

Assignment 2

2017-05-03

Question 1: Inference for the Poisson parameter λ :

- (a) Given a general Gamma(a,b) prior, derive the posterior distribution of λ , given data (x_1, \dots, x_n) , followed by the posterior predictive distribution of (z_1, \dots, z_m) .
- (b) Assuming Gamma(a,b) priors for two Poisson parameters λ_1 and λ_2 , derive the posterior for $\phi = \lambda_1/\lambda_2$. (Hint: use nuisance parameter $\mu = \lambda_2$).
- (c) Derive the Jeffreys prior for (ϕ, μ) , and the corresponding marginal posterior for ϕ .
- (d) Derive the reference prior for (ϕ, μ) , and the corresponding marginal posterior for ϕ .
- (e) Based on (c), consider the posterior for ϕ based on uniform priors for λ_1 and λ_2 , and comment on when inference based on this posterior could be quite different from that based on the reference posterior from (e). Give a data example, in terms of resulting intervals. (Hint: the above posteriors are related to known pdfs, and by transformation may be simplified even further.).

Question 2: Inference for variance components:

- (a) Use e.g. SAS (PROC VARCOMP) to perform a classical analysis of the data in Table 5.1.4 of Box & Tiao (1973), based on finding point estimates only.
- (b) Use WinBUGS for a Bayesian analysis of (a), and find reasonable point and interval estimates for σ_1^2 and σ_2^2 . Include graphs, including one of the joint posterior. [6 marks]
- (c) Box & Tiao also studied a 3-component model (Table 5.3.1).
 - i. Derive central credible intervals, for the 3 individual components, based on Table 5.3.3.
 - ii. Use WinBUGS to do the same, and include graphs.
 - iii. While not going as far as Box & Tiao's Figure 5.3.2, produce a graph of the joint posterior of σ_2^2 and σ_3^2 , and one of σ_2^2/σ_3^2
- (d) Box, Hunter & Hunter (1976, Chapter 17.3) studied a pigment paste example with three components, focusing on point estimates only. Use WinBUGS again to perform a Bayesian analysis. Include graphs.