Exercise 2

Consider the linear advection equation:

$$\frac{\partial C}{\partial t} + u_0 \frac{\partial C}{\partial x} = 0$$

In this exercise, you are going to solve this equation numerically using three different methods. The domain has size $L = 2500km$ and periodic boundaries. Divide the domain in grid cells with $\Delta x = 25km$, having index j (for the time index we use n).
Consider the following initial concentration:

$$C(0,x) = \begin{cases} 0, & x < 1125 \\ C_0, & 1125 \leq x \leq 1375 \\ 0, & x > 1375 \end{cases}$$

With x-values given in kilometres. Let $\mathrm{u}_0 = 10\ ms^{-1}$.

Perform this simulation using three different numerical schemes. You will discover that some schemes exhibit numerical diffusion and others numerical dispersion, or both. This exercise is called the “Molenkamp test”, after Molenkamp (1968). Use the following schemes:

- Euler *forward* in time and *upwind* in space (repeated below)
- Lax-Wendroff scheme
- Spectral method

$$\frac{C_j^{n+1} - C_j^n}{\Delta t} + u_0 \frac{C_j^n - C_{j-1}^n}{\Delta x} = 0$$

Hand in this exercise no later than Tuesday, May 16th.

Molenkamp, Charles R (1968). “Accuracy of finite-difference methods applied to the advection equation”. In: *Journal of Applied Meteorology and Climatology* 7.2, pp. 160–167.