MATH2320 Assignment 9

Due: 5pm Tuesday 18th October 2016.

Benjamin Moran

If you have any questions about this assignment, please see or email your lecturer for this course, Dr Daniel Sutherland ⟨ Daniel.Sutherland@newcastle.edu.au ⟩.

Questions which review material from Week 8 (Monday) and Week 9 (Friday):

1. Let V be a finite-dimensional inner-product space. Let U be a subspace of V, and $(e_1, \ldots, e_m)$ an orthonormal basis of U. Recall that the orthogonal projection of V onto U is the operator $P_U \in \mathcal{L}(V)$ defined by

$$P_U v = \langle v, e_1 \rangle e_1 + \cdots + \langle v, e_m \rangle e_m.$$

a. Use the above definition to prove that $U^\perp \subseteq \text{ null } P_U$.
b. Use the above definition to prove that null $P_U \subseteq U^\perp$, and hence that null $P_U = U^\perp$.
c. Use the above definition to prove that $P_U^2 = P_U$. That is, $P_U(P_U v) = P_U v$ for all $v \in V$.

2. Let $V = C[-\pi/2, \pi/2]$ be the real vector space of continuous real-valued functions on the interval $[-\pi/2, \pi/2]$ with inner product

$$\langle f, g \rangle = \frac{1}{\pi^3} \int_{-\pi/2}^{\pi/2} f(x)g(x)dx.$$

(a) Show that $\big(k_1(3x + \pi), k_2(4x - \pi), k_3(12x^2 - \pi^2)\big)$ is an orthonormal list of vectors in V scalars k_1 , k_2 and k_3 are

$$k_1 = \sqrt{\frac{4}{7}}, \quad k_2 = \sqrt{\frac{3}{7}}, \quad k_3 = \frac{\sqrt{5}}{2\pi}$$

(b) Verify the following identities for this inner-product space:

$$\begin{aligned} \langle bx + c, cos(x) \rangle &= \frac{2c}{\pi^3} && \text{for} b, c \in \mathbb{R}; \\ \langle ax^2 + c, cos(x) \rangle &= \frac{(2c)\pi^2 - 8)a}{2\pi^3} + \frac{2c}{\pi^3} && \text{for} a, c \in \mathbb{R}; \end{aligned}$$

Introductory level question relating to Week 10:

3. Suppose V is the inner-product space from Question 2. Let

$$e_1 = k_1(3x + \pi), \qquad e2 = k_2(4x - \pi), \qquad e_3 = k_3(12x^2 - \pi^2),$$

so that (e_1, e_2, e_3) is the orthonormal list of vectors from Question 2(a). Let $v = cos(x) \in V$ and consider the subspace $U = P_2(\mathbb{R})$ of V.

(a) Explain why (e_1, e_2, e_3) is an orthonormal basis of U.

(b) Use the identities from Question 2(b) to verify the following inner products:

$$\langle v, e_1 \rangle = \frac{2k_1}{\pi^2}, \quad \langle v, e_2 \rangle = -\frac{2k_2}{\pi^2}, \quad \langle v, e_3 \rangle = -\frac{4(12 - \pi^2)k_3}{\pi^3}.$$

(c) Recall that for $u \in U$, the expression $\|v - u\|$ is minimised by choosing $u = P_U v$. Verify that the orthogonal projection of v onto the subspace U is given by $P_U v = u_0 + u_1 x + u_2 x^2$, where

$$u_0 = \frac{3(20 - \pi^2)}{\pi^3}, \qquad u_1 = 0, \qquad u_2 == -\frac{60(12 - \pi^2)}{\pi^5}.$$

(d) Plot the continuous functions $u(x)$ and $v(x)$ over the interval $[-\pi/2, \pi/2]$.

Submitting your assignment (due 5pm Tuesday 18th October 2016)

Submit your assignment in hardcopy in your Demonstrator's pigeonhole in the Assignment boxes near the Maths Clinic, on the opposite wall to the Maths Clinic, left of the door to V109. We also ask that you scan your written work and submit it on the MATH2320 UoNline/Blackboard site as a backup and proof of submission, not as a substitute. ***Note that we still require the hardcopy submitted in the Assignment Box for your Demonstrator to mark, even if you have submitted a backup on Blackboard.***