Morphodynamics of Tidal Systems
Analyzing Water Levels in Python

Lasse Vulto (4685784), Ida Haven (3490475), Bennet Weiss (7349106)

November 10, 2023

Answer the following questions:

1. **Make a plot of the measured water levels during one year, and several subplots in which you indicate the following: (Use a maximum of 4 figures and keep your text brief.)**

   (a) **Diurnal inequality, the spring-neap cycle, the yearly variation in magnitude of the diurnal and semidiurnal tide.**

   Figure 1 shows the water level in Cuxhaven for the year 2018. The dataset is complete for the entire year, with the exception of the 18th to the 20th of Septemper. For the 18th and 19th September some data is missing and no data is available for 20th of September.

   The diurnal inequality is shown in Figure 2, in which the data of the 6th of May is shown. The diurnal inequality can be seen in the difference in amplitude between the first and second peak, since the amplitude of the second period is larger than the amplitude of the first period.

   Figure 3 shows the tidal signal from Figure 1 for the months of May to September and an estimate of the spring-neap cycle. The spring-neap cycle can most clearly be in the summer months as the data is least noisy here. The spring tide occurs during full moon and new moon, when the moon sun and earth are aligned and the gravity of both moon and sun enhance each other. Neap tide occurs during first and third quarter when the sun, moon and earth form a right angle and and the combined gravity of moon and sun dampens the tides.

   The semi-diurnal cycle can be seen as the double wave signal during the day, with the diurnal cycle as the difference between the first and the second wave signal during the day. Based on Figure 2 the difference between the first and second wave (order of magnitude of 0.1 m) is small compared to the amplitude of the semi-diurnal wave (order of magnitude of 1 m). Since the magnitude of the semi-diurnal tide is an order of magnitude bigger, we conclude that the tide in Cuxhaven is mainly semi-diurnal. This fits with the theory shown in Lecture 2.

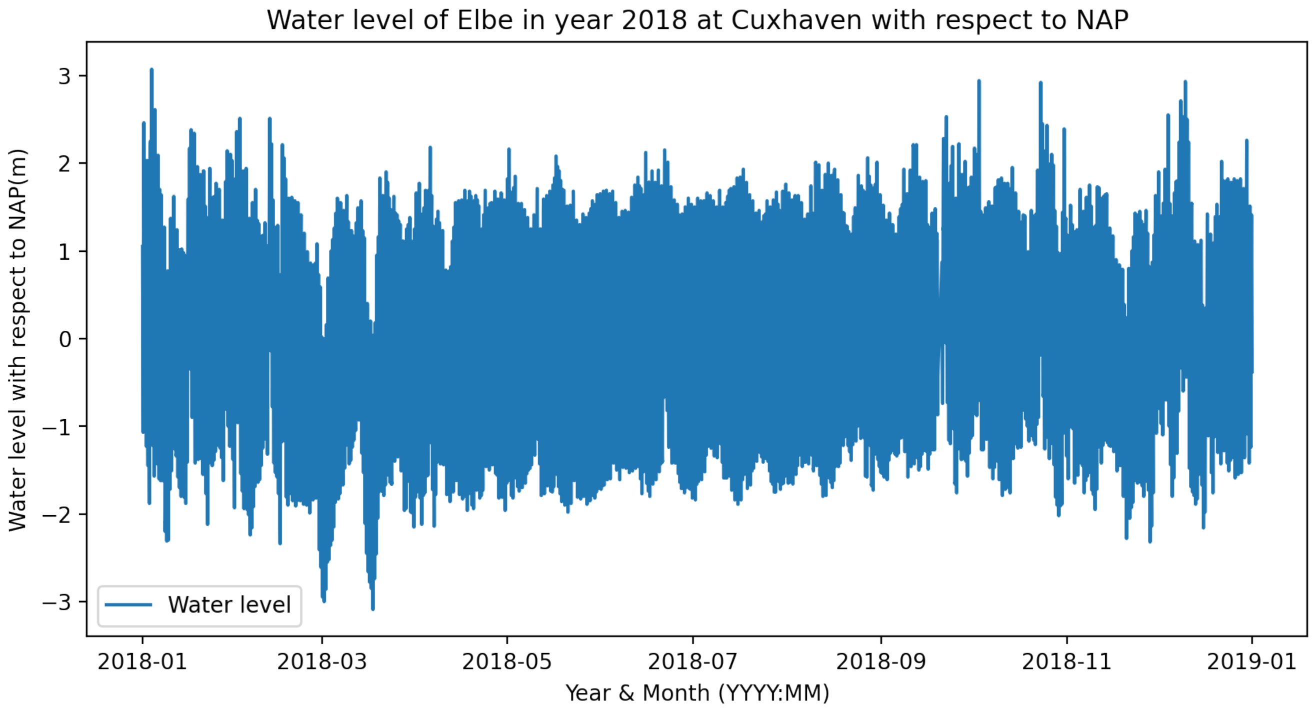

Figure 1: Water level of the Elbe in 2018 at Cuxhaven with respect to NAP

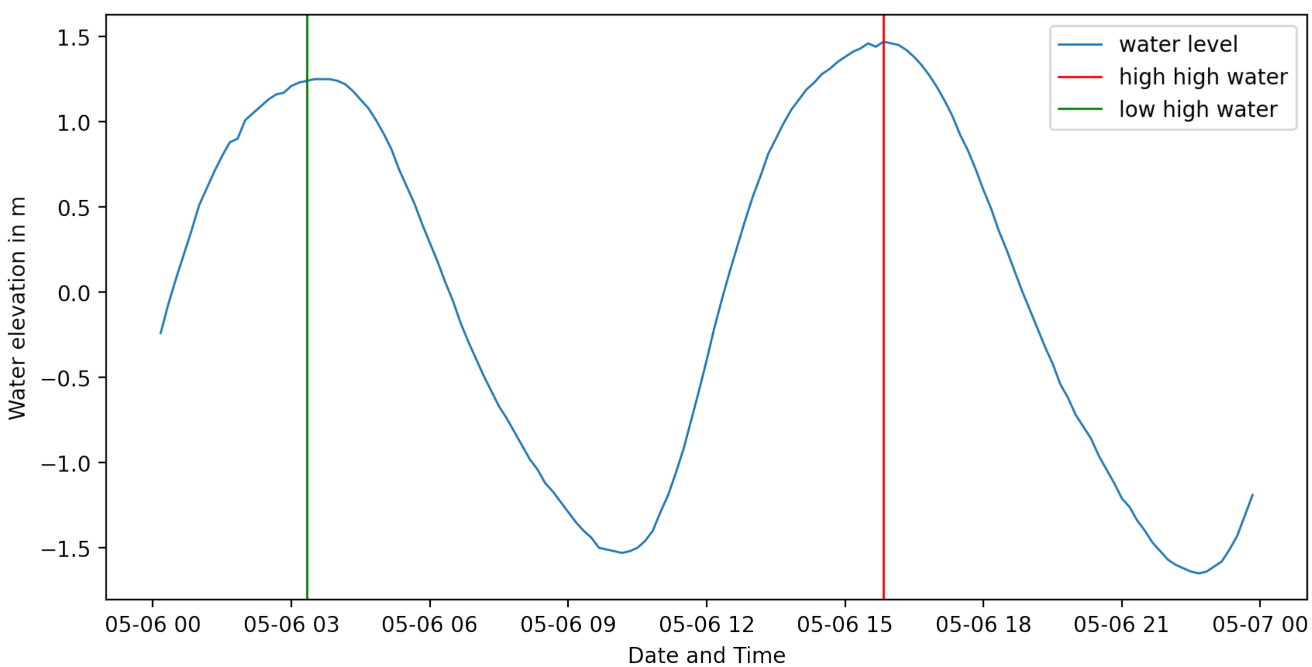

Figure 2: 24 Hours water cycle of the Elbe on 6 May 2018 from 00:10 to 23:50

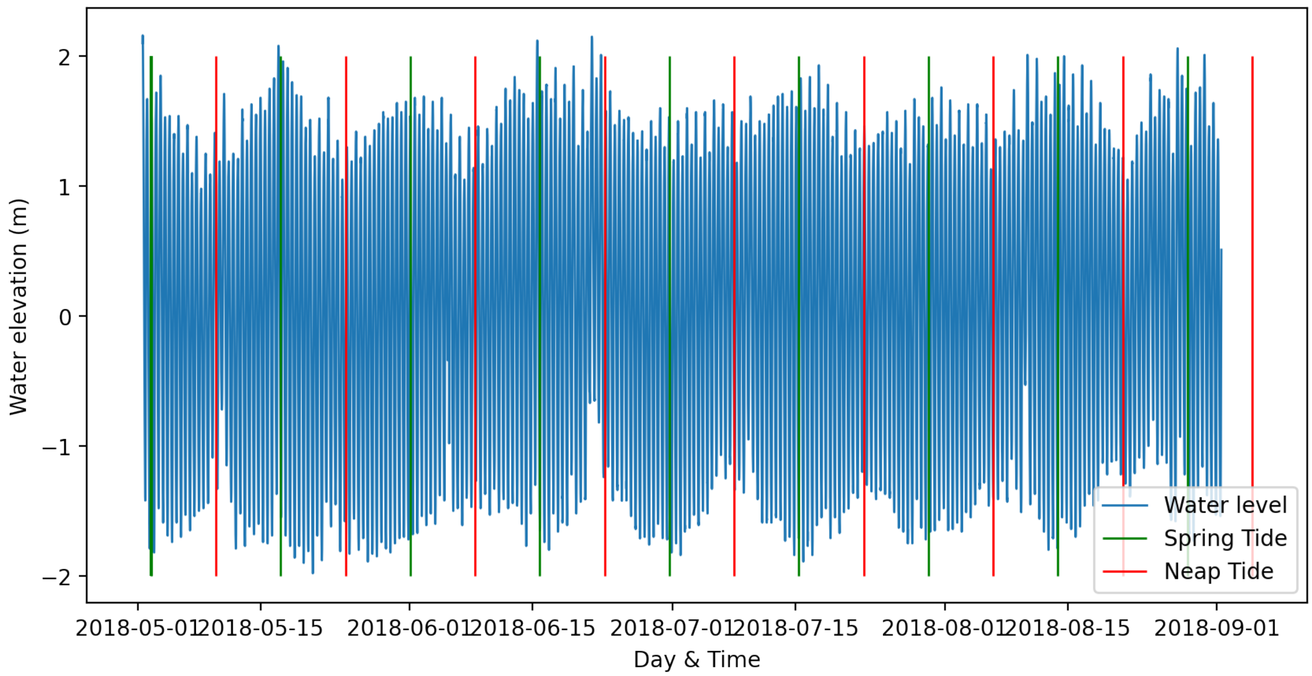

Figure 3: 4 Month water level cycle of the Elbe in 2018

(b) **Asymmetry of tidal signal: average HW level compared to LW level (compare them to yearly averaged water level Zmean), time water levels are above Zmean versus time that water levels are below Zmean.**

The average water level of the Elbe in 2018 is 0.092 m above NAP. That the average sea level is around 0.1 m and not 0 can be explained by the average sea level raise, which is also around 0.1 m with respect to NAP (sea). The average high high water level is 1.613 m and the average low low water level -1.562 m above NAP. In the year 2018 there are 361.208 days of data recorded of which 185.312 days the Elbe was above average and 175.896 days below average water level. Thus on yearly basis the Elbe is 10 more days above than below average water level. This is relatively normal number of difference. This can have to do with external influences like the wind, the bathymetry of the shelf and the estuary and in general asymetry of the tides. Meaning that the tides come into the estuary faster than they leave the estuary.

(c) **The moments in the year with strong nontidal behavior (storm surge, strong set down)** In general, the signal in Figure 1 seems somewhat noisy. This could be explained by the measurement location; Cuxhaven, which is located at the mouth of the Elbe estuary. Since the Elbe estuary is connected to the North Sea, which is very shallow near the Dutch and German coastline. Furthermore, the currents and thus also the tidal currents are influenced by the presence of the landmasses of Denmark, Norway and the United Kingdom. The variation in the amplitude seems to be the strongest during the winter period. This can also be seen in Figure 4. One explanation could be that it is due to tidal variation as a result of a shorter distance between the earth and the sun. Another, and possibly the dominant, explanation could be the non-tidal factors, such as weather variability like zonal wind stress and a varying discharge of the Elbe during the winter months (2).

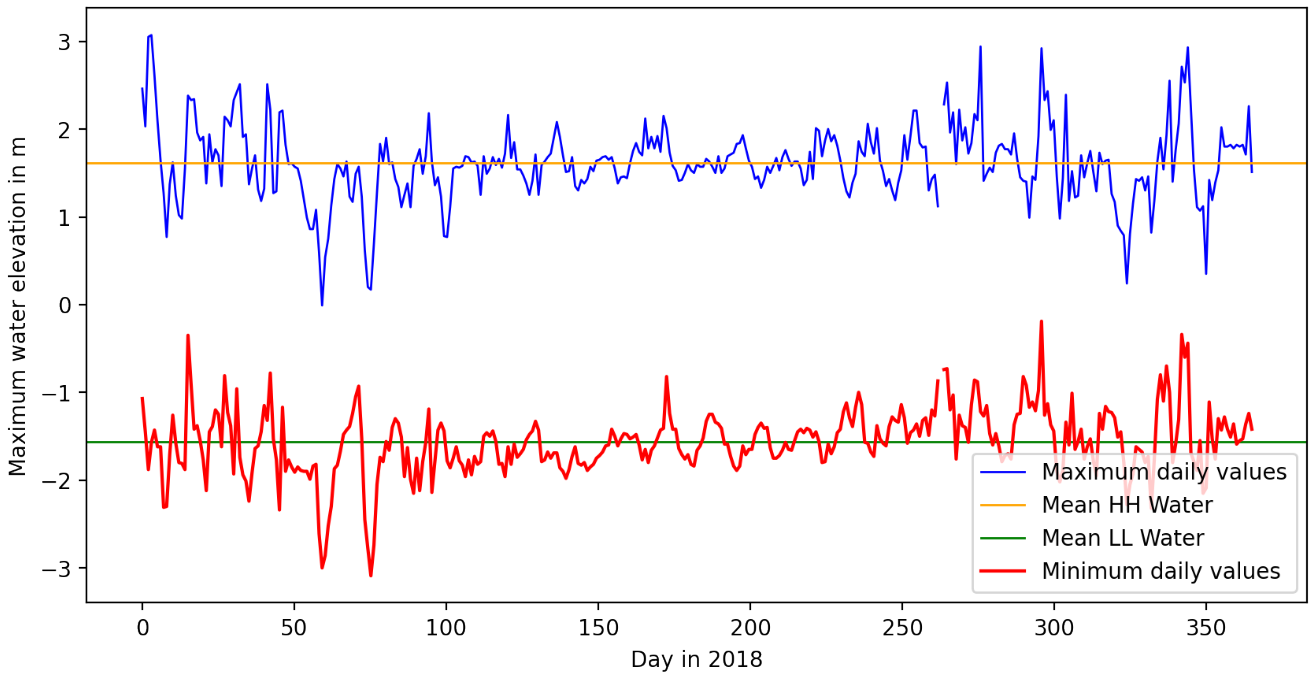

Figure 4: Daily maximum high water and low water of the Elbe in 2018 (Januari 1st till December 31st)

2. **Make a harmonic fit to the full year data.**

   (a) **Make a table with calculated amplitudes and phases of the tidal constituents. Also include the value of the mean water level (a0) and quantify the quality of the fit. (Insert your table here.)** To choose the relevant constituents for the fit, a Fast-Fourier transform was used which can be seen in Figure 5. Using this Fourier analysis an estimation of the most important constituents for the fit was made. Based on Figure 5 the most important constituents are those with a period of around 12 h and additionally those with a period of 4, 6, 24 and 26 h. This fits with the expected semi-diurnal, diurnal and higher harmonics constituents.

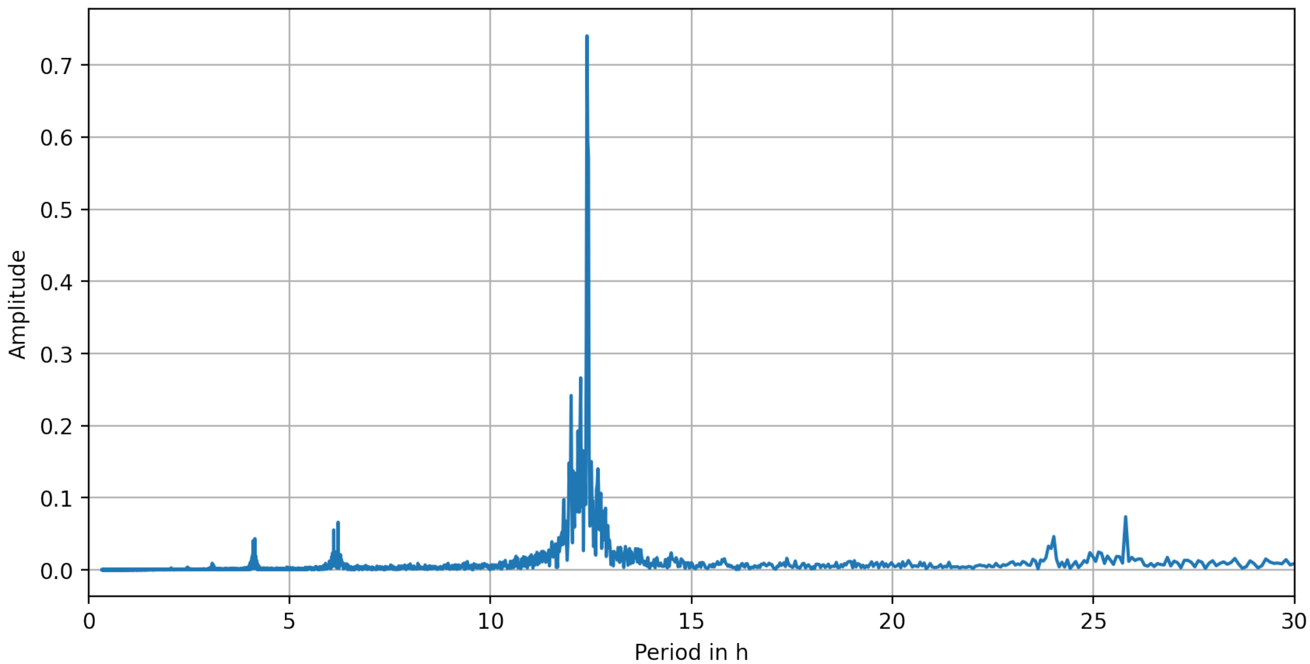

Figure 5: Fast-Fourier Transform of the water 2018 level data for periods up to 30 h. Peaks indicate important sinusoidal constituents of the signal.

In Table 1 the constituents used for the fit are shown. All the constituents in the table passed the Rayleigh criterion: $(\omega_2 - \omega_1)\,T_{obs} > 2\pi$ where the observed time is 1 year, and ω_i is the angular frequency of the constituent. All combinations of frequencies were checked. The mean water level is 0.092 m and the RMS is 0.41 m. From the table, it can be seen that the constituent with the largest amplitude is the M_2 tide. This is also reflected in the Fourier transformation where also the strongest signal is one with a period of 12.42 h.

Table 1: Constituents, amplitude and phase of the constituents used for simulating the tides in Cuxhaven

Constituents	**Amplitude (m)**	**Phase (rad)**
$\mathbf{M_2}$	1.373164	-0.784589
$\mathbf{K_1}$	0.064383	0.658362
$\mathbf{M_4}$	0.115182	2.743362
$\mathbf{M_6}$	0.073172	-0.444361
$\mathbf{S_2}$	0.335179	0.914361
$\mathbf{N_2}$	0.196421	-1.364509
$\mathbf{K_2}$	0.092831	-2.318160
λ_2	0.043511	2.860884
$\mathbf{O_1}$	0.093716	-2.358208
$\mathbf{M_m}$	0.027671	1.982821
$\mathbf{S_{sa}}$	0.045057	-0.200837
$\mathbf{S_a}$	0.146217	-1.561837
$\mathbf{M_{sf}}$	0.039477	1.350640
$\mathbf{M_f}$	0.090786	0.285733

(b) **Plot the best fit together with the measured water levels and indicate the moments you have good correspondence between fit and data and the moments the quality is less good.**

The combination of the measured data and the best fit of the modeled tides is shown in Figure 8. As mentioned before, the measured water level is quite noisy. Therefore the best fit still cannot explain all the variation in water level. In general, the phase of the modeled and measured data, match up very well. However there is still quite a bit of variation in the difference between the amplitude of modeled and measured tides. The smallest difference in amplitude is seen during summer (05-2018 to 09-2018) and the largest difference during winter. This is also seen in Figure 7, where a few day selection is shown where the fit is very good, which was during summer, and the fit differs quite a bit, which is during winter. This also fits with the general trend seen in the yearly data, where there are larger variations in the water level during the winter.

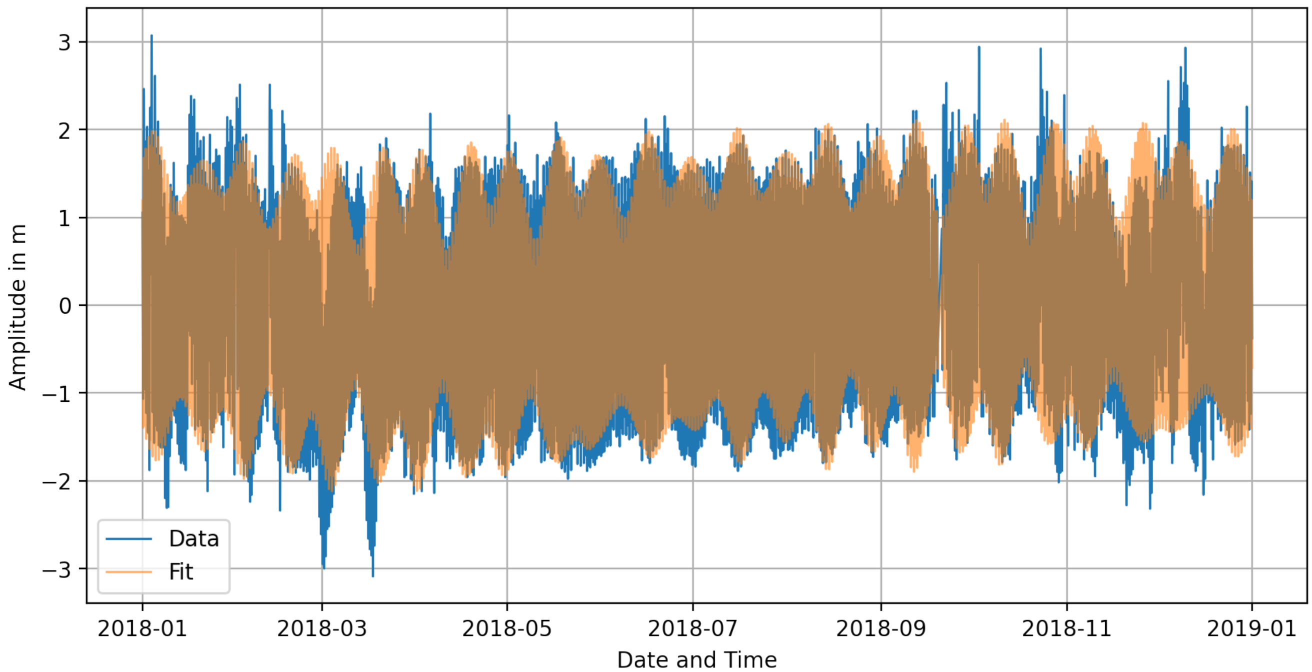

Figure 6: Water level measured in Cuxhaven in 2018 and the fitted tidal prediction of the water level

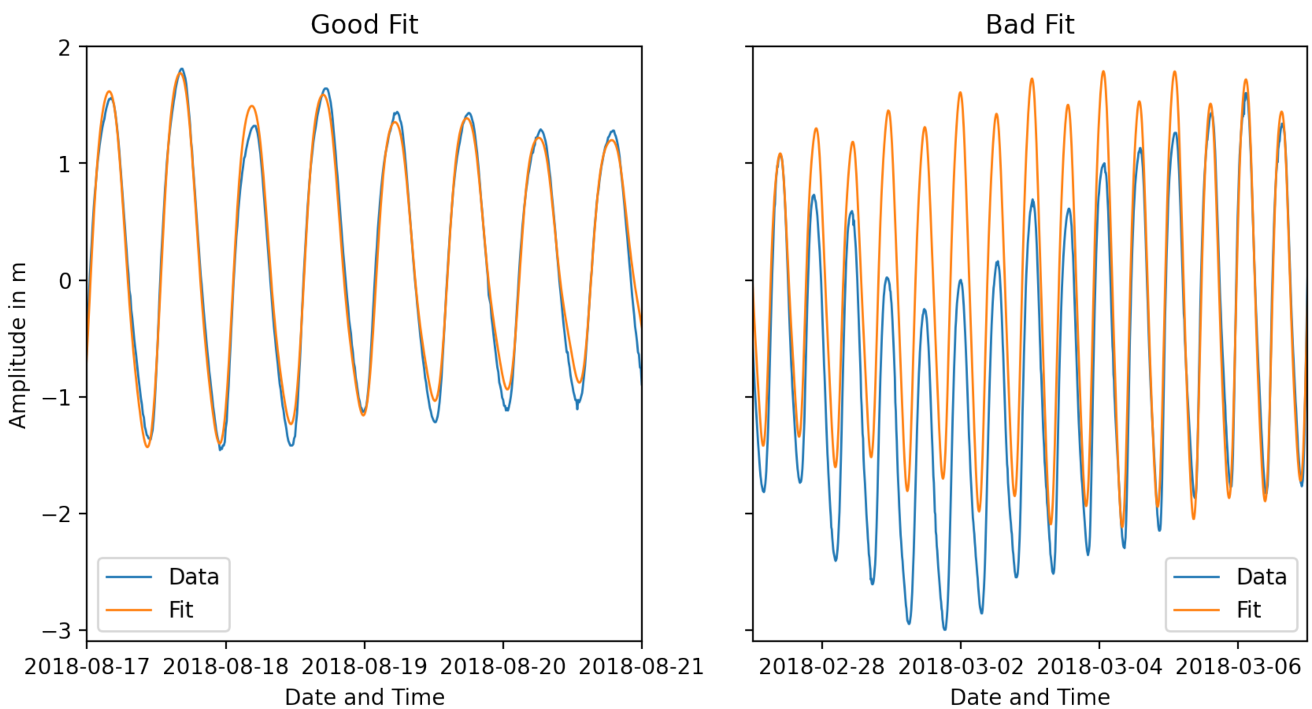

Figure 7: Few day selection of measured- and modeled tidal signal in Cuxhaven 2018 where the fit is very good and where the fit is not as good.

(c) **Plot the difference between observed and fitted data and discuss the reason for the mismatch**

between measured and fitted water levels.

Figure 8 shows the difference between the fit and the actual water level data measured in Cuxhaven. The mismatch could either be caused by which constituents are used for the fitting or by non-tidal causes. As reasoned in the next question the chance that the mismatch is caused by either not implementing enough constituents or implementing the wrong constituents is very small. Furthermore, in question 1a the hypothesis was mentioned that the high variability in sea water level could be caused by the lunar cycle or non-tidal factors. By incorporation the solar cycle (Sa) in the constituents this hypothesis is actually tested. Although the solar cycle does explain part of the variation seen during winter, there is still quite a mismatch. This mismatch is therefore likely to be caused by non-tidal behaviour like wind-stress (which is higher during winter), difference in discharge from the Elbe and storm surges or let downs.

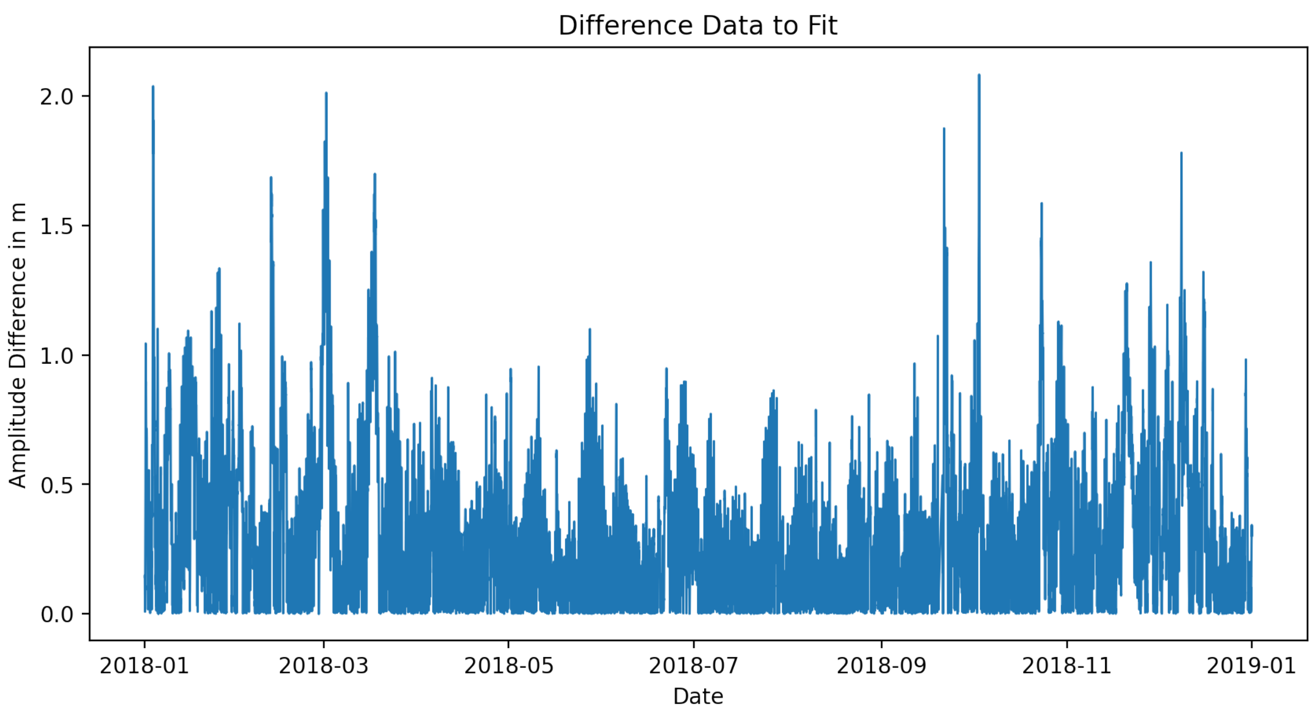

Figure 8: Absolute difference between the water level measured in Cuxhaven in 2018 and the fitted tidal prediction of the water level

(d) **Indicate how much the fit can be improved by adding more tidal constituents.** To determine whether the constituents we used really improve the fit and how much they improve it, the RMS of the fit is plotted as a funciton of the number of constituents in Figure 9. Here, it can be seen that the addition of the K1 constituent leads to the largest improvement of the model after which the improvement of the additional constituents leads to a smaller and smaller improvement. Therefore, we think that adding more constituents will not lead to significant improvements of the fit. Furthermore since the addition of all the constituents used leads to the improvement of the fit we also assume that based on this analysis and the Fourier analysis all the constituents are relevant for this location. Additionally, the Fourier-Analysis proves that no significant constituents with a period of below 30 hours were left out in the final fit.

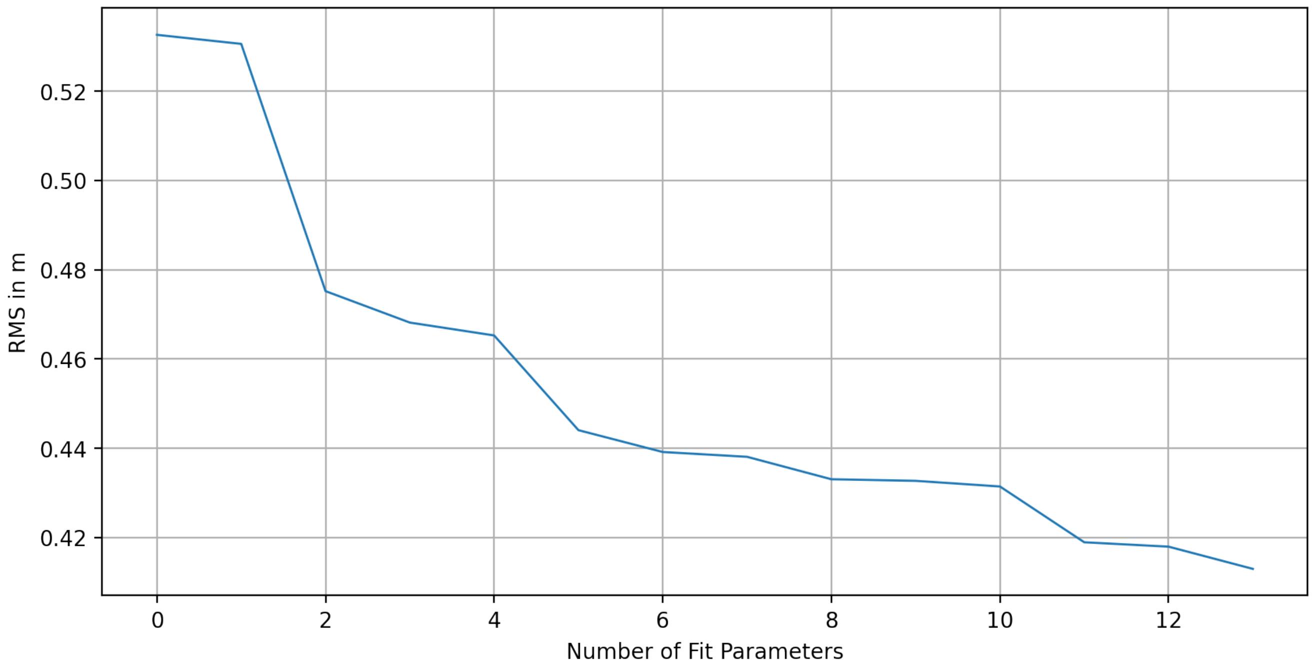

Figure 9: Root mean square error (RMS) of the fit of the tidal signal against the number of constituents using the order of Table 1.

3. **Do the running 25 hour fit.**

   (a) **Make a plot of the amplitude of the diurnal and semidiurnal tidal constituents as a function of time. Interpret the results and answer: (1) why semi-diurnal amplitude varies at a monthly and on a yearly time scale. (2) why diurnal component varies throughout the month and year. In case this variation is not visible in your plot, explain why this is the case. (Use a maximum of 1 figure (combine the plots) and keep your text brief.)**

   In Figure 10 the amplitude of the running 25 h mean of the diurnal and semi-diurnal component is shown. In this figure it can be seen that the semi-diurnal component varies on a twice monthly timescale. This behaviour corresponds to the spring-neap cycle we also discussed earlier. Furthermore it is generally observable that one peak of the spring-neap cycle is higher than the other peak. While this behaviour is most prominent in the summer months due to the lower noise, it can be explained by the perigee - apogee cycle of the moon as it rotates around the earth on an ellipse. Similarly, one also expects to see a yearly cycle due to the declination angel of the sun and the distance of the earth to the sun. Although the signal seems to go down after September, which fits with the expected declination angel, a clear pattern cannot be distinguished in the signal. As this effect is relatively small, it gets lost in the signal which is heavily influenced by non-tidal effects. For the diurnal signal also a twice monthly and monthly signal can be seen, which is also caused by the spring-neap cycle and the perigee - apogee cycle of the moon. The signal of the diurnal cycle is in anti-phase with the signal of the semi-diurnal cycle. This can be explained by equation 1 shown in Lecture 2. Here, you can see that signal of the diurnal cycle (T1) goes via the cosine and the semi-diurnal via the sine (T2). Just like with the semi-diurnal component you also expect to see a yearly pattern in the diurnal pattern. However due to the noisiness (as explained before) of the signal of the diurnal component, it is not possible to see this. The outliers in the amplitude of the diurnal signal correspond with the mean water level peaks, so they are probably not caused by a tidal processes, but by storm surges.

$$-\Omega_p = \frac{3}{2} a g \frac{m_l}{m_e} \left(\frac{a}{R_l}\right)^3 \begin{bmatrix} \frac{3}{2}\left(\sin^2 d_l - \frac{1}{3}\right)\left(\sin^2 \phi_p - \frac{1}{3}\right) \\ +\frac{1}{2}\sin 2d_l \sin 2\phi_p \cos C_p \\ +\frac{1}{2}\cos^2 d_l \cos^2 \phi_p \cos 2C_p \end{bmatrix} \quad \begin{matrix} \mathrm{T0} \\ \mathrm{T1} \\ \mathrm{T2} \end{matrix} \tag{1}$$

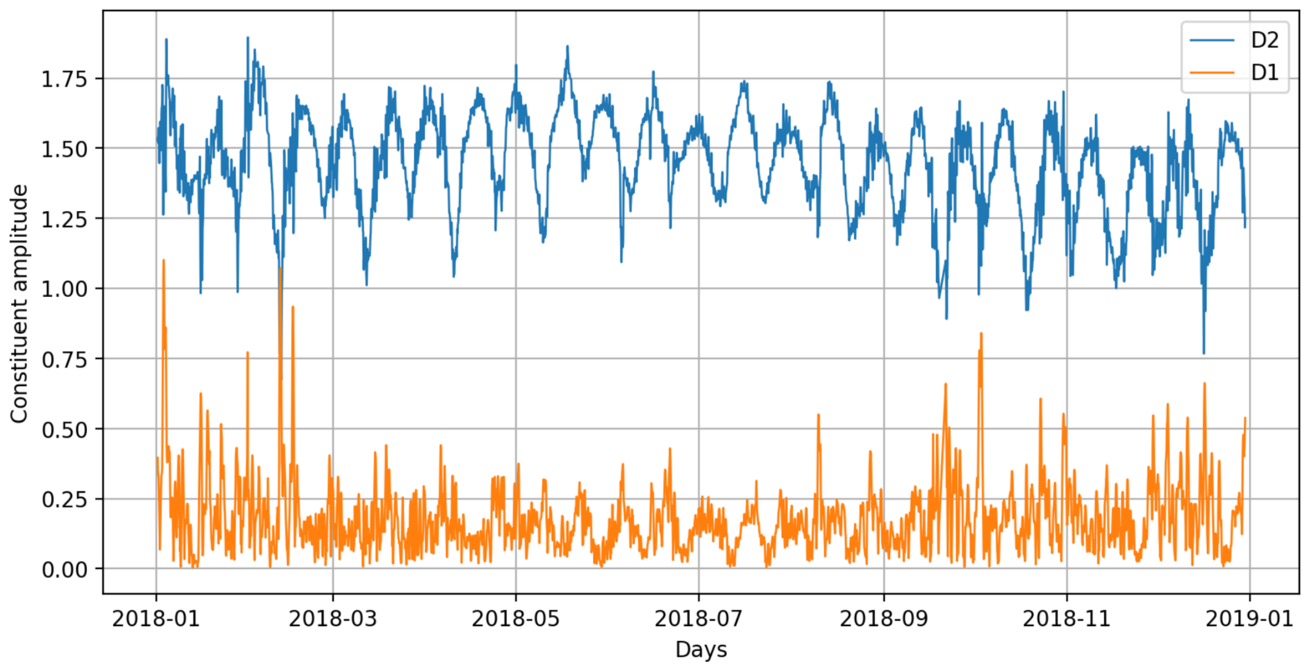

Figure 10: Semi-diurnal(D2) and Diurnal(D1) for the year 2018 data, with a data point every 2 hours.

(b) **Plot the mean water level as a function of time and explain what you observe. (Use a maximum of 1 figure and keep your text brief.)**

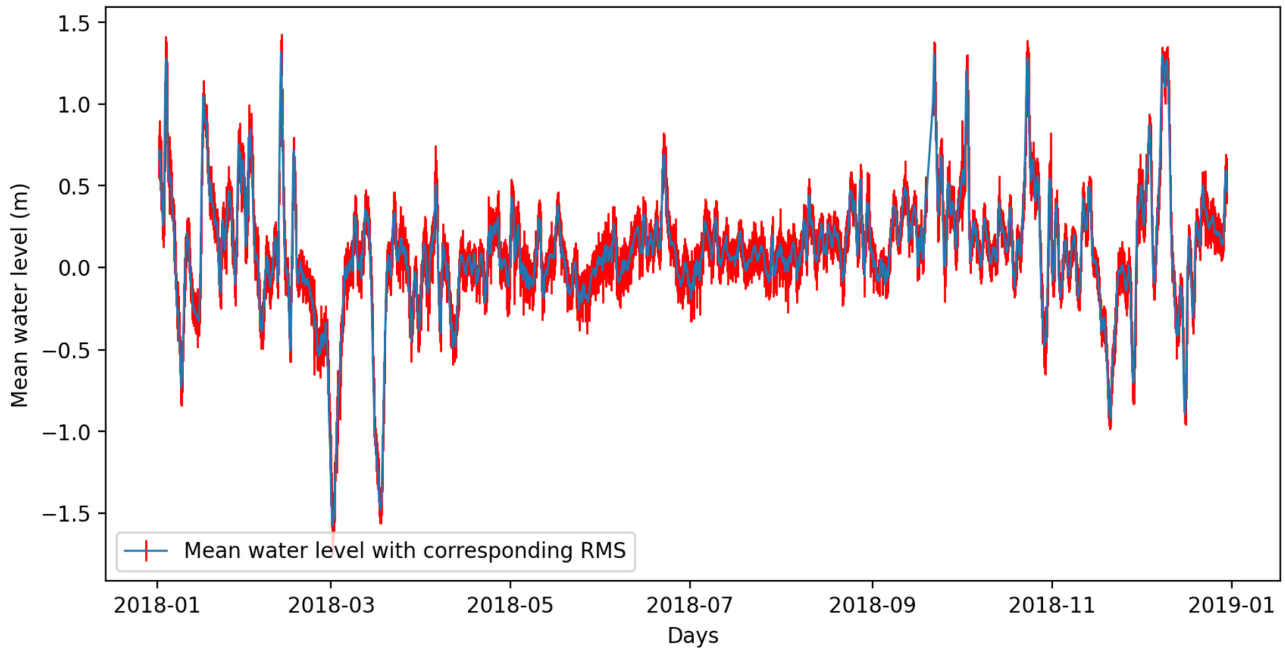

Figure 11: The mean water level (a0) of 2018 data, with a data point every 2 hours. The RMS is included as errorbar.

In Figure 11 the average 25 h mean water level is shown. Not a very clear trend can be seen, apart from the peaks and lows probably from storm surges pushing the water to or from the coast. As discussed before, the mean water level is more constant during the summer months than in the winter.

(c) **Do D4 and D6 amplitude depend on the magnitude of the semi-diurnal component? Explain. (Use a maximum of 1 figure and keep your text brief.)** In order to check if the D4 and D6 amplitude depend on the semi-diurnal component we plotted them against each other, as can be seen in Figure 12. From this figure it can be seen that there seems to be a linear correlation between the D4 and D6 components and the semi-diurnal component. This makes sense since they are higher harmonics of the semi-diurnal component. Consequently, as the semi-diurnal component increases, the higher harmonics are expected to increase as well.

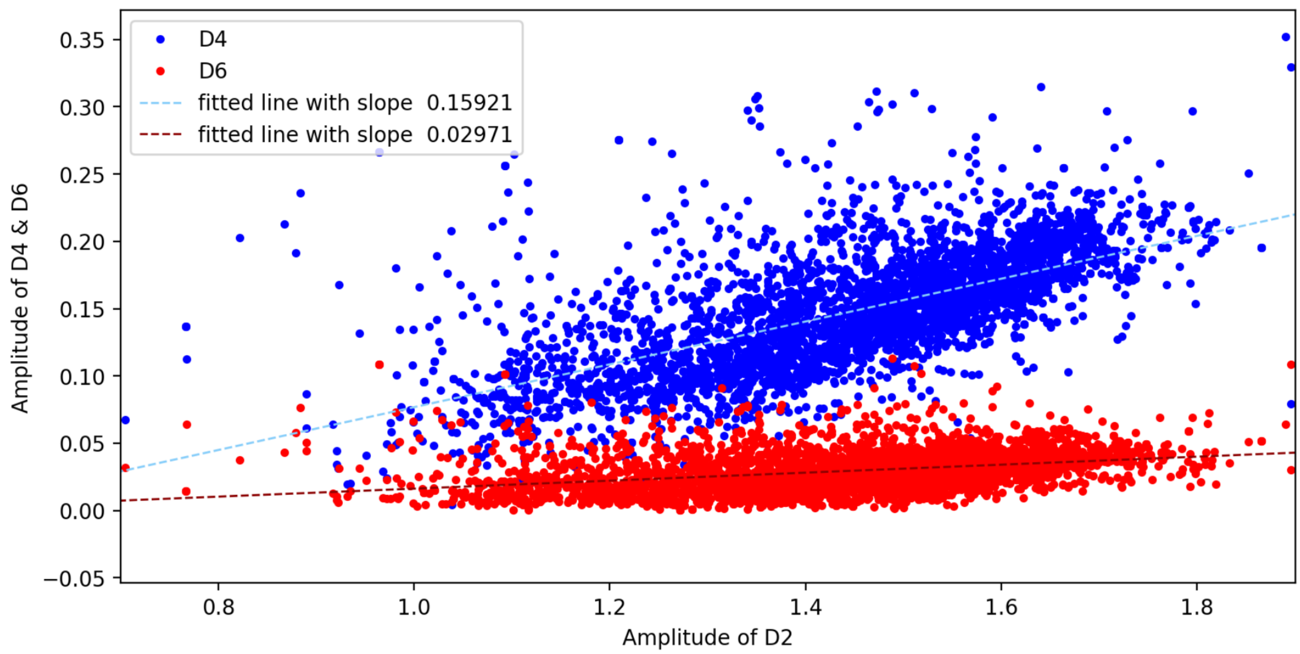

Figure 12: The D4 and D6 constituents plotted as a function of the D2 constituent. A linear function is fitted through the data points as indicated by the dashed lines with the slope as given in the legend.

References

[sea] Bekijk zeespiegelstijging in klimaatdashboard - klimaatadaptatie. https://klimaatadaptatienederland.nl/actueel/actueel/nieuws/2022/zeespiegelstijging-klimaatdashboard/. (Accessed on 11/21/2022).

[2] Dangendorf, S., Mudersbach, C., Wahl, T., and Jensen, J. (2013). Characteristics of intra-, inter-annual and decadal sea-level variability and the role of meteorological forcing: The long record of cuxhaven - ocean dynamics. *SpringerLink*.