psi']\, \mathcal{U}_I\, \phi$. Thus, we have $P(C\, \mathcal{U}_I\, \phi)$.
- Inductive case $\phi\, \mathcal{U}_I\, C$: Similar to the above case.
- Inductive case $C\, \mathcal{R}_I\, \phi$: Assuming $P(C)$, we take an arbitrary trace π and time-point t, assume $\pi, t \vDash C[\psi]\, \mathcal{R}_I\, \phi$, and want to prove $\pi, t \vDash C[\psi']\, \mathcal{R}_I\, \phi$. By the semantics of $\mathcal{R}$ there are two cases:
  1. For all $i \in I$ we have $\pi, t + i \vDash \phi$, thus we have $\pi, t + i \vDash C[\psi']$, and so we have $\pi, t \vDash C[\psi']\, \mathcal{R}_I\, \phi$.
  2. There exists an $i \in I$ such that $\pi, t + i \vDash C[\psi]$, and that for all $j \in [0, i] \cap I$ we have $\pi, t + j \vDash \phi$. Taking arbitrary i and j, by the assumptions we have that $\pi, t + i \vDash C[\psi']$ and $\pi, t + j \vDash \phi$, and then by the semantics we have $\pi, t \vDash C[\psi']\, \mathcal{R}_I\, \phi$.

  Thus, in both cases we have $P(C\, \mathcal{R}_I\, \phi)$.
- Inductive case $\phi\, \mathcal{R}_I\, C$: Similar to the above case. □

We show that, depending on which temporal operator is used, by expanding or contracting its interval we can weaken or strengthen the surrounding formula.

Definition 4 (Right-bound modifications of intervals). *Let $I = [a, b]$ be an interval. For any $i \in \mathbb{N}$, a right-bound modification of I is either a right-bound extension $[a, b + i]$, or, provided $i \leq b - a$, a right-bound contraction $[a, b - i]$. A right-bound modification is* strict *if $i > 0$. The set of all right-bound modifications of I is denoted $\mathcal{B}_R(I)$.*

Lemma 5 (Weakening of $\mathcal{U}$ interval). *Let ϕ and ψ be MTL formulae, and I and I' be intervals, where I' is a right-bound extension of I. Then, $\phi \,\mathcal{U}_I\, \psi \sqsubseteq \phi \,\mathcal{U}_{I'}\, \psi$.*

Proof. We assume that I' is a right-bound extension of I, and so, taking an arbitrary trace π and time-point t, we assume $\pi, t \vDash \phi \,\mathcal{U}_I\, \psi$ and want to prove $\pi, t \vDash \phi \,\mathcal{U}_{I'}\, \psi$. We know that there exists an $i \in I$ such that $\pi, t + i \vDash \psi$, and that for all $j \in [0, i) \cap I$ we have $\pi, t + j \vDash \phi$. Taking arbitrary i and j, we have that $i, j \in I'$, and so $\pi, t \vDash \phi \,\mathcal{U}_{I'}\, \psi$. Thus, $\phi \,\mathcal{U}_I\, \psi \sqsubseteq \phi \,\mathcal{U}_{I'}\, \psi$. □

Lemma 6 (Weakening of $\mathcal{R}$ interval). *Let ϕ and ψ be MTL formulae, and I and I' be intervals, where I' is a right-bound contraction of I. Then, $\phi \,\mathcal{R}_I\, \psi \sqsubseteq \phi \,\mathcal{R}_{I'}\, \psi$.*

Proof. We assume that I' is a right-bound contraction of I, and so, taking an arbitrary trace π and time-point t, we assume $\pi, t \vDash \phi \,\mathcal{R}_I\, \psi$ and want to prove $\pi, t \vDash \phi \,\mathcal{R}_{I'}\, \psi$. By the semantics of $\mathcal{R}$ there are two cases:

1. For all $t' \in I$ we have $\pi, t + t' \vDash \psi$. Then, as $I' \subseteq I$, we know that for all $t'' \in I'$ we have $\pi, t + t' \vDash \psi$, and so $\pi, t \vDash \phi \,\mathcal{R}_{I'}\, \psi$.
2. There exists a $t' \in I$ such that $\pi, t + t' \vDash \phi$ and for all $t'' \in I \cap [0, t']$, we have $\pi, t + t'' \vDash \psi$. As I' is a right-bound contraction of I, there are two further cases:
   (a) If $t' \in I'$, then we still have that $\pi, t + t' \vDash \phi$ and for all $t'' \in I' \cap [0, t']$, we have $\pi, t + t'' \vDash \psi$, and so $\pi, t \vDash \phi \,\mathcal{R}_{I'}\, \psi$.
   (b) If $t' \notin I'$, then $I' \cap [0, t'] = I'$ and so we know that for all $t'' \in I'$, we have $\pi, t + t'' \vDash \psi$, and so $\pi, t \vDash \phi \,\mathcal{R}_{I'}\, \psi$.

Thus, in all cases we have that $\phi \,\mathcal{R}_I\, \psi \sqsubseteq \phi \,\mathcal{R}_{I'}\, \psi$. □

Often in CEGIW, recursive calls will generate a set of intervals from which either the strongest or weakest must be chosen. We show that there is a total order of implication over the set of right-bound modifications of an interval, which allows us to make that choice.

Lemma 7 (Extension-weakening order of right-bound modifications). *Let I be an interval. Then, $\mathcal{B}_R(I)$ has a total order $\supseteq$, where for all MTL contexts C, MTL formulae ϕ and ϕ', and all $I', I'' \in \mathcal{B}_R(I)$, if $I'' \supseteq I'$ then we have $C[\phi \,\mathcal{U}_{I'}\, \phi'] \sqsubseteq C[\phi \,\mathcal{U}_{I''}\, \phi']$.*

Proof. For any pair of intervals I' and I'' in $\mathcal{B}_R(I)$ we can order the resulting subformulae by applying Lemma 5 to get $\phi \,\mathcal{U}_{I'}\, \phi' \sqsubseteq \phi \,\mathcal{U}_{I''}\, \phi'$ (or the reverse), and then order the full formulae with their contexts by applying Theorem 3 to get $C[\phi \,\mathcal{U}_{I'}\, \phi'] \sqsubseteq C[\phi \,\mathcal{U}_{I''}\, \phi']$ (or the reverse). □

Lemma 8 (Contraction-weakening order of right-bound modifications). *Let I be an interval. Then, $\mathcal{B}_R(I)$ has a total order $\subseteq$, where for all MTL contexts C, MTL formulae ϕ and ϕ', and all $I', I'' \in \mathcal{B}_R(I)$, if $I'' \subseteq I'$ then we have $C[\phi \mathcal{R}_{I'} \phi'] \sqsubseteq C[\phi \mathcal{R}_{I''} \phi']$.*

Proof. Similar to the proof of Lemma 7, but uses Lemma 6 to order $\phi \mathcal{R}_{I'} \phi'$ and $\phi \mathcal{R}_{I''} \phi'$. □

3 Algorithm for Interval Weakening

CEGIW is split into two levels. First, there is a function *weaken* (Algorithm 1) that, given an MTL formula ϕ, an interval I in ϕ to weaken, and a counterexample trace π, weakens I such that the new formula ϕ' holds on π (Section 3.1). However, this does not guarantee that ϕ' holds on the model itself, and so we need to repeat the process, finding a new counterexample trace for ϕ' and weakening the interval again. The second part of CEGIW is this iterative process that finds counterexample traces by model checking (Section 3.2).

3.1 Weakening on a Counterexample

Model checkers generally produce a specific type of infinite counterexample trace, called a *lasso trace*.

Definition 9 (Lasso traces). *A trace π is* lasso *if it can be separated into a finite prefix π_{pre} and an infinitely repeating finite suffix π_{suf}, forming*

$$\pi = \pi_{\text{pre}}(\pi_{\text{suf}})^{\omega}. \tag{7}$$

This restricts us to a subset of infinite traces that can be finitely represented. The finite length of a lasso trace is then defined as $|\pi| = |\pi_{\text{pre}}| + |\pi_{\text{suf}}|$.

We show that we can prove properties of an entire infinite lasso trace using only a finite *covering interval*. Without this, we may need to iterate over the entire infinite trace, impacting completeness.

Definition 10 (Covering intervals). *The suffix-covering interval of π, defined with respect to an interval $[a, b]$, is*

$$\text{cov}_{\pi}([a, b]) = [a, \min(b, \text{end}_{\pi}(a))] \tag{8}$$

where we specify a finite end *of the infinite trace with*

$$\text{end}_{\pi}(a) = \begin{cases} |\pi| & \text{if } a < |\pi_{\text{pre}}| \\ a + |\pi_{\text{suf}}| - 1 & \text{otherwise.} \end{cases} \tag{9}$$

Lemma 11 (Lasso trace coverage). *Let ϕ be an MTL formula, π be a lasso trace, and $a \in \mathbb{N}$. If for all $t \in [a, \text{end}_{\pi}(a)]$ we have $\pi, t \vDash \phi$, then for all $t' \in \mathbb{N}$ with $t' \geq a$ we have $\pi, t' \vDash \phi$.*

Algorithm 1: Weakening within a context C

```
1  function Weaken(C, ψ △_{I_orig} ψ', π, t)
2      if C = [−] then
3          if △ = U then
4              return WeakenUDirect(ψ, ψ', I_orig, π, t)
5          else // △ = R
6              return WeakenRDirect(ψ, ψ', I_orig, π, t)
7      ...
8      else if C = C U_J φ then
9          return WeakenULeft(C, φ, J, ψ △_{I_orig} ψ', π, t)
10     else if C = φ U_J C then
11         return WeakenURight(φ, C, J, ψ △_{I_orig} ψ', π, t)
12     else if C = C R_J φ then
13         return WeakenRLeft(C, φ, J, ψ △_{I_orig} ψ', π, t)
14     else if C = φ R_J C then
15         return WeakenRRight(φ, C, J, ψ △_{I_orig} ψ', π, t)
```

Proof. We assume that for all $t \in [a, \text{end}_\pi(a)]$ we have $\pi, t \vDash \phi$, and, taking an arbitrary $t' \in \mathbb{N}$ with $t' \geq a$ want to prove that $\pi, t' \vDash \phi$. There are two cases. Firstly, if $t' < |\pi|$ then we know that this is within the $[a, \text{end}_\pi(a)]$ range and so we have $\pi, t' \vDash \phi$. Otherwise, if $t' \geq |\pi|$, we split π into its prefix π_{pre} and infinitely repeating suffix π_{suf}, and want to prove that $\pi_{\text{pre}}(\pi_{\text{suf}})^\omega, t' \vDash \phi$. As $t' \geq |\pi|$, we can split it into $t' = |\pi_{\text{pre}}| + n \cdot |\pi_{\text{suf}}| + m$ for some $n, m \in \mathbb{N}$ with $n \geq 1$ and $m < |\pi_{\text{suf}}|$.

$$\begin{aligned} & \pi_{\text{pre}}(\pi_{\text{suf}})^\omega, |\pi_{\text{pre}}| + n \cdot |\pi_{\text{suf}}| + m \vDash \phi \\ \implies & (\pi_{\text{suf}})^\omega, n \cdot |\pi_{\text{suf}}| + m \vDash \phi \\ \implies & (\pi_{\text{suf}})^\omega, m \vDash \phi \\ \implies & \pi_{\text{pre}}(\pi_{\text{suf}})^\omega, |\pi_{\text{pre}}| + m \vDash \phi \end{aligned} \tag{10}$$

We know that $|\pi_{\text{pre}}| + m$ is in the $[a, \text{end}_\pi(a)]$ interval, so we have $\pi, t' \vDash \phi$. □

We also define the *optimality* of weakenings, used to prove that CEGIW will not produce an interval weakening that is any stronger than it needs to be.

Definition 12 (Optimality of right-bound extensions and contractions). *An interval I' is an optimal right-bound extension (resp. contraction) of an interval I with respect to a context C, MTL formulae ψ and ψ', a temporal operator $\triangle \in \{\mathcal{U}, \mathcal{R}\}$, trace π, and time-step t, if*

$$\pi, t \vDash C[\psi \triangle_{I'} \psi'] \tag{11}$$

and either (a) $I = I'$, or (b) there exists no strict right-bound contraction (resp. extension) I'' of I' such that $\pi, t \vDash C[\psi \triangle_{I''} \psi']$.

The entrypoint of CEGIW is Algorithm 1, which recurses following the inductive structure of the MTL context grammar. The proof of correctness and

Algorithm 2: Directly weakening interval of $\mathcal{U}$

```
1 function WeakenUDirect(ψ_l, ψ_r, [a, b], π, t)
2     for i ← a to end_π(a) do
3         if π, t + i ⊨ ψ_r then
4             return [a, max(b, i)]
5         if π, t + i ⊭ ψ_l then
6             break
7     return None
```

optimality follows the same inductive structure, with base cases for directly weakening the intervals of $\mathcal{U}_I$ and $\mathcal{R}_I$ (Lemmas 13 and 14), and inductive cases for weakening within both operators on either side (Lemmas 15 to 18).

Intuitively, to weaken a $\mathcal{U}$ formula $\psi_l \; \mathcal{U}_I \, \psi_r$ in Algorithm 2, the algorithm considers how the interval can be adjusted so that the formula becomes satisfied. Starting from time t, if the formula does not hold under the original interval, the only admissible weakening is to extend the right bound, thereby allowing additional time for the right subformula ψ_r to become true while the left subformula ψ_l continues to hold. The algorithm therefore extends the right bound incrementally until either the $\mathcal{U}$ formula holds on the given trace or no further extension is possible. In the former case, it returns the smallest such extension, yielding an optimal weakening; in the latter case, it reports that no interval weakening exists.

Lemma 13 ($\mathcal{U}$ base case). *Let I be an interval, ψ_l and ψ_r MTL formulae, and π a lasso trace. Then, for all timepoints $t \in \mathbb{N}$ with*

$$I' = WeakenUDirect(\psi_l, \psi_r, I, \pi, t), \tag{12}$$

either I' is an optimal right-bound extension of I such that $\pi, t \vDash \psi_l \; \mathcal{U}_{I'} \, \psi_r$, or $I' = None$, in which case there exists no such interval.

Proof. We take an arbitrary t. Our proof for Algorithm 2 uses the loop invariant that $\forall j \in [a, \text{end}_\pi(a)]$ with $j < i$ (where $I = [a, b]$), we have that $\pi, t+j \nvDash \psi_r$ and $\pi, t+j \vDash \psi_l$. On first entry to the loop there is no such j, so this is trivially true. On reaching the end of the loop body, we know that $\pi, t+i \nvDash \psi_r$ and $\pi, t+i \vDash \psi_l$, and so in combination with the loop invariant we know that $\forall j \in I$ where $j \leq i$, we have $\pi, t + j \nvDash \psi_r$ and $\pi, t + j \vDash \psi_l$. Thus, the loop invariant is preserved. Suppose at the start of iteration i the loop invariant holds. If $\pi, t + j \vDash \psi_r$ on Line 3 then we return $I' = [a, \max(b, i)]$. This is an optimal right-bound extension of I and we have that $\pi, t \vDash \psi_l \; \mathcal{U}_{I'} \, \psi_r$.

If $None$ is returned, then either we broke out of the loop early because for some $i \in [a, \text{end}_\pi(a)]$ we have $\pi, t + i \nvDash \psi_l$ at Line 5, or we ran the loop to completion. In the first case, we know that $\pi, t+i \nvDash \psi_r$ as this is checked before at Line 3, and so combining with the loop invariant we know that ψ_r never

held up until ψ_l stopped holding, and so there is no right-bound extension I' for which $\pi, t \vDash \psi_l \, \mathcal{U}_{I'} \, \psi_r$.

In the second case, by the loop invariant we have that for all $i \in [a, \text{end}_\pi(a)]$ we have $\pi, t+i \vDash \psi_l$ and $\pi, t+i \nvDash \psi_r$. By Lemma 11 we then have the same for all $i \in \mathbb{N}$ with $i \geq a$, and so there exists no right-bound extension I' of I that satisfies $\pi, t \vDash \psi_l \, \mathcal{U}_{I'} \, \psi_r$. □

Lemma 14 ($\mathcal{R}$ base case). *Let I be an interval, ψ_l and ψ_r MTL formulae, and π a lasso trace. Then, for all timepoints $t \in \mathbb{N}$ with*

$$I' = WeakenRDirect(\psi_l, \psi_r, I, \pi, t), \tag{13}$$

either I' is an optimal right-bound contraction of I such that $\pi, t \vDash \psi_l \, \mathcal{R}_{I'} \, \psi_r$, or $I' = None$, in which case there exists no such interval.

Proof. Full proof and pseudocode is available in our repository. □

We prove the inductive cases with an MTL context C, an interval I, MTL formulae ψ and ψ', a temporal operator $\triangle \in \{\mathcal{U}, \mathcal{R}\}$, and a lasso trace π. We use the induction hypothesis $P(C)$, that for all timepoints $t \in \mathbb{N}$ with $I' = Weaken(C, \psi \triangle_I \psi', \pi, t)$, if I' is an interval then $\pi, t \vDash C[\psi \triangle_I \psi']$, and

1. If $\triangle = \mathcal{U}$, then I' is an optimal right-bound extension of I;
2. If $\triangle = \mathcal{R}$, then I' is an optimal right-bound contraction of I.

If $I' = None$, then there exists no such interval in each case.

Intuitively, when weakening within the left subformula of a $\mathcal{U}$ operator in Algorithm 3, we must ensure that the left subformula holds at every relevant timestep until the right subformula becomes true. Starting from time t, the algorithm therefore examines each timestep $t+i$ within the original interval and determines the interval weakening required for the left subformula to hold on the given trace at that point. Because the left operand of $\mathcal{U}$ is interpreted universally over the interval, the overall weakening must be strong enough to satisfy all such requirements. The algorithm therefore selects the weakest interval that subsumes all interval weakenings computed for individual timesteps. If no such interval exists, or if weakening fails at any timestep, the algorithm reports that no valid weakening can be found.

Lemma 15 ($\mathcal{U}$-left inductive case). *Let C be an MTL context, I and J intervals, ϕ, ψ, and ψ' MTL formulae, $\triangle \in \{\mathcal{U}, \mathcal{R}\}$ a temporal operator, and π a lasso trace. If $P(C)$ holds, then so does $P(C \, \mathcal{U}_J \, \phi)$.*

Proof. For Algorithm 3 we assume the inductive hypothesis $P(C)$ and want to prove $P(C \, \mathcal{U}_J \, \phi)$. We take an arbitrary t and distinguish two cases, according to whether $\triangle$ is $\mathcal{U}$ or $\mathcal{R}$. In either case, by the induction hypothesis each recursive call evaluates to either $None$ or an optimal interval I' related to I by the corresponding relation (right-bound extension or contraction respectively) such that $\pi, t + t' \vDash C[\psi \triangle_{I'} \psi']$.

We use the loop invariant that, for all $j \in \text{cov}_\pi(J)$ with $j < i$, we have that $\pi, t + j \nvDash \phi$ and that $I' = Weaken(C, \psi \triangle_I \psi', \pi, t + j)$ is an interval such that $\pi, t + j \vDash C[\psi \triangle_{I'} \psi']$. On first entry to the loop there is no such

Algorithm 3: Weakening within $\mathcal{U}$ on the left

```
1  function WeakenULeft(C, φ, [a,b], ψ △_{I_orig} ψ', π, t)
2      b_fin ← min(b, end_π(a))
3      intervals ← [ ]
4      for i ← a to b_fin do
5          if π, t + i ⊨ φ then
6              if i = a then
7                  return I_orig
8              return interval in intervals with maximal absolute difference to
                   I_orig
9          I ← Weaken(C, ψ △_{I_orig} ψ', π, t + i)
10         if I = None then
11             return None
12         append I to intervals
13     return None
```

j, so this is trivially true. On reaching the end of the loop body, we know that $Weaken(C, \psi \triangle_I \psi', \pi, t+i) \neq None$ from Line 10, and so by the induction hypothesis the recursive call must have produced a suitable interval I'. As we also know that $\pi, t+i \not\models \phi$ from Line 5, the loop invariant is thus preserved for $j \leq i$. Suppose at the start of iteration i the loop invariant holds. If $Weaken(C, \psi \triangle_I \psi', \pi, t+i) = None$ at Line 10 then by the induction hypothesis we know that there is no suitable interval I'' for which $\pi, t+i \models C[\psi \triangle_{I''} \psi']$, and by the loop invariant that there is no $j < i$ for which $\pi, t+j \models \phi$. Thus, there is no suitable interval I'' for which $\pi, t \models (C\, \mathcal{U}_J\, \phi)[\psi \triangle_{I''} \psi']$. If $\pi, t+j \models \phi$ at Line 5 then we split on whether it is our first iteration or not. If $i = a$ (where $J = [a, b]$) then we know that $\pi, t+a \models \phi$, and so any interval will work. We simply return the original interval I_{orig}.

Otherwise, by the loop invariant we know that for all $j \in \text{cov}_\pi(J)$ with $j < i$ — of which there must be at least one as $i > a$ — we have an interval I' such that $\pi, t+j \models C[\psi \triangle_{I'} \psi']$. Applying Lemma 7 if $\triangle = \mathcal{U}$, or Lemma 8 if $\triangle = \mathcal{R}$, we obtain a maximum interval I'' such that for all $j \in \text{cov}_\pi(J)$ with $j < i$ we have $\pi, t+j \models C[\psi \triangle_{I''} \psi']$. If $\text{cov}_\pi(J) = J$ then we have

$$\pi, t \models (C\, \mathcal{U}_J\, \phi)[\psi \triangle_{I''} \psi']. \tag{14}$$

Otherwise, if $\text{cov}_\pi(J) = [a, \text{end}_\pi(a)]$, then by Lemma 11 for all $k \in \mathbb{N}$ with $k \geq a$ we have $\pi, t+k \models C[\psi \triangle_{I''} \psi']$, and so the above holds here too. □

Lemma 16 ($\mathcal{U}$-right inductive case). *Let C be an MTL context, I and J intervals, ϕ, ψ, and ψ' MTL formulae, $\triangle \in \{\mathcal{U}, \mathcal{R}\}$ a temporal operator, and π a lasso trace. If $P(C)$ holds, then so does $P(\phi\, \mathcal{U}_J\, C)$.*

Lemma 17 ($\mathcal{R}$-left inductive case). *Let C be an MTL context, I and J intervals, ϕ, ψ, and ψ' MTL formulae, $\triangle \in \{\mathcal{U}, \mathcal{R}\}$ a temporal operator, and π a lasso trace. If $P(C)$ holds, then so does $P(C\, \mathcal{R}_J\, \phi)$.*

Lemma 18 ($\mathcal{R}$-right inductive case). *Let C be an MTL context, I and J intervals, ϕ, ψ, and ψ' MTL formulae, $\triangle \in \{\mathcal{U}, \mathcal{R}\}$ a temporal operator, and π a lasso trace. If $P(C)$ holds, then so does $P(\phi\, \mathcal{R}_J\, C)$.*

Full proofs for Lemmas 16 to 18 are available in our repository. We use these supporting lemmas to prove the correctness and optimality of CEGIW.

Theorem 19 (Correctness for weakening). *Let C be an MTL context, I and J intervals, ϕ, ψ, and ψ' MTL formulae, $\triangle \in \{\mathcal{U}, \mathcal{R}\}$ a temporal operator, π a lasso trace, and $t \in \mathbb{N}$ be a timepoint. Let $I' = Weaken(C, \psi \triangle_I \psi', \pi, t)$. If I' is an interval then $\pi \vDash C[\psi \triangle_{I'} \psi']$, and*

1. *If $\triangle = \mathcal{U}$, then I' is an optimal right-bound extension of I;*
2. *If $\triangle = \mathcal{R}$, then I' is an optimal right-bound contraction of I.*

If $I' = None$, then there exists no such interval in each case.

Proof. We use the same induction hypothesis $P(C)$ defined for the preceding inductive lemmas. The non-temporal inductive cases are omitted for brevity.

Base case $[-]$: By Lemma 13 if $\triangle = \mathcal{U}$, and Lemma 14 if $\triangle = \mathcal{R}$, we have $P([-])$.
Inductive case $C\, \mathcal{U}_J\, \phi$: Assuming $P(C)$, by Lemma 15 we have $P(C\, \mathcal{U}_J\, \phi)$.
Inductive case $\phi\, \mathcal{U}_J\, C$: Assuming $P(C)$, by Lemma 16 we have $P(\phi\, \mathcal{U}_J\, C)$.
Inductive case $C\, \mathcal{R}_J\, \phi$: Assuming $P(C)$, by Lemma 17 we have $P(C\, \mathcal{R}_J\, \phi)$.
Inductive case $\phi\, \mathcal{R}_J\, C$: Assuming $P(C)$, by Lemma 18 we have $P(\phi\, \mathcal{R}_J\, C)$. □

The time complexity of Algorithm 1 is $O(|\pi|^{\mathtt{td}(\phi)})$. where π is the counterexample trace and $\mathtt{td}(\phi)$ is the *temporal depth* of the MTL formula ϕ, i.e. the maximum number of nested temporal operators along any path in the syntax tree.

3.2 Iterative Weakening

Assume that we have an MTL formula ϕ that has a temporal subformula $\psi \triangle_I \psi'$ with an interval I that we want to weaken. ϕ can be split into $\psi \triangle_I \psi'$ and the surrounding MTL context C, such that ϕ is logically equivalent to $C[\psi \triangle_I \psi']$. Assume we also have a transition system $\mathcal{M}$. Using a model checker, we check whether ϕ holds on $\mathcal{M}$. If it holds then we are done, but if not, we will receive a counterexample trace π through $\mathcal{M}$ for which $\pi \nvDash C[\psi \triangle_I \psi']$. We can then weaken on this counterexample with

$$I' = Weaken(C, \psi \triangle_I \psi', \pi, 0). \tag{15}$$

By Theorem 19, if $I' = None$, then there exists no weakening I'' of I such that $\pi \vDash C[\psi \triangle_{I''} \psi']$, and so the same holds for the model $\mathcal{M}$. Otherwise, I' is an interval such that $\pi \vDash C[\psi \triangle_{I'} \psi']$. However, $C[\psi \triangle_{I'} \psi']$ does not necessarily hold on $\mathcal{M}$, and so we model check again, creating an iterative loop that ends when we either produce an interval I'' that is a weakening of I such that $C[\psi \triangle_{I''} \psi']$ holds on $\mathcal{M}$, or show that no such weakening exists.

4 Evaluation

We evaluate how *effective* interval weakening is in understanding the temporal behaviour of specifications, and how applicable it is to real-world requirements. To this end, we investigate the following research questions:

RQ1: How can CEGIW be used to explore and diagnose timing margins in MTL specifications during early design? (Section 4.1)

RQ2: To what extent do existing real-world requirements provide practical targets for interval weakening, and are such weakenings meaningful in their application domains? (Section 4.2)

Choosing a model checker. There are no industrial-strength model checkers for MTL with pointwise semantics [9, 2], yet many efficient tools exist for linear temporal logic [11, 22] (LTL). Thus, we translate MTL formulae into LTL using the *next* (X) operator [7, Remark 5.15] and use an LTL model checker. During preliminary investigation, it was found that symbolic LTL model checkers such as nuXmv [11] and Spin [22] typically generate minimal counterexample traces. Weakening intervals with these usually only increments or decrements the bound rather than modifying it by a larger amount, which increases the number of calls made to the model checker dramatically. Our implementation uses nuXmv in bounded model checking (BMC) mode, producing multiple counterexamples for a specific bound length, finding the optimal interval for each of them and returning the weakest, making it more likely that we make fewer calls to the model checker. For our implementation we require the user to choose the BMC bound, but theoretical completeness can be preserved as completeness thresholds do exist for BMC [14].

4.1 Demonstration of CEGIW (RQ1)

To address **RQ1**, we use an example based on a model of a foraging robot swarm [25]. Robots are located in an arena and do a random walk to find food, which they then carry back to their home. Here they recharge and then repeat their foraging task. We model a single robot with a state machine, depicted abstractly in Fig. 1. In the concrete transition system $\mathcal{M}$ the robot can remain in a given state for a configurable amount of time before it is forced to move to a next state. The transition system is specified concretely using SMV, the language of the nuXmv model checker [11]. We would like to prove that, after leaving the `resting` state, the robot will return to `resting` in at most 3 time units, represented by

$$\mathcal{M} \vDash \square(\texttt{resting} \rightarrow \Diamond_{[1,3]}\texttt{resting}) \tag{16}$$

and translated from MTL to LTL as

$$\mathcal{M} \vDash \square(\texttt{resting} \rightarrow X(\texttt{resting} \vee X(\texttt{resting} \vee X(\texttt{resting})))). \tag{17}$$

If this does not hold in the transition system, we would like to weaken the interval to produce a new, weaker property that does hold. Using CEGIW, we find in

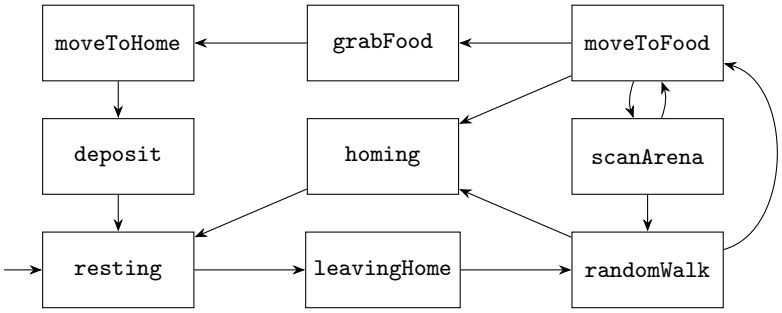


Fig. 1: Abstract state transition system for the robot's foraging behaviour. The robot begins in a resting state, then searches for, collects, and deposits food.

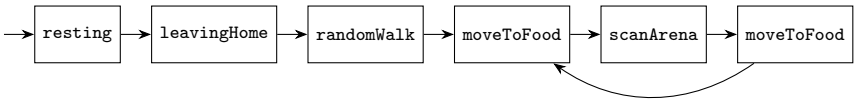


Fig. 2: Infinite lasso counterexample trace for the property in Eq. (16).

the first iteration that no suitable weakening of the interval exists based on the counterexample in Fig. 2, which shows an infinite loop between the `scanArena` and `moveToFood` states. This suggests a mistake in the modelling of the system, as in the real world the robot's battery would run out of charge. We amend the design by including in the requirements the notion of a battery that decreases as transitions are taken. While we are in `randomWalk`, `scanArena`, or `moveToFood` — in other words, searching for food — we monitor the battery level, and if it decreases below a certain threshold we abort and return home to recharge. The modified state transition system is depicted in Fig. 3. We check our desired property (Eq. (16)) against our amended model, and can see in Fig. 4a that in four iterations of CEGIW we extended the interval, and ended with the optimal interval which was then verified to hold in the system. So, the optimal property that holds in our amended system is

$$\mathcal{M} \vDash \square(\texttt{resting} \rightarrow \Diamond_{[1,20]}\texttt{resting}). \tag{18}$$

Another property we are interested in is not the maximum time that the robot can spend away from home, but the *minimum*. We wish the robot to spend at least 20 time units away from home, formalised as

$$\mathcal{M} \vDash \square((\texttt{resting} \wedge \Diamond_{[1,1]}\neg\texttt{resting}) \rightarrow \square_{[1,20]}\neg\texttt{resting}). \tag{19}$$

Again, we check this against our modified model and can see in Fig. 4b that it took only one iteration to contract the interval, reaching the optimal interval which was then verified to hold in the system as

$$\mathcal{M} \vDash \square((\texttt{resting} \wedge \Diamond_{[1,1]}\neg\texttt{resting}) \rightarrow \square_{[1,3]}\neg\texttt{resting}). \tag{20}$$

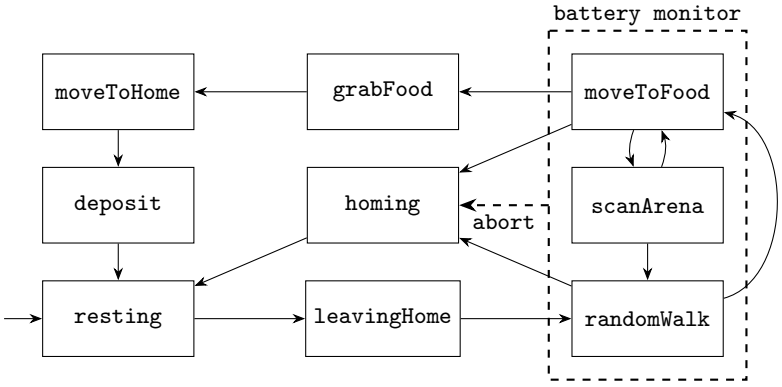
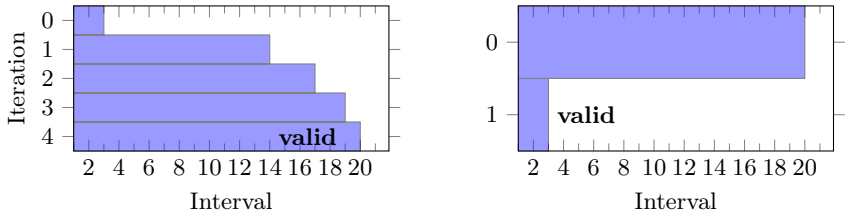


Fig. 3: Abstract state transition system for the robot's modified foraging behaviour. The battery monitor is represented by the dashed section on the right.



(a) Interval extension for Eq. (16). (b) Interval contraction for Eq. (19).

Fig. 4: Iterative interval weakening to generate optimal, valid intervals.

By using CEGIW, we first identified that the original specification had a design flaw that allowed unwanted infinite loops. While a traditional model checker would conclude that the given specification does not hold, it cannot itself deduce that *no* weakening exists. After modifying the specification, we then deduced both the maximum time that the robot can stay away from home, as well as the minimum time. CEGIW can thus provide value in both analysing existing requirements and supporting system modelling.

4.2 Applicability of Interval Weakening to Real-World Requirements (RQ2)

In this section, we analyse existing requirements from a number of real-world case studies to assess how often interval weakening is applicable, and how weakened timing bounds can be interpreted in their respective domains. These requirements are formalised using the Formal Requirements Elicitation Tool (FRET) [20] and are written in FRETISH, a structured natural language that can be translated to MTL [21]. FRETISH requirements can have a `timing` field, on which we can use interval weakening to weaken the requirement itself. The number of require-

Table 1: Interval-weakenable requirements in FRET case studies.

Case study	Total requirements	Weakenable requirements
Mechanical lung ventilator [16]	121	57
Autonomous drone [28]	62	19
Lift-plus-cruise aircraft [27]	49	29
Aircraft engine controller [17]	42	0
Inspection rover [8]	15	1
Grasping for debris removal [18]	20	0
Robotic patterns [29]	36	11
LMCP challenges [26]	74	7
Total	**419**	**124**

```
upon ControlLoopStart System shall
within 12 milliseconds satisfy
ControlLoopFinish
```

(a) Autonomous drone requirement REQ018 describing the maximum time the control loop can take to complete.

```
if powerFailure System shall for
120 minutes satisfy !off
```

(b) Mechanical lung ventilator requirement FUN37 describing how long the system must stay on after power failure.

Fig. 5: Example FRETISH requirements from the case studies. Both are taken from systems that are fully implemented and operational in real-world settings.

ments that can be weakened using interval weakening per case study is shown in Table 1. As an example, the requirement in Fig. 5a from the autonomous drone case study [28] uses the `within 12 milliseconds` timing which specifies that if the condition holds in one state, then the consequent must hold within the next twelve states (assuming that state transitions correspond to a millisecond of time passing). The MTL translation of this timing corresponds to the MTL temporal operator $\Diamond_{[0,12]}$, and so the requirement corresponds to

$$\Box(\texttt{ControlLoopStart} \rightarrow \Diamond_{[0,12]}(\texttt{ControlLoopFinish})). \tag{21}$$

Under system degradation, for example if the onboard communications network is degraded so that commands take longer to reach control surfaces, we may not be able to guarantee this and so would have to weaken the property by extending the interval, giving more time for the system to run its control loop, with an example weakening in

$$\Box(\texttt{ControlLoopStart} \rightarrow \Diamond_{[0,24]}(\texttt{ControlLoopFinish})). \tag{22}$$

An example requirement from the mechanical lung ventilator case study [16] is shown in Fig. 5b, and as the FRETISH timing `for 120 minutes` corresponds to the MTL temporal operator $\Box_{[1,120]}$, the corresponding MTL property is

$$\Box(\texttt{powerFailure} \rightarrow \Box_{[1,120]}(\neg\texttt{off})). \tag{23}$$

This is a regulatory requirement [23] and so, if it does not hold in the degraded system, it is critical to know by exactly how much it is violated. We may only be able to guarantee that the ventilator will stay on for at most 90 minutes after

powerFailure, producing the weakening

$$\square(\texttt{powerFailure} \rightarrow \square_{[1,90]}(\neg\texttt{off})). \tag{24}$$

Of the 127 interval-weakenable requirements in Table 1, 116 can be weakened by interval extension as in Eq. (22), and 11 by interval contraction as in Eq. (24).

Several case studies in Table 1 have few or no requirements that can be weakened with interval weakening. These requirements are typically liveness properties specified with the `eventually` timing, which cannot be weakened further, or safety properties specified with the `always` timing, for which interval weakening would not be appropriate. For example, from the grasping for debris removal case study [18],

```
SV shall always satisfy !collide(SV, TGT).
```
(25)

To answer **RQ2**, we have shown that interval weakening is applicable to a substantial proportion of existing temporal requirements, and that such weakenings have meaningful interpretations in safety-critical domains. We have also answered **RQ1** by using CEGIW to identify problems in a specification, and then deduce useful timing properties in the fixed system.

5 Conclusion

We present CEGIW, a novel algorithm for weakening intervals in MTL properties of degraded systems, and prove its correctness and optimality. We demonstrate how CEGIW can be used during the design phase to understand system limitations under degradation, and explore how the formalised requirements of a number of real-world systems may be weakened against real implementations. This shows the applicability of CEGIW in the design of safety-critical systems for understanding the impacts of system degradation.

Future work. A current limitation is that we only weaken on the right-hand-side of intervals, when both left- and right-bound modifications can produce valid weakenings. Restricting to only right-bound modifications creates a total order over the search space, so there is always a single optimum when multiple choices exist. Expanding to both left- and right-bound modifications creates a partial order over generated intervals, and so choosing between intervals is much less obvious. Future work will also explore other types of weakening, making syntactic changes to formulae beyond intervals.

Availability. The implementation of CEGIW, full proofs, extended pseudocode, and all case study artefacts are available in our public repository[1]. Scripts are provided to reproduce all tables and examples reported in Section 4.

References

[1] A. Abreu, N. Macedo, and A. Mendes. "Exploring Automatic Specification Repair in Dafny Programs". In: *International Conference on Automated Software Engineering Workshops*. 2023.

[1] https://github.com/benmandrew/CEGIW

[2] S. Akshay, P. Contractor, P. Gastin, R. Govind, and B. Srivathsan. *Efficient Verification of Metric Temporal Properties with Past in Pointwise Semantics*. https://arxiv.org/abs/2510.14699v1. 2025.
[3] R. Alur, R. Bodik, G. Juniwal, M. M. K. Martin, M. Raghothaman, S. A. Seshia, R. Singh, A. Solar-Lezama, E. Torlak, and A. Udupa. "Syntax-Guided Synthesis". In: *Formal Methods in Computer-Aided Design*. 2013.
[4] R. Alur and T. A. Henzinger. "Real-Time Logics: Complexity and Expressiveness". In: *Information and Computation* 104.1 (1993).
[5] R. Alur, S. Moarref, and U. Topcu. "Counter-Strategy Guided Refinement of GR(1) Temporal Logic Specifications". In: *Formal Methods in Computer-Aided Design*. 2013.
[6] B. M. Andrew. "Weakening Goals in Logical Specifications". In: *Rigorous State-Based Methods*. 2026.
[7] C. Baier and J.-P. Katoen. *Principles of Model Checking*. 2008.
[8] H. Bourbouh, M. Farrell, A. Mavridou, I. Sljivo, G. Brat, L. A. Dennis, and M. Fisher. "Integrating Formal Verification and Assurance: An Inspection Rover Case Study". In: *NASA Formal Methods*. 2021.
[9] T. Brihaye, G. Geeraerts, H.-M. Ho, A. Milchior, and B. Monmege. "Efficient Algorithms and Tools for MITL Model-Checking and Synthesis". In: *International Conference on Engineering of Complex Computer Systems*. 2018.
[10] M. Brizzio, M. Cordy, M. Papadakis, C. Sánchez, N. Aguirre, and R. Degiovanni. "Automated Repair of Unrealisable LTL Specifications Guided by Model Counting". In: *Genetic and Evolutionary Computation Conference*. 2023.
[11] R. Cavada, A. Cimatti, M. Dorigatti, A. Griggio, A. Mariotti, A. Micheli, S. Mover, M. Roveri, and S. Tonetta. "The nuXmv Symbolic Model Checker". In: *Computer Aided Verification*. 2014.
[12] J. Cerqueira, A. Cunha, and N. Macedo. "Timely Specification Repair for Alloy 6". In: *Software Engineering and Formal Methods*. 2022.
[13] E. Clarke, O. Grumberg, S. Jha, Y. Lu, and H. Veith. "Counterexample-Guided Abstraction Refinement". In: *Computer Aided Verification*. 2000.
[14] E. Clarke, D. Kroening, J. Ouaknine, and O. Strichman. "Completeness and Complexity of Bounded Model Checking". In: *Verification, Model Checking, and Abstract Interpretation*. 2004.
[15] J. M. Cobleigh, D. Giannakopoulou, and C. S. Păsăreanu. "Learning Assumptions for Compositional Verification". In: *Tools and Algorithms for the Construction and Analysis of Systems*. 2003.
[16] M. Farrell, M. Luckcuck, R. Monahan, C. Reynolds, and O. Sheridan. "FRETting and Formal Modelling: A Mechanical Lung Ventilator". In: *Rigorous State-Based Methods*. 2024.
[17] M. Farrell, M. Luckcuck, O. Sheridan, and R. Monahan. "FRETting About Requirements: Formalised Requirements for an Aircraft Engine Controller". In: *Requirements Engineering: Foundation for Software Quality*. 2022.

[18] M. Farrell, N. Mavrakis, A. Ferrando, C. Dixon, and Y. Gao. "Formal Modelling and Runtime Verification of Autonomous Grasping for Active Debris Removal". In: *Frontiers in Robotics and AI* 8 (2022).
[19] L. Gazzola, D. Micucci, and L. Mariani. "Automatic Software Repair: A Survey". In: *International Conference on Software Engineering*. 2018.
[20] D. Giannakopoulou, T. Pressburger, A. Mavridou, J. Rhein, J. Schumann, and N. Shi. "Formal Requirements Elicitation with FRET". In: *International Working Conference on Requirements Engineering: Foundation for Software Quality* (2020).
[21] D. Giannakopoulou, T. Pressburger, A. Mavridou, and J. Schumann. "Automated Formalization of Structured Natural Language Requirements". In: *Information and Software Technology* 137 (2021).
[22] G. J. Holzmann. "The Model Checker SPIN". In: *IEEE Transactions on Software Engineering* 23.5 (1997).
[23] ISO. *Particular Requirements for Basic Safety and Essential Performance of Critical Care Ventilators*. 80601-2-12. 2023.
[24] R. Koymans. "Specifying Real-Time Properties with Metric Temporal Logic". In: *Real-Time Systems* 2.4 (1990).
[25] W. Liu and A. F. T. Winfield. "Modeling and Optimization of Adaptive Foraging in Swarm Robotic Systems". In: *The International Journal of Robotics Research* 29.14 (2010).
[26] A. Mavridou, H. Bourbouh, D. Giannakopoulou, T. Pressburger, M. Hejase, P.-L. Garoche, and J. Schumann. "The Ten Lockheed Martin Cyber-Physical Challenges: Formalized, Analyzed, and Explained". In: *International Requirements Engineering Conference*. 2020.
[27] T. Pressburger, A. Katis, A. Dutle, and A. Mavridou. "Authoring, Analyzing, and Monitoring Requirements for a Lift-Plus-Cruise Aircraft". In: *Requirements Engineering: Foundation for Software Quality*. 2023.
[28] O. Sheridan, L. B. Becker, M. Farrell, M. Luckcuck, and R. Monahan. "Sharper Specs for Smarter Drones: Formalising Requirements with FRET". In: *Requirements Engineering: Foundation for Software Quality*. 2025.
[29] G. Vázquez, A. Mavridou, M. Farrell, T. Pressburger, and R. Calinescu. "Robotics: A New Mission for FRET Requirements". In: *NASA Formal Methods*. 2024.