Program Design

Invasion Percolation: Tuning

Our program works correctly

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At least, it passes all our tests

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Next step: figure out if we need to make it faster

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Next step: figure out if we need to make it faster

Spend time on other things if it's fast enough

How do we measure a program's speed?

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Average of many running times on various grids

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How do we *predict* running time on bigger grids?

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Use *asymptotic analysis*

How do we measure a program's speed?

Average of many running times on various grids

How do we *predict* running time on bigger grids?

Use *asymptotic analysis*

One of the most powerful theoretical tools in computer science

5	3	7	2	6	1	1	3	4
8	5	6	5	7	2	3	6	2
2	5	8	7	5	5	6	5	9
5	2	6	4	9	3	9	6	5
4	6	8	8	5	9	7	3	9
7	6	4	5	1	2	6	8	5
5	4	2	5	8	5	5	5	8
5	7	5	1	5	3	8	5	5
4	5	1	9	7	8	6	5	1

N

N

$N \times N$ grid has

N^2 cells

5	3	7	2	−1	1	1	3	4
8	5	6	5	−1	2	3	6	2
2	5	8	7	−1	5	6	5	9
5	2	6	4	−1	3	9	6	5
4	6	8	8	−1	9	7	3	9
7	6	4	5	1	2	6	8	5
5	4	2	5	8	5	5	5	8
5	7	5	1	5	3	8	5	5
4	5	1	9	7	8	6	5	1

N

N

$N \times N$ grid has

N^2 cells

Must fill at least N

to reach boundary

5	3	7	2	−1	1	1	3	4
8	−1	−1	−1	−1	−1	−1	−1	2
2	−1	−1	−1	−1	−1	−1	−1	9
5	−1	−1	−1	−1	−1	−1	−1	5
4	−1	−1	−1	−1	−1	−1	−1	9
7	−1	−1	−1	−1	−1	−1	−1	5
5	−1	−1	−1	−1	−1	−1	−1	8
5	−1	−1	−1	−1	−1	−1	−1	5
4	5	1	9	7	8	6	5	1

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$N \times N$ grid has

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Must fill at least N

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Can fill at most

$(N-2)^2+1 = N^2-4N+5$

5	3	7	2	−1	1	1	3	4
8	−1	−1	−1	−1	−1	−1	−1	2
2	−1	−1	−1	−1	−1	−1	−1	9
5	−1	−1	−1	−1	−1	−1	−1	5
4	−1	−1	−1	−1	−1	−1	−1	9
7	−1	−1	−1	−1	−1	−1	−1	5
5	−1	−1	−1	−1	−1	−1	−1	8
5	−1	−1	−1	−1	−1	−1	−1	5
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N

N

$N \times N$ grid has

N^2 cells

Must fill at least N

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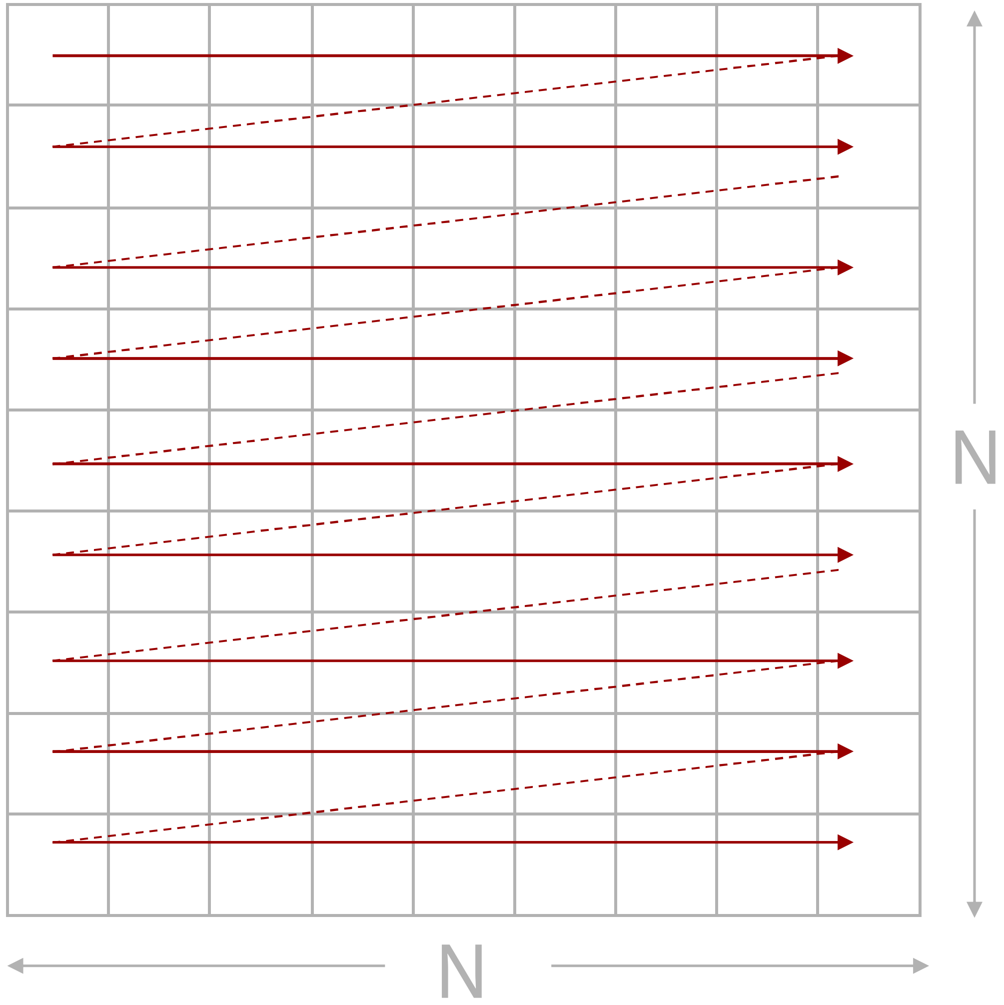
Can fill at most

$(N-2)^2+1 = N^2-4N+5$

For large N, this is

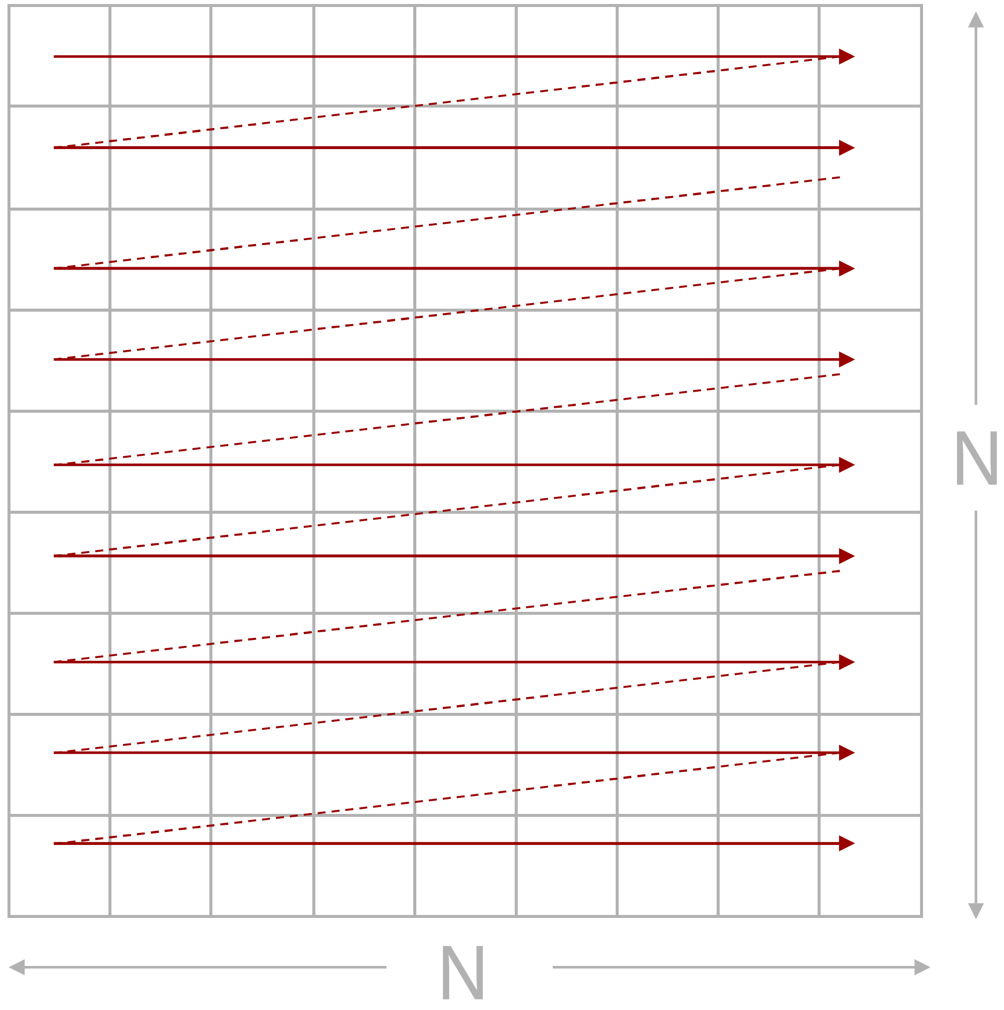
approximately N^2



Program looks at

N^2 cells to find the

next cell to fill



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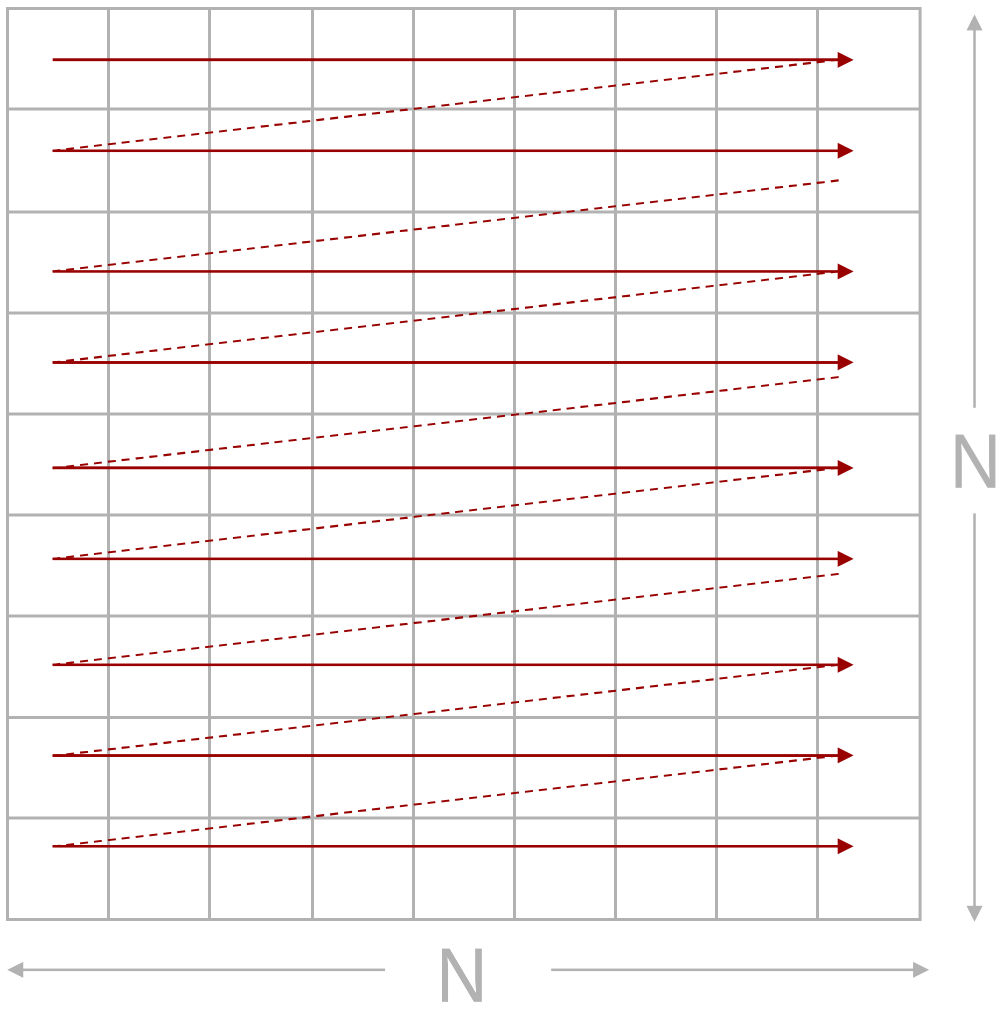

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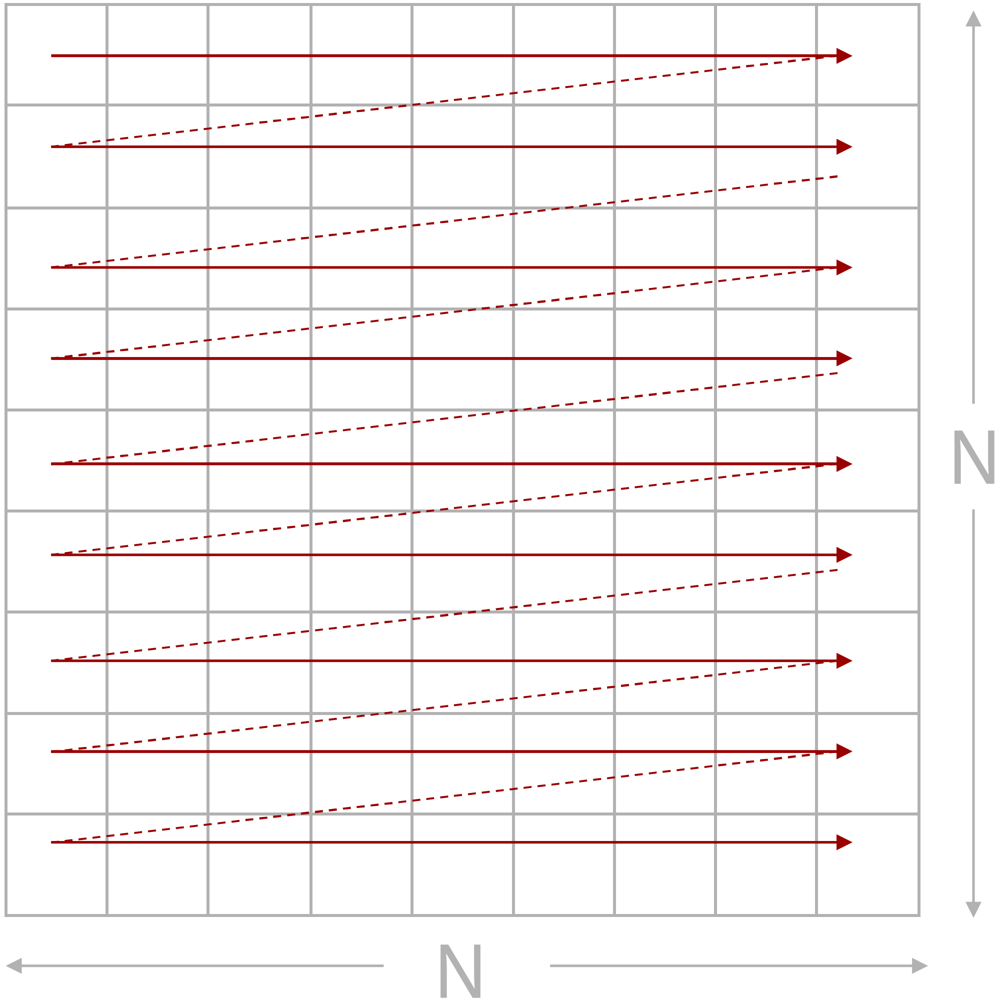
Worst case: $N^2 \cdot N^2$ or N^4 steps



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Ouch

Averaging exponent to $N^{3.5}$ doesn't make sense...

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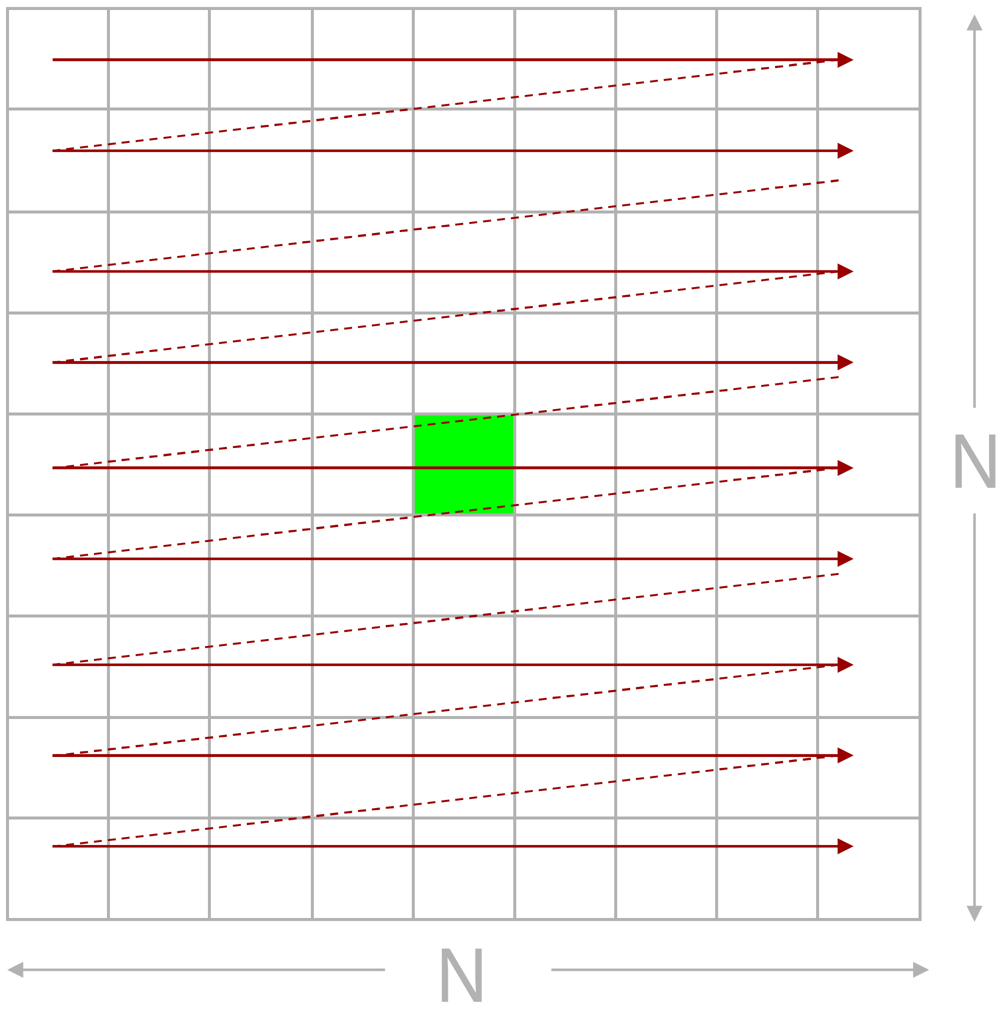
Grid Size	Time
N	T
2N	11.3 T
3N	46.7 T
4N	128 T

Averaging exponent to $N^{3.5}$ doesn't make sense...

...but it will illustrate our problem

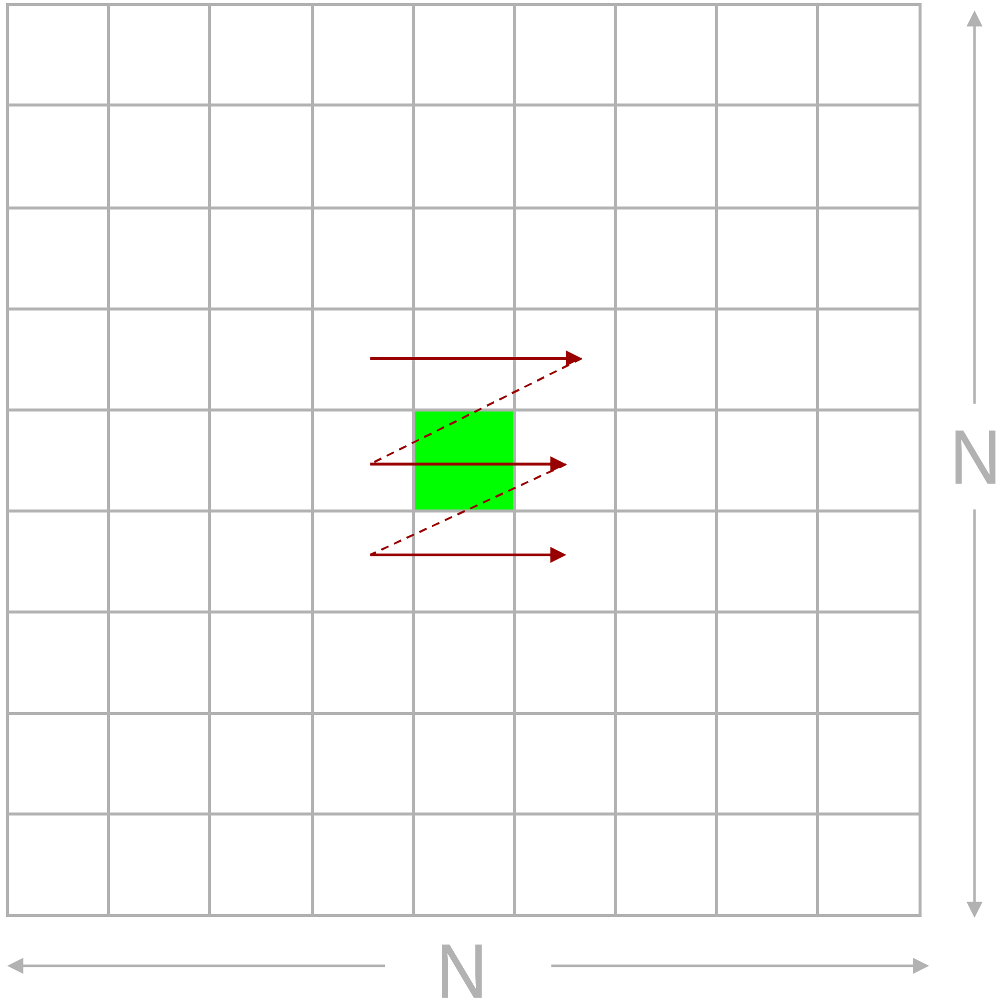
Grid Size	Time	Which Is...
N	T	1 sec
2N	11.3 T	11 sec
3N	46.7 T	47 sec
4N	128 T	2 min
10N	3162 T	52 min
50N	883883 T	1 day
100N	10^7 T	115 days



Idea #1:

Why are we looking

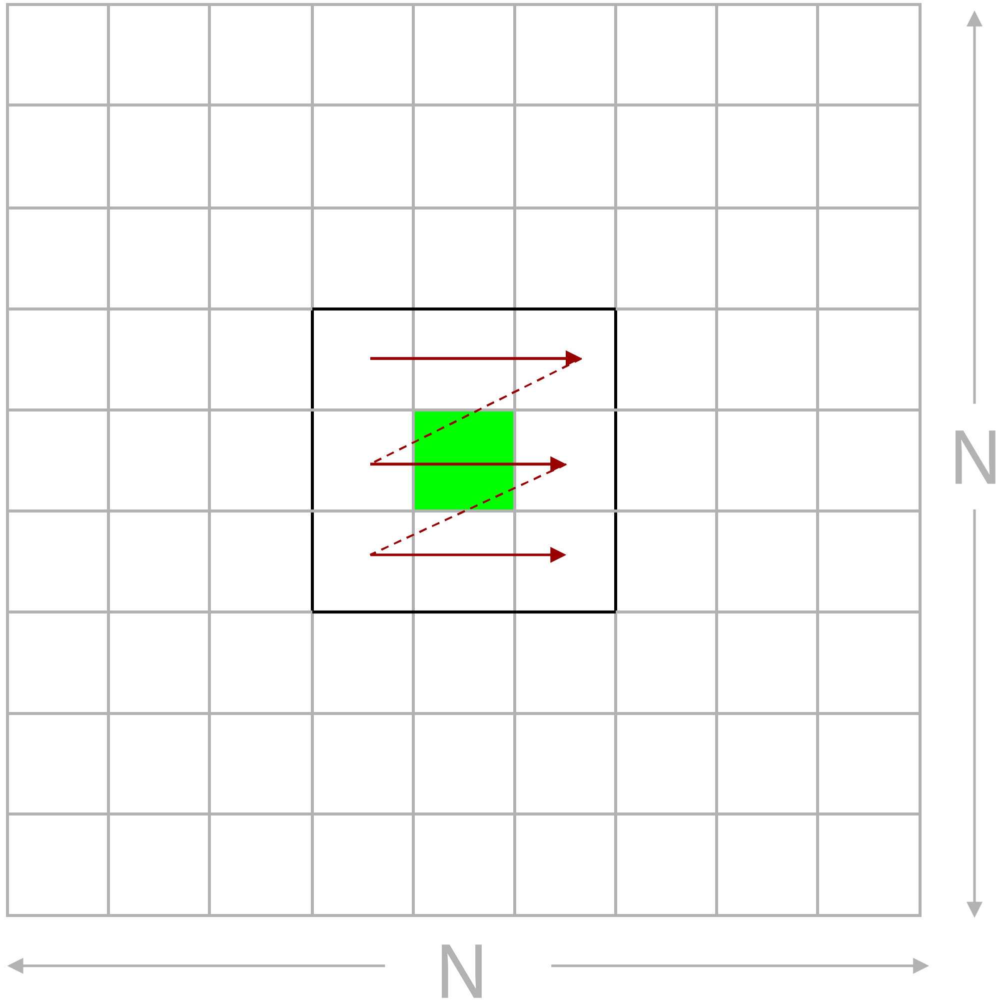
at all the cells?



Idea #1:

Why are we looking at all the cells?

Just look at cells that *might* be adjacent

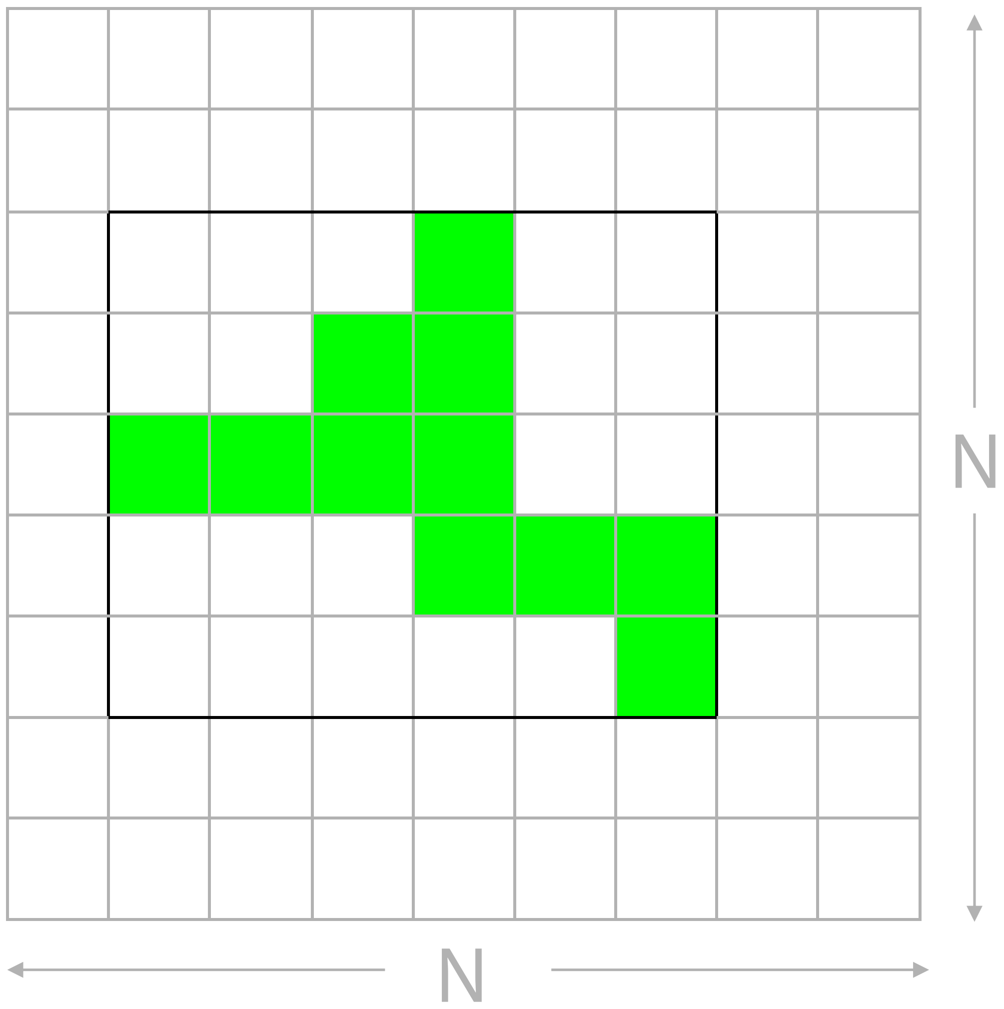


Idea #1:

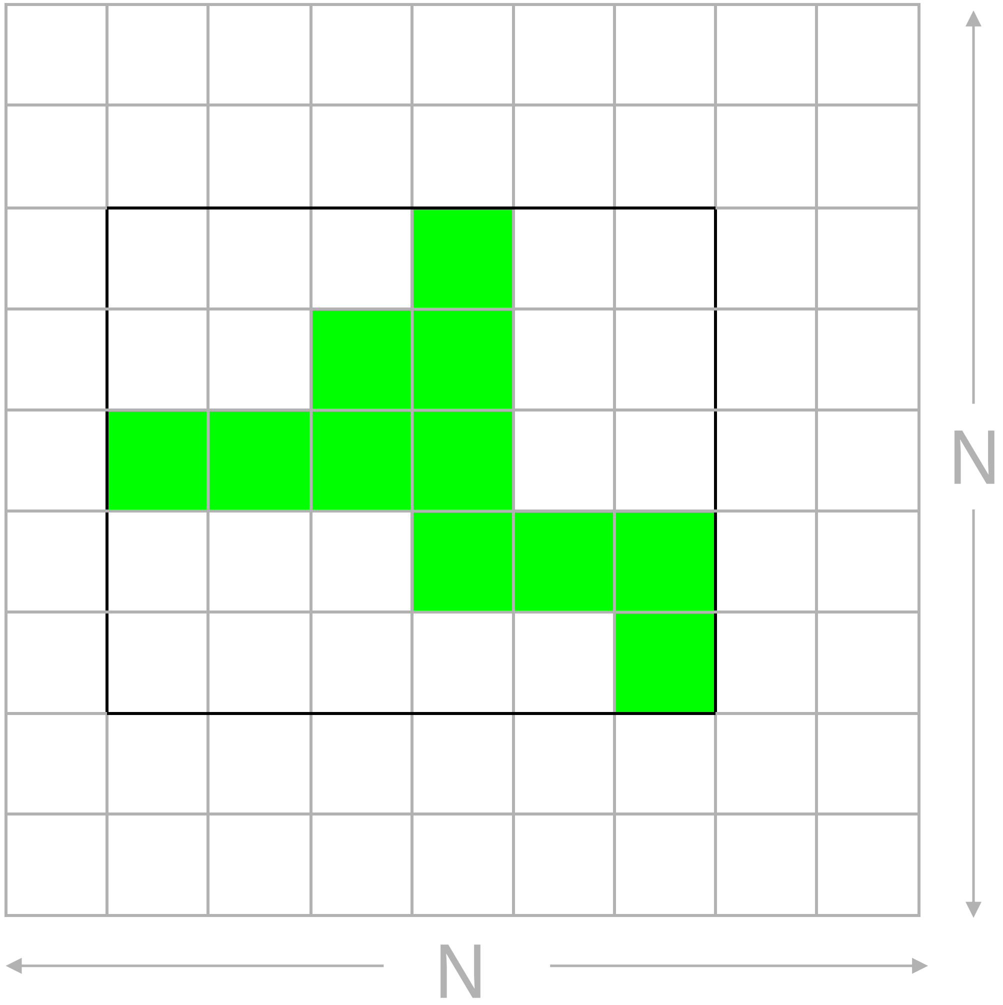
Why are we looking at all the cells?

Just look at cells that *might* be adjacent

Keep track of min and max X and Y and loop over those



But on average, the filled region is half the size of the grid



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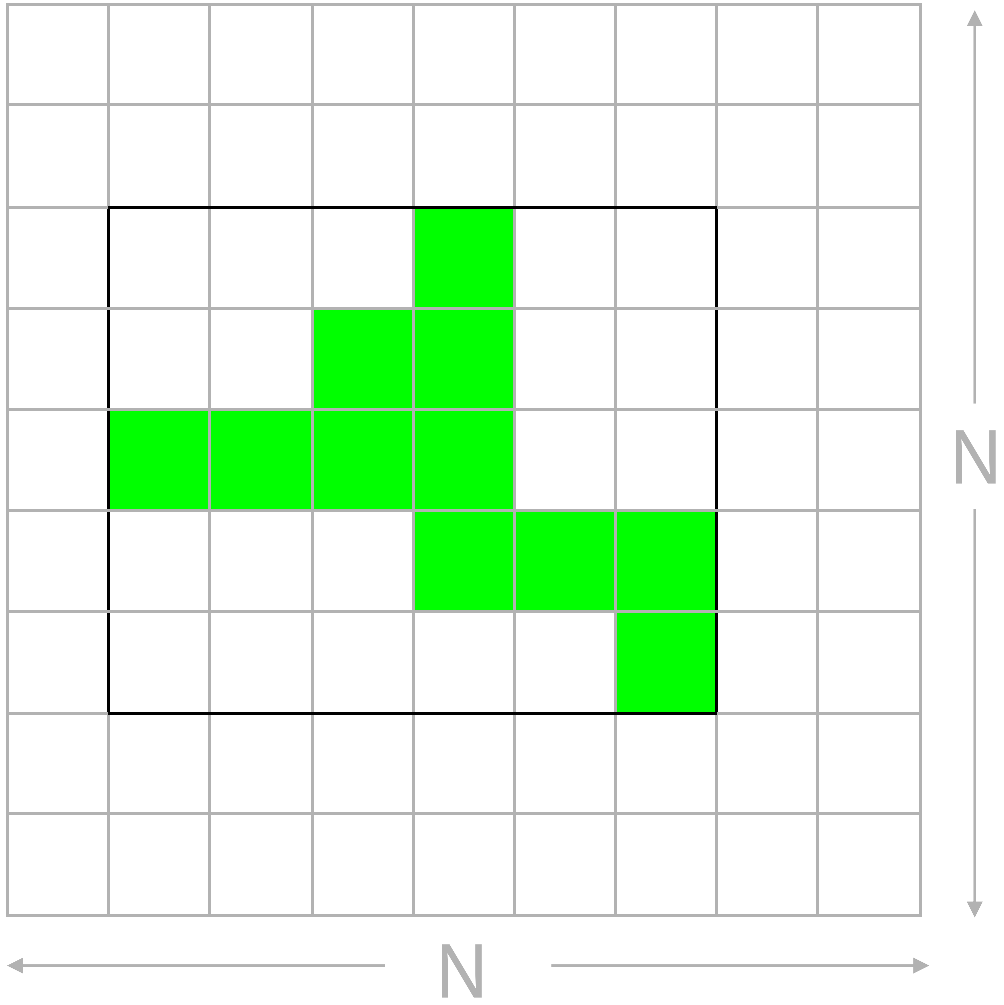
$N/2 \cdot N/2 = N^2/4$



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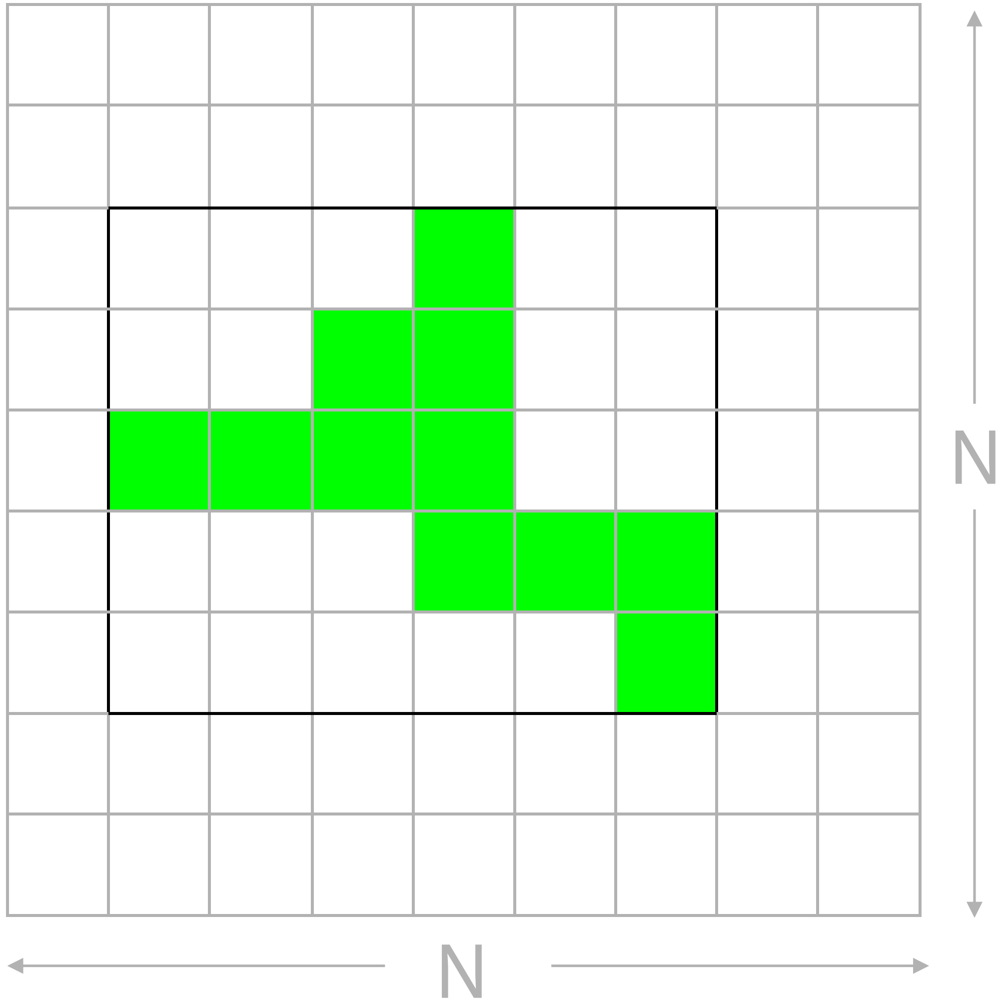
115 days → 29 days



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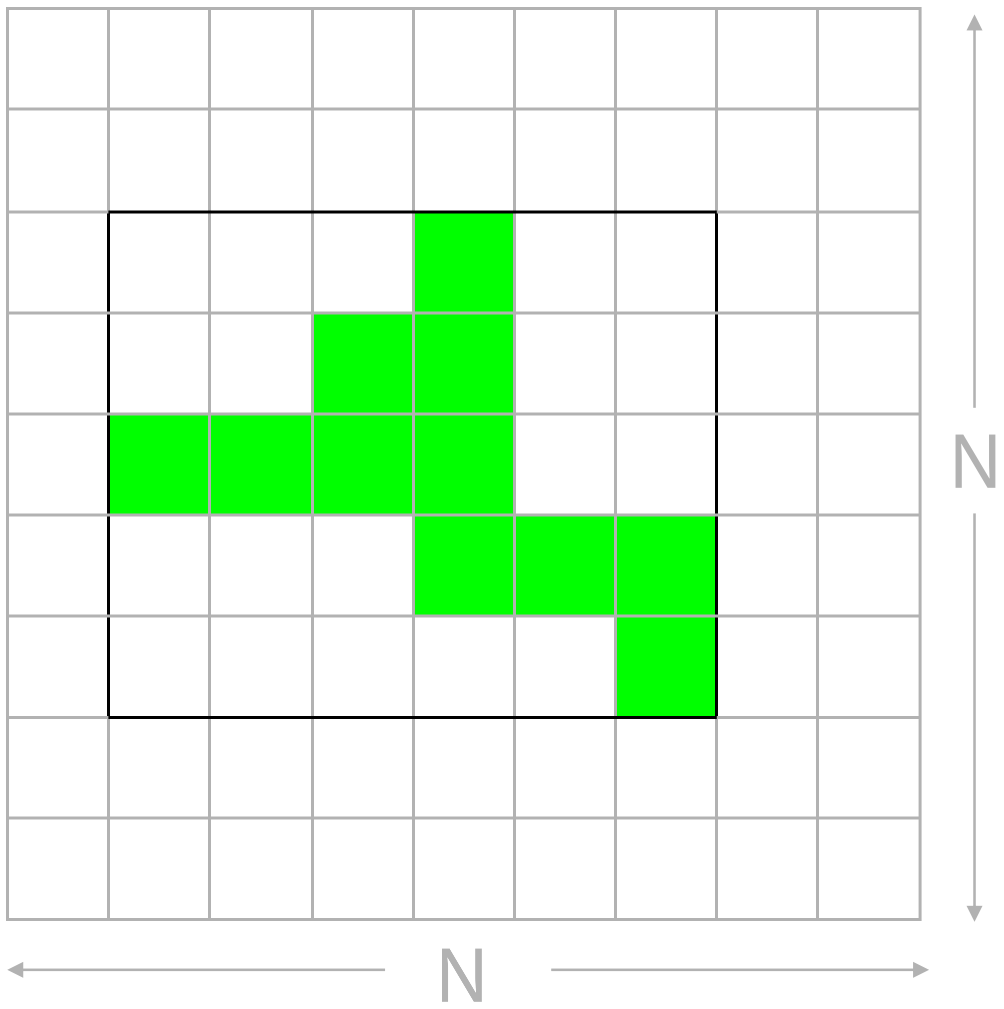
148N×148N grid eats that up



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$N/2 \cdot N/2 = N^2/4$

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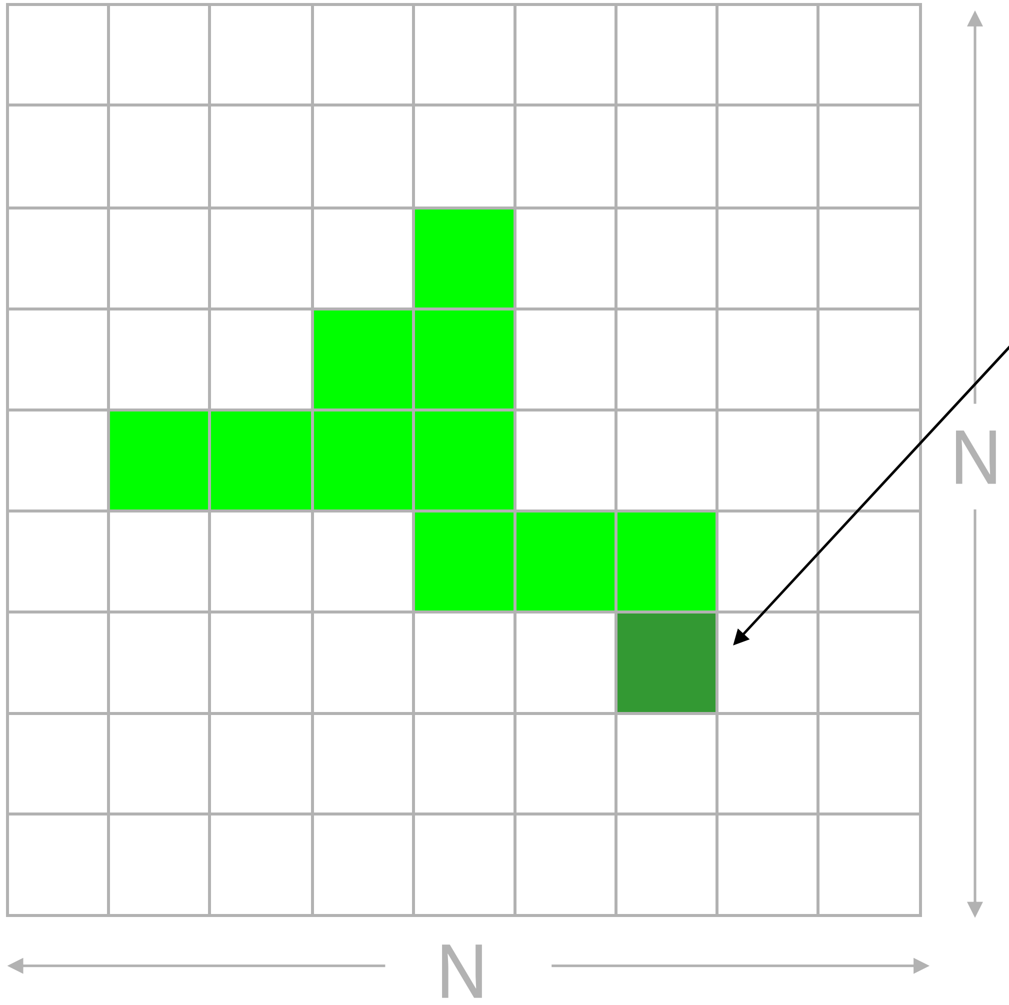
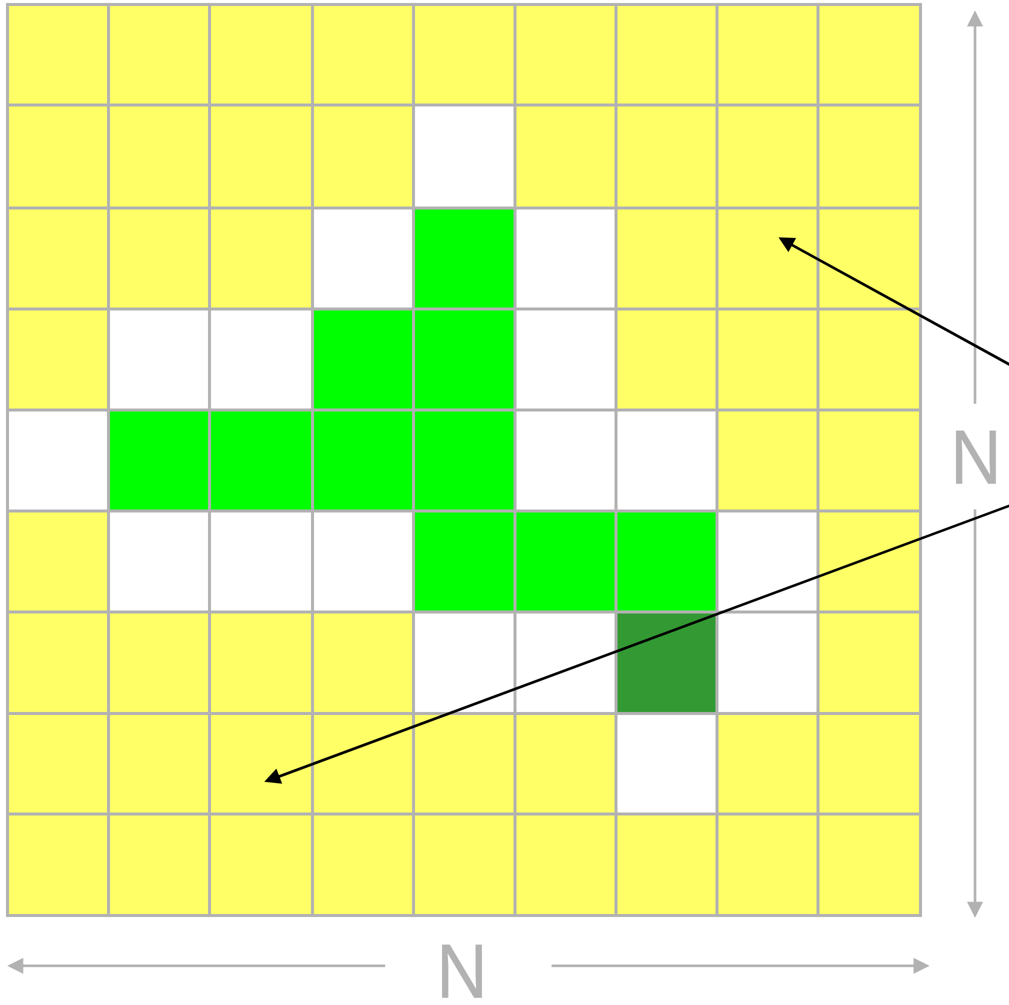
148N×148N grid eats that up

Need to attack the *exponent*

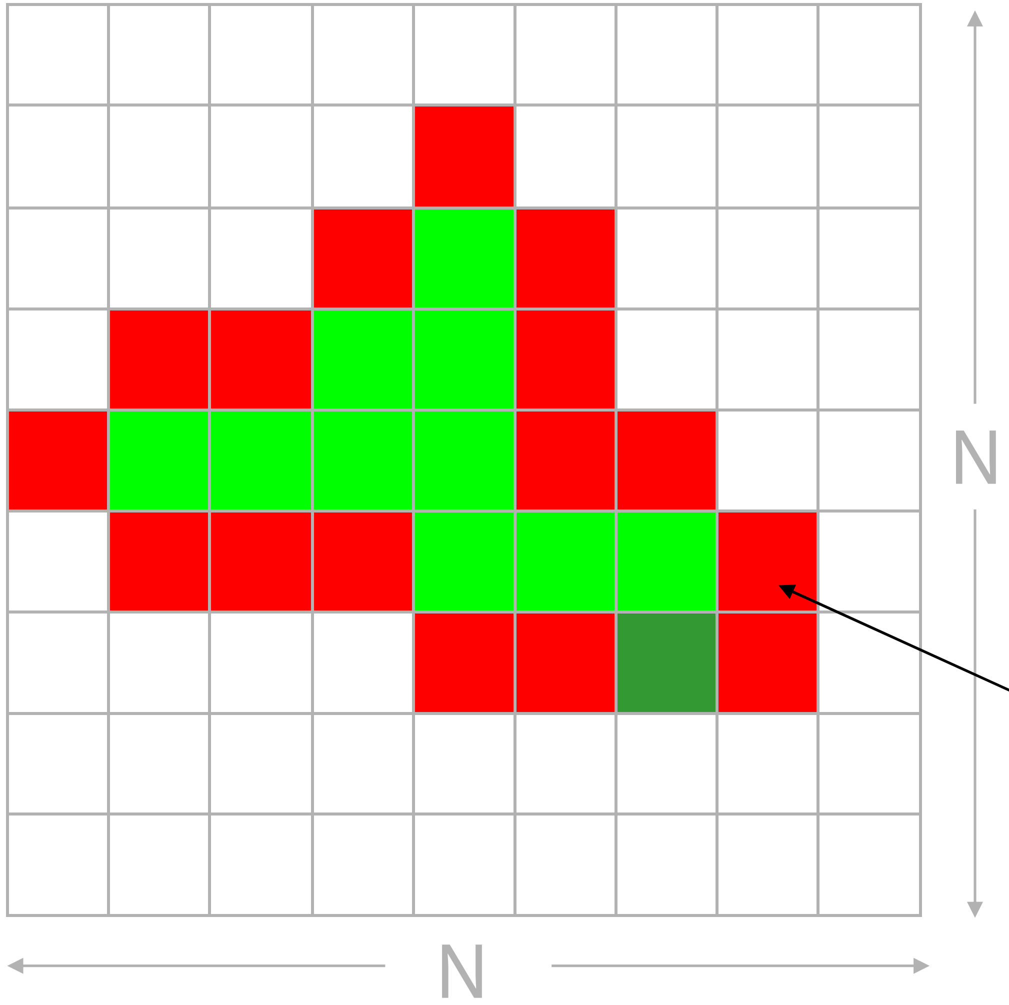


If we just added this cell...



If we just added this cell...

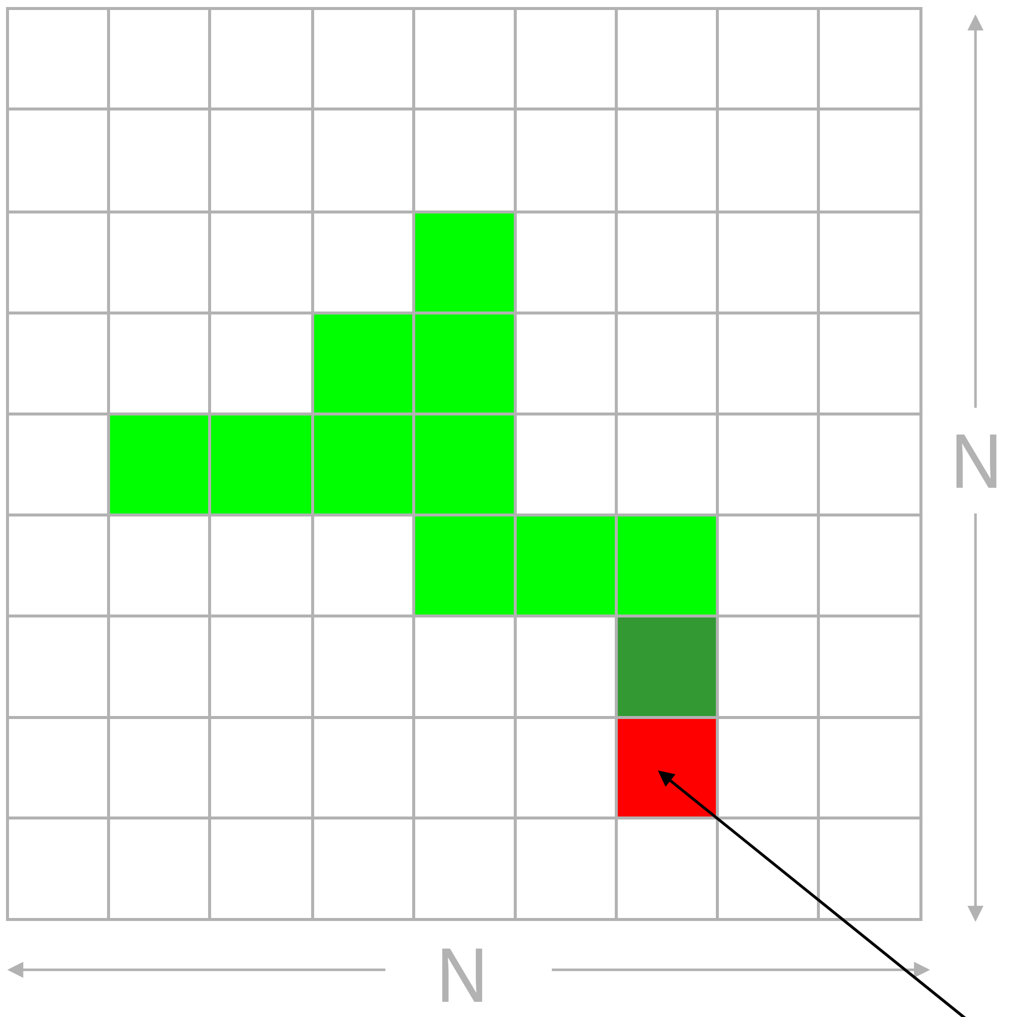
...why check these cells again?



If we just added this cell...

...why check these cells again?

We should already know that these are candidates



If we just added this cell...

...why check these cells again?

We should already know that these are candidates

This is the only new candidate cell

Big idea: trade memory for time

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Record redundant information in order to save re-calculation

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Each time a cell is filled, check its neighbors

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Each time a cell is filled, check its neighbors

If already in the list, do nothing

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Otherwise, add to list

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Insert into list so that low-valued cells at front

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Each time a cell is filled, check its neighbors

If already in the list, do nothing

Otherwise, ~~add~~ to list

Insert into list so that low-valued cells at front

Making it easy to choose next cell to fill

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	5	9	7	3	9	4
7	6	4	5	1	2	6	8	5	3
5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

List of cells on edge is initially empty

```
edge = []
```

	0	1	2	3	4	5	6	7	8	
	5	3	7	2	6	1	1	3	4	8
	8	5	6	5	7	2	3	6	2	7
	2	5	8	7	5	5	6	5	9	6
	5	2	6	4	9	3	9	6	5	5
	4	6	8	8	-1	9	7	3	9	4
	7	6	4	5	1	2	6	8	5	3
	5	4	2	5	8	5	5	5	8	2
	5	7	5	1	5	3	8	5	5	1
	4	5	1	9	7	8	6	5	1	0

List of cells on edge is initially empty

Fill center cell and add its neighbors

```
edge = [(1,4,3), (8,3,4), (9,4,5), (9,5,4)]
```

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	1	2	6	8	5	3
5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
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Fill center cell and add its neighbors

value

```
edge = [(1,4,3), (8,3,4), (9,4,5), (9,5,4)]
```

coordinates

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	-1	2	6	8	5	3
5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

Take first cell from list and fill it

```
edge = [(8,3,4), (9,4,5), (9,5,4)]
```

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	-1	2	6	8	5	3
5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

Take first cell from list and fill it

Add its neighbors to the list

```
edge = [(2,5,3), (5,3,3), (8,4,2), (8,3,4),
        (9,4,5), (9,5,4)]
```

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	-1	-1	6	8	5	3
5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

Take, fill, and add again

```
edge = [(5,3,3), (5,5,2), (6,6,3), (8,4,2), (8,3,4),
          (9,4,5), (9,5,4)]
```

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
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5	4	2	5	8	5	5	5	8	2
5	7	5	1	5	3	8	5	5	1
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Take, fill, and add again

Don't re-add cells that are already in the list

```
edge = [(5,3,3), (5,5,2), (6,6,3), (8,4,2), (8,3,4),
        (9,4,5), (9,5,4)]
```

0	1	2	3	4	5	6	7	8	
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In case of ties, find all cells at the front of the list with the lowest value...

```
edge = [(5,3,3), (5,5,2), (6,6,3), (8,4,2), (8,3,4),
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4	6	8	8	-1	9	7	3	9	4
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5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

In case of ties, find all cells at the front of the list with the lowest value...

...and select and fill one of them

```
edge = [(3,5,1), (5,3,3), (5,5,2), (6,6,3), (8,4,2),
        (8,3,4), (9,4,5), (9,5,4)]
```

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```
def insert(values, new_val):
  '''Insert (v, x, y) tuple into list in right place.'''
  i = 0
  while i < len(values):
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Works even if list is empty...

...or new value greater than all values in list

If list has K values…

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...insertion takes on average K/2 steps

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Not much of an improvement on $N^{3.5}$

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But there's a much faster way to insert

To look up a name in the phone book...

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...open it in the middle

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If the name there comes after the one you want,

go to the middle of the first half

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Then repeat this procedure in that half

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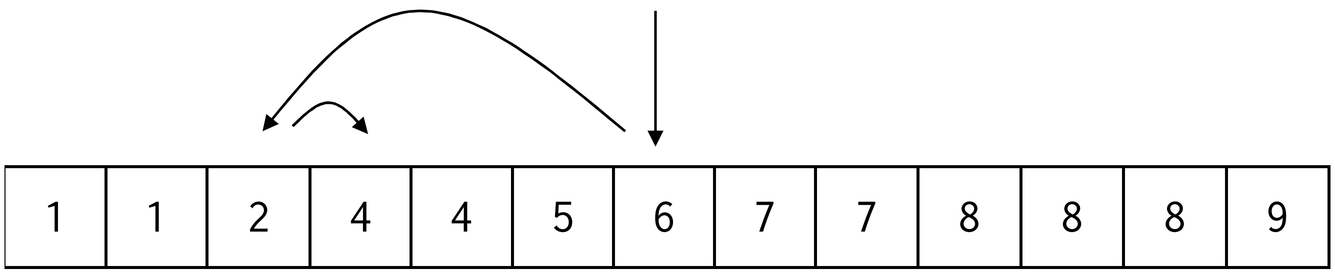
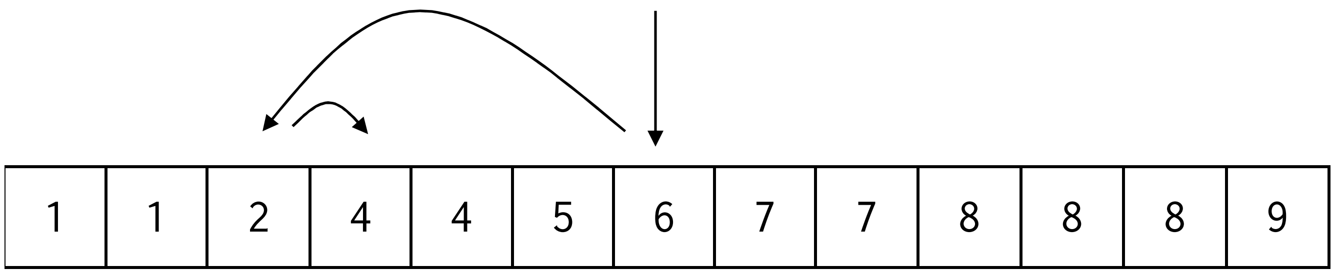


How fast is this?



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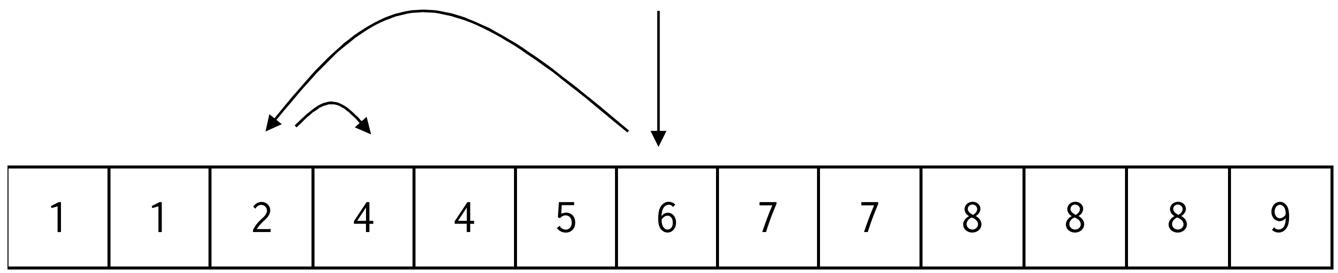
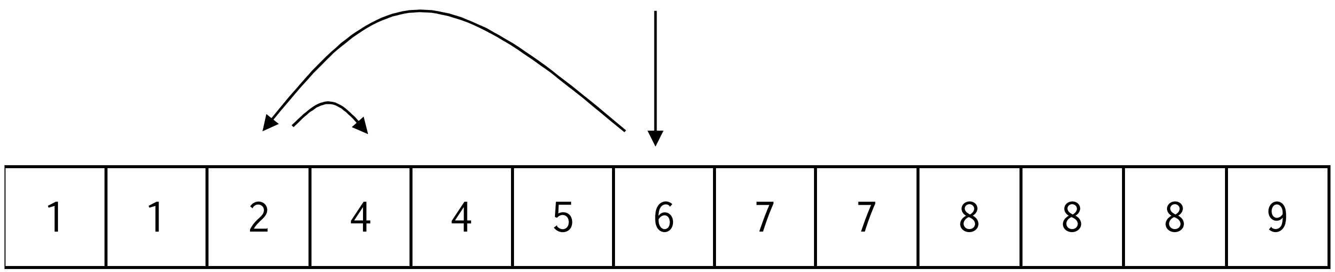
One probe can find one value



How fast is this?

One probe can find one value

Two probes can find one value among two (2^1)



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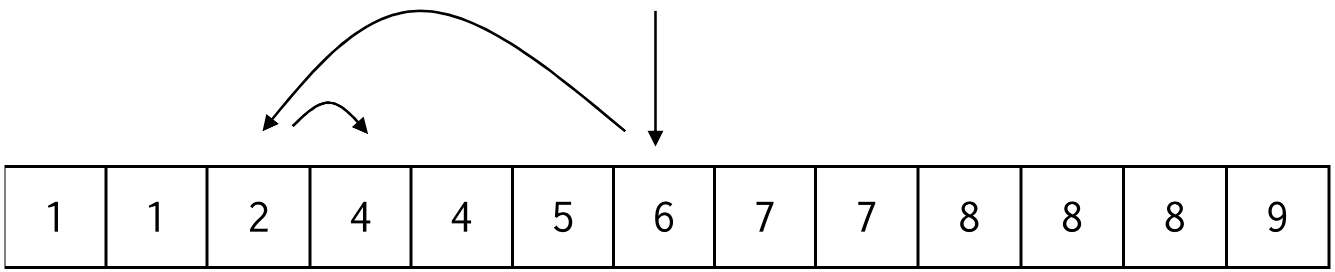
Two probes can find one value among two (2^1)

Three probes can find one value among four (2^2)



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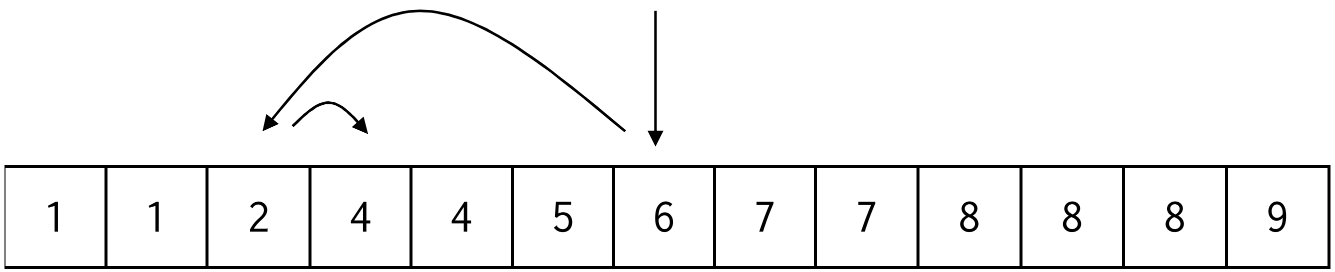

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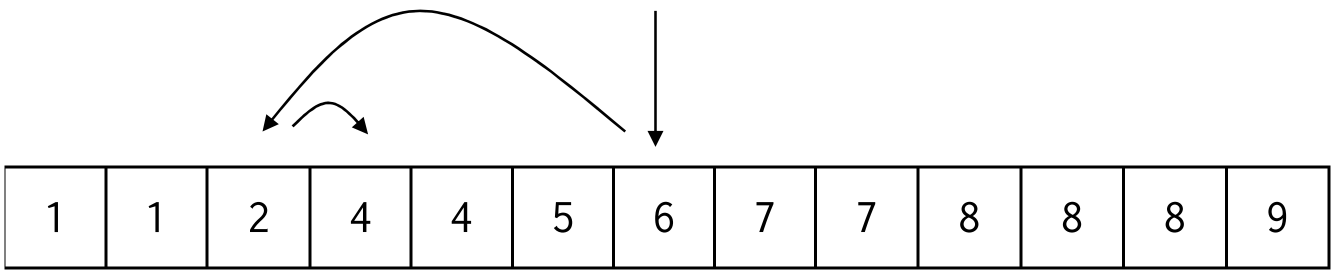

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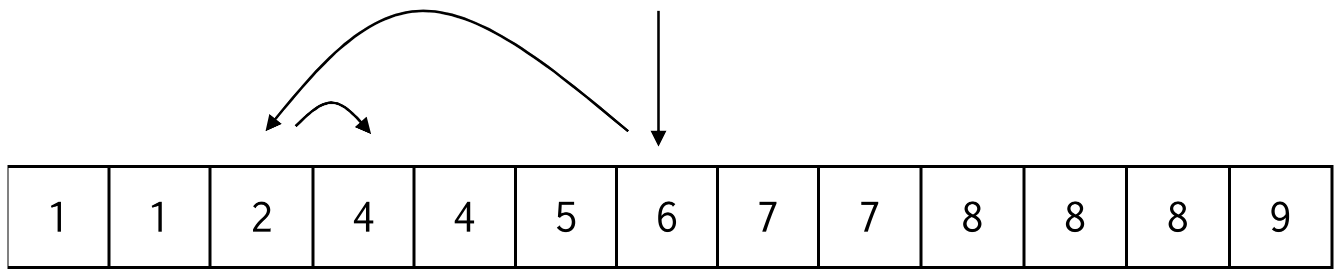
$\log_2(N)$ probes: one among N values



0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	-1	-1	6	8	5	3
5	4	2	5	8	-1	5	5	8	2
5	7	5	1	5	3	8	5	5	1
4	5	1	9	7	8	6	5	1	0

Running time is

$N^{1.5} \cdot \log_2(N^{1.5})$

1	1	2	4	4	5	6	7	7	8	8	8	9

0	1	2	3	4	5	6	7	8	
5	3	7	2	6	1	1	3	4	8
8	5	6	5	7	2	3	6	2	7
2	5	8	7	5	5	6	5	9	6
5	2	6	4	9	3	9	6	5	5
4	6	8	8	-1	9	7	3	9	4
7	6	4	5	-1	-1	6	8	5	3
5	4	2	5	8	-1	5	5	8	2
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4	5	1	9	7	8	6	5	1	0

Running time is

$N^{1.5} \cdot \log_2(N^{1.5})$

Or $N^{1.5}\log_2(N)$ if we get rid of constants

1	1	2	4	4	5	6	7	7	8	8	8	9

That changes things quite a bit

Grid Size	Old Time	Which Was...	New Time	Which Is...
N	T	1 sec		
2N	11.3 T	11 sec	2.8 T	3 sec
3N	46.7 T	47 sec	8.2 T	8 sec
4N	128 T	2 minutes	16 T	16 sec
10N	3162 T	52 minutes	105 T	2 min
50N	883883 T	1 day	1995 T	33 min
100N	10^7 T	115 days	6644 T	2 hours

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And the gain gets bigger as N increases

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Python implementation is in `bisect` library

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5	4	2	5	8	5	5	5	8
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Generating the grid

takes N^2 steps

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5	3	7	2	6	1	1	3	4
8	5	6	5	7	2	3	6	2
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5	2	6	4	9	3	9	6	5
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If we fill these cells...

We can do even better

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Generating the grid

takes N^2 steps

If we fill these

cells...

...we only ever look

at these cells...

We can do even better

				9				
			8	−1	9			
			5	−1	−1	6		
		2	5	8	−1	−1	5	
	7	−1	−1	−1	−1	8		
	5	−1	9	7	8	6		

Generating the grid takes N^2 steps

If we fill these cells...

...we only ever look at these cells...

...so why bother generating values for these ones?

Store grid as a dictionary

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```

And of course use `set_value(grid, x, y, V)` as well

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Trading human time for machine performance

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Difference is *not* $N^{0.5}$

As N gets large, $N^2 - N^{1.5} \approx N^2$

I.e., without this change, total runtime would be

$N^2 + N^{1.5}\log_2(N) \approx N^2$

Moral of the story:

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Biggest performance gains always come from changing algorithms and data structures

The story's other moral:

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Write and test a simple version first,

then improve it piece by piece

(re-using the tests to check your work)

created by

Greg Wilson

June 2010