Complexity of quantum dynamics: Where are we with RUCs?

Benjamin McDonough

(Dated: 5 May 2025)

1. INTRODUCTION

In 2019, Google shocked the scientific community with a splashy article in Nature declaring that it's 53-qubit quantum computer had done something that would take a state-of-the-art classical computer over 10 millenia[1]. The claim was quickly refuted by a team at IBM, arguing that an algorithm based on tensor network contraction deferral would simulate the same circuits on Summit, the supercomputer at Oak Ridge National Lab, in only 2.5 days[2]. To add to this debate, a new paper came out demonstrating that short-depth noisy circuits could be efficiently simulated using tensor network methods, becoming ArXiv's most cited paper of 2020[3]. Despite intending to address Google's experiment, the method this paper introduced was not able to simulate the level of entanglement achieved by Google. Two years later, another paper was released demonstrating a tensor-network algorithm on a classical computer that beat the fidelity of Google's quantum computer with a runtime of only 5 days on a small cluster of 60 GPUs[4]. Shortly thereafter, a team led by Jian-Wei Pan performed a similar experiment on Zuchongzhi, a superconducting processor with 60 qubits, completing a computation with updated estimated classical runtime of 5 millenia[5]. But despite the fact that its larger size put it comfortably out of range of previous classical algorithms, the declaration of quantum supremacy remained controversial.

Quantum supremacy experiments continue to be plagued by competition from their classical counterparts, using clevery-divised algorithms that exploit low fidelities, sample complexity, and limited entanglement to overturn the community lore on what is a computationally hard problem. Notably, IBM's simulation of a kicked Ising model on a 127-qubit processor claimed to demonstrate a near-term computation advantage[6], but this work was quickly (and somewhat embarrassingly) debunked by a team that managed to produce near-exact results in 3 seconds on a single CPU[7], and another algorithm which is able to efficiently simulate the circuits to late times on an arbitarily large system using computational power equivalent to a mobile phone[8].

Despite these cautionary tales, it seems that the journal Nature will always be willing to throw caution to the wind when it comes to quantum supremacy. Two years after the Zuchongzhi experiment, Google published a new result using dual-unitary circuits to observe finite-size signatures of a measurement-induced transition between an area-law and volume-law entangling phase on 70 qubits[9], although the phrase "quantum supremacy" appears nowhere in the paper. Then just last year, Google published another paper claiming to observe a noise-induced phase transition, accompanied by a stronger claim that this problem is outside the reach of classical computational power[10].

It seems like the perfect time to step back and take stock of the consensus on the complexity of RUCs; computational complexity is an incredibly nuanced question. It is now established that noisy circuits can be efficiently simulated classically, but with an overhead that may still be prohibitively large[11,12]. Circuits sporting low-entanglement are efficiently simulable on current computers by MPO-type algorithms[3,13]. One must also ensure that an observer could distinguish between the output of the quantum and classical computer with only a polynomial number of samples. This paper will attempt to address the state-of-the-art understanding of the hardness of the random unitary circuit sampling problem and its application to current experiments.

2. COMPLEXITY THEORY OF RUC SAMPLING

The core of quantum complexity theory is the conjecture that quantum computing violates the Extended Church-Turing Thesis (ECTT), which holds that a probabilistic Turing machine can simulate any reasonable model of computation with at most polynomial overhead. If quantum simulation is hard, then this would violate the ECTT. While there are certainly things that happen in nature that seem hard to simulate, in order to precisely understand the gap in complexity, this hardness needs to extend to a well-formulated class of problems. RUC sampling is an eminently reasonable problem, because it is the most generic quantum circuit one could execute on a quantum computer, and quantum computers are certainly good at simulating themselves.

2.1. HOG and cross-entropy

First, we will discuss the work of Aaronson et al., which helped guide the early quantum supremacy experiments[14]. Due to the Holevo bound on the information of a single quantum measurement, despite the dynamical complexity, each shot in an RUC experiment returns at most 1 bit of information. This leaves open the following question: Could a classical algorithm fool an observer who only has a polynomial amount of time to compare ther results? Remarkably, the sampling problem can be reasonably related to the problem of computing the probability amplitudes. First, the problem needs to be precisely defined:

Definition 1 (Heavy Output Generation (HOG)).

Given a circuit C, generate k output strings at least $\frac{2}{3}$ of which have greater than the median probability in the output distribution $p_C(n) = |\langle 0|C|n\rangle|^2$.

Definition 2 (Quantum Threshhold Assumption (QUATH))**.** *There is no polynomial-time classical algorithm which takes an input circuit C on N qubits and guesses whether $p_C(0)$ is larger than the median of $\{p_C(n)\}_n$ with success probability greater than $\frac{1}{2} + \Omega(2^{-N})$.*

Clearly, random guessing would achieve a success probability of $\frac{1}{2}$. The notation Ω means "asymptotically lower-bounded by a constant multiple of". This assumption seems reasonable but non-obvious, given that $\Omega(2^{-N})$ is a vanishingly small difference to separate complexity classes. It is straightforward to see that a Schrodinger or Feynman simulation of the evolution is harder than 2^N, so neither violate QUATH, and Aaronson attempts to create an algorithm that thwarts it, to no avail. A quantum computer is immediately well-suited to this problem:

Proposition 1. *A quantum computer can solve HOG with probability $1 - \exp(-\Omega(k))$.*

The proof idea is quite elegant but some of the details are tedious, so we will sketch it; because the Porter-Thomas distribution has heavy tails, each bitstring sampled is likely to be more probable than the median probability. By a Chernoff bound, the probability that less than $\frac{2}{3}$ of them are heavy decreases exponentially in the number of sampled bitstrings. This way, raw sampling of bitstrings from a noiseless quantum computer solves HOG. By contrast,

Proposition 2. *Assuming QUATH, then there is no classical algorithm which solves HOG in polynomial time with 99% probability.*

The proof is by contradiction; If such an algorithm A did exist, then pick a random bitstring z and sample k outputs. Picking one of these outputs z_i at random, the probability that it is heavy is $.99 \times \frac{2}{3}$. Comparing this with the initial guess z, we simply check if $z = z_i$, in which case we guess that z is in fact heavy, otherwise, we guess it is not. It is easy to show that this algorithm succeeds with probability $\frac{1}{2}+\Omega(2^{-N})$, violating QUATH.

This early work began laying the foundations for RUC experiments, but the assumption of the hardness of QUATH is quite strong. To remedy this, Bouland et al. published another result in advance of the Google experiment which both proved the hardness of RUC sampling on weaker complexity theoretic assumptions by providing the first worst-to-average-case reduction, and additionally proved an important result regarding the cross-entropy benchmark, a metric commonly used[15].

Worst-case hardness results had already been known for quantum circuits. Specifically, there were known examples of quantum circuits for which accurately sampling the output with a classical computer was known to be #P-hard[16], which involves counting the number of solutions to an NP-hard problem. By a theorem due to Toda, a polynomial-time algorithm with #P oracle, a class written as $\mathrm{P}^{\#\mathrm{P}}$, can efficiently solve any problem in the polynomial hierarchy.

Definition 3 (Polynomial hierarchy)**.** *A boolean function g is in* P *if it is computable in polynomial time, and a function f is in* NP *if $f(x) = \bigvee_y g(x,y)$ where $g \in$ P. This pattern extends; the k^{th} level of the hierarchy are those functions $f(x) = \bigvee_{y_1}\bigwedge_{y_2}\cdots\bigvee_{y_k} g(x,y_1,y_2,\ldots,y_k)$, where $\bigvee$ and $\bigwedge$ are AND and OR, and the sequence of k quantifiers is arbitrary.*

If the k^{th} level is equal to the $(k+1)^{\text{th}}$ level, then the k^{th} level is equal to all successive levels. This is known as the "collapse" of the polynomial hierarchy[17].

The worst-to-average case reduction proceeds as follows: Let $G = G_1 \ldots G_n$ be a worst-case unitary circuit for the problem of random circuit sampling. If we insert random unitary gates U_i interleaved in the circuit, we obtain $H = U_1 G_1 \ldots U_n G_n$, and we would expect this to be an average-case circuit due to the uniqueness of the Haar measure. Letting $U_i(\theta) \equiv \mathrm{e}^{-\mathrm{i}\log(U_i)\theta} U_i$, we can construct a parameterized circuit $G(\theta) = U_1(\theta)G_1 \ldots U_n(\theta)G_n$. Thus $G(\theta)$ interpolates between H, the average-case circuit, and G, the worst-case circuit. Then for any finite m, we can truncate $\mathrm{e}^{-\mathrm{i}\log(U_i)\theta} \approx \sum_{j=0}^{m} \frac{(\log(U_i)\theta)^n}{j!}$ to obtain the circuit $\widetilde{H}(\theta)$, which is a low-degree polynomial in the parameters $\theta_1, \ldots, \theta_n$ that approximates G. Then polynomial interpolation can be used to compute the probability amplitudes of G within an error that scales as $2^{-\operatorname{poly}(n)}$, showing that the average-case circuit must also be $\#P$-hard. In this way, the following theorem is proven:

Theorem 1. *On a processor architecture with n qubits for which computing $|\langle 0|G|x\rangle|^2$ within additive error $2^{-\operatorname{poly}(n)}$ is #P-hard, it is #P-hard to exactly compute $\frac{3}{4} + \frac{1}{\operatorname{poly}(n)}$ of the probabilities $|\langle x|C|0\rangle|^2$ where C is drawn from the RUC measure.*

The authors then provide strong evidence for, but are not able to prove, that this result also holds when approximating the amplitudes to an additive inverse exponential error 2^{-n^d}. In particular, the worst-to-average case breaks because the polynomial interpolation algorithm is not stable to noise. Under the assumption of this stronger approximation result, they also show that the existence of an approximate sampler, one that samples from a distribution that is close to the true one in total veriational distance, would collapse the polynomial hierarchy. Furthermore, it is shown that the approximate sampling problem is equivalent to HOG.

A proof of the worst-to-average case reduction for the approximate estimation algorithm was not found until several years later by Movassaugh[18,19]. The main issue is

that the Taylor series truncation has undesirable properties for interpolation. Movassaugh's work developed the notion of Cayley path interpolation on the unitary group, which is based on QR-decomposing a convex combination relating two unitary matrices of interest. Using this improved interpolation technique, he was able to prove

Theorem 2. *Given a quantum processor with n qubits for which it is #P-hard to compute the outputs of a circuit to within an additive error of 2^{-n^c} for any $c > 0$, it is also #P-hard to compute the output of $\frac{3}{4}$ of RUCs to within an additive error of $2^{-\Omega(n\log(n))}$.*

This almost closes a big loophole in the robustness issue, however the approximation error still vanishes with respect to $2^{-n}/\operatorname{poly}(n)$, representing a constant multiplicative error as n grows.

Another major contribution the work of Bouland et al.[15] is to prove a guarantee regarding the cross-entropy metric. Given the device distribution P_{dev} and the ideal distribution P_C, the cross-entropy is defined as the surprisal of P_C averaged over the distribution P_{dev}:

$$\operatorname{CE}(P_{\text{dev}}, P_C) = -\sum_x P_{\text{dev}}(x)\log(P_C(x)) \qquad (1)$$

In general, a large cross-entropy says nothing about the closeness of two distributions. However, the following theorem is shown:

Theorem 3. *If $H(P_{dev}) \geq H(P_C)$, then $|\operatorname{CE}(P_{dev}, P_C) - H(P_{dev})| < \epsilon$ implies $\|P_{dev} - P_C\| < \sqrt{\epsilon}$.*

where $\|\cdot\|$ is the total variational distance. This says that biased noise may violate this assumption, and only noise such as depolarizing noise that has the maximal-entropy distribution as a fixed point will guarantee the effectiveness of the cross-entropy benchmark. The cross-entropy benchmark is experimentally tractable because one can draw a sample of bitstrings, and then numerically compute the ideal surprisal for each. The cross-entropy is self-averaging in the sense that it converges very quickly with the number of samples.

2.2. Concentrate on Anticoncentration

One of the main goals of work in random unitary circuits has been establishing anticoncentration theorems[20].

Definition 4. *If μ is a measure valued in quantum circuits on n qubits, then we say that the distribution $P_C(x) = |\langle x|C|0\rangle|^2$ anticoncentrates if there exist constants $\gamma, \kappa > 0$ for all x, n such that*

$$\mathbb{P}_{C\sim\mu}(P_C(x) \geq \frac{1}{\kappa 2^n}) > 1 - \gamma \qquad (2)$$

Anticoncentration is an important property simply because all of the complexity-theoretic guarantees about computing probability amplitudes involve an additive error tolerance, which becomes meaningless if the measure asymptotically vanishes on some large subset of the outputs. A simple union bound shows the following theorem[15]:

Lemma 1. *Let μ be the measure of RUCs for a fixed processor architecture, and suppose μ anti-concentrates. If there exists an algorithm that estimates P_C to additive error $\epsilon 2^{-n}$ for a fraction $1-\delta$ of μ, then this can be used to estimate P_C to a multiplicative error $\kappa\epsilon$ for a fraction $1-\delta-\gamma$ of μ.*

This is important because, as shown in Thm. 3, we can verify that a sampling experiment approximates the distribution to within an additive error. The engine behind collapsing the polynomial hierarchy lies in the Stockmeyer theorem[21]:

Theorem 4 (Stockmeyer)**.** *Suppose we have a function $f : \{0,1\}^n \to \{0,1\}$ which can be evaluated in polynomial time. Then there is a BPP^{NP} algorithm which estimates $\mathbb{P}_{b\in\{0,1\}^n}[f(b) = 1]$ within a multiplicative error $(1+\epsilon)$ for any $\epsilon > 0$.*

This algorithm can be leveraged to use sampled results to approximate the distribution to within any tolerance, collapsing the polynomial hierarchy.

Previous work on anticoncentration first used the anticoncentration property for a unitary 2-design, and then proved that an RUC approximately attains this property within a circuit depth $\mathrm{O}(n)$[20]. Subsequent work was able to show that RUCs only require $\Omega(n\log(n))$ gates, and thus $\Omega(\log(n))$ circuit depth before they anticoncentrate[22].

3. THE FIRST RUC SAMPLING EXPERIMENTS

The first landmark in quantum supremacy experiments based on RUC sampling was Google's experiment in 2019[1], based on earlier proposals[23,24]. This experiment ran RUC samples on up to 53 qubits, taking over 1 million samples on tens of instances for each experiment. The computational goal was to achieve a high score on the linear cross-entropy:

$$F_{XEB} = 2^n\langle p(z)\rangle_z - 1 \qquad (3)$$

where the expectation value is taken over samples from the device. To understand why this is expected to estimate the fidelity, we introduce the Porter-Thomas distribution. First, consider a D-dimensional complex space. We can see that the measure $\mathrm{d}\psi = \frac{D}{\pi}\exp\left(-D|\psi|^2\right)\mathrm{d}z$ defines a unitary-invariant measure if we sample each component of a quantum state $\vec{\psi}$ with respect to this measure. By a change of variables $p = |\psi|^2$, we have $\mathrm{d}p = D\exp(-Dp)\mathrm{d}x$. States sampled from this measure are also approximately normalized:

$$\sum_i |\psi_i|^2 \to D\int p\mathrm{d}p = 1 \qquad (4)$$

Clearly, a translationally invariant, normalized measure is unique, so we have discovered the unique measure defining random quantum states, known as the Porter-Thomas distribution. Note that we are treating p itself as a random variable, and for each bitstring z, $p(z)$ is a sample of that random variable. Assuming depolarizing noise with a rate γ, our distribution takes the form

$$\rho_{\text{exp}} = \exp(-\gamma l)\rho_{\text{ideal}} + (1 - \exp(-\gamma l))\mathbb{1}/D \tag{5}$$

where $\mathbb{1}/D$ is the maximally mixed state. Using the Porter-Thomas distribution, we compute

$$F_{\text{XEB}} \to D^2 \operatorname{Corr}_z(p_{\text{exp}}(z), p_{\text{PT}}(z)) \tag{6}$$
$$= \exp(-\gamma l) + (1 - \exp(-\gamma l))/D \tag{7}$$

This makes F_{XEB} is an approximate fidelity estimator as $D \to \infty$. It has been suggested[3] that the cross-entropy is a bad estimator of fidelity when the ideal state is far from the Porter-Thomas distribution, and while this is true, it is not for the reason commonly suggested in the literature. The universal features of the Porter-Thomas distribution are not explicitly important, and we could simply replace F_{XEB} with $\operatorname{Corr}(p_{\text{exp}}, p_{\text{ideal}})/\operatorname{Var}(p_{\text{ideal}})$, and determine $\operatorname{Var}(p_{\text{ideal}})$ computationally, even if p_{ideal} is not universal! However, the Porter-Thomas distribution does have a property that makes it especially well-suited; The long tail of the distribution results from random interference the same way speckles appear in a laser pattern from scattering off a rough objective. These speckles dominate the correlator, so in fact, it is difficult to produce a distribution that is highly correlated with the ideal distribution. This provides a great deal of robustness to the noise model assumptions when including both biased and coherent noise.

Google's experiment, although later shown to be farther from the quantum supremacy regime than originally believed, was nevertheless a landmark moment in our ability to control quantum systems. Despite the simplicity of the noise assumption, the decay in fidelity was observed to be incredibly consistent with the fidelities obtained from single-qubit benchmarking. Furthermore, one should observe that computing F_{XEB} requires classical computation which is not available in the quantum supremacy regime. To overcome this, the experiment was performed with simplified circuits of the same depth and on the same number of qubits, with no two-qubit gates straddling a particular cut in the system. This limits the entanglement, and makes the deep circuits on all 53 qubits tractable. Remarkably, the single-qubit error model and error estimated from circuits with limited entanglement continue to agree with exact simulations at different systems sizes and up to 14 circuit layers, so it is reasonable to extrapolate this agreement to the supremacy regime where exact simulations are no longer available. The one major caviat in this experiment is that as the number of qubits is increased, the fidelity will fall as $\sim \exp(-nl\gamma_1)$ where γ_1 is the single-qubit depolarizing rate. Since the circuit depth should be proportional to system size, this means $F \sim \exp(-n^2\gamma_1)$, which is comfortably outside the scope of existing hardness theorems explained previously. The fidelity achieved at 53 qubits and 20 cycles is $F \sim 10^{-3}$. While an impressive technological feat, it is not a stretch of the imagination that a classical computer could compete with this low fidelity.

4. NOISE AND ALGORITHMIC ADVERSARIES

Since Google's experiment, our understanding of quantum complexity has continued to evolve through smarter classical algorithms and no-go theorems. The main takeaway is that a constant noise rate largely spoils the computational complexity, with particularly strong results when the noise rate is high. This relies on the relationship between noise and entanglement. The Google experiment essentially assumed that the state produced was the ideal, highly entangled state, plus a maximally mixed distribution, and samples from this distribution would still be incredibly hard to reproduce classically if a large enough fraction were drawn from the entangled error-free state. In retrospect, this was an optimistic perspective. Shortly following the experiment, Zhou et al. showed that a matrix product state simulation with restricted bond dimension exhibits the same exponential scaling as the experimentally observed error[3].

Definition 5. *A matrix product state is a state $|\psi\rangle$ of the form*

$$|\psi\rangle = \sum_{\vec{m}} \operatorname{Tr}\left[A^{(m_1)} \dots A^{(m_n)}\right] |\vec{m}\rangle \tag{8}$$

where $A^{(i)}$ is a $\chi \times \chi$ tensor, and χ is known as the bond dimension.

With a MPO, as the system size in increased, the cost of matrix contraction and singular value decompositions across each of the bonds is polynomial in n for a fixed χ, and the simulation is exact if $\chi = 2^n$. It was found numerically that the fidelity increases polynomially in the bond dimension up to a fidelity barrier f_∞, at which point the cost of improving the fidelity further increases exponentially. Finite-bond-dimension MPSs span a very small region of the Hilbert space, so this established an error barrier beyond which the computational power of quantum circuits largely vanish. This result was refined by Cheng et al.[13], who showed that physical noise models can be incorporated into a finite-bond-dimension matrix product density operator (MPDO). Earlier work by Noh et al. had taken a different approach, simulating a local noise model with matrix product operators and monitoring a quantity called the MPO entanglement entropy, which combines the spread of the Schmidt coefficients across each of the bonds. They found that a finite noise rate set a characteristic system size beyond which this entropic measure stopped growing, signaling that adding more qubits would not provide meaningful computational speedup[25].

Lastly, we would like to discuss an approach which may provide the strongest caviat to the complexity-theoretic arguments in the presence of noise. This algorithm is Pauli-path truncation[11,12]. The key idea is that the evolution of an operator may be written as a path integral in the Pauli basis. This path integral consists of sequences of terms containing identity and non-identity Pauli operators. Pure depolarizing noise maps non-identity Pauli operators to zero, meaning that paths with a higher "weight", i.e. with more non-identity terms, are exponentially suppressed by any level of depolarizing noise. The consequence is that truncating Pauli paths at a finite weight provides an accurate polynomial-time algorithm for simulating deep circuits with depolarizing noise. The caviat is that, while other algorithms based on MPOs are designed to be computationally practical, this algorithm may have a very large overhead. While this is only proven for pure depolarizing noise, it is likely that it extends to other noise models as well.

5. NEXT GENERATION CIRCUIT SAMPLING

Clearly the naive approach of sampling random quantum states and comparing them to simulation leaves a complexity-theoretic gap through which skeptics of near-term quantum primacy can easily pass. To address this, over the past few years Google has set in motion the next generation of quantum supremacy experiments[9,10] based on noise-induced phase transitions. This work largely seeks to respond to the assertion that a quantum process is only leveraging a small portion of the Hilbert space because the output can be spoofed by a finite-bond-dimension MPO. The "order parameter" that is used to define the phase transition is F^d/F_{XEB}, where $F = \mathrm{e}^{-\epsilon n}$ is the single-layer fidelity and d is the circuit depth. Entanglement generation is modeled using a "weak link" between two subsystems, which consists of a single entangling gate connecting two halves of the system applied every T cycles. The competition between this entanglement generation and noise is then monitored through a noise-injection protocol consisting of inserting random rotations before single-qubit gates. A simplified model for F_{XEB} is developed;

$$F_{\text{XEB}} \approx 2\lambda^{d/T}\mathrm{e}^{-\epsilon n d/2} + \mathrm{e}^{-\epsilon n d} \tag{9}$$

In the strong noise regime corresponding to a large ϵn, the first term dominates, while the second term dominates in the low-noise regime, leading to a phase transition in the order parameter from unity to zero. This transition is analytically predicted to occur around $\epsilon n = \frac{4\log(2)}{T}$, which agrees will with the experimental behavior. The observation of an entangled phase leads them to conclude that the subsequent supremacy experiment, which is carried out on 70 qubits and a circuit depth of 26 cycles, lies in the regime where entanglement generation outpaces the error rate. This experiment still occurs in a very low-fideltiy limit, $F \approx 10^{-3}$ after 26 cycles. While the observation of a phase transition is quite interesting, the physical connection between the order parameter and an entangling phase still seems slightly unclear after reading the paper. In the end, it seems like the phase transition is a separate consideration from the quantum supremacy experiment, which employs fidelity estimation much like the one in 2019.

6. CONCLUSION

This review has illustrated some of the many nuances in the complexity of RUCs; the subject is rife with caviats, marginal comparisons, and unproven assumptions. We have seen that perfect classical simulation of RUCs is hard; near-exact simulation is also hard in the average case, but the hardness of approximate simulation is not known. Noise is the enemy of complexity, and any constant depolarizing noise rate spoils the asymptotic complexity of a quantum computation. However, many of these algorithms come with large overheads, and quantum advantage may still be safe in the near term with the latest experiments on 70 qubits. Algorithmically, it seems there are many avenues to explore, but these should involve short-depth circuits, restricted classes of gates such as IQP circuits, biased noise models, or continuous-time dynamics. Such fidelity estimation experiments have already been carried out in neutral-atom simulators[26], and it is an exciting challenge to develop tools that put Feynman's original conjecture about the complexity of simulating quantum systems on more rigorous footing. The most challenging open question in the complexity theory of RUCs seems to be the robustness to a constant relative error, which could be applicable to the hardness of constant-depth circuits.

While the current results are likely to disappoint complexity theorists, the true winners are the physicists. The control of quantum systems demonstrated in these supremacy experiments is remarkable, and seeing dynamical entanglement transitions probed in an actual experimental platform is incredibly exciting. If anything, supremacy or not, this is reason enough to continue the dialogue between quantum and classical algorithms, and push the boundaries of what is possible in experiments.

7. ACKNOWLEDGEMENTS

We would like to thank Prof. Rahul Nandkishore for sharing his academic interests and enlightening insights into the modern theory of quantum dynamics.

[1] F. Arute, K. Arya, R. Babbush, D. Bacon, J. C. Bardin, R. Barends, R. Biswas, S. Boixo, F. G. Brandao, D. A. Buell, *et al.*, "Quantum supremacy using a programmable superconducting processor," Nature **574**, 505–510 (2019).

[2] E. Pednault, J. A. Gunnels, G. Nannicini, L. Horesh, and R. Wisnieff, "Leveraging secondary storage to simulate deep 54-qubit sycamore circuits," arXiv preprint arXiv:1910.09534 (2019).

[3]Y. Zhou, E. M. Stoudenmire, and X. Waintal, "What limits the simulation of quantum computers?" Physical Review X **10**, 041038 (2020).
[4]F. Pan and P. Zhang, "Simulating the sycamore quantum supremacy circuits," arXiv preprint arXiv:2103.03074 (2021).
[5]Q. Zhu, S. Cao, F. Chen, M.-C. Chen, X. Chen, T.-H. Chung, H. Deng, Y. Du, D. Fan, M. Gong, *et al.*, "Quantum computational advantage via 60-qubit 24-cycle random circuit sampling," Science bulletin **67**, 240–245 (2022).
[6]Y. Kim, A. Eddins, S. Anand, K. X. Wei, E. Van Den Berg, S. Rosenblatt, H. Nayfeh, Y. Wu, M. Zaletel, K. Temme, *et al.*, "Evidence for the utility of quantum computing before fault tolerance," Nature **618**, 500–505 (2023).
[7]H.-J. Liao, K. Wang, Z.-S. Zhou, P. Zhang, and T. Xiang, "Simulation of ibm's kicked ising experiment with projected entangled pair operator," arXiv preprint arXiv:2308.03082 (2023).
[8]J. Tindall, M. Fishman, E. M. Stoudenmire, and D. Sels, "Efficient tensor network simulation of ibm's eagle kicked ising experiment," Prx quantum **5**, 010308 (2024).
[9]"Measurement-induced entanglement and teleportation on a noisy quantum processor," Nature **622**, 481–486 (2023).
[10]A. Morvan, B. Villalonga, X. Mi, S. Mandra, A. Bengtsson, P. Klimov, Z. Chen, S. Hong, C. Erickson, I. Drozdov, *et al.*, "Phase transitions in random circuit sampling," Nature **634**, 328–333 (2024).
[11]T. Schuster, C. Yin, X. Gao, and N. Y. Yao, "A polynomial-time classical algorithm for noisy quantum circuits," arXiv preprint arXiv:2407.12768 (2024).
[12]D. Aharonov, X. Gao, Z. Landau, Y. Liu, and U. Vazirani, "A polynomial-time classical algorithm for noisy random circuit sampling," in *Proceedings of the 55th Annual ACM Symposium on Theory of Computing* (2023) pp. 945–957.
[13]S. Cheng, C. Cao, C. Zhang, Y. Liu, S.-Y. Hou, P. Xu, and B. Zeng, "Simulating noisy quantum circuits with matrix product density operators," Physical review research **3**, 023005 (2021).
[14]S. Aaronson and L. Chen, "Complexity-theoretic foundations of quantum supremacy experiments," arXiv preprint arXiv:1612.05903 (2016).
[15]A. Bouland, B. Fefferman, C. Nirkhe, and U. Vazirani, "On the complexity and verification of quantum random circuit sampling," Nature Physics **15**, 159–163 (2019).
[16]M. J. Bremner, R. Jozsa, and D. J. Shepherd, "Classical simulation of commuting quantum computations implies collapse of the polynomial hierarchy," Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences **467**, 459–472 (2011).
[17]A. W. Harrow and A. Montanaro, "Quantum computational supremacy," Nature **549**, 203–209 (2017).
[18]R. Movassagh, "The hardness of random quantum circuits," Nature Physics **19**, 1719–1724 (2023).
[19]Y. Kondo, R. Mori, and R. Movassagh, "Quantum supremacy and hardness of estimating output probabilities of quantum circuits," in *2021 IEEE 62nd Annual Symposium on Foundations of Computer Science (FOCS)* (IEEE, 2022) pp. 1296–1307.
[20]D. Hangleiter, J. Bermejo-Vega, M. Schwarz, and J. Eisert, "Anticoncentration theorems for schemes showing a quantum speedup," Quantum **2**, 65 (2018).
[21]L. Stockmeyer, "Proceedings of the fifteenth annual acm symposium on theory of computing, stoc'83," (1983).
[22]A. M. Dalzell, N. Hunter-Jones, and F. G. Brandão, "Random quantum circuits anticoncentrate in log depth," PRX Quantum **3**, 010333 (2022).
[23]S. Boixo, S. V. Isakov, V. N. Smelyanskiy, R. Babbush, N. Ding, Z. Jiang, M. J. Bremner, J. M. Martinis, and H. Neven, "Characterizing quantum supremacy in near-term devices," Nature Physics **14**, 595–600 (2018).
[24]C. Neill, P. Roushan, K. Kechedzhi, S. Boixo, S. V. Isakov, V. Smelyanskiy, A. Megrant, B. Chiaro, A. Dunsworth, K. Arya, *et al.*, "A blueprint for demonstrating quantum supremacy with superconducting qubits," Science **360**, 195–199 (2018).
[25]K. Noh, L. Jiang, and B. Fefferman, "Efficient classical simulation of noisy random quantum circuits in one dimension," Quantum **4**, 318 (2020).
[26]A. L. Shaw, Z. Chen, J. Choi, D. K. Mark, P. Scholl, R. Finkelstein, A. Elben, S. Choi, and M. Endres, "Benchmarking highly entangled states on a 60-atom analogue quantum simulator," Nature **628**, 71–77 (2024).