# Morphodynamics of Tidal Systems

## Tides in basins and estuaries

Lasse Vulto (4685784), Ida Haven (3490475), Bennet Weiss (7349106)

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**The questions in this exercise are formulated to guide your analysis of tides in estuaries and basins. In the answers to your questions make sure you show that you understand the main issues. Do not focus on generating as many plots as possible.**

**Do a harmonic analysis of the modeled sea surface elevations (Z) and flow velocity (U). Determine the tidally averaged, M2 and M4 components (amplitude and phase of each constituent), and for case D also diurnal components if your system has a strong diurnal signal. Also determine relative phase difference between M2 water levels and M2 flow velocities.**

**Part 1: Dynamics of rectangular tidal basins**

**We start with a rectangular basin (having constant width) of** 1 km **wide.**

**Discuss and explain your results:**

A. **Take a basin of 20 km length. Take** $Cd = 2.5 * 10^{-3}$**. Take** $\Delta x = 500\,\mathrm{m}$**. Prescribe an M2 tide at the seaward side with an amplitude of 1 meter. Vary the still water depth between 2 and 10 meters in steps of 1 meter. Make sure the time step is small enough by checking the Courant number for the case with largest depth** (CFL $< 1$)**. This will also result in a good time step for the shallower cases.**

   1) **Study the sensitivity of the amplitude of the M2 tide as a function of space for the different depths. Explain the dependence of tidal amplitude on depth. For which depths are you close to the pumping model solution (Short basin limit in paper by Friedrichs)? Also remember that for pumping model the phase of M2 water level should be uniform in the basin.**

      First, the situation with the parameters as given above was performed. While it was ensured that the Courant number (with $\Delta t = 30\,\mathrm{s}$) is always smaller than one, this simulation provided us, amongst other parameters, with the development of the sea surface elevation $\eta$ in space and time. To ensure that the data we are working with originates from a time where the numerical simulation was already stable, only data from after 5 full tidal M2 periods was considered. For attribution of the sea surface level change to the different tidal constituents, a fitting algorithm similar to that of the last project was performed. For each position $x$ such a harmonic fit was done, where the M2 tides as well as its higher harmonics M4 and M6 were considered. This was then done for every simulation with water levels from 2 to 10 m and the result is summarized in figure 1. Here, it is visible that the M2 amplitude drops drastically in the basin for shallow waters while it even rises slightly for deeper estuaries. As the tidal wave only experiences friction at the boundaries of the basin, friction becomes proportionally more important for shallower basins with lower water volume. This explains why the maximum tidal amplitude gets so low for the shallow basins. At the same time, shallower basins have a lower phase speed and thus waves can become higher as they are compressed horizontally.

This explains why the intermediate-depth basins in fact develops the highest M2 amplitude as they are in a kind of sweet spot between effects of not too much friction and slow enough phase speeds. Another explanation can be that for waves at intermediate depth the reflected wave increases the M2 amplitude as well. This is not observed for the most shallow case due to friction and for the deeper cases the M2 amplitude decreases since the reflected amplitude spreads out over the deeper water depth.

An analysis of the M2 Phase, performed in a similar way as in the last project, is displayed in figure 2. While the phase is not constant for a shallow basin up to a depth of 6 m, it remains approximately constant for deeper basins. In summary, for deep water above approximately 7 m, the M2 amplitude as well as the phase is, in first approximation, not a function of position anymore. This indicates that these solutions are close to a standing wave as predicted in (Friedrichs, 2010, p. 34-35) as the short basin limit.

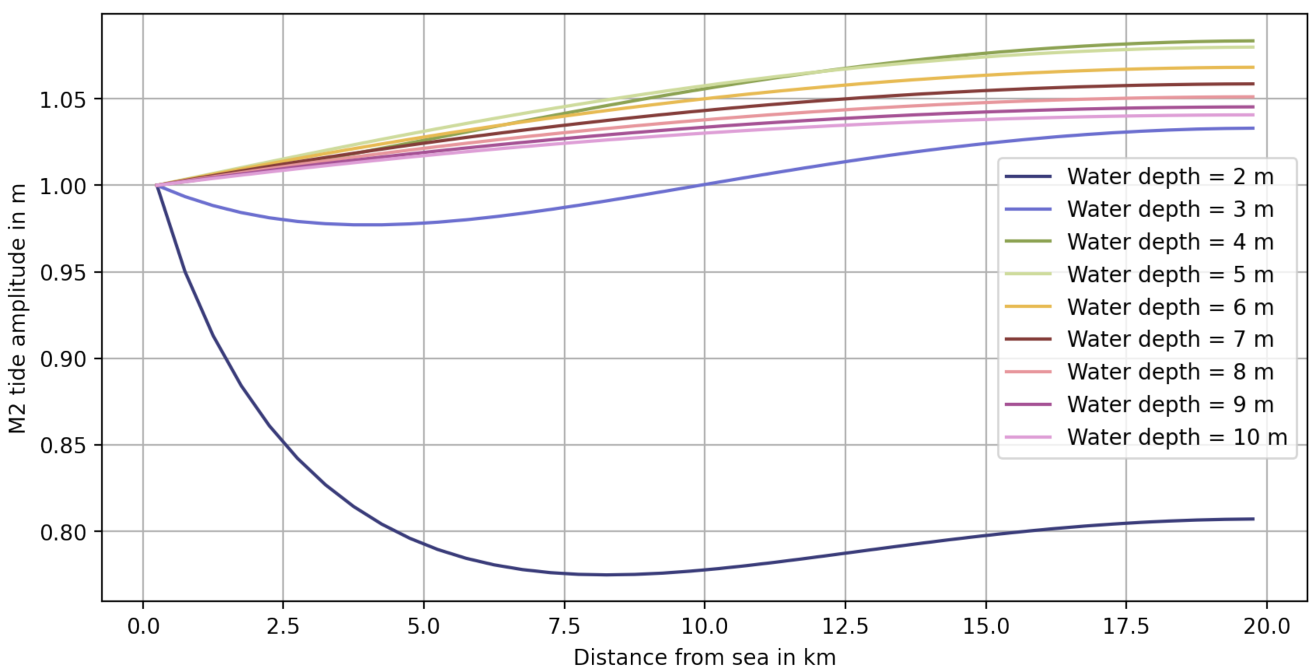


Figure 1: Amplitude of the M2 tide as a function of distance of the sea in a simulated rectangular basing for different water depths.

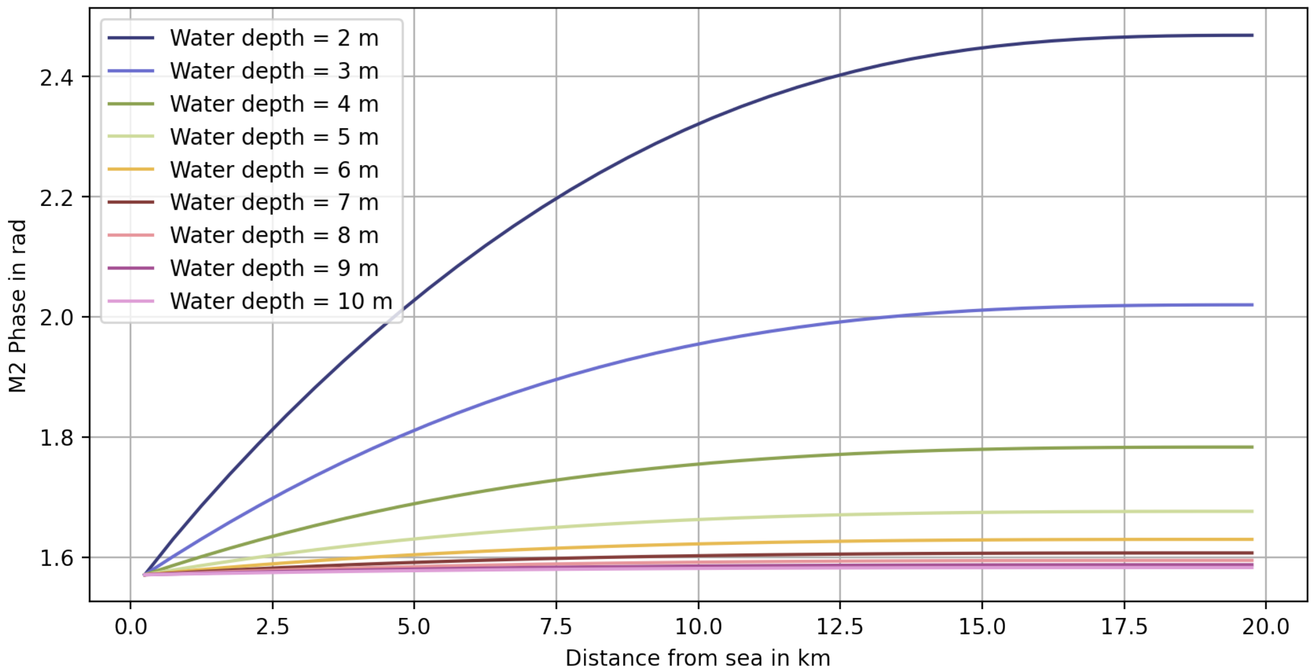


Figure 2: Phase of the M2 tide as a function of distance of the sea in a simulated rectangular basing for different water depths.

2) **Determine the deformation of the tide for the shallowest case (2 m) and deepest case (10 m) by determining the amplitude of the M4 water level. Study these as function of position in the basin. Explain the difference between a deep and shallow basin.**

   The spatial development of the amplitude of the M4 tide for a simulated 2 m and 10 m basin is depicted in figure 3. From this figure, it follows that the tide is deformed stronger in the shallow basin than in the deeper one as the M4 tide is significantly smaller for the deeper basin. This leads to strong asymmetries of the tides, especially in the shallower basin, where the tide rises much faster than it falls.

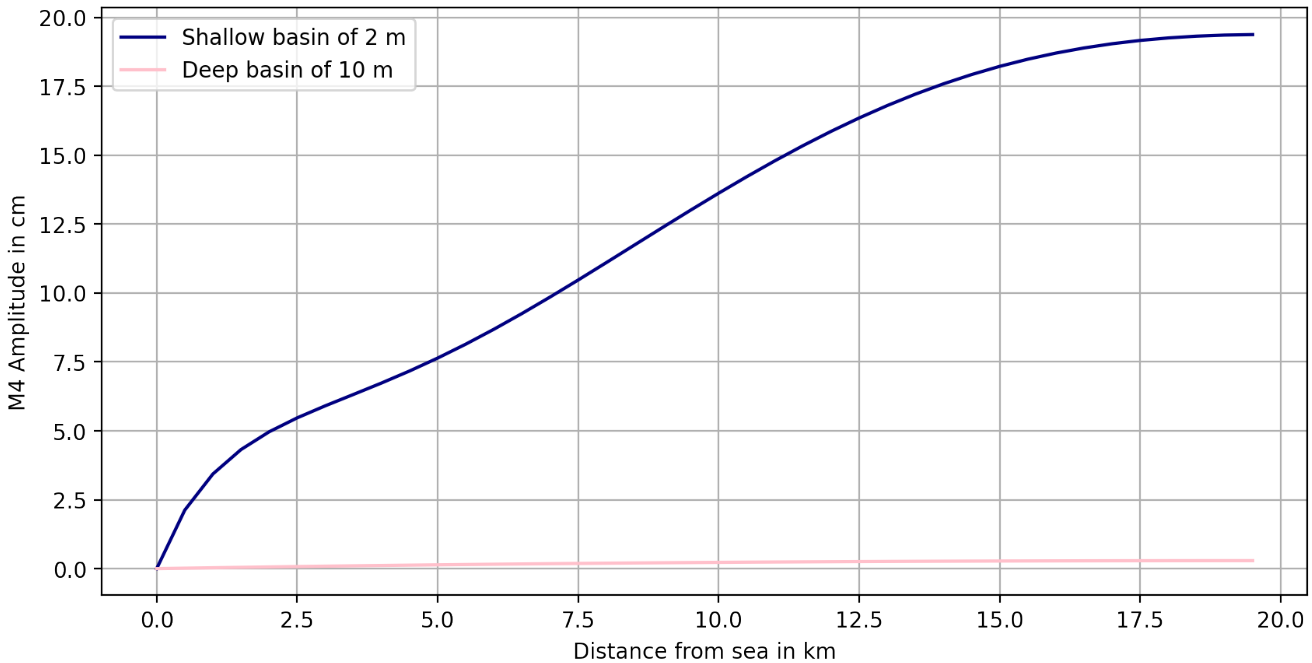


Figure 3: Amplitude of the M4 tide amplitude for a shallow (purple) and a deep (rose) basin as a function of distance to the sea.

Deformations in the form of higher harmonics, such as the M4 tide, have their origin in non-linearities of the governing equations. In our simulation we treated the estuary width, which in our case is the same as the channel width, as well as the depth as a constant, so non-linearities cannot occur here. Another important cause for production of higher harmonics is friction, which is non-linear in flow velocity. While friction itself is the same for both simulations, friction relative to water depth, which is explicitly part of the governing equations, is not. As the water depth decreases, the relative significance of frictions increases and with that the non-linearities of the solution do, too.

The M4 amplitude rises as it gets further away from the sea as the non-linearities have then had a longer time to deform the tide.

3) **For the shallowest case: Determine the amplitude of the mean (=averaged over a tidal cycle), the M2 and the M4 tidal currents in the basin and determine the relative phase difference between the M2 and M4 tidal currents. Also determine for each position whether peak ebb or peak flood currents are largest. Explain (i) the presence of the mean tidal currents and (ii) explain why peak ebb or peak flood currents are largest (for different positions in the basin).**

From the simulation of the shallow 2 m basin we also get information about the current velocities as function of time and position. Fitting the M2 and M4 constituents as well as a mean velocity to that leaves us with the fitted parameters as depicted in figure 4. Here, it can be seen that the M2 velocity amplitude (blue), which starts at over 1 m/s, decreases down to zero at the end of the basin. The M4 velocity amplitude (orange), which has its origin in the M2 tide, first increases as non-linear terms become more dominant. Deeper in the basin, however, the M4 velocity amplitude decreases again as the M2 velocity amplitude becomes too small to generate significant higher harmonics. The mean velocity is negative at the beginning, increases deeper in the basin and approaches zero towards the end.

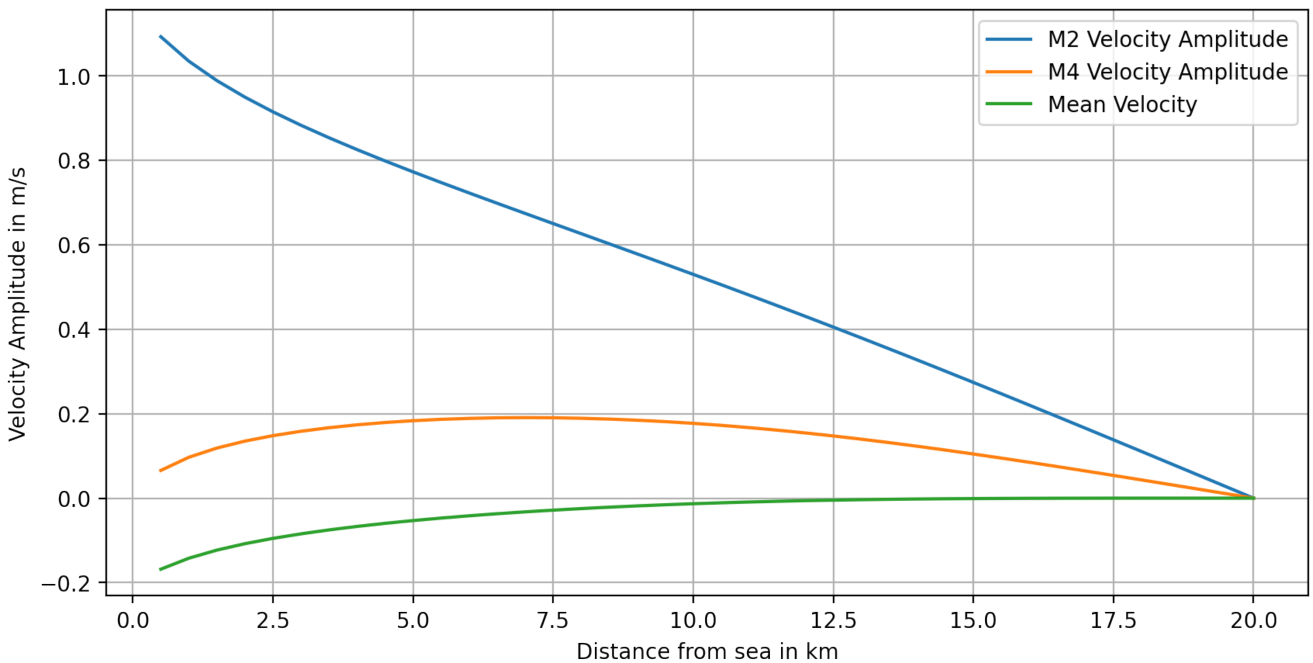


Figure 4: M2 and M4 Amplitude of the velocity profile as well as mean velocity as function of distance to the sea for the shallowest basin (2m).

To analyse tidal asymmetry and skewness of the signal we turn to a phase analysis. In figure 5, the phases of the M2 (orange) and M4 (blue) velocity signal as well as their phase difference (green) is plotted. It can be seen that the phase difference is slightly negative, but to a good approximation $0°$. This translates to higher flood velocities that are maintained for a shorter period of time than the slower ebb velocities. Ultimately, this leads to a skewed signal which means that this basin induces a flood dominated tidal behaviour. The slight anticlockwise deviation from $0°$ additionally introduces asymmetries in which the time from peak ebb to peak flood flow is longer than the other way around. As the phase difference is relatively constant across the whole basin (constant angle in phase plot), the asymmetry and skewness are constant and thus the peak flood flow is larger than the peak ebb flow in the whole basin.

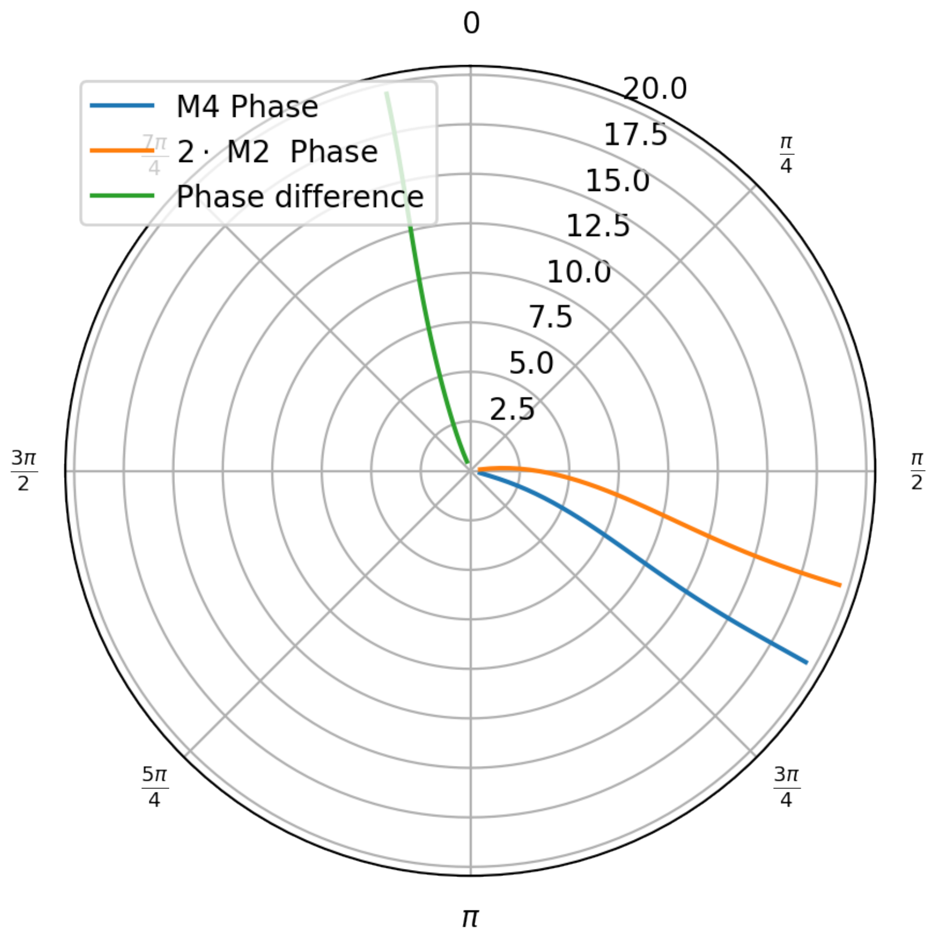


Figure 5: Phases of M2 and M4 tide as well as the difference plotted in polar coordinates. The distance to the sea is represented by the radius and has units of kilometers.

That also explains the presence of a nonzero mean flow in figure 4 (green). Due to higher peak flood currents, more water can be transported at the same time during peak flood than at peak ebb and thus the water level stays longer below zero than above. This then translates to a slightly negative mean water level which is observed in the plot.

B. **Take a basin with depth of 10 meters, $\Delta x = 1000\,\mathrm{m}$. Choose an appropriate time step. Prescribe an M2 tide at the seaward side with amplitude of 1 meter.**

1) **Vary the length of the basin between 10 km and 150 km in steps of 20 km and take $Cd = 0.2 \cdot 10^{-3}$. Study the sensitivity of the amplitude of the M2 tide as function of space for the different lengths of the basin. Explain your results. In which case is the basin resonant? Compare to what you expect theoretically.**

To speed up the algorithm, the time step was increased to $\Delta t = 60\,\mathrm{s}$, which still satisfies the condition of the Courant number being smaller than 1. This time, a moderate friction was introduced to ensure non-diverging results and different basin lengths were examined. Once again, the spacial evolution of the M2 amplitude was of interest. This is plotted for different basin lengths in figure 6. Note that the different lines have different lengths because of the different basin lengths. For smaller basin lengths, the M2 amplitude represents the tidal pumping solution as discussed in section 1). It is clearly visible that the M2 amplitude reaches a maximum at the end of the basin with a length of 110 km (red line). This has its origin in the quarter wave resonance, where the length $L$ of the basin is related to the tidal wavelength $\lambda$ by

$$L = \lambda/4. \tag{1}$$

The phase speed of the wave can be approximated to $c = \sqrt{gH}$, where $g$ is the gravitational acceleration and $H = 10\,\mathrm{m}$ is the depth of the basin. With that, the wavelength can be calculated through

$$\lambda = cT_{M2}, \tag{2}$$

where $T_{M2}$ is the tidal period of the M2 wave. It turns out that a basin with a length of $L \approx 110.7\,\mathrm{km}$ would achieve most tidal resonance, which is very close to the basin length of the red line of figure 6. Basins with lower or higher lengths would produce less tidal amplification and have thus lower associated M2 amplitudes. This matches with the results of our simulation in figure 6 where the M2 amplitudes rises with increasing basin lengths until it reaches a maximum for $L = 110\,\mathrm{km}$ and then drops again with even higher basin lengths.

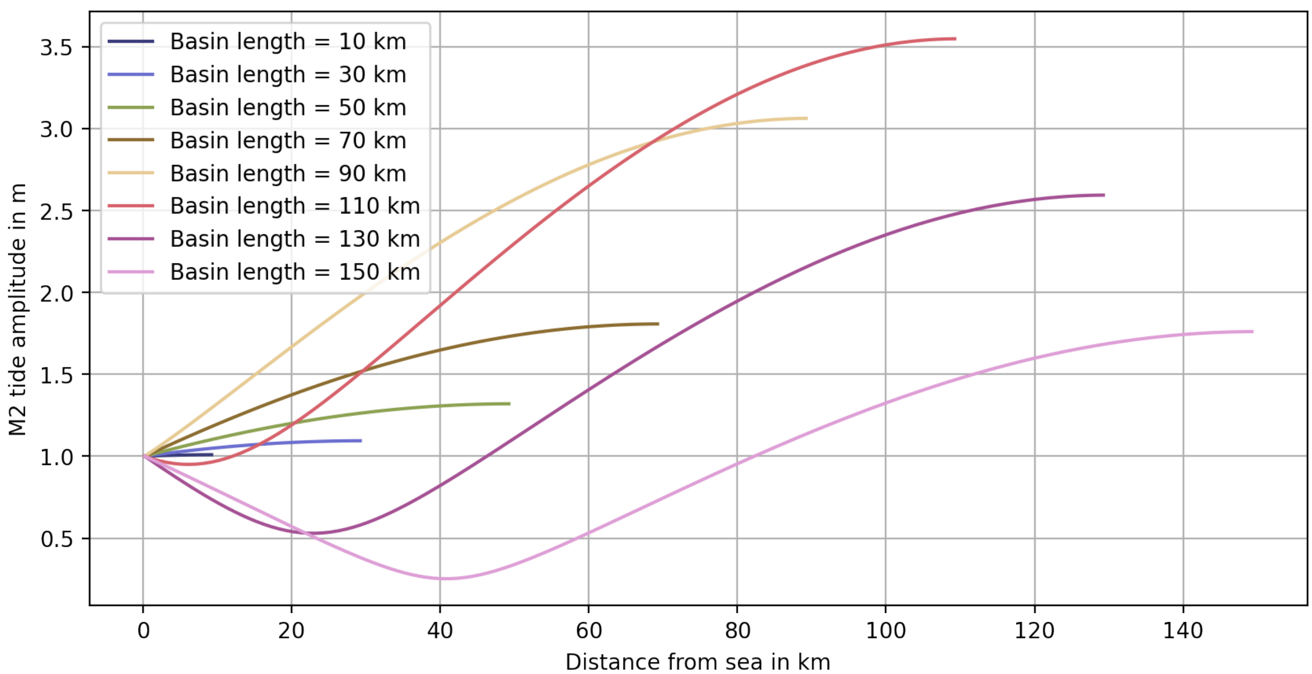


Figure 6: Amplitude of the M2 tide as a function of distance of the sea in a simulated rectangular basin of different lengths.

2) **Take a basin length of 110 km and study the M2 water level amplitude in the basin for $Cd = \left[0.2 \cdot 10^{-3}, 0.5 \cdot 10^{-3}, 10^{-3}, 2 \cdot 10^{-3}, 5 \cdot 10^{-3}\right]$. Explain your results. What happens to the wave length of the tide due to friction? In case of moderate friction, is the M2 tide resonant for larger basin lengths or smaller basin lengths compared to the case of**

**negligible friction? Give two reasons why the amplitude of the M2 water levels at the end of the basin is smaller for increasing values of Cd.**

To investigate the influence of friction to our tidal waves, we look at the basin with a length of 110 km which we found to be the most resonant case before. We let the simulation run multiple times with different drag coefficients which results in the M2 amplitudes as given in figure 7.

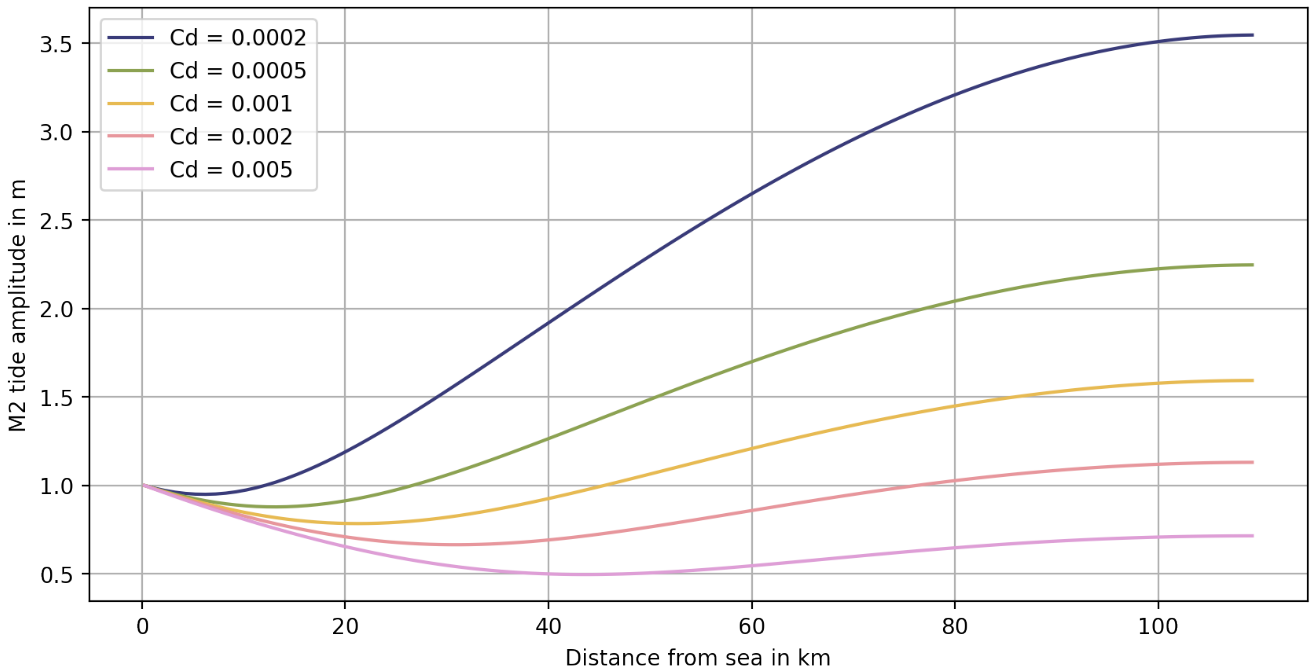


Figure 7: Amplitude of the M2 tide as a function of distance of the sea in a simulated rectangular basin with different drag coefficients.

From there, it is evident, that the maximum M2 amplitude is higher with reduced friction. As the phase speed of the tidal wave decreases with increased friction this also decreases the wavelength of that wave in the basin according to equation 2 and ultimately also reduces the basin length $L$ at which resonance occurs according to equation 1. Consequently, for moderate friction we would expect shorter resonant basin lengths than for negligible friction. This is also one reason why the amplitude of the M2 water levels at the end of the basin is smaller for increasing values of the drag coefficient. With increasing Cd, the basin length matches less and less the resonance length and thus the amplitude decreases. Another effect playing a role here is the energy loss of the wave due to friction which decreases the maximum amplitude of the M2 wave.

C. **Closure of Zuiderzee. Take a basin depth of 5 m, $Cd = 2 \cdot 10^{-3}$, $\Delta x = 1000\,\mathrm{m}$. Study M2 sea surface height amplitude and phase for the case that the basin is 110 km long (before closure) and for the case that the basin is 40 km long. Explain why for the shorter tidal basin the Tidal Prism is larger than the case that the basin was long. Take two effects into account: The tidal**

**wave propagation and resonance effects. The tidal prism (TP) is calculated as:**

$$TP = 0.5 \int_0^T |Q(x = 0, t)| dt$$

Lastly, we want to study how the closure of the Zuiderzee influences the tides. For that, we simulate a basin with a length of 110 km, which should simulate the Zuiderzee before closure, and a length of 40 km, representing the estuary after closure. The simulated behaviour of the M2 amplitude is shown in figure 8.

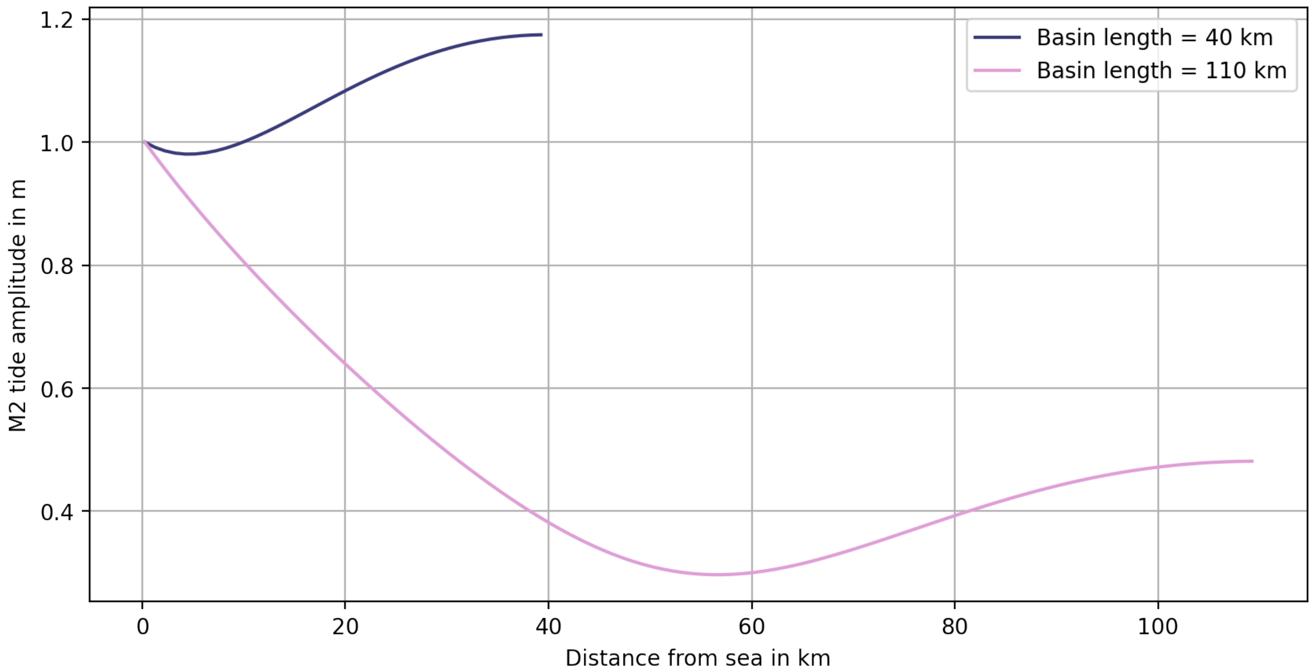


Figure 8: Amplitude of the M2 tide as a function of distance of the sea in a simulated 5 m deep rectangular basin with of different lengths.

It can be clearly seen that the amplitudes of the shorter basin are much higher than for the longer one. Consequently, the closed Zuiderzee is much closer to a resonant case than the open Zuiderzee. That the closed Zuiderzee in fact almost forms a standing wave can also be seen in figure 9, where the phase of the wave is plotted as function of distance to the sea.

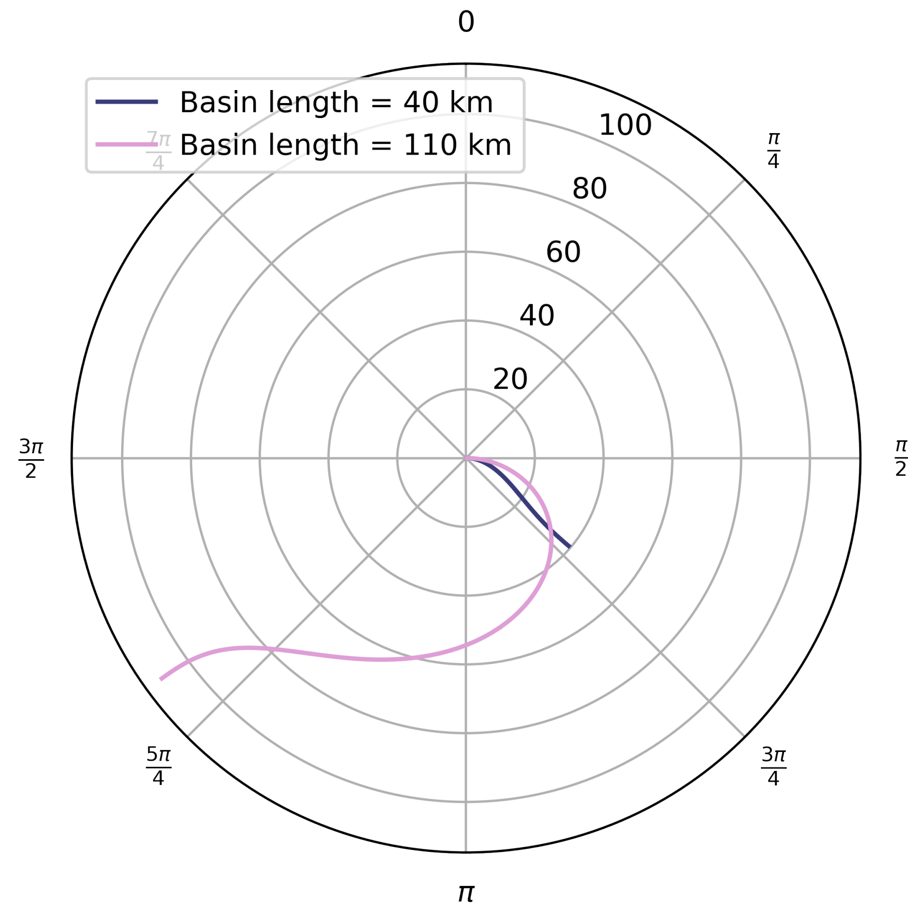


Figure 9: Phase of the M2 tide as a function of distance of the sea in a simulated 5 m deep rectangular basin with of different lengths. The distance to the sea is represented by the radius and has units of kilometers.

While the face of the long basin (pink) changes a lot with distance, the phase of the short basin (purple) stays almost constant. This indicates that the tides here are indeed close to a standing wave where high tide (and low tide) is reached at the same time everywhere in the basin.

The tidal prism (TP) measures the amount of water flowing through the entry of the basin between high tide and low tide. For the short basin we find $TP_{short} \approx 87\,\mathrm{Mm}^2$ while the longer basin has a tidal prism of about $TP_{long} \approx 62\,\mathrm{Mm}^2$. At first glance, this is surprising since the longer basin has a water volume which is more than twice as large as that of the shorter basin. There are, however, two main reasons why the TP is in fact bigger for the shorter basin than the longer one. Firstly, as we discussed earlier, the shorter basin is closer to the resonant case which is also visible by the larger tidal amplitudes. These higher amplitudes also need more water to be flowing in and out of the basin which increases the tidal prism. Additionally, as also discussed when we looked at the phase of the M2 amplitude, the almost standing wave that formed in the shorter basin requires high tide everywhere in the basin at the same time. This also increases the TP as all the water generating the difference of high and low tide has to completely flow in and out of to the basin in each tidal period. In conclusion, the resonance and wave propagation effects both increase the TP of the short basin so much that it overcompensates the shorter length of the closed Zuiderzee.

**Part 2: Dynamics of estuaries**

**For this case we take a converging channel width, as often observed in real estuaries. The width is modeled as: $B(x) = B_0 e^{-x/L_b}$. Where $B_0$ is the width at the entrance of the estuary and $L_b$ is the e-folding length scale of the width. At some point the channel gets a constant width $B_{river}$ determined by the dynamics of the river.**

**Discuss and explain your results:**

D. **Solve the tidal dynamics in your own estuary as explained in the instructions.**

1. **Vary the bed level and try to find the depth for which the estuary is an equilibrium estuary. Show and discuss your results for three depths: equilibrium depth, a much shallower system (take 66% of the equilibrium depth) and much deeper system (take 150% of equilibrium depth).**

   For the Elbe estuary $B_0 = 5$ km, $B_{river} = 0.3$ km and $x_r = 120$ km is used. From $L_b = -x_r/\log(B_{river}/B_0)$ it follows that $L_b = 42\,652.9$ m. The average values of D1 and D2 are taken from the last problem set. These amplitudes were found to be 0.18 and 1.45 m, respectively. From this it follows that $D2 > 3 \cdot D1$ and a semi-diurnal tide is prescribed. The bed level leading to the smallest variation in M2 water level amplitude over the first 75% of the estuary is $H0 = 8.69$ m, which is thus the equilibrium depth. This value is obtained by least-squares fitting, namely via the `scipy.optimize.curve_fit` function.

   i) **Explain the sensitivity of M2 water level amplitude to depth.**
   In figure 10 the amplitude of the M2 tide is shown for the three different depths. Here it can be seen that for the equilibrium depth the amplitude indeed shows the least variation and is closest to the incoming M2 amplitude at the sea (black). For the deeper system the amplitude of the tide is higher and the amplitude is lower for the shallow system. For the equilibrium depth increase of energy by channel convergence should be exactly balanced by the loss due to bed friction. Since the amplitude does not stay exactly the same this is not fully the case in this system, but is a good approximation.
   For the shallower case friction is the dominant force, leading to a decrease in amplitude. The reverse is the case for the deeper case, where friction is less important and channel convergence overcompensates the friction, leading to an increase in amplitude.
   Summarizing, we can say that for the more shallow system friction is the dominant effect for the M2 water level whereas for the deep system channel convergence dominates.

   ii) **Explain the increase in tidal amplitude at the landward end of the estuary.**
   In figure 10 it can be seen that the amplitude for all three depths increases landwards from $x_r$ onwards. This seems counter-intuitive as the convergence of the estuary stops here and thus cannot balance the friction anymore. Consequently, this should lead to energy-loss and the amplitudes should decrease. A possible explanation to why that is not the case might lie in wave reflection. As the tidal wave reaches the end of the basin, it is reflected and might interfere constructively with the incoming waves. This leads to a an increase of the M2 amplitude at the end of the basin.

   iii) **Determine and explain the phase speed (=propagation speed of the wave; note that phase behaves as -kx, with k the wave number and that c=angular velocity/wave number) of D2 tide (and D1 tide if you have mixed tide) and its sensitivity to water depth. Also discuss the deviations from c=sqrt(gh).**
   In order to determine the phase speed of the waves, first the phase of the M2 wave of the three different depths are plotted (see figure 11). Here it can be seen that for all three depths, the phase increases approximately linearly up to $x = 0.75x_r$. For the three depths

a linear fit is done in this range in which the slope is equal to k. The linear fit found by *scipy.optimize* is also shown in the legend of figure 11. Using $C_{\text{model}} = \frac{\omega}{k}$ then the phase speed is determined, where $\omega = \frac{2\pi}{T}$ and $T$ is the semi-diurnal period in seconds (44 700 s). The theoretical phase speeds are calculated according to $C_{\text{theory}} = \sqrt{gh}$. This then leads to the following phase speeds for the different depths:

| **H0 in m** | **$C_{\text{model}}$ in m/s** | **$C_{\text{theory}}$ in m/s** |
|---|---|---|
| **5.74** | 6.59 | 7.50 |
| **8.69** | 10.27 | 9.24 |
| **13.04** | 15.68 | 11.31 |

In an equilibrium estuary, we would expect the phase speed of deep, straight and of shallow, converging estuaries to be the same. As we are close (yet not equal) to an equilibrium estuary for $H0 = 8.69$ m, the relative difference between modeled and theoretical phase speed is lowest here. For other water depth, the modeled phase speed deviate farther from $\sqrt{gh}$ as they are not close to equilibrium anymore. Here, their phase speed could be more accurately calculated using theory for shallow, funnel-shaped estuaries where $c \propto h \not\propto \sqrt{h}$. Consequently, the theory where $c = \sqrt{gh}$ becomes less accurate the more the situation deviates from an equilibrium estuary (Friedrichs, 2010, p. 47-48). Another explanation that explains the difference in $C_{\text{model}}$ from $C_{\text{theory}}$ could be that the fitted line in figure 11 is a less good fit for the bigger H, since this directly influences the wave number and thus the phase speed.

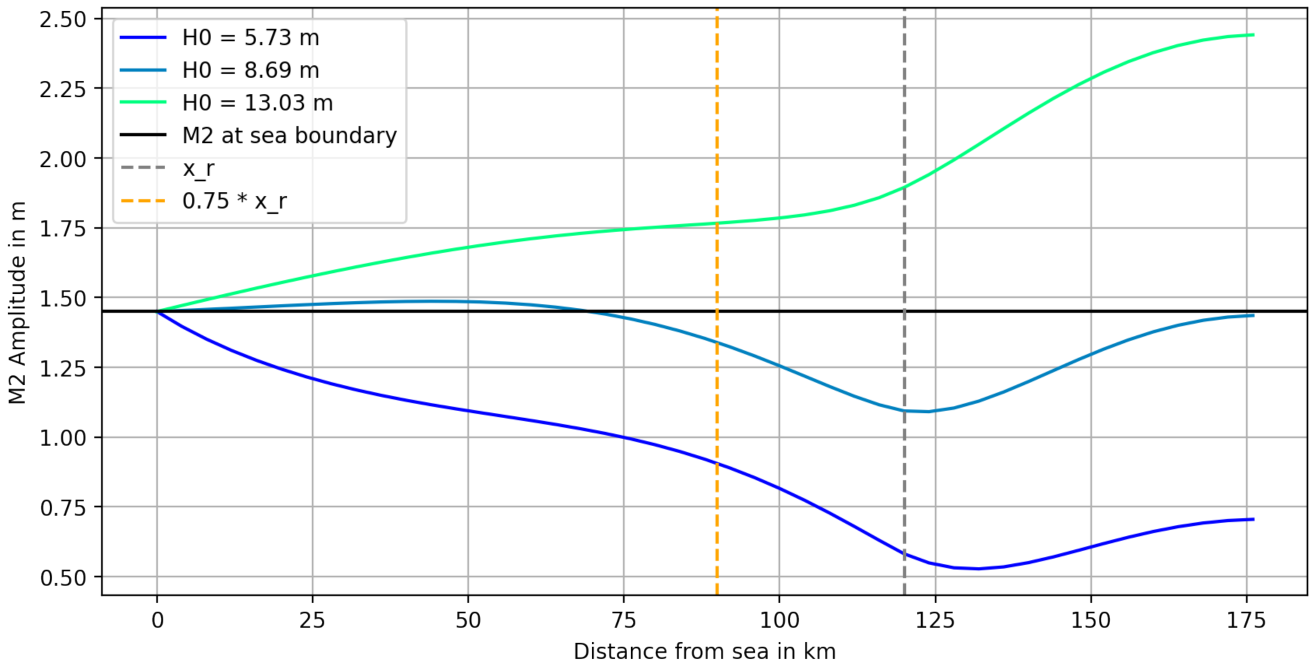


Figure 10: Amplitude of the M2 tide as function of distance from the sea in a simulated Elbe basin for the equilibrium depth (H0=8.68 m), 66% of the equilibrium depth (H0=5.73 m) and 150% of the equilibrium depth (H0=13.02 m). With the dotted grey line the end of the estuary and the beginning of the river is marked.

2. **For the case with equilibrium depth, compare the modeled wave number (k), propagation speed of wave (C), velocity magnitude (U), and phase difference ($\phi$) between water levels and flow velocities with the theoretically predicted values presented in section 3.10 in Friedrichs (2010).**

   The method to determine the wave number and the propagation speed of the wave from the model is described in Section iii). For the velocity magnitude the mean in the section until x=0.75 x_r of the M2 amplitude of the velocity is used. For the phase difference the difference between the phase based on the water level and velocity is used. This is then also averaged for the section until x=0.75 x_r.

   For the analytical calculations the equations from Friedrichs (2010) are used. Since there are no tidal areas $w = b$ and $Lw = Lb$. The left side of equation 3 then reduces to $C = \sqrt{gh}$. This is then calculated by plugging in the values for g and the equilibrium depth. Using this value of C, the $\omega$ of Section iii) and Equation 3 then the value of k is calculated. With Equation 4, combined with the average D2 amplitude $a$ of 1.45 m the analytical magnitude of the velocity is determined. Next, using equation 5, the friction factor r is determined, which is then used for calculating $\phi$ using Equation 6.

$$c = \frac{\omega}{k} = \left(\frac{ghw}{b}\right)^{1/2} \quad \text{and} \quad c = \frac{\omega}{k} = \frac{ghw L_w{}^{-1}}{rb} \tag{3}$$

$$U = \frac{a\omega b}{hkw\left[1 + (kL_w)^{-2}\right]^{1/2}} \tag{4}$$

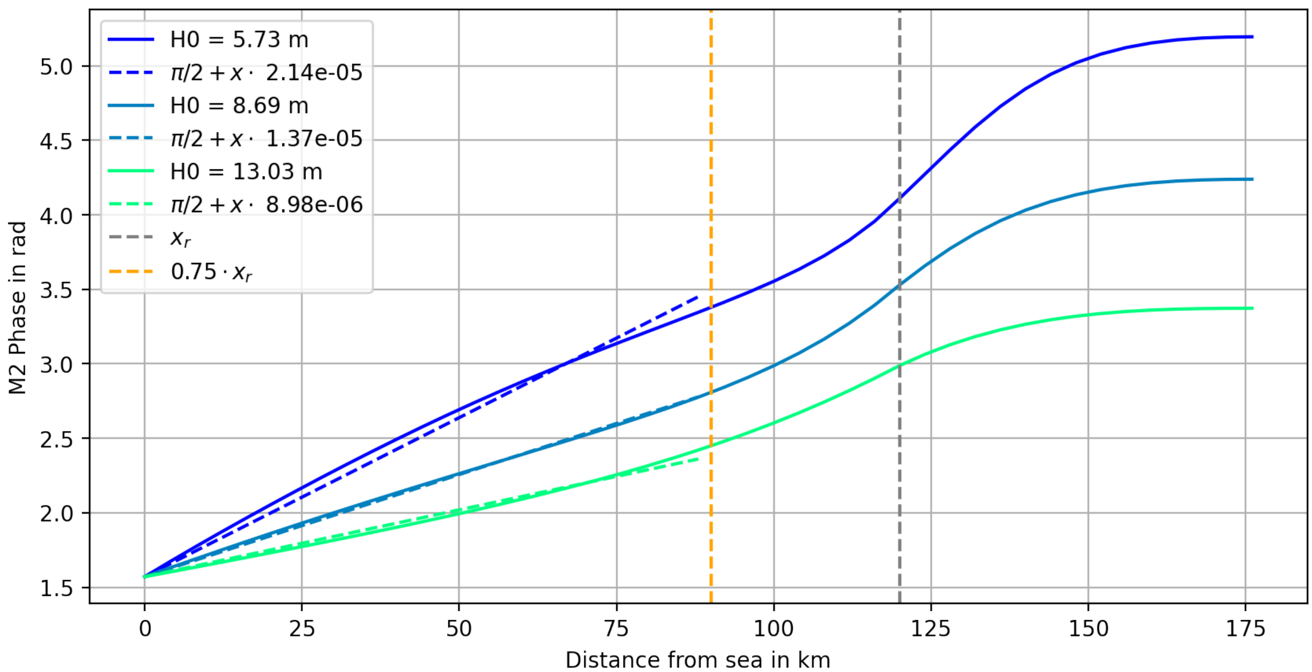


Figure 11: Phase of the M2 tide as a function of distance from the sea in a simulated Elbe basin for the equilibrium depth (H0=8.68 m), 66% of the equilibrium depth (H0=5.73 m) and 150% of the equilibrium depth (H0=13.02 m). With the dotted lines a linear fit of the phase is denoted in which the slope is the wave number (k). With the dotted grey line the end of the estuary and the beginning of the river is marked.

$$r = \frac{c_d}{< h >} \frac{8U}{3\pi} \tag{5}$$

$$\phi = -\arctan\left(\frac{L_w{}^{-1}}{k}\right) = -\arctan\left(\frac{r}{\omega}\right) \tag{6}$$

This then results in the following values:

| **Variable** | **Model** | **Theory** |
|---|---|---|
| **C** ($ms^{-1}$) | 10.25 | 9.23 |
| **k** ($m^{-1}$) | 1.37e-05 | 1.52e-05 |
| **U** ($ms^{-1}$) | 0.82 | 0.84 |
| **Phi (rad)** | -0.95 | -0.97 |

In general the values of the model and the analytical solutions are very comparable. This means that equations governing a long, intermediate depth equilibrium basin (=theory) align well with the analytical solution of the model of the Elbe estuary (=model). Thus, this theory is a good approximation for the model. The biggest differences are seen in the value of C and k. This is probably because of the fact that the modelled estuary is actually is not fully in equilibrium as is assumed in the theory. There are still some small deviations of the M2 amplitude, even for the equilibrium depth. Additionally, as k was fitted, it carries its uncertainty to all of the modelled parameters. Furthermore, deviations of the model and theory could in part be caused by assumptions made in the theory. For instance, $\Delta b / < b > \ll 1$ might be a relatively rough approximation.

3. **For the case of equilibrium depth: determine the magnitude of the mean flow along the estuary. Explain your results.**

   In figure 12 the tidally averaged flow velocity is shown.

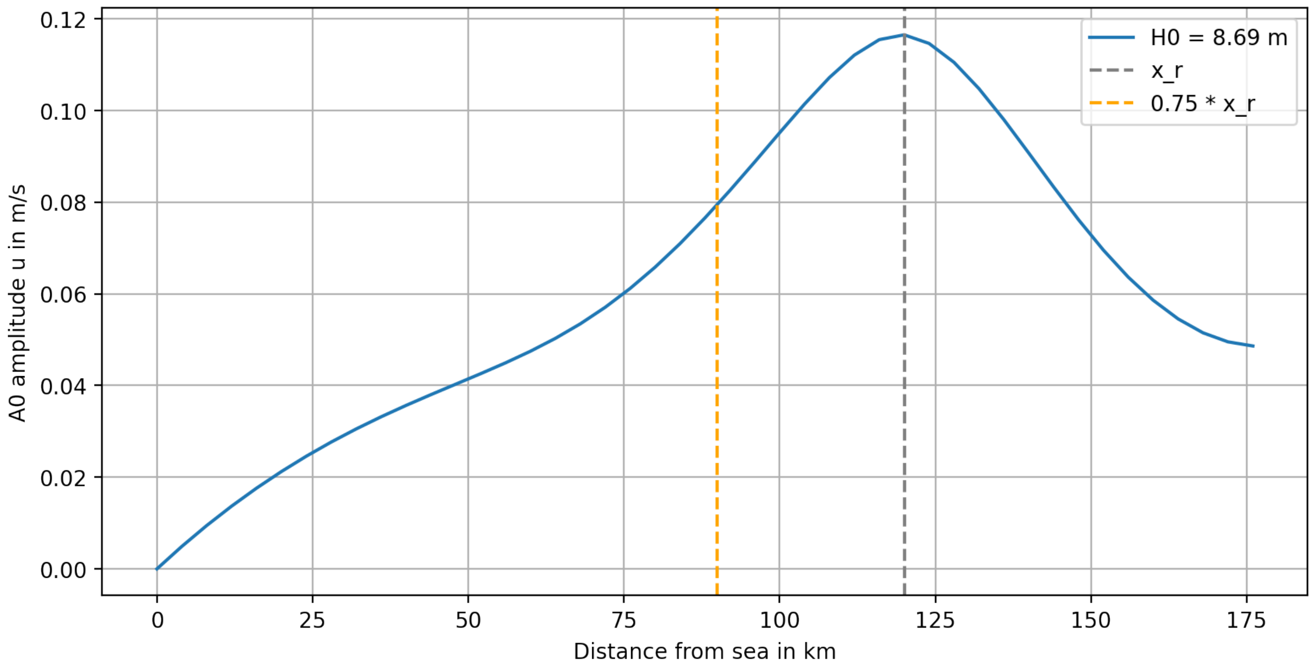


Figure 12: Tidally averaged velocity as a function of distance from the sea in a simulated Elbe basin for the equilibrium depth (H0=8.68 m). With the dotted grey line the end of the estuary and the beginning of the river is marked.

A positive mean flow velocity is observed along the estuary. This mean is steadily increasing until $x_r$, where the estuary transitions into a river. This behaviour is explained by a skewed signal. Therefore, with a positive mean flow velocity dominant in the estuary, a dominant ebb current is present. After $x_r$ the mean velocity decreases again because the channel is not convergent anymore.

4. **Take values of your optimized case. Study the sensitivity of (i) M2 water level amplitude, (ii) M2 velocity amplitude and (iii) propagation speed of tidal wave to e-folding length scale of channel width (Take range between Lb 50% larger and $L_b$ 33% smaller). When changing $L_b$, also change $B_0$ by using that $B_0 = B_{river} * \exp(\mathrm{xr}/\mathrm{Lb})$. Explain your results.**
Using the value of the equilibrium depth (H0=8.68 m), the model is rerun using three values of Lb. The original value of Lb=42 652.85 m, 67% of the original Lb (Lb=28 150.88 m) and 150% of the original Lb (Lb=63 979.27 m).

   i) The M2 water level amplitude for the different e-folding length scales is shown in figure 13.

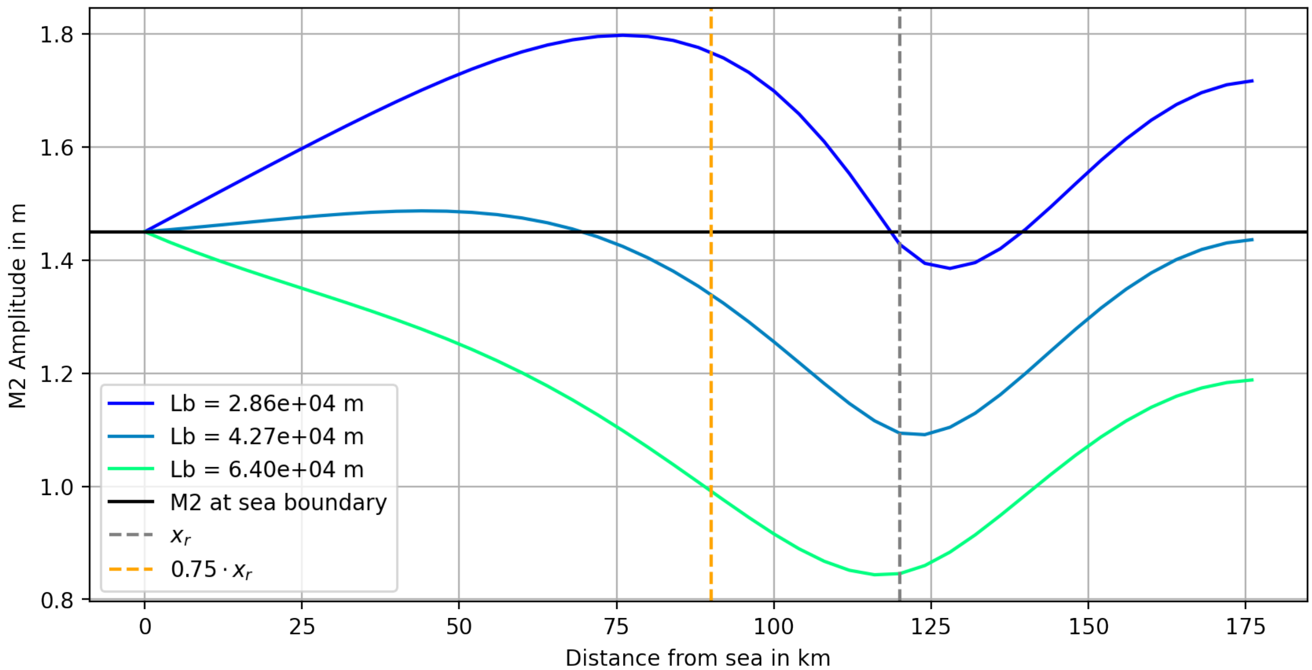


Figure 13: Water level amplitude of the M2 tide as function of distance from the sea in a simulated Elbe basin for the original e-folding length scale (Lb=42 866.12 m), 67% of the original Lb (Lb=28 150.88 m) and 150% of the original Lb (Lb=63 979.27 m). With the dotted grey line the end of the estuary and the beginning of the river is marked.

As expected, with $L_b = 100\%$ the M2 Amplitude is relatively constant till 75% $x_r$, since this is closes to an equilibrium estuary. For 150% of $L_b$, the amplitude of M2 is decreasing faster than for 100% of $L_b$. This is in line with expectation, since the e-folding length heavily increased here. The channel convergence is not strong enough to balance the energy loss by friction and thus the amplitude drops. According to Friedrichs, if the the e-folding length is greater than $\frac{c}{r}$, the amplitude decays Friedrichs (2010). In fig 13 we can see that this is the case. Consequently, if the e-folding length is less than $\frac{c}{r}$, the amplitude grows. We can see in figure 13 that this is the case for the 75% $L_b$. Here, the channel convergence overcompensates friction and the amplitude increases.

ii) The M2 velocity amplitude for the different e-folding length scales is shown in figure 14.

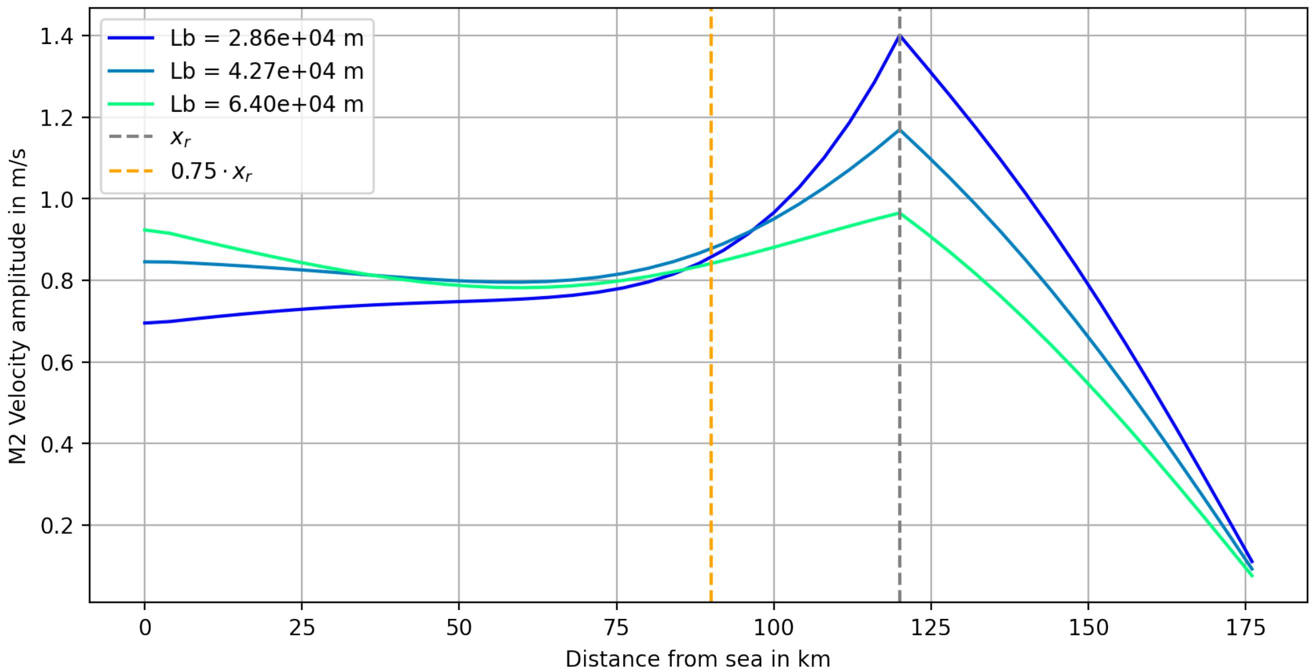


Figure 14: Velocity amplitude of the M2 tide as function of distance from the sea in a simulated Elbe basin for the original e-folding length scale (Lb=42 866.12 m), 67% of the original Lb (Lb=28 150.88 m) and 150% of the original Lb (Lb=63 979.27 m). With the dotted grey line the end of the estuary and the beginning of the river is marked.

For the 67% simulation, the M2 velocity amplitude increases more towards the end of the converging basin than the 100% simulation,while and the velocity amplitude for the 150% simulation is even lower. This again has its origin in the channel convergence, as the stronger converging estuaries have more energy to support faster flow velocities. They are needed to move enough water for the change in amplitude observed in figure 13.

iii) Here the same method as in Section iii) is used. In figure 15 the phase is shown for the different e-folding length scales combined with the linear fits and the different values of k. This then leads to the following wave velocities:

| **Lb (m)** | **C model** ($ms^{-1}$) | **C theory** ($ms^{-1}$) |
|---|---|---|
| 28150.88 | 14.05 | 9.23 |
| 42866.12 | 10.25 | 9.23 |
| 63979.27 | 9.07 | 9.23 |

The value for the theoretical value of C is constant, since the model used has a constant $h$. $C_{\text{theory}}$ is calculated by $C_{\text{theory}} = \sqrt{g \cdot h}$, and is therefore the same for all e-folding length.

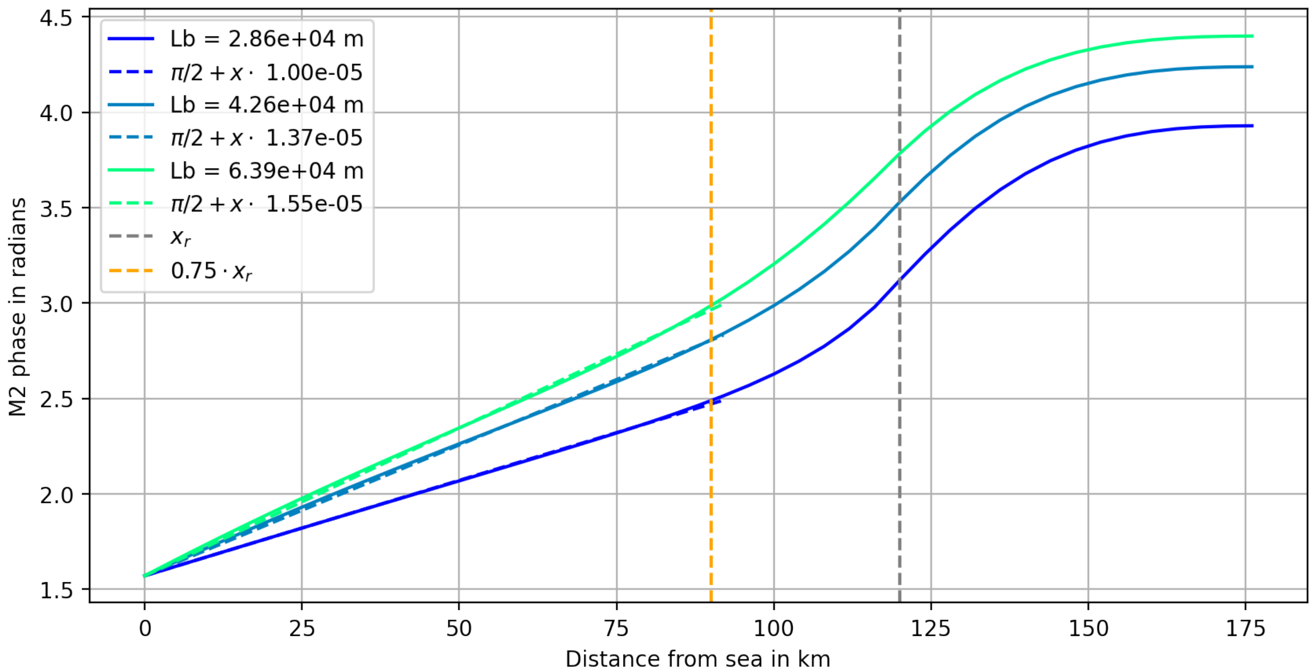


Figure 15: Phase of the M2 tide as function of distance from the sea in a simulated Elbe basin for the original e-folding length scale (Lb=42 866.12 m), 67% of the original Lb (Lb=28 150.88 m) and 150% of the original Lb (Lb=63 979.28 m). With the dotted lines a linear fit of the phase is denoted in which the slope is the wave number (k). With the dotted grey line the end of the estuary and the beginning of the river is marked.

It can be seen that the M2 phase is increasing with length in the basin for all different e-folding length. Though, for the shortest e-folding length the slope is the smallest, meaning this phase progresses relatively slower. The biggest e-folding length system has the highest slope and thus this phase increases relatively fastest. This is compatible with the theory as the phase speed is proportional to $Lb^{-1}$ and thus increases with lower e-folding lengths.

Furthermore, it can be seen that the wave velocities of the model with 100% and 150% simulation are the most similar to the theory. This makes sense since the equation to calculate the theoretical wave speed is for deep, straight estuaries and the longer the e-folding length scale is, the less important the convergence is. Thus leading to a wave speed more similar to the theory.

## References


Friedrichs, C. T. (2010). *Barotropic tides in channelized estuaries*, page 27–61. Cambridge University Press.