Exercise 3

Solve the linear one-layer shallow water equations numerically:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}$$

$$\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

Use a periodic spatial domain with length $L = 10^4\ km$. Set $H = 1000\ m$ and $g = 10\ ms^{-2}$. Initially, the fluid is at rest and a perturbation in the free surface height, h, is inserted in the middle of the domain at $x = 0$:

$$h(0,x) = \begin{cases} H, & x < -5000 \\ H + \eta_0 \exp\left(-\left(\frac{x}{a_0}\right)^2\right), & -5000 \leq x \leq 5000 \\ H, & x > 5000 \end{cases}$$

With x-values given in kilometres. Let $\eta_0 = 50\ m$ and $a_0 = 200\ km$. Do the integrations for different values of the grid distance, Δx, and with multiple time-stepping schemes.

The philosophy of these experiments is similar to that of the Molenkamp test. You should observe two perturbations in the free surface propagating in both directions with constant phase velocity without loss of form (no dispersion). Due to the periodic boundary conditions, the waves will meet after a certain time and the state of this time, in terms of the height of the free surface, h, should be exactly as at $t = 0$.

a) What is the phase speed of the gravity waves on the free surface?
b) At what time will the waves be at the same position as initially?
c) Varying the total amount of grid points from 100 to 1000, and with a time step $\Delta t = 50\ s$, integrate the equations and reflect on your results in light of "the overview of errors in the numerical simulation of gravity waves" from the lectures.
d) Can you improve on the results by reducing Δt?
e) Repeat exercise (c) with a staggered grid in space with the same resolution. In which way does this improve the numerical solution?

Hand in this exercise no later than Tuesday, May 16th.