# Inattentive Consumers and Imperfect Competition

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**Abstract**

Price transparency policies are often promoted as a simple way to protect consumers from exploitative pricing schemes which take advantage of their imperfect perceptions of price, whether due to partial attention or imperfect recall. The intuition of these policies is that more accurate perceptions of price makes consumers more elastic to price changes, which should decrease prices that firms offer. We offer a formal framework of the implications of behavioral consumers on a simple monopoly setting and show that the aforementioned intuition tells only one half of the story. There is a secondary effect which relates to consumers' expectations of price, and this effect can be such that it makes price transparency policies cause a monopolist *increase* their price. Further, we show that cost pass-through—a well-studied empirical object—can serve as a sufficient statistic to determine the impact of price transparency policies. Next, we extend this framework to the problem of joint price and quality choice and provide a rigorous definition of the phenomenon colloquially known as 'shrinkflation'; we then analyze its causes and response to quality transparency policies.

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# 1 Introduction

Price is arguably the central quantity in a market economy, yet the usual asssumption that consumers perfectly perceive price is often violated. Whether due to behavioral biases like imperfect recall or partial attention, consumer perceptions of prices often diverge from reality. One may intuit that consumers' imperfect perceptions of price gives firms more room to exploit these consumers, who may underreact to price changes or who may fail to anticipate hidden fees or other non-salient elements of the price. This intuition motivates the popularity of price transparency policies, like mandatory listing of fee-inclusive prices up front or banning hidden fees altogether. Price transparency policies—whether they restrict hidden fees or ban .99 cent pricing—share the common feature that they improve consumers' perceptions of and sensitivities to price. When consumers perceive price more accurately, their beliefs are not only closer to reality but they are also more responsive to changes.

We model perceptions of price following the extensive behavioral literature (see Gabaix 2019 or Chetty et al. 2009) as $p^s = mp + (1-m)p^d$, where $m \in [0,1]$ is the parameter that controls both the accuracy of beliefs and their sensitivity to changes. Note that at $m = 0$, $p^s = p^d$, i.e. when people pay no attention they believe the price is its usual or expected value; when $m = 1$, $p^s = p$. The sensitivity of perceived price to changes in the true price ($\frac{dp^s}{dp}$) is simply $m$. Thus we use $m$ as a modelling tool to capture the level of sensitivity and accuracy of consumers' beliefs. Therefore if we vary this parameter, we can investigate the impacts of price transparency policies, or any other intervention that impacts perceptions in a similar manner.

Given that we model these policies as an increase in the $m$, meaning that they increase the accuracy of price perceptions and sensitivity to price changes, one may assume that the impact of price transparency policies on firms' pricing is obvious. It seems intuitive that in the face of consumers who are more sensitive to changes in price and have more accurate beliefs about price, a monopolist would certainly decrease their price. However, as we show, improving consumer perceptions is not equivalent to simply making consumers' demand more elastic to price changes. As a result, the intuition that price transparency policies must help consumers often fails. For example, if consumers expect prices to be very high, this pessimism can exert downward pressure on the monopolist's price. An increase in the accuracy and sensitivity of consumers' beliefs about price serves to relieve that downward pressure on the firm, and thus a price transparency policy—which is meant to protect consumers and lower prices—can actually have the opposite effect, leading to a *higher* price.

This paper generally asks questions of the form above, i.e. how policies which raise $m$ (and thus improve consumer perceptions) will impact a monopolist's behavior. To do so, we generally characterize the behavior of a monopolist who faces consumers who imperfectly perceive the price of the good they are purchasing. Our setting is maximally simple: we consider a constant marginal cost monopolist who faces an identical group of consumers who are all equally inattentive. We then consider the impacts of a policy which improves consumers' perceptions of price, and we assume that the policy has a uniform impact on all consumers. We adopt this approach because we do not seek to analyze cases where consumer heterogeneity allows for price discrimination, but instead seek to demonstrate under the simplest conditions possible the key mechanisms that characterize monopoly with consumer inattention.

To analyze monopoly with behavioral consumers, we must construct the primitives of the economic environment with the relevant behavioral modifications. We introduce a notion of the 'behavioral demand curve', which is simply the usual demand function evaluated at the perceived, rather than actual, price. Left digit bias provides an easy example of why this approach is useful: left digit bias consumers' demand for a product with price 2.99 may be very similar (in levels) to the demand of rational consumers for the same product priced at 2.65. That is, we suppose that behavioral consumers' demand follows a traditional demand function that is simply evaluated at a (usually) incorrect price. After describing the differences between the behavioral and traditional demand function, we ask and answer a set of basic questions about how the monopolist will behave in this environment.

The first question is the most natural: what price will the monopolist choose? We describe this using a *behavioral inverse elasticity rule*, which illustrates how the traditional inverse elasticity rule is modified in this behavioral setting. The most important question is: is the 'behavioral' price, i.e. that which follows the behavioral inverse elasticity rule, higher than the fully rational price? In order to answer this question, we immediately turn to characterizing the key comparative static of how optimal price varies with attention/accuracy $m$. Analyzing this allows us to answer whether or not the price faced by behavioral consumers is higher than the price that would be faced by a counterfactual group of consumers who are all completely rational.

Equivalently, this analysis provides the conditions under which an intervention that causes higher accuracy and sensitivity to price changes will lead to a lower monopoly price. Such an intervention has two effects, one direct and one indirect: the direct effect of improving perceptions of price leads to higher sensitivity, which puts strictly downward pressure on the monopolist. Since the firm faces consumers who are more sensitive to price, they have an incentive to decrease their price. However, a second, indirect effect can exert pressure in the other direction—if consumers expected prices to be high and the true price is low, then increasing the accuracy of their beliefs can incentivize the monopolist to raise their prices, since consumers' perception of price decreased.

Although the analysis involves the discussion of new, behavioral/perceptual mechanisms, the underlying mathematical properties can be related to more traditional objects, like cost pass-through. In fact, we demonstrate that the same conditions that determine whether cost pass-through is above or below unity is the same characteristic which determines if higher expected prices cause the monopolist to increase or decrease their price. As is shown, this finding helps to inform the economic mechanisms behind the impact of higher $m$ on monopoly price. Further, we show that cost pass-through can be a sufficient statistic for the sign of attentional pass-through, thus allowing researchers to infer the impacts of policies that increase attention from empirical estimates of cost pass-through.

We then turn to focusing on the welfare of a different agent in the market: the firm. Just as lower attention does not always mean higher prices, nor does lower attention mean higher profits. Instead, firms only benefit from lower consumer attention if consumers' expected price is higher than the true price. For example, a firm who charges \$3.00 for an item would prefer that consumers be more attentive if their expectation of price is \$5.00, since more attention means a lower perceived price, which raises demand.

Next, we turn to the isomorphic problem of optimal choice of quality when facing inattentive consumers. We show how a monopolist will choose the quality of their good, and illustrate the conditions under which a 'better reputation' (i.e. higher expected quality) of the firm leads to free-riding, i.e. lower quality, or reputational reinforcement, i.e. higher quality. This problem is generally very similar to that of pricing, and the same mathematical conditions are relevant.

Having analyzed the optimal choice of price and quality separately, we turn to the problem of their simultaneous choice. We study a monopolist who chooses both quality and price, where price is perfectly perceived and quality imperfectly perceived, in order to formally characterize and investigate the phenomenon colloquially known as 'shrinkflation'. This phenomenon is typically presented as firms decreasing the quantity/quality of a good (thus 'shrinking') instead of changing the price, which is more salient. We model the need to decrease quality or increase price as a response to a cost shock, and so we define shrinkflation as asymmetric cost pass-through to quality vs. price as a result of inattention. We then demonstrate that in the case of convex costs and a simple assumption of how price and quality enter the (arbitrary) demand function, there is less pass-through to quality when consumers are more attentive. In other words, we show that our definition of shrinkflation is decreasing in consumer attention.

Finally, we return to the topic of monopoly pricing with inattentive consumers, but we suppose that attention is endogenous to price. That is, we not only assume that higher prices have the traditional demand response, but also that higher prices may increase consumers' attention to price itself. For example, firms may fear that raising prices in response to cost shocks will frustrate consumers who assign greed or other negative forces to the price increase, and thus will be more attentive to price changes in the future. This topic uses tools from behavioral economic modeling that are more recent, and far more involved, than those in the previous sections. Whereas the previous sections consider the key comparative statics of price and attention, this analysis compares price and *cognitive costs*, i.e. the psychic cost to consumers who wish to observing price. We provide an analysis of when lower cognitive costs should lead to lower prices, since this allows consumers to more easily observe price.

This work sits at the intersection of industrial organization and behavioral economics. This paper takes as a starting point some canonical notions of industrial organization as found in Tirole (1988) and Perloff and Salop (1985) to demonstrate how famous formulae change when consumers are behavioral. Further, we connect insights about the importance of the curvature of demand as found in Bulow and Pfleiderer (1983) and Weyl and Fabinger (2013) to explain how the functional form of demand is a critical ingredient to relating consumer rationality to optimal monopoly prices. Finally, we also rely on theoretical advances in Gabaix et al. (2016) to analyze how competition is dampened by behavioral consumers in the setting of random utility demand oligopoly.

The characterization of consumer rationality comes from canonical expressions for 'anchoring and adjustment' as described in Chetty et al. (2009), Gabaix (2014), and Gabaix (2019). Further, this paper adds to a research agenda that demonstrates how traditional economic theory is augmented when behavioral consumers are introduced; papers like Farhi and Gabaix (2020), Gabaix (2020), and Gabaix (2014) exemplify this literature.

In the last twenty years, an entirely new literature of behavioral industrial organization has emerged, with important theoretical treatments in Gabaix and Laibson (2006) and Spiegler (2011) providing simple and intuitive models of naive consumers. More recently, empirical analyses like Lacetera et al. (2012), List et al. (2023), and structural work in Strulov-Shlain (2022) have focused on the importance of left digit bias in a variety of industries.

Although there are many popular models that include behavioral consumers, such as that found in Gabaix and Laibson (2006), there is not yet a general model which can incorporate a wide range of misperceptions of price into a single, tractable model. This paper seeks to sketch out how fundamental elements of industrial organization—demand elasticity, optimal pricing, competition, etc.—change when consumers are allowed to be imperfectly rational.

The rest of the paper is structured as follows: Section 2 gives a simple example of how attention relates to price to give intuition, Section 3 provides definitions for necessary concepts and lemmas that illustrate how inattentive demand differs from rational demand, Section 4 provides the central propositions that describe the relationship between consumer rationality and optimal prices (as well as other market outcomes), Section 5 extends this framework to a firm choosing quality rather than price, Section 6 models a firm that chooses both quality and price, allowing for a characterization of 'shrinkflation', Section 7 returns to the problem of monopoly pricing, but endogenizes consumer attention, and Section 8 concludes.

## 2 A Motivating Example

We begin with the simplest possible example to demonstrate the key idea of the paper, and also to demonstrate that its main mechanisms do not require any exotic conditions. Suppose that consumers perceive the price as a linear combination of the true and some default price: $p^s = mp + (1-m)\,p^d$, and further suppose that demand is linear: $D(V(p^s)) = \max\{0, V(p^s)\}$, where the market valuation $V(p^s) = A - \alpha p^s$. In the traditional case, we'd simply have that $V(p)$, such that $D = \max\{0, A - \alpha p\}$. But here, consumer demand is a function of the *perceived* price, and so we have that the profit function is:

$$\pi(p; m, p^d) = (p-c)\max\left\{0, A - \alpha\left(mp + (1-m)\,p^d\right)\right\} \tag{1}$$

Therefore the optimal price for the monopolist facing this demand curve is (assuming sufficiently large $A$):

$$p^* = \frac{A - \alpha p^d}{2\alpha m} + \frac{p^d + c}{2} \tag{2}$$

Note that at $m = 1$, i.e. full attention to price, the above pricing scheme reduces to the traditional one, i.e. $p^* = \frac{A+\alpha c}{2\alpha}$. The first characteristic of this pricing schedule to describe is how it behaves when the default, or expected, price $p^d$ changes. It is easy to see that:

$$\frac{dp^*}{dp^d} = \frac{m-1}{2m} < 0 \tag{3}$$

In other words, a higher default price puts downward pressure on the optimal price: as consumers expect a higher price, the firm must compensate by charging a lower true price. The key question, then, is how the optimal price will change when $m$ is increased from some $m \in (0,1)$. We see that it depends on how large is $p^d$:

$$\frac{dp^*}{dm} = -\frac{A - \alpha p^d}{2\alpha m^2} \tag{4}$$

The sign of this is easy to interpret, as the numerator $A - \alpha p^d$ gives the market valuation $V(p^s)$ when $m = 0$, i.e. when consumers pay no attention to true price. If the valuation at $p^s = p^d$ is positive, then the monopolist's price will decrease when there is more attention. However, if the default is sufficiently high that $A/\alpha < p^d$, then increasing attention will *increase* the monopolist's price. This is because, as shown in Equation 3, a higher default exerts downward pressure on the optimal price. By increasing attention, this downward pressure is relaxed, thus allowing the firm to price higher. The condition given simply states how large the default must be for this effect to counteract the increase in sensitivity caused by higher $m$: the default must be sufficiently high that at the default $p^d$ the valuation would be negative and thus demand zero.

This example is the simplest possible one, and it illustrates some of the key mechanisms behind the relation of monopoly price to attention. We now move to sketching the model more generally, and we introduce definitions and derive the conditions that determine the relation between monopoly pricing and consumer attention.

# 3 Primitives

For any variable of interest $x$—which will be quality or price—we will employ a formulation of imperfect perception from behavioral economics that exhibits a feature called *anchoring and adjustment*. This formulation can serve as the end result of an extremely wide range of microfounded biases, like rational inattention, left digit bias, imperfect recall, or even rational behavior, like Bayesian updating. Starting at some default belief or expectation of the value of $x$— called the 'reference point' or 'anchor'—people adjust to the true value of $x$. The extent to which people will shift away from their anchor to the true value can be parametrized by an 'attention parameter', denoted $m$. We can represent this adjustment as a convex combination of the anchor $x^d$ ($d$ for default) and the true value $x$. The subjective perception of $x$, denoted $x^s$ is the result of a convex combination:

$$x^s = mx + (1-m)\,x^d \tag{5}$$

We assume that $m \in [0,1]$, where $m = 0$ means no attention to the true value of $x$, and $m = 1$ means full attention to the true value of $x$. Thus, note that $x^s = x$ for $m = 1$, and $x^s = x^d$ for $m = 0$.

This representation of $x^s$ is sufficiently general to capture a wide range of misperceptions and economic contexts. Although the attention parameter $m$ can be endogenized – see Gabaix (2014) or Sims (2003) – we abstract away from the process by which attention is formed at first, and endogenize attention in Section 7. There are many real-world examples of the phenomena that this paper intends to address; here are a few:

- Consumers underreact to changes in rightmost digits of prices (known as left-digit bias); this leads to .99 cent pricing. Recent work highlights the importance of this; see Strulov-Shlain (2022).

- Consumers face a base fee and an add-on; they may only partially anticipate the add-on fee, such as with banking or concert tickets. See Gabaix and Laibson (2006) or Ellison (2005).

- Consumers underreact to changes in the size of an item in comparison to changes in the price; this allows for firms to change the size of an item rather than raise price in times of rising production costs. This is commonly called 'shrinkflation'. Experimental evidence that consumers do not notice said size changes is provided in Pignatelli and Solano (2020).

- If consumers make mistakes when mentally processing numbers, they have exhibit underreaction to changes in price; see Gabaix (2019).

- Consumers may use information about a given brand's flagship good to make an inference about a different good in the product line. For example, one may assume that any Mercedes vehicle is of high quality since its best cars are well-made.

Now turning to the relevant market context, we can suppose that there exists a true price $p$, and consumers have a default or expected price $p^d$, which can be microfounded as an average price, the last price they saw, etc. For a given attention parameter $m \in [0,1]$, we have that the subjective perception of price is $p^s = mp + (1-m)\,p^d$.

Given this misperception of price, it is necessary to characterize how this misperception coheres with a demand function. For this, we introduce a definition; denote the *rational demand function* as $D^r(p)$ and the *inattentive demand function* as $D^s(p)$. We can relate the two concepts in the following way:[1]

**Definition. Inattentive Demand**
*For a given functional form of demand, $D^r(p)$, the inattentive demand function is the rational demand function evaluated at the subjective perception of price, $p^s$:*

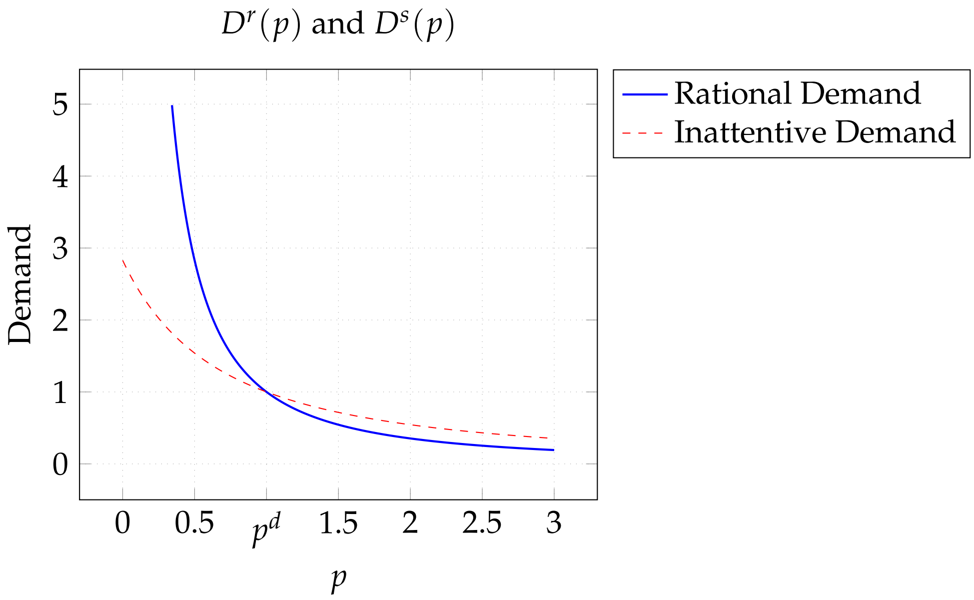
$$D^s(p) = D^r(mp + (1-m)p^d) \tag{6}$$

For example, if demand is $D^r(p) = \exp(-p)$, then $D^s(p) = \exp\big(-\big(mp + (1-m)p^d\big)\big)$.

Naturally, inattentive demand will be lower than rational demand when $p^s > p$, and it will be higher than when $p^s < p$; this condition reduces to when $p > p^d$.[2] Here, we see that the rational demand is higher below the default price, and inattentive demand is higher above the default price. We can illustrate the inattentive and rational demand functions for $D = p^{-\psi}$:

[1]Note that this is similar to the definition of behavioral Marshallian demand as defined in Gabaix (2019), but his application of that definition assumes $p^d = 0$.

[2]We can easily demonstrate this fact, and will do so for the case where $p^d > p$. Suppose the true price is lower than the default; we wish to show that the rational demand will be higher than the inattentive demand: $p^d > p \implies (1-m)\,p^d > (1-m)\,p \implies mp + (1-m)\,p^d > (1-m)\,p + mp \implies p^s > p \implies D^r(p^s) < D^r(p) \iff D^s(p) < D^r(p)$ Where the last line follows from the fact that $D^r$ is decreasing in its argument.




Parameter values: $m = .5$, $p^d = 1$, $D = p^{-\psi}$, $\psi = 1.5$

As stated, rational demand is lower when the true price is above the default, and rational demand is higher when the true price is below the default. Note that at the default, although the demand functions take on the same value, their slopes differ; thus, it's worth understanding the nature of demand sensitivities too. The derivative of inattentive demand with respect to price is given by the following equation:

$$\frac{\partial D^s}{\partial p} = \frac{\partial D^r}{\partial p^s}\frac{\partial p^s}{\partial p} = \frac{\partial D^r}{\partial p^s} \cdot m$$

Note that if $m = 1$, meaning that there is full attention and therefore $p^s = p$, then we would have right hand side be $\frac{\partial D^r}{\partial p}$; if there is full attention, the behavioral and rational demand functions exhibit identical behavior. However, suppose that $m < 1$; in this case, things are not so simple:

**Lemma 1.** ***Inattentive Demand Sensitivity to Price***
*Inattentive demand sensitivity to price is given by:*

$$\frac{\partial D(p^s)}{\partial p}|_{m<1} = \frac{\partial D(mp + (1-m)p^d)}{\partial p^s} \cdot m \tag{7}$$

*And it is not globally larger or smaller (in magnitude) than the rational demand sensitivity.*

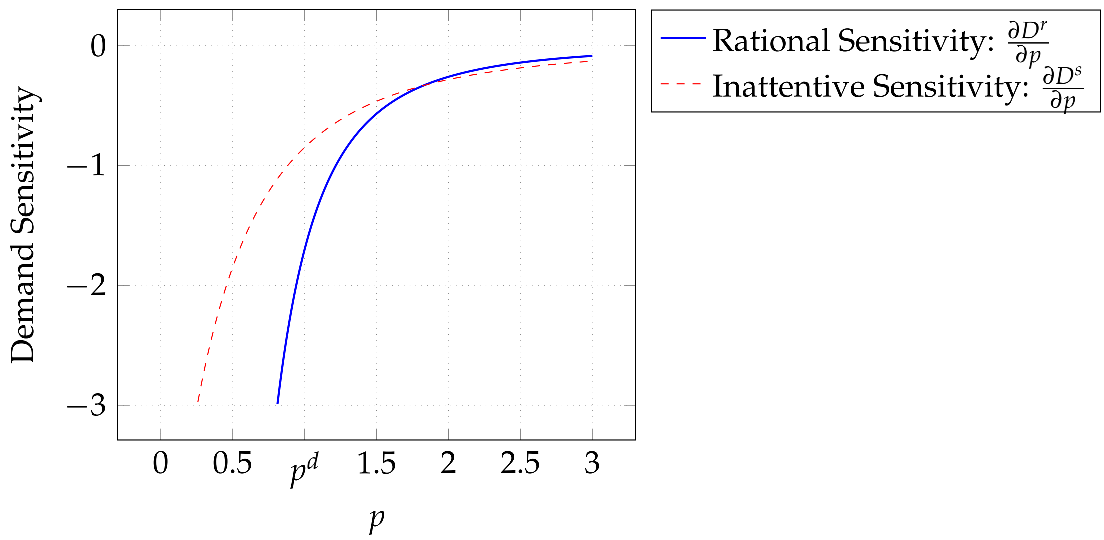
There are two forces that make the inattentive demand sensitivity distinct from the rational demand sensitivity. Firstly, the argument in $D$ is $p^s = mp + (1-m)p^d \neq p \; \forall p \neq p^d$, so the derivatives are evaluated at different points. Secondly, we see that the inattentive demand function has sensitivity to price equal to the rational demand sensitivity evaluated at $p^s \neq p$ and is dampened by inattention via the term $m < 1$. Note further that we arrive at the representation of behavioral Marshallian demand that is given in Gabaix (2019) if we take the special case where the behavioral consumers' default is the true price ($p = p^d$):

$$\frac{\partial D(p^s)}{\partial p}\Big|_{m<1,p^d=p} = \frac{\partial D(p)}{\partial p} \cdot m$$

All of this can be made more explicit with an example. Suppose that $D^r = p^{-\psi}$ where $\psi > 1$ is price elasticity; then, it follows that

$$\frac{\partial D^r}{\partial p} = -\psi p^{-\psi-1}$$
$$\frac{\partial D^s}{\partial p} = -\psi \left(mp + (1-m)\, p^d\right)^{-\psi-1} \cdot m$$

We can see this represented in the following figure:




It is clear that at $p = p^d$, the behavioral sensitivity is dampened (closer to zero) by a factor of $m = .5$. However, the inattentive demand function having lower sensitivity to changes in the price is not true along the entire domain of prices. It is clear that there is an entire range of prices over which the rational consumers are less sensitive than the behavioral consumers; the point at which the two functions are equal is given by $p'$:

$$p' = \frac{(1-m)\, p^d}{m^{\frac{1}{\psi+1}} - m} \approx 1.83$$

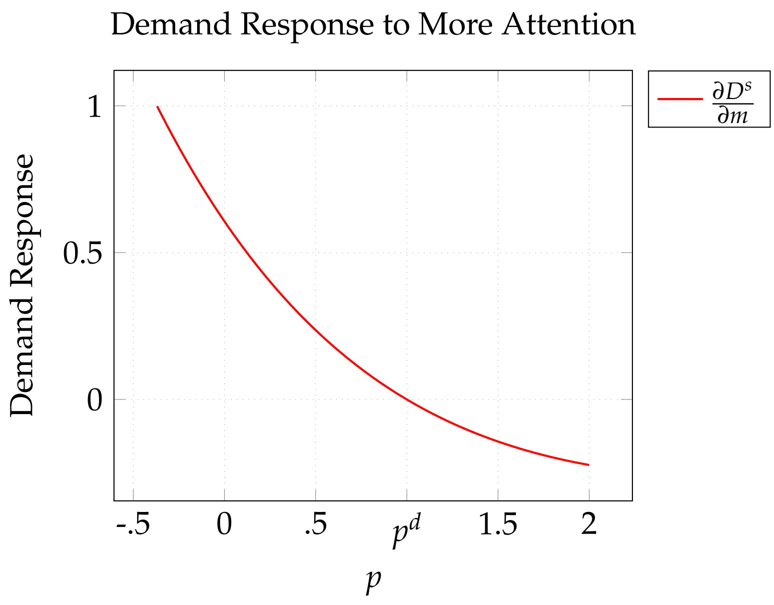
This is a simple example to demonstrate that the inattentive demand function is not simply a less sensitive version of the rational demand function, although it does behave that way around the default $p^d$. This will have important implications later in the analysis.

So far, we have only discussed the relationship of demand to price and changes in price. However, changes in attention itself can generate changes in demand as well, and we demonstrate that the relationship between demand and attention is highly intuitive:

**Lemma 2.** ***Inattentive Demand and Changes in Attention***
*Holding price fixed, inattentive demand's response to a change in attention is given by the following:*

$$\frac{\partial D^s}{\partial m} = \frac{\partial D^r(p^s)}{\partial p^s} \cdot \left(p - p^d\right) \tag{8}$$

Since $\frac{\partial D}{\partial p^s} < 0$, then it is clear that the effect of attention on demand depends entirely on whether or not the true price is above or below the default. This is natural – if consumers' default is that an apple's price is $p^d = 1$, but they begin to notice that actually $p = 1.99$, then this should lead to a drop in demand, since $p^s \to p > p^d$ as $m \to 1$. Using $D^r = e^{-p}$ as an example, we can see how demand changes with attention, noting that the change is zero at the default $p^d = 1$:




So, if the true price is higher than the reference price, we would expect that since higher attention means the perceived price gets larger, there would be decrease in demand. Indeed, this is what we see above: the line is above 0 for $p < p^d$ (more attention means lower perceived price, so higher demand), and the line is below 0 for $p > p^d$ (more attention means higher perceived price, so lower demand).

Now that we have defined the primitives of the inattentive demand function, we can investigation how traditional notions in industrial organization change with behavioral consumers.

# 4 Optimal Pricing with Consumer Inattention

## 4.1 Prices and Attention

Suppose a monopolist faces consumers whose perception of price is $p^s = mp + (1-m)\, p^d$; how does this monopolist price optimally? If the profit function is of the following form:

$$\pi = (p-c)\, D(mp + (1-m)p^d) \tag{9}$$

where the demand function has the property that $1/D(p^s)$ is convex in $p^s$,[3] then we have the following lemma:

**Proposition 1.** ***Behavioral Inverse Elasticity Rule***
*For a demand function $D$ that satisfies the property that $1/D(p)$ is convex, then the optimal price is given by the following equation:*

$$p^* = c - \frac{D(p^s)}{D'(p^s)} \cdot \frac{1}{m} = c + \frac{\mu(p^s)}{m} \tag{10}$$

Where $\mu(p^s) \equiv -D(p^s)/D'(p^s)$ denotes the markup term, where $m < 1$ amplifies said markup. Given that $1/m$ increases the optimal price, one would suspect that increasing attention should always decrease price; however, $m$ also features in the markup term $\mu(p^s)$. Although the above expression may look similar to the traditional inverse elasticity, if we write it in terms of elasticities, their differences become clear. It is well known (Tirole 1988) that in a traditional monopoly setting (i.e. $m = 1$), we have that price is given by:

$$p^* = c \cdot \frac{\epsilon}{\epsilon + 1} \tag{11}$$

Where $\epsilon = \frac{dD}{dp} \cdot \frac{p}{D}$. Now defining the elasticity to perceived price as $\epsilon^s \equiv \frac{dD}{dp^s} \cdot \frac{p^s}{D}$, we have that the monopolist's price is given by:

**Corollary 1.** ***Behavioral Inverse Elasticity Rule, Part 2***
*Rewriting Equation 10 in terms of elasticities gives:*

$$p^* = c \cdot \frac{\epsilon^s}{\epsilon^s + 1} - \left(\frac{1-m}{m}\right) \cdot \frac{p^d}{\epsilon^s + 1} \tag{12}$$

The use of writing the pricing rule as in Corollary 1 is that it helps to demonstrate and emphasize the differences with the typical pricing rule. Equation 10

[3]Note that this weak condition implies quasi-concavity of the profit function; see Chapter 6 of Anderson et al. (1992)

Changes in attention $m$ enter this equation in multiple ways, and the relationship of attention to optimal prices is not obvious. We will investigate how attention changes optimal price, called here *attentional pass-through.*

**Proposition 2.** ***Attentional pass-through: how higher consumer attention impacts monopoly pricing***
*Denoting* $\mu' = d\mu/dp^s$ *where* $\mu' < 0$ *if D is log-concave, then:*

$$\frac{dp^*}{dm} = \frac{\mu'}{1-\mu'} \cdot \left(\frac{c-p^d}{m}\right) - \frac{\mu}{m^2} \tag{13}$$

**Explanation of result** Finding the sign of Equation 13 depends on two things: a) the log-curvature of demand ($\mu'$) and b) the default/expected price $p^d$. The term $-\mu/m^2$ is always negative, and always exerts downward pressure on price. If we assume the most reasonable parameter values, i.e. that demand is log-concave (so $\mu' < 0$) and expected prices are above marginal cost ($p^d > c$), then the effect of attention on price is ambiguous. This is because the first term is positive and the second term is negative. The economic intuition of Proposition 2 is simple to explain. Returning to Equation 10, the change of price with attention is:

$$\begin{aligned}
\frac{dp^*}{dm} &= \frac{1}{m}\left[\frac{d\mu(p^s)}{dm} - \frac{\mu(p^s)}{m}\right] \\
&= \frac{1}{m}\left[\frac{d\mu(p^s)}{dp^s}\frac{dp^s}{dm} - \frac{\mu(p^s)}{m}\right] \implies \\
\frac{dp^*}{dm} &= \frac{1}{m\,(1-\mu')}\left[\underbrace{\mu'\left(p^*-p^d\right)}_{2} - \underbrace{\frac{\mu(p^s)}{m}}_{1}\right]
\end{aligned} \tag{14}$$

By assumption, $\mu' < 1$ and so the term outside of the brackets does not determine the sign of pass-through. Then the sign of the effect is determined by two effects: 1. The direct effect 2. The perceptual effect. The first effect is unambiguous—increasing consumer's sensitivities to price changes decreases price. The second effect summarizes how a change in perceived price relates to a higher or lower markup term $\mu$. An increase in $m$ changes the perception of price by the quantity $p^* - p^d$. Once $p^s$ changes, the way that the change in the perceived price impacts the optimal price depends entirely on whether or not $\mu$ is increasing in $p^s$.

Suppose that the consumer overestimated the true price, and so $p^* < p^d$, and that demand is log-concave and thus $\mu' < 0$. Then, $\mu' \cdot (p^* - p^d) > 0$ and so the overall effect of increasing attention on price is ambiguous. If instead the consumer underestimated the price (keeping log-concave demand), then the effect of more attention would be surely negative.

Since optimal price is still on the right side of Equation 14, we can substitute in its value (given in Equation 10) to arrive at the expression given in Equation 13. That latter formulation is useful because it demonstrates how the relation of the primitives—demand curvature, default price, and marginal cost— of the model determines the sign of attentional pass-through. Importantly, it speaks to the feasibility of consumers' expectations: if consumers expectations are so unrealistic that $p^d < c$, then firms facing log-concave demand curves will decrease their price when there is more attention. Conversely, if consumers' beliefs are closer to accurate and thus $p^d > c$, this reality in turn hurts them since attentional pass-through could then be positive.

We can explore some examples to make the mechanisms clear.

**Attentional pass-through with logit demand** First, we begin with the example of logit demand of the form:

$$D_1(p^s) = \frac{\exp(q - p^s)}{\exp(q - p^s) + 1}$$

The optimal price given $\pi = (p - c)\, D_1(p^s)$ is:

$$p_1^* = c + \frac{1}{m\,(1 - D(p^s))}$$

And since $\mu_1' = -D(p^s)/(1 - D(p^s)) < 0$ means that $D_1$ is log-concave, then we have:

$$\begin{aligned}\frac{dp_1^*}{dm} &= \frac{D(p^s(p^*))}{m} \cdot \left(p^d - c\right) - \frac{1}{m^2 \cdot (1 - D(p^s(p^*)))} \\ &= \frac{mD(p^s) \cdot (1 - (D(p^s))) \cdot \left(p^d - c\right) - 1}{m^2 \cdot (1 - D(p^s))}\end{aligned}$$

The first line matches exactly the general expression given in Proposition 2, and the second line simply rearranges to emphasize that the expected price $p^d$ must be much larger than marginal cost $c$ in order for attentional pass-through to be positive. Noting that attention and demand are both between zero and one, we see that $mD(p^s) \cdot (1 - D(p^s)) << 1$ and so $p^d - c$ must be large in order for the left term in the numerator to be greater than one.

**Attentional pass-through with exponential demand** Suppose now that demand is given by:

$$D_2(p^s) = \exp(-p^s)$$

Then for $\pi = (p - c)D_2(p^s)$, we have that price is given by:

$$p_2^* = c + \frac{1}{m}$$

Since $\mu = 1$, $\mu' = 0$ and so the perceptual effect (effect 2 in Equation 14) is irrelevant. This leaves us only with the term $-\mu/m^2$, which clearly follows $dp_2^*/dm$.

**Attentional pass-through with linear demand** Revisiting the motivating example in Section 2, we suppose that

$$D_3(p^s) = A - \alpha p^s$$

Linear demand is log-concave with $\mu_3' = -1$, and so again plugging in from the expression given by Proposition 2:

$$\begin{aligned}\frac{dp_3^*}{dm} &= \frac{1}{2}\left(\frac{p^d - c}{m}\right) - \left(\frac{A - \alpha p^d \cdot (1-m) - \alpha m c}{2\alpha m^2}\right) \\ &= -\frac{A - \alpha p^d}{2\alpha m^2}\end{aligned}$$

Again the first expression exactly matches that from Proposition 2, and the second expression is the same as that derived in the motivating example shown in Equation 4. The sign of attentional pass-through is given in the following table, which shows the results and their dependence on expected price $p^d$ and the curvature of demand:

Table 1: Relationship between consumer attention and price

| | Log-concave demand | Log-convex demand |
|---|---|---|
| $p^d > c$ | Ambiguous | $\frac{dp^*}{dm} < 0$ |
| $p^d < c$ | $\frac{dp^*}{dm} < 0$ | Ambiguous |

The previous analysis, specifically the examples of logit and linear demand, suggest that somewhat large gaps between marginal cost and default price must exist for attentional pass-through to be positive. We can formalize that intuition with the following:

**Corollary 2.** ***Attentional pass-through with expectations of perfect competition***
*If consumers have a default price as $p^d = c \equiv p^{pc}$ where $p^{pc}$ is the perfectly competitive price, then attentional pass-through is always negative:*

$$\frac{dp^*}{dm}\Big|_{p^d = p^{pc}} = \frac{D(p^s)}{D'(p^s)} \cdot \frac{1}{m^2} < 0 \tag{15}$$

Thus, when consumers assume that a market is perfectly competitive when it is not, then improving their perception of prices will unequivocally decrease prices. This is surprising, because this statement requires no restrictions on the demand function—specifically at $p^d = c$, log-curvature is irrelevant, and more attention always implies lower prices.

Revisiting the example of linear demand, we see that attentional pass-through evaluated at $p^d = c$ gives $dp^*/dm = -(A - \alpha c) / \left(2\alpha m^2\right)$. The condition $A - \alpha c > 0$ must hold, because this represents demand when firms price at marginal cost; if there is no demand at the lowest possible price, then there could not be a market for that good in the first place. As a result, attentional pass-through is negative for the example of linear demand when consumers expect price to be equal to marginal cost.

Although we have explained the mechanisms behind the changes in price to more attention, the answer to a basic question may still not be clear: can fully rational consumers face higher prices than would their behavioral counterparts? If consumers were all perfectly rational ($m = 1$), could the price they face be higher than in a counterfactual world where consumers were all behavioral ($0 < m < 1$)? We can give a local answer by finding how price changes when attention slightly decreases away from $m = 1$.

**Corollary 3.** ***When a little inattention helps consumers***
*Slightly inattentive agents would face lower prices than fully rational agents if the following condition holds:*

$$-\frac{dp^*}{dm}|_{m=1} = \frac{\mu'}{1-\mu'} \cdot \left(\frac{p^d - c}{m}\right) + \frac{\mu}{m^2} < 0 \tag{16}$$

Again we see that under the most plausible conditions—log-concave demand ($\mu' < 0$) and realistic expectations ($p^d > c$)—the above condition can be met. This result is unexpected; although one may be willing to believe that optimal price is not always increasing in attention, it is extremely surprising that fully rational agents are not those that would face the lowest monopoly price.

**Connection between cost pass-through and consumer expectations** A large literature has studied the impact of a cost shock on firm pricing; the way that optimal price responds to a unit increase in cost is called *cost pass-through*. We show here that the properties of the demand function that determine values of cost pass-through are the same ones that help to determine attentional pass-through. We do so by first illustrating how marginal cost $c$ and default price $p^d$ play similar roles in the profit function. Then, we show that in the same way that demand curvature determines how optimal price responds to a shock to marginal costs, so too does demand curvature determine how optimal price respond to a shock to default prices.

Marginal costs $c$ and expected prices $p^d$ play a somewhat similar role in the profit function, since $p^d$ serves as a sort of 'perceptual cost' which is (like marginal cost) not a function of the price itself, but which of course impacts the optimal price. Recalling that $p^s = mp + (1-m)p^d$, we note that the second term $(1-m)p^d$ is not a function of the true price and so serves only to decrease demand, and thus profits. Higher marginal cost does not decrease demand, but decreases marginal revenue and thus profit. Said formally:

**Lemma 3.** ***Optimal profit falls with higher marginal costs and higher default prices***
*Optimal profit falls with higher values of marginal cost and expected price $p^d$. Denote $\pi^* \equiv \pi(p^*(c, p^d))$, then by the Envelope Theorem:*

$$\frac{d\pi^*}{dc} = -D(p^*) < 0$$
$$\frac{d\pi^*}{dp^d} = (p^* - c)\, D'(p^s)\, (1-m) < 0$$

The role of $c$ and $p^d$ are similar in that higher values decrease optimal profits. Their similarities, however, do not end there. It is well-known that cost pass-through is given by the following equation:

$$\rho \equiv \frac{dp^*}{dc} = \frac{1}{1-\mu'(p^*)} \tag{17}$$

Since $\mu'(p) < 1$ by assumption (since it guarantees quasi-concavity of the profit function), then log-concavity ($\mu' < 0$) implies that $\rho < 1$, and log-convexity ($\mu' > 1$) implies $\rho > 1$. So if a firm faces a log-concave demand function, then a one unit increase in cost results in a less than one unit increase in price, whereas a log-convex demand function implies a greater than one unit increase in price. Accordingly, Rochet and Tirole (2011) call log-concave demand functions 'cost-absorbing' and log-convex ones 'cost-amplifying', since the burden of the cost increase is put onto the consumer with log-convex demand and the burden is put onto the firm with log-concave demand.

A similar logic about how the log-curvature of demand determines whether the firm or the consumer bears the burden of a change in a parameter holds when examining $p^d$:

**Proposition 3.** ***Optimal price and price expectations***
*The impact of an increase in expected price $p^d$ on optimal price is given by:*

$$\frac{dp^*}{dp^d} = \frac{\mu'}{1-\mu'} \cdot \frac{1-m}{m} \tag{18}$$

Whereas log-concave demand functions have been called 'cost-absorbing' and log-convex ones 'cost-amplifying', we see here that log-curvature lends itself to a similar categorization when analyzing expected prices. If demand is log-concave ($\mu' < 0$), then price falls with higher $p^d$; if demand is log-convex, then price increases with a higher default price. Just as log-concave demand functions imply that firms bear the burden of higher costs (since price increases by less than one unit), so too does the firm facing a log-concave demand function bear the burden of higher expected prices, since $dp^*/dp^d < 0$. Naturally, log-convex demand follows the same pattern; just as higher costs mean higher prices, so too does higher $p^d$ imply higher optimal price. Therefore we call log-convex demand *self-fulfilling*, since consumers who expect higher prices will indeed cause higher prices, and we call log-concave demand self-defeating, since higher expected prices drives down the optimal price.

Finally, we can easily use the way that higher expectations impacts price (as given in Equation 18) to give more intuition to attentional pass-through:

**Corollary 4.** ***Attentional pass-through and consumer expectations***
*Plugging in $dp^*/dp^d$ from Equation 18 into the expression for attentional pass-through, we have:*

$$\frac{dp^*}{dm} = \frac{dp^*}{dp^d} \cdot \frac{c - p^d}{1 - m} - \frac{\mu}{m^2} \tag{19}$$

The expression above is more intuitive to interpret because the ratio $\mu'/(1 - \mu')$, whose meaning is somewhat opaque, is replaced with the far more legible $dp^*/dp^d$. Instead of expositing the sign of attentional pass-through with references to log-curvature, we can now explain that if higher expectations of price put downward pressure on the firm ($dp^*/dp^d < 0$), then realistic expectations ($p^d > c$) make the sign of attentional pass-through ambiguous.

**Empirical relevance** The mechanisms of attentional pass-through have now been explained, and it is necessary to state in what types of markets one would expect different forms of attentional pass-through. It seems clear that more attention will typically make price go down. Empirical verification of this may seem quite difficult, but the discussion of cost pass-through makes this exercise far easier. Recalling the expression for cost pass-through given in Equation 17 and that for attentional pass-through in Equation 13, we have:

**Proposition 4.** ***Cost pass-through as a sufficient statistic for attentional pass-through***
*Denoting cost pass-through $dp^*/dc = \rho$, attentional pass-through can be rewritten as:*

$$\frac{dp^*}{dm} = \frac{\mu}{m^2} \left[ (1 - \rho) \cdot \left( \frac{p^d - c}{p^* - c} \right) - 1 \right] \tag{20}$$

The expression in Equation 20 relates attentional pass-through to cost pass-through. This is useful because cost pass-through has been estimated in many markets, and so empirical estimates $\hat{\rho}$ can help to inform the likely impact of attentional interventions, like price transparency policies. For example, if a market typically exhibits unit pass-through ($\hat{\rho} = 1$), then one can almost sure that policies that raise $m$ will lead to lower prices, since the perceptual effect will become irrelevant with $\rho \approx 1$.

Log-concavity is a popular assumption in empirical settings (see Kang and Vasserman 2024 for examples and a discussion) and so for the moment we make that assumption, i.e. $\mu' < 0$. Further, assume that expectations are realistic, i.e. $p^d > c$. With the aforementioned assumptions can rewrite the inequality $dp^*/dm > 0$ in a revealing fashion:

$$\frac{dp^*}{dm} > 0 \iff \rho < 1 - \frac{p^* - c}{p^d - c} \tag{21}$$

One of the main challenges that an econometrician hoping to empirically implement the ideas so far is that one would need to estimate the attention parameter $m$, which is often hard to do without experimental data. However, the above inequality holds regardless of the value of $m$, eschewing the burden of estimating it. Indeed, the inequality given in (21) relates quantities which are often empirically estimated or observed in the Empirical Industrial Organization literature, i.e. $\rho$, $p^*$, and $c$. From there, it is quite easy to interpret the condition given. The ratio $(p^* - c)/(p^d - c)$ is simply the ratio of the true and expected markup; if $p^* = p^d$, then the inequality is $\rho < 0$, which is not possible. Given that we assumed $\rho < 1$ for this discussion, we see that it is more possible that the inequality is satisfied as $p^d$ grows larger than $p^*$. This condition emphasizes that inaccuracy of expectations is key to get positive attentional pass-through. For example, consumers likely know grocery prices quite well and so their expectations $p^d \approx p^*$, whereas infrequent purchases for items whose prices vary widely, like concert tickets, could likely have large gaps between $p^*$ and $p^d$.

As a result, we can characterize the types of markets which would likely benefit from policies which raise attention $m$ to price, whether through policies that ban add-on fees or make prices all-inclusive (i.e. not separate charges which the consumer has to add together) or another policy instrument. From Corollary 4, we see that attentional pass-through is almost certainly negative if either $\rho \approx 1$ or $p^d \approx p^*$, i.e. if the market has unit pass-through or if expectations are generally accurate. One would expect that expectations are accurate if price variation is low and frequency of purchase is high, and so policies to raise consumer attention would likely be useful in markets with small, frequent purchases like everyday consumer goods. If cost pass-through is either far from 1 or expectations are especially inaccurate, then it is more difficult to determine if an attentional intervention would be successful.

## 4.2 Rationality and the Resolution to Consumer-Firm Tensions

The analysis in Section 4 suggests that there may be some tension between the interests of the firm and the benefits to the consumer. Whereas the previous discussion focused on the behavior of optimal price and attention, we instead focus here on the behavior of *profit* and attention. In a sense, price levels do not matter to the monopolist – prices are only a step in the path to profit. So, although we may have implicitly assumed that a lower price resulting from more consumer attention hurts the monopolist, there is of course no reason to believe that this is true, as it amounts to an assumption about the value function $\pi^*$, which depends on the functional form of $D(p)$. As a result, we can present the conditions under which firms and consumers alike benefit from more rationality.

To do so, we first make a very weak assumption: $m$ is not 0. If someone is agreeing to buy a product, it is not plausible that they pay literally no attention to price,[4] so it may be of more use to think that $m \in [\underline{m}, 1]$ for some $\underline{m} > 0$, like $m = .05$ or $m = .1$. The reason that this matters is that at $m = 0$, the monopolist can price at infinity; but once we exclude unrealistically low levels of attention, then an unintuitive result becomes clear: more consumer attention is often more profitable for the firm.

By the Envelope Theorem, it is easy to show that $d\pi^*/dm = (p^* - c)\, D'(\cdot) \cdot \left(p^* - p^d\right)$, and thus we have the following proposition:

**Proposition 5.** ***When firms benefit from more consumer attention***
*Firms would have higher profits if consumers were more attentive if the following inequality holds:*

$$p^d > p^* \tag{22}$$

Therefore firms prefer more attention if consumers' expectation of price is above the firm's true price.

## 4.3 Symmetric oligopoly

Although it is not trivial to determine whether or not markups are higher with inattention because of possibly conflicting effects from the log-curvature of demand and direct dampening of the markup, we can make a conclusive statement about the behavior of markups as the number of firms increases. For tractability, we consider random utility demand in a symmetric oligopoly setting as originally outlined in Perloff and Salop (1985), as the formulation for markups and their asympotitic behavior as given by Gabaix et al. (2016) are particularly straightforward. We give a review of the canonical random utility formulation and the resultant markups from Gabaix et al. (2016) in Appendix B.1. Because we focus mostly on asymptotics here, we use 'prices' and 'markups' interchangeably.

We can make an easy extension from the results in Gabaix et al. (2016) to demonstrate how behavioral markups are always higher than rational markups in a symmetric equilib-

[4]The case of $m = 0$ also makes the problem trivial and uninteresting in other ways, and so it is a polar case best left ignored.

rium. We will see that the factors that lead to uncertainty about whether or not the behavioral markup is larger than the rational markup disappears in the symmetric equilibrium, and instead we find that each firm is given unequivocally more market power by the consumer's dampened response to price.

Reformulating the discussion around random utility models with behavioral perceptions of price, we make the simplifying assumption that consumers' reference price is the same for every firm; this means that they take the reference price to be some attribute of the market, and not of any single firm. This is a good approximation when consumers are interested in deviations from a market average or some other market-wide quantity, but not if their default is different for every firm. Given this modification, we can restate the demand function in the following way:

$$D_i\left(p_1,\ldots,p_n;p^d,m\right) = \mathbb{P}\left(X_i - mp_i + (1-m)p^d \geq \max_{j\neq i}\left\{X_j - mp_j + (1-m)p^d\right\}\right) \tag{23}$$

$$= \mathbb{P}\left(X_i + m\left(p_j - p_i\right) \geq \max_{j\neq i}\left\{X_j\right\}\right) \tag{24}$$

Now looking at the case where firm $i$ posts price $p_i$ and all other firms post price $p$, we get the following equations for demand and demand sensitivity:

$$D\left(p_i,p;n,m,p^d\right) = \int_{w_l}^{w_u} f(x)F^{n-1}\left(x+m\left(p-p_i\right)\right)dx \tag{25}$$

$$\frac{\partial D\left(p_i,p;n,m,p^d\right)}{\partial p_i} = -m(n-1)\int_{w_l}^{w_u} f(x)^2 f\left(x+m\left(p-p_i\right)\right)F^{n-2}\left(x+m\left(p-p_i\right)\right)dx \tag{26}$$

Now restricting attention to a symmetric equilibrium ($p_i = p$) for tractability (as in Gabaix et al. (2016) and Anderson et al. (1992)), the demand expression simply becomes $1/n$. Therefore we can express markups, denoted as $\mu^b$ where the $b$ denotes the markups are with the behavioral modification:

$$\mu^b = \frac{1}{m\cdot n(n-1)\int_{w_l}^{w_u} f(x)^2 f\left(x\right)F^{n-2}\left(x\right)dx} = \frac{\mu^r}{m} \tag{27}$$

Therefore markups are stated to be a function of i) the attentional parameter $m$ ii) the distribution of taste shocks $F(x)$ and iii) the number of firms. So, for a fixed distribution of taste shocks, these markups are some function of $m$ and $n$. Further, the fact that inattention enters the markup only multiplicatively means that we can easily extend this formulation to relate it to the asymptotic behavior of markups with respect to the number of firms:

**Proposition 6.** ***Inattention limits the reach of competition***

*In a symmetric equilibrium with random utility demand, behavioral markups are strictly larger* *than rational markups, and behavioral asymptotic markups are higher than traditional asymptotic markups. For a given distribution of random taste shocks, if*

$$\frac{D^r(p,n)}{\partial D^r/\partial p} = \mu_n^r \implies \frac{D^s(p,n)}{\partial D^s/\partial p} = \frac{\mu_n^r}{m} \tag{28}$$

*And if*

$$\lim_{n\to\infty} \mu_n^r \equiv \mu_\infty \implies \lim_{n\to\infty} \mu_n^b = \frac{\mu_\infty}{m} \tag{29}$$

*Thus, behavioral markups are always higher than rational markups. Further, the markup elasticity of number of firms is $\gamma/m$, where $\gamma$ is the tail index of the distribution of preferences.*

The proofs are a basic extension of Gabaix et al. (2016) and can be found in Appendix B.2.[5] The reason that we can make such a strong statement about the size of behavioral markups relative to the size of fully rational markups is the fact that we are in the context of a symmetric equilibrium with a random utility model, so price levels do not determine the size of the markups – only the distribution of random taste shocks determines markups. Because prices themselves cancel out, all that remains is markups as a function of the number of firms and the factor $\frac{1}{m}$ which unambiguously amplifies prices.

Taking the example where demand is given by the random utility model with Gumbel noise, we get the following rational markups which are then contrasted with demand when price is misperceived:

$$\mu^r_{\text{Gumbel}} = \frac{n}{n-1} \to 1 \tag{30}$$

$$\mu^b_{\text{Gumbel}} = \frac{n}{m\,(n-1)} \to \frac{1}{m} \tag{31}$$

If we repeat the above exercise with markups resulting from uniform noise, we can plot the difference between the $\mu^r$ and $\mu^b$ under each distribution to make the point clear:

[5] Technically, this statement is only true for distributions for which the tail index $\gamma$ is negative. As noted in Gabaix et al. (2016), distributions with positive tail indices have markups that *grow* with the number of firms in the market. Somewhat like upward sloping demand, we ignore this case.

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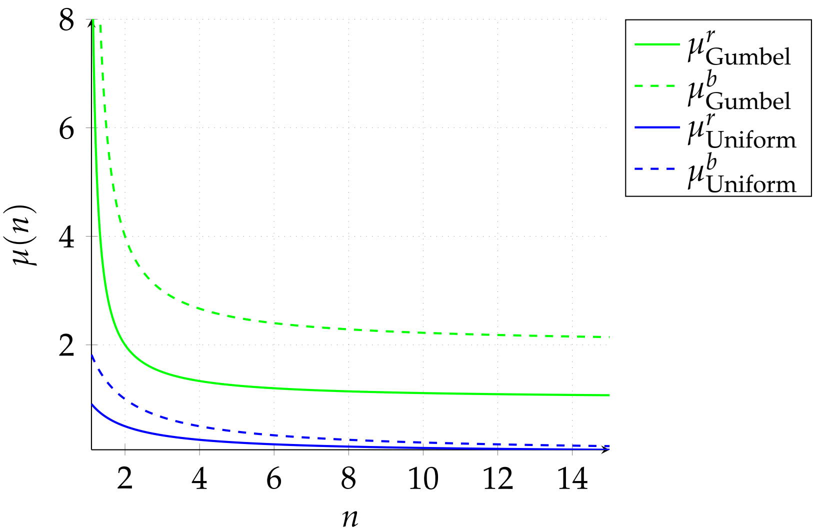



We have thus far explained how changes in attention affect price and how the presence of consumer inattention dampens the competitive forces stemming from the introduction of firms into a market. But it is also useful to investigate more precisely how we can relate changes in attention to changes in consumer, rather than firm, behavior.

# 5 Optimal Quality Choice with Inattention

## 5.1 Inattentive Demand for Quality

We now extend the previous framework to deal with when firms choose the quality of a good rather than the price. The results are symmetric and demonstrate how quality markdowns, rather than price markups, will behave under different levels of attention. Suppose the monopolist faces a demand function where the consumer misperceives quality as $q^s = mq + (1-m)\,q^d$. Then, if they have a linear cost function $C(q) = b$, we have a profit function of the form:

$$\pi = (p - bq)\,D(mq + (1-m)q^d) \tag{32}$$

Facing this profit function, the optimal quality is given by:

$$q^* = \frac{p}{b} - D\left[\frac{\partial D}{\partial q^s}\right]^{-1} \cdot \frac{1}{m} \tag{33}$$

Where again convexity of $1/D(q^s)$ is sufficient for quasi-concavity of the profit function, as demonstrated in Appendix A.4.1. Thus we can revisit the question of attentional pass-through but now in reference to quality:

**Proposition 7.** ***Attentional pass-through for quality***

$$\frac{dq^*}{dm} = \left[\frac{\mu'}{1-\mu'}\left(q^{pc} - q^d\right) + \frac{D(q^s)}{dD/dq^s}\right] \cdot \frac{1}{m} \tag{34}$$

Which is analogous to that which we found for price, as in equation **??**. Now that we have established that quality choice with inattention is analogous to the case of price, then we can move to a more applicable question: how do firms that offer multiple goods of varying quality optimize when some goods are more prominent than others?

## 5.2 Flagship and Peripheral Goods

Suppose a firm produces two goods, where one is the central or flagship product, whereas the second good is a peripheral, or auxiliary good. For example, a vehicle manufacturer may have a model of car that is their most well-known and celebrated, while also producing lesser known, less popular models. Denote the flagship good as having characteristics $\boldsymbol{x}^d = (p^d, q^d)$ for quality and price, while the peripheral good has quality $q$ and price $p$.

Importantly, however, consumers see the two goods as asymmetrically related: they infer something about the quality of the peripheral good from the flagship product, but not the other way around. So, the perceived quality of the central good, for some adjustment parameter $m$, is given by the following equation:

$$q^s = mq + (1-m)\, q^d \tag{35}$$

So when $m = 0$, people assume that the peripheral good's quality is the same as the flagship product, while when $m = 1$, the quality of the central good is irrelevant. However, we can assume that usually $m \in (0,1)$, meaning that consumers think that the peripheral good has quality that is somewhat similar to the flagship product. Importantly, there is no misperception of the central good's quality – consumers know the true quality of the central good, but not of the peripheral one. From this asymmetry, the rest will follow.

The monopolist makes profit from each good:

$$\pi(\boldsymbol{x^d}, \boldsymbol{x}) = (p - bq)D(mq + (1-m)\, q^d, p) + (p^d - bq^d)D^d(q^d, p^d) \tag{36}$$

So, note that the demand for the peripheral good (left) has the convex combination between peripheral and central quality, whereas the demand for the central good (right) contains no such confusion. The optimal choice of quality for the peripheral good is:

$$q^* = \underbrace{\frac{p}{b}}_{\text{Competitive quality}} - \underbrace{\frac{D}{m}\left[\frac{dD}{dq^s}\right]^{-1}}_{\text{Quality Markdown}} \tag{37}$$

In contrast, the optimal choice of quality for the central good contains an extra term:

$$q^d = \underbrace{\frac{p^d}{b}}_{\text{Competitive quality}} + \underbrace{\left(\frac{p-bq}{b}\right)\left(\frac{dD/dq^s}{dD/dq^d}\right)\cdot(1-m)}_{\text{Centrality Markup}} - \underbrace{D^d\left[\frac{dD^d}{dq^d}\right]^{-1}}_{\text{Quality Markdown}} \tag{38}$$

These very simple derivations can be found in Appendix A.4.

We have just demonstrated how a firm will endogenously supply more quality to the central good and less to the peripheral good due to the asymmetry in demand dependencies, and that the extent of this depends on the attentional parameter $m$.

Next, we wish to demonstrate how the optimal choice of the peripheral good changes when firms are endowed with different central good qualities. That is, suppose that different companies have different 'histories' or reputations. Consumers believe that when they observe some new good's quality $q_i$ from firm $i$, that this good has a quality level related to that of firm $i$'s reputation, $q_i^d$. So, do firms with better reputations have higher or lower peripheral quality?

It immediately follows (see Appendix A.4) that *reputational pass-through* is given by the following:

**Proposition 8.** ***Reputational Pass-Through***
*As a firm's reputation $q^d$ increases, the firm changes their peripheral good's quality following this expression:*

$$\frac{dq^*}{dq^d} = \frac{1-m}{m}\mu' \tag{39}$$

*Where $\mu' < 0$ if $D(q^s)$ is log-concave.*

It is immediately clear that for any log-concave demand function, optimal quality is decreasing in default quality. So, if Mercedes faces a log-concave demand function (multinomial logit, for example) when offering their cars, then as the reputation of their most famous car goes up, they have an incentive to undercut the quality of their auxiliary vehicles.

Now that we have drawn an analogy of attentional pass-through from price to quality and to the multiproduct monopolist, we can now analyze the situation in which the firm chooses both quality and price.

## 6 Shrinkflation

**Background and motivation** When firms face cost shocks, they often change their price to compensate for the loss in revenue caused by a higher cost. However, prices are not the only dimension of a product that can change due to a change in costs—firms will often change other features of the product, like its size or its quality, in order to decrease costs. Thus the firm has two ways to increase profit when costs increase: set a higher price, or offer a lower quality.

The phenomenon of firms offering lower quality is colloquially known as 'shrinkflation', since firms slightly decrease the size of products without changing their price. Suppose that the monopolist faces a profit function with convex costs of quality:

$$\pi(p,q) = \left(p - \alpha q^2\right) D(\theta q^s - p) \tag{40}$$

Where $D(u)$ is the demand function which is increasing in the perceived consumer utility $u = \theta q^s - p$, and $\alpha$ is the cost parameter. Further, $q^s = mq + (1-m)q^d$ and $f(q^s) = \theta q^s$ converts the consumer's valuation for perceived quality into a dollar value. We assume that consumers have full attention to price, but the interpretation is similar if we instead had $m \equiv m_q/m_p$, i.e. $m$ is attention to quality relative to the attention to price. We assume convex costs because with linear costs and linear $f(q^s)$, there are either infinitely many or zero solutions to the optimal price-quality pair.

Denote the pass-through rates of cost to quality and attention as:

$$\rho_q \equiv \frac{dq^*}{d\alpha} \tag{41}$$

$$\rho_p \equiv \frac{dp^*}{d\alpha} \tag{42}$$

The intuition for shrinkflation is that it should be larger, i.e. there should be more relative pass-through to quality than price, when quality is harder to perceive before purchasing. Thus although there can be differential pass-through for many reasons (wholesaler-retailer negotiations, different costs of changing production procedures vs. listed prices, etc.) we will focus on how the relative pass-through rates change as consumers become more or less attentive. To do so, we first introduce the following definition of shrinkflation:

**Definition. Shrinkflation**
Shrinkflation is the ratio of pass-through rates to quality and price, which vary with consumer attention to quality $m$:

$$\overline{\rho}(m) \equiv \frac{\rho_q}{\rho_p} \tag{43}$$

Now that we have provided a definition of shrinkflation, we examine a simple example to see how $\overline{\rho}$ behaves when attention to quality $m$ changes.

### 6.1 Motivating example with linear demand

Suppose that the monopolist chooses both $p$ and $q$ and faces a linear demand curve and quadratic cost function:

$$\pi(p,q) = \left(p - \alpha q^2\right) \cdot \left(A - p + \theta q^s\right) \tag{44}$$

Where $q^s = mq + (1-m)q^d$. Then we have that the optimal price and quality are (all derivations are in Appendix A.7):

$$q^* = \frac{m\theta}{2\alpha} \tag{45}$$

$$p^* = \frac{1}{2}\left[A + \theta\left(1-m\right)q^d + \frac{3\left(m\theta\right)^2}{4\alpha}\right] \tag{46}$$

This means that the pass-through rates are:

$$\rho_q = \frac{dq^*}{d\alpha} = -\frac{m\theta}{2\alpha^2} \tag{47}$$

$$\rho_p = \frac{dp^*}{d\alpha} = -\frac{3\left(m\theta\right)^2}{8\alpha^2} \tag{48}$$

Therefore we have that shrinkflation $\overline{\rho}$ is:

$$\overline{\rho} = \frac{4}{3m\theta} \tag{49}$$

Finally, we see that shrinkflation is decreasing as consumers are more attentive to quality:

$$\frac{d\overline{\rho}}{dm} = -\frac{4}{3m^2\theta} < 0 \tag{50}$$

This example demonstrates how the provided definition of shrinkflation returns tractable expressions in simple settings and has sensible comparative statics. We now move to a broader characterization of shrinkflation.

### 6.2 General formulation

Having motivated the underlying economic logic with examples, we can now move to more generally characterize shrinkflation. We take the monopolist's profit function to have convex costs and consumer's linear valuation for quality:

$$\pi(p,q) = \left(p - \alpha q^2\right) \cdot D(\theta q^s - p) \tag{51}$$

Where $D(u)$ is the demand function which is increasing in the perceived consumer utility $u = \theta q^s - p$, and $\alpha$ is the cost parameter. As shown in Appendix A.7.2, the optimal choices of price and quality are given by:[6]

$$p^* = \frac{(\theta m)^2}{4\alpha} + \frac{D(p^*, q^s(q^*))}{D'(p^*, q^s(q^*))} \tag{52}$$

$$q^* = \frac{\theta m}{2\alpha} \tag{53}$$

As is sensible, we see that a higher value of $m$ means a higher choice of quality $q^*$. However, consumer welfare is given only by the relative choices in price vs. quality, and so we also note that higher quality implies higher price. In addition, the pass-through rates (as found using the Implicit Function Theorem) are:

$$\frac{dp^*}{d\alpha} \equiv \rho_p = \left(\frac{\theta m}{2\alpha}\right)^2 \cdot \frac{2\mu' - 1}{1 - \mu'} \tag{54}$$

$$\frac{dq^*}{d\alpha} \equiv \rho_q = -\frac{\theta m}{2\alpha^2} \tag{55}$$

For $\mu' < 0$, pass-through to both quality and price is negative. The key conceptual argument, of course, is that only relative pass-through rates matter. If quality goes down slightly but price drops significantly, then this may be beneficial to the consumer. Importantly, if $.5 < \mu' < 1$ then price increases and quality decreases. We can generally express the ratio of pass-through of quality to price as:

$$\overline{\rho} = \frac{2\left(\mu' - 1\right)}{\theta m \left(2\mu' - 1\right)} \tag{56}$$

Then, we have the key proposition:

**Proposition 9.** ***Less shrinkflation with more attention***
*For constant $\mu'$ across values of $p$ and $q$, we have that the ratio of pass-though rates $\overline{\rho} = \rho_q/\rho_p$ is decreasing in magnitude with more attention:*

$$\frac{d\left|\overline{\rho}\right|}{dm} < 0 \tag{57}$$

[6] These solutions assume that the interior solution is not the one that puts demand and revenue equal to zero.

We focus on the magnitude of $\overline{\rho}$ rather than its absolute size because there are two cases. The first case is that $\mu' < 1/2$, and so $\rho_q,\ \rho_p < 0$, which means that $\overline{\rho} > 0$. Then, $d\overline{\rho}/dm < 0$ means that the rate of pass-through to quality is getting smaller relative to the pass-through to price. The second case is that $\mu' > 1/2$, in which case $\rho_q < 0$ and $\rho_p > 0$, meaning that $\overline{\rho} < 0$. Thus $d\overline{\rho}/dm > 0$, i.e. $\overline{\rho}$ is getting closer to zero. In both cases, there is less relative pass-through to quality than to price as attention increases.

# 7 Endogenous Attention

## 7.1 Motivating example: monopoly with sparse max consumer

Previous sections assumed that attention was exogenously given and could be perturbed by a policymaker. However, attention is endogenously produced: it is a result of a trade off between the costs and benefits of paying attention to something. The previous section yielded fairly simple expressions that explain whether or not increasing attention $m$ will decrease monopoly price. In this section, we instead treat $m \equiv m(p, p^d, \kappa)$, where $\kappa$ is a *cognitive cost* parameter, i.e. how costly it is for an individual to pay attention to economic variables like prices.

The question in this section is: when will decreasing consumers' cost $\kappa$ of observing/paying attention to price lead to a decrease in monopoly price? The analysis, although analogous to the previous section, is far more complicated when attention is endogenous. In order to provide intuition for the sorts of mechanisms relevant to this context, we provide a concrete example of monopoly pricing when facing a consumer with endogenous attention. Specifically, we make use of (a slight variant of) the sparse max algorithm provided in Gabaix (2014).

**Endogenous attention: sparse max** As described in Gabaix (2014), the sparse max individual's attention to price will be a trade-off between costs of cognition $\kappa$ and deviation from the default price $|p - p^d|$.[7] To derive the exact trade-off between benefits and cost of information, we have to assume a functional form on utility: So, we suppose that the individual has utility function:

$$U(Q) = y + \frac{A}{\alpha}Q - \frac{Q^2}{2\alpha} \implies \tag{58}$$

$$Q^*(p^s) = A - \alpha p^s \tag{59}$$

[7] We use absolute value rather than quadratic loss because we cannot analytically solve the case of quadratic loss. With absolute value, we can split things into cases and solve from there.

Where again $p^s = mp + (1-m)p^d$ and $Q^*$ is demand. The difference from the previous section is that $m$ will be endogenous, i.e. responsive to the market environment. This reflects that changes in price will not only generate traditional demand responses, but it will also trigger consumers to be more attentive to price, i.e. increase $m$. As shown in Appendix XXX, this quadratic utility function implies the following form for attention:

$$m^*(p;\kappa,p^d) = \max\left\{0, 1 - \frac{\kappa}{|p-p^d|\,A\alpha}\right\} \tag{60}$$

For $m > 0$, attention is increasing in difference between $p$ and $p^d$, i.e. deviations from the usual price. Further, attention is decreasing in $\kappa$ and increasing in both price sensitivity $\alpha$ and (some measure of the) valuation of the good $A$. Importantly, attention *can* be zero in this context—the previous section had assumed that $m$ would be positive and constant everywhere. It is easy to see that attention is positive iff $|p-p^d| > \kappa/A\alpha$; consumers only pay attention if deviations from the usual price are sufficiently large. As stated, those deviations have to be larger to get positive attention when $\kappa$ is larger, and the deviations can be smaller if $A$ or $\alpha$ are large. This creates an inattentive interval, as seen in Figure 1:

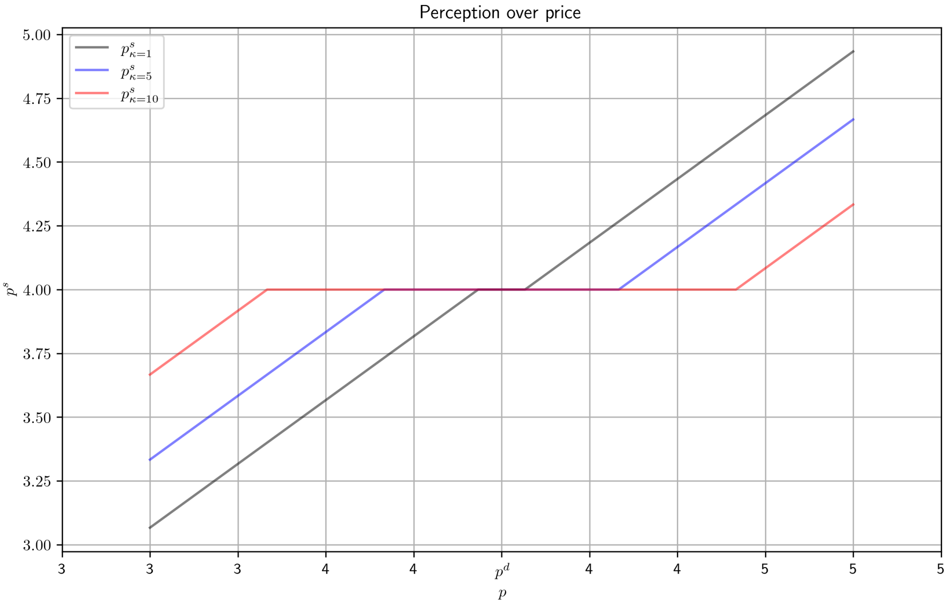



Figure 1: Perceived prices

We plot the perceived price for different values of $\kappa$. Each function displays an inattentive band around $p^d$ whose size increases in $\kappa$. Outside of this band, perceptions are linear and increasing in price.

Importantly, this means that attention $m$ is *not* monotonically increasing in $p$—it is increasing in $p$'s deviation from $p^d$. This can be seen in the following figure:

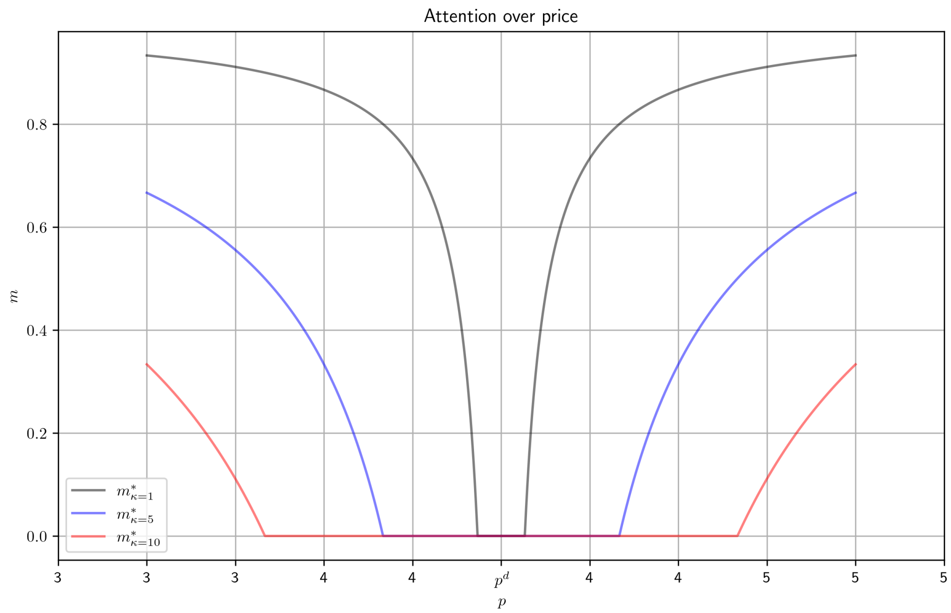



Figure 2: Attention

The inattention bands mean that demand is *not* strictly decreasing in price, which violates the traditional assumption. Although $D'(p^s) < 0$, we see that $dD(p^s(m(p,\kappa)))/dp = 0$ when $p \in (p^d - \kappa/A\alpha, p^d + \kappa/A\alpha)$, since $dm/dp = 0$. The fact that demand is no longer always decreasing in price means that there are kinks in the profit function. We see that profit is:

$$\pi(p, p^d, \kappa) = \begin{cases} (p-c)\left(A - \alpha p - \frac{\kappa}{A}\right) & \text{if } p < p^d - \frac{\kappa}{A\alpha} \\ (p-c)\left(A - \alpha p^d\right) & \text{if } p \in \left(p^d - \frac{\kappa}{A\alpha}, p^d + \frac{\kappa}{A\alpha}\right) \\ (p-c)\left(A - \alpha p + \frac{\kappa}{A\alpha}\right) & \text{if } p > p^d + \frac{\kappa}{A\alpha} \end{cases} \tag{61}$$

The kinks in the profit function can be seen easily:

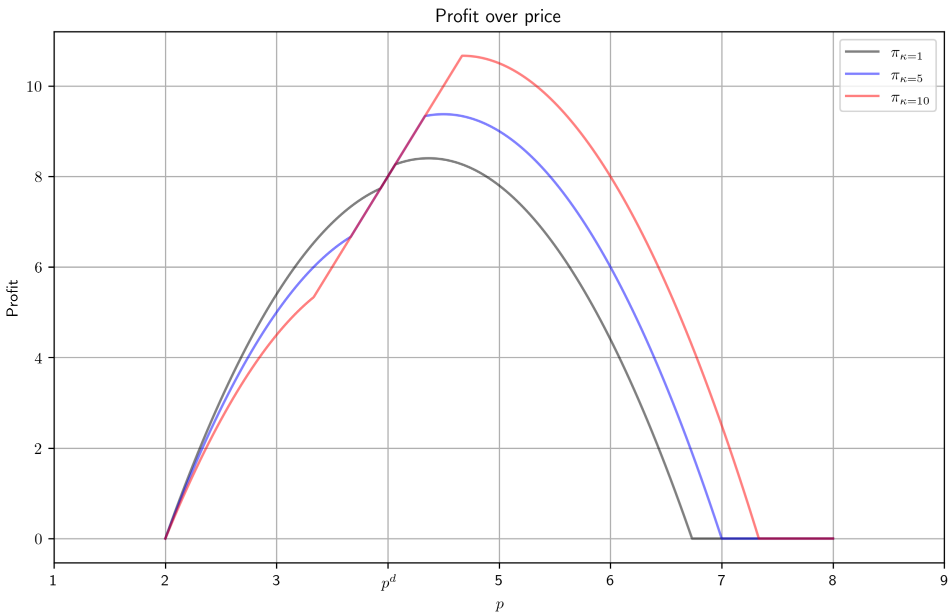



Figure 3: Profit

The above figure shows some stuff.

Naturally, there are candidate solutions within each region of prices; there are candidate interior solutions of the form:

$$p^*_> = \frac{A + \alpha c}{2\alpha} + \frac{\kappa}{2A\alpha} \tag{62}$$

$$p^*_< = \frac{A + \alpha c}{2\alpha} - \frac{\kappa}{2A\alpha} \tag{63}$$

Where $p^*_>$ denotes the interior solution for values of $p > p^d + \kappa/A\alpha$, i.e. above the inattentive region, and $p^*_<$ denotes the interior solution below the inattentive region, i.e. for $p < p^d - \kappa/A\alpha$. Because the profit function is strictly concave in the attentive (i.e. $m > 0$) regions, then we know that only one of the possible interior solutions can be satisfied. Denote $\gamma \equiv 2A\alpha \cdot \left[p^d - \frac{A+\alpha c}{2\alpha}\right]$; then we have that:

$$p^*_> > p^d + \frac{\kappa}{A\alpha} \iff \kappa < -\gamma \tag{64}$$

$$p^*_< < p^d - \frac{\kappa}{A\alpha} \iff \kappa < \gamma \tag{65}$$

Since $\kappa > 0$ and $\gamma$ is either negative or positive, then only one of the above conditions can hold. Again, this is sensible given that the function is strictly concave when $m > 0$, and so it is not possible that two interior solutions are obtained. Suppose that $\gamma > 0$; then since $\kappa$ is positive, it is not possible that $\kappa < -\gamma$, and thus it is not possible that $p^*_>$ lies in its assumed region. As a result, it must be true that $p^*_<$ is the interior solution. The case of $\gamma < 0$ holds symmetrically.

We can interpret the sign of $\gamma$ easily: since its sign is determined by whether or not $p^d > (A + \alpha c)/2\alpha$, we see that the key relation is between the expected price and the fully rational monopoly price. If consumers expect to face a price higher than what they would've been charged absent any cognitive frictions (i.e. $\kappa = 0$), then $\gamma > 0$ and so the interior solution is given by $p^*_<$. Thus which interior solution is obtained for a given set of parameters depends only on the relation between $p^d$ and $p^*_{\kappa=0}$.

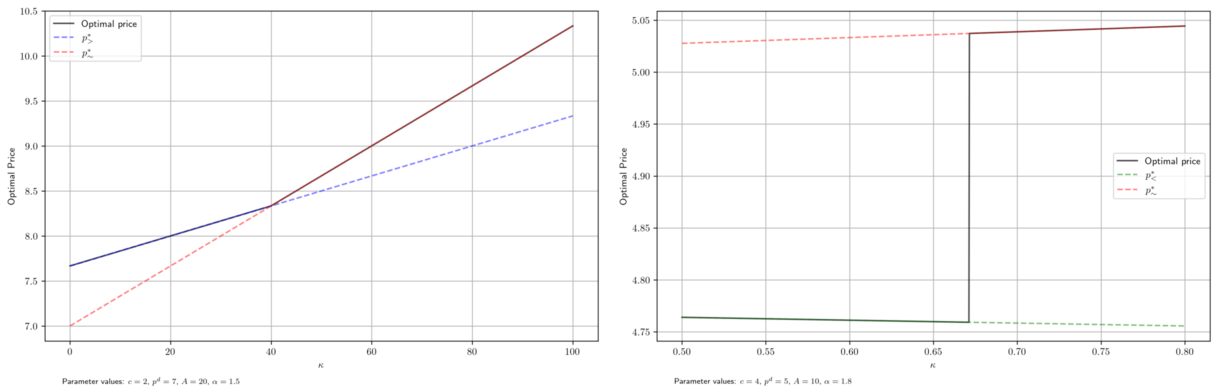
Now that we have demonstrated when each interior solution will be obtained, we next need to identify when that interior solution's price is preferred to the boundary price $p^d + \kappa/A\alpha$. As shown in the appendix, the optimal price schedule is given by:

$$p^* = \begin{cases} \frac{A+\alpha c}{2\alpha} - \mathrm{sgn}(\gamma) \cdot \frac{\kappa}{2A\alpha} & \text{if } \kappa \leq \zeta \\ p^d + \frac{\kappa}{A\alpha} & \text{if } \kappa > \zeta \end{cases} \tag{66}$$

The definition of $\zeta$ is as follows. If $\gamma < 0$ and so $p^*_>$ is a local maximum, then define $\zeta = -\gamma$. If $\gamma > 0$ and so $p^*_<$ is the local maximum, then we can implicitly define $\zeta$ as the value of $\kappa$ which satisfies the equation $\pi^*_< / \pi^*_\sim = 1$. Recall that the traditional monopoly price is $p = (A + \alpha c)/2\alpha$, and so $p^*_>$ and $p^*_<$ differ from that price by a value of $\kappa/2A\alpha$. Interestingly, $p^*_<$ is *below* the traditional monopoly price, whereas $p^*_>$ is above the traditional monopoly price. Whether or not $p^*_\sim$ is above or below the traditional price depends on the value of $p^d$.

The underlying conditions surrounding the optimal pricing rules are somewhat involved, but we present two figures of the optimal pricing schedule as an example of the two relevant cases:

Figure 4: Optimal pricing schedules




The point of this exercise is to show that even in a very simple setup, it is not obvious a) whether or not the monopolist will price above or below the traditional monopoly price or b) if $p^*$ is increasing or decreasing in $\kappa$. This example serves to demonstrate some of the key elements of monopoly pricing against endogenously inattentive consumers, and now we seek to characterize the economic environment more generally.

## 7.2 Fundamentals with Endogenous Attention

**Attentional function** To achieve a microfoundation of attention, we will instead treat $m \in [0,1]$ as a function of a few variables: price $p$, default $p^d$, and cognitive cost $\kappa$. We make the following set of assumptions:

**Assumption 1. Attention is increasing in deviation of price and decreasing in cognitive cost**

We assume that $m \equiv f(p, p^d, \kappa)$ such that

$$\frac{\partial f}{\partial \left|p - p^d\right|} \geq 0 \tag{67}$$

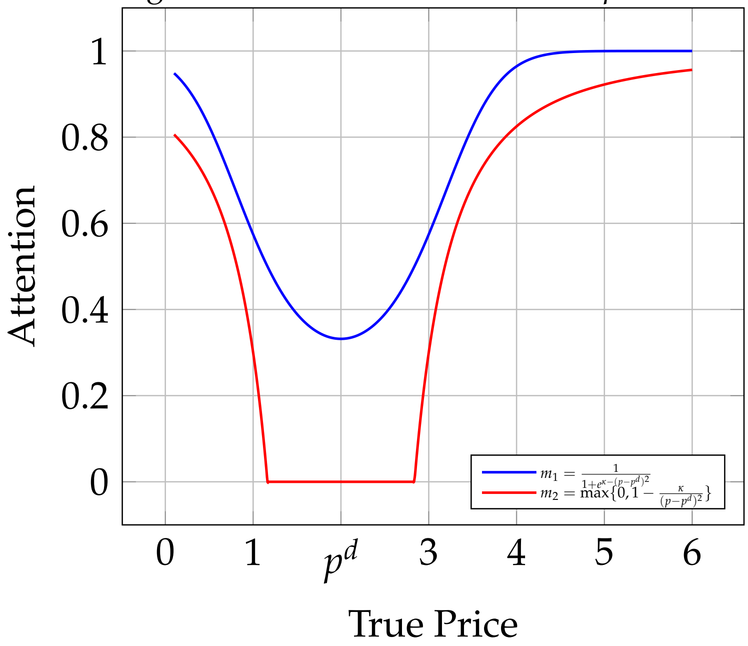
$$\frac{\partial f}{\partial \kappa} \leq 0 \tag{68}$$

For example, we can consider the sparse max attention function (with quadratic loss, rather than absolute value), or we could consider:

$$m_2 = \frac{1}{1 + e^{\kappa - \left(p - p^d\right)^2}} \tag{69}$$

Both of which are bounded between 0 and 1. For $p \approx p^d$, consumers pay relatively little attention to price, and for large deviations from their expectation $p^d$ they pay maximal attention. We can see that here:

Figure 5: Attention over values of $p$




Parameter values: $p^d = 2$, $\kappa = .7$

**Attention and price** For values sufficiently close to the default, attention is very low, and for values very far from the default, attention is near or equal to one. We see that the value of $dm^*/dp$ is not linear for either function, and nor is it monotonic. As $p$ increases, attention is decreasing when $p < p^d$ and increasing when $p > p^d$. Further, these two examples display non-linear attention in $p$; if we replaced the sparse max attention $m_2$ with $(p - p^d)^2$ by $|p - p^d|$, then it $m$ would be piecewise linear in $p$, as demonstrated in the previous section.

**Perceptions and true price** Maintaining the assumption that $p^s$ is a convex combination of the true price and the default, we have that:

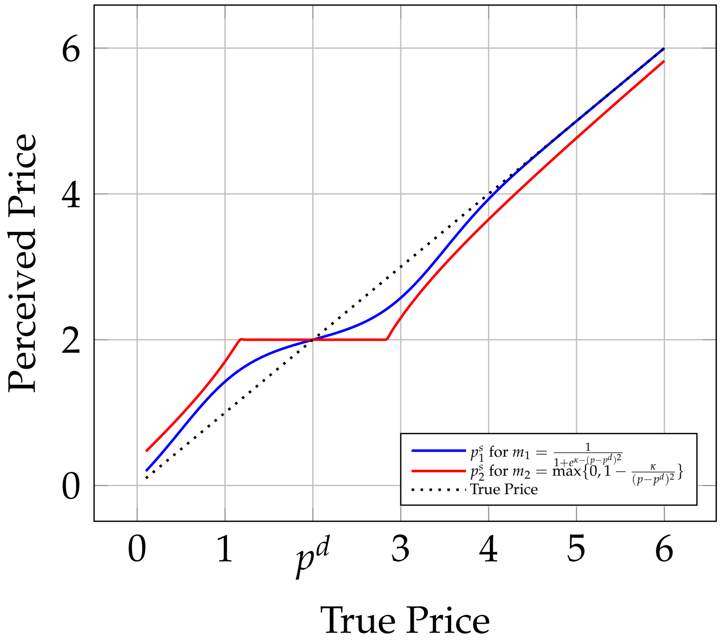
$$p^s(m(\kappa, p), p, p^d) = m(p, \kappa) \cdot \left(p - p^d\right) + p^d \tag{70}$$

**Lemma 4.** *Perceived price is weakly increasing in true price*

$$\frac{dp^s}{dp} = \frac{dm}{dp} \cdot \left(p - p^d\right) + m \geq 0 \tag{71}$$

The sign of $dp^s/dp$ comes from the assumption that $m$ is increasing in the deviation between $p$ and $p^d$ (see Assumption 1); if $p < p^d$, then $dm/dp < 0$ since price is moving closer to its usual value, and so their product is positive. If $p > p^d$, then $dm/dp > 0$ because price is moving further from the usual price, and so the product is again positive. However, some attentional functions also exhibit 'inattentive regions' (like the variant of sparse max presented in the previous section), and so in those $m = 0$ regions we have that $dp^s/dp = 0$. The behavior of perceived price over true prices can be seen:

Figure 6: Perceived Price for Different Attention Functions




Parameter values: $p^d = 2$, $\kappa = .7$

**Endogenously inattentive demand and price** Maintaining the assumption that $D^r$ (as defined in Definition 3) is strictly decreasing, we see that endogenously inattentive demand is weakly decreasing in true price:

$$\frac{dD(p^s)}{dp} = D'(\cdot)\frac{dp^s}{dp} \leq 0 \tag{72}$$

So, demand is weakly decreasing in price, which follows immediately from Lemma 4. Importantly, however, if $p$ is such that $m = 0$ (i.e. the price is in an 'inattentive region'), then we have that $dD(p^s)/dp = 0$. In traditional settings, demand is strictly decreasing—here, it is only weakly decreasing. As will become clear in the next paragraph, this imposes a new and perfectly intuitive restriction on the interval of optimal prices.

**Monopoly pricing with endogenous attention** Given that $m(p)$, it is now necessary to rederive the monopolist's optimization problem. Facing a profit function of the form:

$$\pi(p; p^d, m(\kappa, p)) = (p - c)\, D(m(\kappa, p)\left(p - p^d\right) + p^d) \tag{73}$$

Then we have that first order condition is given by:

$$p^* = c - \frac{D(p^s)}{D'(p^s)} \cdot \left[\frac{\partial p^s}{\partial p}\right]^{-1} \tag{74}$$

$$= c + \frac{\mu(p^s)}{m(p,\kappa) + \frac{\partial m}{\partial p} \cdot (p^* - p^d)} \tag{75}$$

Where again we denote the markup $-D(p^s)/D'(p^s)$ as $\mu$. If $m$ is *not* a function of $p$ (as in Section XXX), then we simply have the usual $p^* = c + \mu/m$.

Since $p^* - c \geq 0$ at optimum and $\mu > 0$, we now have a new condition: $dp^s(p^*)/dp > 0$, i.e. the perception of price must be increasing in true price at the optimal price. It is already known that it must be weakly positive, but the requirement of strict positivity is entirely sensible – the firm should not price in a region where consumers don't respond to price increases. If at a candidate solution $p'$ the perception of price is constant, then why not increase the price more? Consumers would see the same price and the firm would make more revenue per sale. Therefore, a solution is only optimal if the perception of price is increasing at that point. This derivation gives the general form of the intuition that we saw in the previous section about not pricing in the 'unresponsive region'.

We deal with second order conditions in Appendix C.2 and demonstrate that a sufficient condition for optimality of price is that $p^s$ be convex in $p$, but note that this is not necessary.

**Attention and cognitive costs** Before analyzing the comparative statics of price w.r.t. cognitive costs $\kappa$, we prepare for that analysis by discussing the relation of cognitive cost to the building blocks of optimal price, i.e. attention and perceived prices. There is a nuanced relationship between attention and cognitive costs $\kappa$. Although we assume that $\partial m/\partial\kappa < 0$, this does not take into account the full economic environment. Again denoting $p^*$ as the optimal monopoly price, we see that:

$$\frac{dm(p^*(\kappa), \kappa)}{d\kappa} = \frac{\partial m}{\partial \kappa} + \frac{\partial m}{\partial p} \cdot \frac{\partial p^*}{\partial \kappa} \tag{76}$$

The first value on the right side is the direct impact of a change in cognitive costs on attention, whose negative sign is an assumption. The negative direct effect, however, is paired with the indirect effect, i.e. $\frac{\partial m}{\partial p} \cdot \frac{\partial p^*}{\partial \kappa}$ whose sign is ambiguous. If $p < p^d$ and optimal price is increasing in $\kappa$, then the indirect effect is negative since $\partial m/\partial p < 0$ and $\partial p^*/\partial \kappa > 0$. This is because the change in $\kappa$ is triggering an increase in price moving $p$ closer to $p^d$. Symmetric arguments hold for other scenarios.

**Perceptions and cognitive costs** Next we wish to analyze how changes in cognitive cost $\kappa$ impacts the perceptions of price $p^s$. We see that for fixed price, the impact is analogous to what we found for exogenous attention in Lemma 2:

$$\frac{\partial p^s}{\partial \kappa} = \frac{\partial m}{\partial \kappa} \cdot \left(p - p^d\right) \tag{77}$$

The complexity enters once we allow for price to be endogenous, i.e. for $p^*(\kappa)$ to change:

$$\begin{aligned}
\frac{dp^s}{d\kappa} &= \frac{d}{d\kappa}\left[m(p^*(\kappa),\kappa) \cdot \left(p^*(\kappa) - p^d\right) + p^d\right] && (78)\\
&= \left[\frac{\partial m}{\partial \kappa} + \frac{\partial m}{\partial p}\frac{dp^*(\kappa)}{d\kappa}\right] \cdot \left(p^* - p^d\right) + m(\cdot) \cdot \frac{dp^*}{d\kappa} && (79)\\
&= \frac{\partial p^s}{\partial \kappa} + \frac{dp^*}{d\kappa} \cdot \frac{dp^s}{dp} && (80)
\end{aligned}$$

Ultimately, we are interested in understanding how optimal price of a monopolist varies over values of cognitive cost $\kappa$; this is the endogenous version of the main proposition that explained *attentional pass-through*.

**Proposition 10.** ***Cognitive Cost Pass-Through***
*The change in a monopolist's optimal price when consumers have higher cost $\kappa$ of observing price $p^*$ is given by:*

$$\frac{dp^*}{d\kappa} = \frac{\left[\frac{\partial p^s}{\partial \kappa}\mu' + \frac{D(p^s)}{D'(p^s)} \cdot \frac{d^2p^s/dpd\kappa}{dp^s/dp}\right]}{\frac{dp^s}{dp}\left[1 - \mu' - \frac{D(p^s)}{D'(p^s)} \cdot \frac{d^2p^s/dp^2}{(dp^s/dp)^2}\right]} \tag{81}$$

Because of the second order condition at the optimum, the denominator is positive. Therefore, the sign of cognitive cost pass-through is given by the numerator. We see then that the sign of the cognitive cost pass-through is given by:

$$\text{sign}\left(\frac{dp^*}{d\kappa}\right) = \text{sign}\left(\frac{\partial p^s}{\partial \kappa}\mu' + \frac{D(p^s)}{D'(p^s)} \cdot \frac{d^2p^s/dpd\kappa}{dp^s/dp}\right) \tag{82}$$

Recall that $\frac{D(p^s)}{D'(p^s)/p^s} < 0$ and $\frac{dp^s(p^*)}{dp} > 0$. The forces at play are exactly the same as those examined in the example section. If demand is log-convex ($\mu' > 0$) and prices are perceived as decreasing ($\frac{\partial p^s}{\partial \kappa}$ and sensitivity to price changes decrease, then we have that cognitive cost pass-through is negative. We can list all possibilities in this table:

| | $2\ \Lambda < 0$ | $\Lambda < 0$ | $0 < \Lambda < 1$ | $0 < \Lambda < 1$ |
|---|---|---|---|---|
| | $\frac{\partial p^s}{\partial \kappa} < 0$ | $\frac{\partial p^s}{\partial \kappa} > 0$ | $\frac{\partial p^s}{\partial \kappa} < 0$ | $\frac{\partial p^s}{\partial \kappa} > 0$ |
| $\frac{\partial p^s}{\partial p \partial \kappa}$ | $\frac{dp^*}{d\kappa} > 0$ | Ambiguous | Ambiguous | $\frac{dp^*}{d\kappa} > 0$ |
| $\frac{\partial^2 p^s}{\partial p \partial \kappa}$ | Ambiguous | $\frac{dp^*}{d\kappa} < 0$ | $\frac{dp^*}{d\kappa} < 0$ | Ambiguous |

Table 2: Cognitive Cost Pass-Through

**Example of cognitive cost pass-through** To fix ideas, we provide a concrete example. Suppose that attention is given by:

$$m_1 = \max \left\{ 0, 1 - \frac{\kappa}{\left(p - p^d\right)^2} \right\} \tag{83}$$

And that demand is given by (for $\psi > 1$):

$$D_1 = e^{-(p^s)^\psi} \tag{84}$$

Neither function exhibits exotic behavior – attention is increasing in price's distance from the default and decreasing in cognitive cost. Demand is decreasing in perceived price.

We have some intuiton from our treatment of $dp^*/dm$ that what matters is the log-curvature of demand and whether or not $p^s$ is increasing in attention. Naturally, if $p^s$ is a function of attention $m$, where $m(p, \kappa)$ is a function of price and cognitive cost $\kappa$, then we should expect to have to investigate how $p^s$ varies with $p$ and $\kappa$ (whereas before we only had to concern ourselves with how $p^s$ varied with $m$). We see that (all derivations can be found in Appendix C.4):

$$\Lambda_1 = \frac{1 - \psi}{\psi \left(p^s\right)^\psi} < 0 \tag{85}$$

$$\frac{\partial p^s}{\partial p} = 1 + \frac{\kappa}{\left(p - p^d\right)^2} > 0 \ \forall \ |p - p^d| > \kappa \text{ and 0 otherwise} \tag{86}$$

$$\frac{\partial p^s}{\partial \kappa} = \frac{1}{p^d - p} \tag{87}$$

It turns out that we will need one more ingredient, which reflects the new interaction between changes in price and cognitive cost; we also need to study how the responsiveness of

$p^s$ to $p$ changes with higher $\kappa$:

$$\frac{\partial^2 p^s}{\partial p \partial \kappa} = \frac{1}{(p - p^d) > 2} > 0 \; \forall \; |p - p^d| > \kappa \text{ and } 0 \text{ otherwise} \tag{88}$$

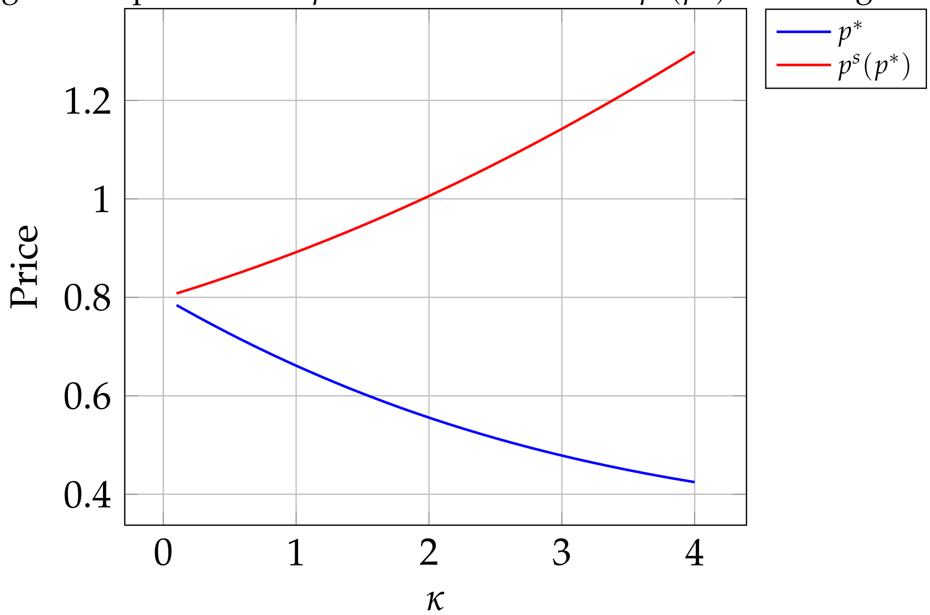
So, we have derived a simple situation where we have log-concave demand, and the attention function is such that:

1. Consumers perceive price as increasing if price is increasing outside of an 'unresponsive' region
2. Consumers perceive price as increasing if cognitive cost $\kappa$ is increasing if $p^d > p$[8]
3. Consumers' perception of price is more sensitive to changes in price when cognitive cost $\kappa$ is higher

We know intuitively that the firm will never price within an attentionally 'unresponsive' region because they could price higher without the consumer noticing (we will be able to derive this in a general setting soon). Thus we know that $\frac{\partial p^s(p^*)}{\partial p} > 0$ at any optimum. We also see that the conditions above mean that the consumer will perceive higher price levels and be more sensitive to price changes as $\kappa$ rises if the default price exceeds the true value. This means that the consumer is perceives price to be increasing, and his perception of price is more sensitive to price changes; thus, we should expect that an increase in cognitive cost should a) increase their perception of price and b) in turn (because $\Lambda_1 < 0$) decrease prices. Indeed, this is what we see:

[8]This is sensible – if consumers are facing higher cognitive costs, they rely on their default more. So, if they rely on their default more and that default value is higher than the true value, then they will perceive price as increasing.

Figure 7: Optimal Price $p^*$ and Perceived Price $p^s(p^*)$ Over Cognitive Costs $\kappa$




Parameter values: $p^d = 5$, $c = .3$, $\psi = 2.3$, $D = e^{-(p^s)^\psi}$, $m = \max\left\{0, 1 - \frac{\kappa}{(p-p^d)^2}\right\}$

Thus we see that even though one may expect that an increase in cognitive costs $\kappa$ would lead to a decrease in monopoly price, the opposite is true in this simple example. However, analytical solutions remain out of reach even for very simple attention and demand functions.

# 8 Conclusion

We have demonstrated that for a very general formulation in which consumers misperceive prices or qualities, making them less behavioral does not necessarily make them better off; this is true whether attention is exogenous or endogenous. Having applied a similar logic to the choice of quality, we then attempt a preliminary investigation of shrinkflation, and demonstrate that it decreases when consumers are more attentive. Further, we demonstrate that in some markets, it may be beneficial for both firms and consumers to have fully rational consumers, and that the assumption that more or less attention implies a trade-off between consumer and producer welfare is misguided. This serves as a preliminary attempt to formalize the behavioral and purely economic mechanisms that cause policies that increase consumers' perceptions of products—whether of price or quality—in order to guide policy interventions in the future.

# A Monopoly with Inattention

## A.1 Attentional Pass-Through

First, we must note that the key derivatives $\frac{\partial D(p^s(p^*(m)))}{\partial m}$ and $\frac{\partial^2 D(p^s(p^*(m)))}{\partial p^s \partial m}$ must be rederived because the optimal price $p^*(m)$ is a function of attention, so the previous equations are no longer complete. So, we can reexpress the derivatives as follows:

$$\frac{\partial D}{\partial m} = \frac{\partial D}{\partial p^s} \cdot \left( p^* - p^d + m \frac{\partial p^*}{\partial m} \right) \tag{89}$$

$$\frac{\partial^2 D}{\partial p^s \partial m} = \frac{\partial^2 D}{\partial p^{s2}} \cdot \left( p^* - p^d + m \frac{\partial p^*}{\partial m} \right) \tag{90}$$

Now deriving attentional pass-through will immediately yield the above two derivatives:

$$\frac{\partial p^*}{\partial m} = \frac{\partial}{\partial m}\left[c - \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m}\right] \tag{91}$$

$$= -\frac{\partial D/\partial m}{\partial D/\partial p^s}\cdot\frac{1}{m} + D\cdot\frac{\partial^2 D/\partial p^s\partial m}{(\partial D/\partial p^s)^2}\cdot\frac{1}{m} + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \tag{92}$$

$$= -\frac{\frac{\partial D}{\partial p^s}\cdot\left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)}{\partial D/\partial p^s}\cdot\frac{1}{m} + D\cdot\frac{\frac{\partial^2 D}{\partial p^{s2}}\cdot\left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)}{(\partial D/\partial p^s)^2}\cdot\frac{1}{m} + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \tag{93}$$

$$= -\left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)\cdot\frac{1}{m} + \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\cdot\frac{\left(p^* - p^d\right)}{m} + \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\cdot\frac{\partial p^*}{\partial m} + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \tag{94}$$

$$2\frac{\partial p^*}{\partial m} = -\frac{p^* - p^d}{m} + \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{\partial p^*}{\partial m}\right) + \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{p^* - p^d}{m}\right) + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \tag{95}$$

$$2\frac{\partial p^*}{\partial m} - \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{\partial p^*}{\partial m}\right) = -\frac{p^* - p^d}{m} + \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{p^* - p^d}{m}\right) + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \tag{96}$$

$$\frac{\partial p^*}{\partial m}[2 - \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}] = \left(\frac{p^* - p^d}{m}\right)\left(\frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2} \implies \tag{97}$$

$$\frac{\partial p^*}{\partial m} = \frac{\left(\frac{p^* - p^d}{m}\right)\left(\frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m^2}}{2 - \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}} \tag{98}$$

$$= \frac{\left(p^* - p^d\right)\Lambda + \frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m}}{1 - \Lambda}\cdot\frac{1}{m} \tag{99}$$

However, we can see that this formulation does not get at the primitives of the model –

we can push further. Now plugging in optimal price $p^* = c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}$ gives:

$$\frac{\partial p^*}{\partial m} = \frac{\left(c - \frac{D(p^s)}{\partial D/\partial p^s} - p^d\right)\left(\frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m}}{2 - \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}} \cdot \frac{1}{m} \tag{100}$$

$$\text{Now defining } \Lambda \equiv \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1 \implies \tag{101}$$

$$\frac{\partial p^*}{\partial m} = \frac{\left(c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m} - p^d\right)\Lambda + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m}}{1 - \Lambda} \cdot \frac{1}{m} \tag{102}$$

$$= \frac{\left(c - p^d\right)\Lambda - \left(\frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}\right)\Lambda + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m}}{1 - \Lambda} \cdot \frac{1}{m} \tag{103}$$

$$= \frac{\left(c - p^d\right)\Lambda + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}(1 - \Lambda)}{1 - \Lambda} \cdot \frac{1}{m} \tag{104}$$

$$= \left[\frac{\Lambda}{1 - \Lambda}\left(c - p^d\right) + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}\right] \cdot \frac{1}{m} \tag{105}$$

Now if we call the perfectly competitive price $p^{pc} = c$, then we have that the equation is:

$$\frac{\partial p^*}{\partial m} = \left[\frac{\Lambda}{\Lambda - 1}\left(p^d - p^{pc}\right) + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}\right] \cdot \frac{1}{m} \tag{106}$$

We can verify the above equation for attentional pass-through by comparing it to the direct derivation of $\frac{\partial p^*}{\partial m}$ when $D = \left(mp + (1-m)p^d\right)^{-\psi}$; see Appendix 113.

## A.2 Optimal price with $D^s = \left(mp + (1-m)\, p^d\right)^{-\psi}$

The derivations for a monopolist facing inattention (whose process of formation is unspecificied) will price when facing $D^s = \left(mp + (1-m)\, p^d\right)^{-\psi}$. The profit function is of the form

$\pi = (p-c)\left(mp + (1-m)\,p^d\right)^{-\psi}$, and so the optimal price is given by:

$$\frac{\partial \pi}{\partial p} = \left(mp + (1-m)\,p^d\right)^{-\psi} - m\psi\,(p-c)\left(mp + (1-m)\,p^d\right)^{-\psi-1} = 0 \tag{107}$$
$$= 1 - \frac{m\psi\,(p-c)}{mp + (1-m)\,p^d} = 0 \tag{108}$$
$$= mp + (1-m)\,p^d - m\psi\,(p-c) = 0 \tag{109}$$
$$= mp - m\psi p = -\left(\psi cm + (1-m)\,p^d\right) \implies \tag{110}$$
$$p^* = \frac{\psi cm + (1-m)\,p^d}{m\,(\psi-1)} \tag{111}$$
$$= \frac{\psi c}{\psi - 1} + \frac{(1-m)\,p^d}{m\,(\psi-1)} \tag{112}$$

Next, it is trivial to check the derivative of the optimal price with respect to $m$:

$$\frac{\partial p^*}{\partial m} = \frac{-p^d}{m(\psi-1)} - \frac{(1-m)p^d}{m^2(\psi-1)} < 0 \tag{113}$$

## A.3 Changes in Perceived Surplus with More Rationality

These calculations are done in order to understand the quantity $\frac{\partial \mathrm{PS}}{\partial m}$: do consumers think they are made better off when they are more rational? First, we derive the simple general equation for any demand function, and then we compute some examples.

### A.3.1 General expression

We have the following expression for perceived surplus, which is denoted PS:

$$\mathrm{PS} = \int_{p^*(m)}^{\overline{p}} D(p^s(p))dp \tag{114}$$

Then, using Leibniz's rule for differentiation under the integral sign, we can find the effect of changes in attention on perceived surplus:

$$\frac{\partial \mathrm{PS}}{\partial m} = D(p^s(\overline{p})) \cdot \frac{\partial \overline{p}}{\partial m} - D(p^s(p^*))\frac{\partial p^*}{\partial m} + \int_{p^*}^{\overline{p}} \frac{\partial D(p^s(p,m,p^d))}{\partial m}dp \tag{115}$$

$$= -D(p^s(p^*))\frac{\partial p^*}{\partial m} + \int_{p^*}^{\overline{p}} \frac{\partial D}{\partial p^s}\left(p - p^d\right)dp \tag{116}$$

### A.3.2 Derivation of examples

First, suppose that $D = (p^s)^{-\psi}$. Then, we have that perceived surplus is:

$$\text{PS} = \int_{p^*(m)}^{\overline{p}} (p^s)^{-\psi} \, dp = \left[ \frac{(p^s)^{1-\psi}}{m(1-\psi)} \right]_{p^*}^{\overline{p}} \tag{117}$$

$$= \frac{(m\overline{p} + (1-m)p^d)^{1-\psi} - (mp^* + (1-m)p^d)^{1-\psi}}{m(1-\psi)} \implies \tag{118}$$

$$\frac{\partial \text{PS}}{\partial m} = \frac{(1-\psi)\,(p^s(\overline{p}))^{-\psi}\left(\overline{p} - p^d\right) - (1-\psi)\,(p^s(p^*(m)))^{-\psi}\left(p^* - p^d + \frac{\partial p^*}{\partial m} \cdot m\right)}{m\,(1-\psi)} \tag{119}$$

$$- \frac{\left[(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}\right](1-\psi)}{m^2(1-\psi)^2} \tag{120}$$

$$= (p^s(\overline{p})^{-\psi} \left( \frac{\overline{p} - p^d}{m} \right) - (p^s(p^*))^{-\psi} \left( \frac{p^* - p^d}{m} + \frac{\partial p^*}{\partial m} \right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{121}$$

We want to check that equation 121 is the same as when we just plug in $D = (p^s)^{-\psi}$ to the general expression given by equation 116. Doing so, we get:

$$\frac{\partial \text{PS}}{\partial m} = -(p^s)^{-\psi} \cdot \frac{\partial p^*}{\partial m} - \int_{p^*}^{\overline{p}} \psi \, (p^s(p))^{-\psi-1} \left(p - p^d\right) dp \tag{122}$$

$$= -(p^s)^{-\psi} \cdot \frac{\partial p^*}{\partial m} + \left[ \frac{p - p^d}{m\,(p^s(p))^{\psi}} - \frac{(p^s(p))^{1-\psi}}{m^2(1-\psi)} \right]_{p^*}^{\overline{p}} \tag{123}$$

$$= -(p^s)^{-\psi} \cdot \frac{\partial p^*}{\partial m} + \left( \frac{\overline{p} - p^d}{m} \right) (p^s(\overline{p}))^{-\psi} - \frac{(p^s(\overline{p}))^{1-\psi}}{m^2(1-\psi)} - \left( \frac{p^* - p^d}{m} \right) (p^s(p^*))^{-\psi} + \frac{(p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{124}$$

$$= \left( \frac{\overline{p} - p^d}{m} \right) (p^s(\overline{p}))^{-\psi} - (p^s(p^*))^{-\psi} \left( \frac{p^* - p^d}{m} + \frac{\partial p^*}{\partial m} \right) + \frac{(p^s(p^*))^{1-\psi} - (p^s(\overline{p}))^{1-\psi}}{m^2(1-\psi)} \tag{125}$$

Thus the general expression is correct. We can further simplify terms.

Using the fact that $p^* = \frac{\psi cm + (1-m)p^d}{m(\psi-1)}$ (equation 112) and $\frac{\partial p^*}{\partial m} = \frac{-p^d}{m^2(\psi-1)}$ (equation 113), we have the following:

$$\frac{\partial \mathrm{PS}}{\partial m} = (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p}-p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{\frac{\psi c m+(1-m)p^d}{m(\psi-1)} - p^d}{m} + \frac{-p^d}{m^2(\psi-1)}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{126}$$

$$= (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p}-p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{\psi c m + (1-m)\,p^d - m(\psi-1)p^d - p^d}{m^2\,(\psi-1)}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{127}$$

$$= (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p}-p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{\psi c m + -m\psi p^d}{m^2\,(\psi-1)}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{128}$$

$$= (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p}-p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{\psi(c-p^d)}{m\,(\psi-1)}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{129}$$

The above expression is not easy to sign, and so instead we draw it for a set of reasonable parameter values in the main text.

## A.4 Optimal quality

### A.4.1 Convexity of $1/D(q^s)$ and quasi-concavity of profit

We demonstrate here that, just as for price, the quasi-concavity of the profit function is ensured with the convexity of $1/D(q^s)$; note that this argument is symmetric to that for price given in Anderson et al. (1992) 6.3.1. Given the profit function

$$\pi(q) = (p - bq)\,D(mq + (1-m)q^d) \tag{130}$$

Then we wish to show that convexity of $1/D(q^s)$ is sufficient to ensure quasi-concavity of the profit function; that is, we want to show that any value of $q$ that satisfies the first order condition, we also have $\partial^2\pi/\partial q^2 < 0$. So, we have that at the FOC:

$$\frac{\partial \pi}{\partial q} = -bD(q^s) + (p - bq)\,\frac{\partial D(q^s)}{\partial q^s}\cdot m = 0 \tag{131}$$

And further we have that the second order condition is:

$$\frac{\partial^2 \pi}{\partial q^2} = -b\frac{\partial D(q^s)}{\partial q^s} \cdot m - b\frac{\partial D(q^s)}{\partial q^s} \cdot m + (p - bq)\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m^2 \tag{132}$$

$$= \frac{\partial^2 \pi}{\partial q^2} = -2b\frac{\partial D(q^s)}{\partial q^s} \cdot m + (p - bq)\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m^2 \tag{133}$$

Next, evaluating the second order condition when the first order condition is zero ($p - bq = \frac{bD(q^s)}{\partial D(q^s)/\partial q^s} \cdot \frac{1}{m}$) gives:

$$\frac{\partial^2 \pi}{\partial q^2} = -2b\frac{\partial D(q^s)}{\partial q^s} \cdot m + \frac{bD(q^s)}{\partial D(q^s)/\partial q^s}\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot \frac{m^2}{m} \tag{134}$$

We want to see what condition under which the above expression is negative, so we will write it as less than zero, and then divide both sides by $\frac{\partial D}{\partial q^s} > 0$:

$$\frac{\partial^2 \pi}{\partial q^2} = -2b \cdot m + \frac{bD(q^s)}{(\partial D(q^s)/\partial q^s)^2}\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m < 0 \tag{135}$$

$$= -bm\left[2 - D(q^s) \cdot \frac{\partial^2 D(q^s)/\partial q^{s2}}{(\partial D(q^s)/\partial q^s)^2}\right] < 0 \tag{136}$$

Thus it is clear that the quantity above is negative if the value in the square brackets is positive. This condition is satisfied if $1/D(q^s)$ is convex.

### A.4.2 Attentional pass-through with quality

Recall that optimal quality choice is given by:

$$q^* = \frac{p}{b} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} \tag{137}$$

Then looking at how this changes with respect to $m$ gives:

$$\frac{\partial q^*}{\partial m} = -\frac{\partial D/\partial m}{\partial D/\partial q^s}\cdot\frac{1}{m} + D\cdot\frac{\partial^2 D/\partial q^s \partial m}{(\partial D/\partial q^s)^2}\cdot\frac{1}{m} + \frac{D}{\partial D/\partial q^s}\cdot\frac{1}{m^2} \tag{138}$$

$$= -\frac{\partial D/\partial q^s}{\partial D/\partial q^s}\cdot\frac{q^* - q^d + m\cdot\frac{\partial q^*}{\partial m}}{m} + D\cdot\frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2}\frac{q^* - q^d + m\cdot\frac{\partial q^*}{\partial m}}{m} + \frac{D(q^s)}{\partial D/\partial q^s}\cdot\frac{1}{m^2} \tag{139}$$

$$= \left(\frac{q^* - q^d}{m}\right)\left[D\cdot\frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} - 1\right] + \frac{\partial q^*}{\partial m}\left[D\cdot\frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} - 1\right] + \frac{D(q^s)}{\partial D/\partial q^s}\cdot\frac{1}{m^2} \tag{140}$$

$$\frac{\partial q^*}{\partial m}\left[1 - \Lambda^q\right] = \left(\frac{q^* - q^d}{m}\right)\Lambda^q + \frac{D(q^s)}{\partial D/\partial q^s}\cdot\frac{1}{m^2} \tag{141}$$

$$\frac{\partial q^*}{\partial m} = \left[\frac{\left(q^* - q^d\right)\Lambda^q + \frac{D}{\partial D/\partial q^s}\cdot\frac{1}{m}}{1 - \Lambda^q}\right]\cdot\frac{1}{m} \tag{142}$$

$$= \left[\frac{\left(\frac{p}{b} - \frac{D}{\partial D/\partial q^s}\cdot\frac{1}{m} - q^d\right)\Lambda^q + \frac{D}{\partial D/\partial q^s}\cdot\frac{1}{m}}{1 - \Lambda^q}\right]\cdot\frac{1}{m} \tag{143}$$

$$= \left[\frac{\left(q^{pc} - q^d\right)\Lambda^q + \frac{D}{\partial D/\partial q^s}\left(1 - \Lambda^q\right)\cdot\frac{1}{m}}{1 - \Lambda^q}\right]\cdot\frac{1}{m} \tag{144}$$

$$= \left[\frac{\Lambda^q}{1 - \Lambda^q}\left(q^{pc} - q^d\right) + \frac{D}{\partial D/\partial q^s}\cdot\frac{1}{m}\right]\cdot\frac{1}{m} \tag{145}$$

### A.4.3 Markdown concavity with quality

We can easily see that quality markdown concavity, which is $\frac{\partial \Lambda^q}{\partial q^s} < 0$ and is analagous to price markup concavity is given by:

$$\frac{\partial \Lambda^q}{\partial q} = \frac{\partial D}{\partial q^s}\cdot\frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} + D(q^s)\cdot\frac{\partial^3 D/\partial q^{s3}}{(\partial D/\partial q^s)^2} - 2D(q^s)\cdot\frac{\left(\partial^2 D/\partial q^{s2}\right)^2}{(\partial D/\partial q^s)^3} \tag{146}$$

Further, notice that:

$$\frac{\partial \Lambda^q(q^s)}{\partial m} = \left( m\frac{\partial q^*}{\partial m} + q^* - q^d \right) \frac{\partial \Lambda^q}{\partial q^s} \tag{147}$$

$$= \left( m \cdot \left( \left[ \frac{\Lambda^q}{1-\Lambda^q} \left( q^{pc} - q^d \right) + \frac{D(q^s)}{\partial D/\partial q^s} \frac{1}{m} \right] \cdot \frac{1}{m} \right) + \frac{p}{b} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} - q^d \right) \frac{\partial \Lambda^q}{\partial q^s} \tag{148}$$

$$= \left( \frac{\Lambda^q}{1-\Lambda^q} \left( q^{pc} - q^d \right) + \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} + q^{pc} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} - q^d \right) \frac{\partial \Lambda^q}{\partial q^s} \tag{149}$$

$$= \left( \frac{q^{pc} - q^d}{1-\Lambda^q} \right) \frac{\partial \Lambda^q}{\partial q^s} \tag{150}$$

So, to get $\frac{\partial \rho^q}{\partial m}$, then we need to evaluate the following:

$$\frac{\partial \rho^q(q^s)}{\partial m} = \frac{\partial}{\partial m} \left[ \frac{-q^{pc}}{b\left(1-\Lambda^q\right)} \right] \tag{151}$$

$$= -\frac{q^{pc}}{b} \frac{\partial \Lambda^q(q^s)/\partial m}{\left(1-\Lambda^q\right)^2} \tag{152}$$

$$= -\frac{q^{pc}}{b(1-\Lambda^q)^2} \left( \frac{q^{pc} - q^d}{1-\Lambda^q} \right) \frac{\partial \Lambda^q}{\partial q^s} \tag{153}$$

$$= \frac{q^{pc}\left(q^d - q^{pc}\right)}{b\left(1-\Lambda^q\right)^3} \frac{\partial \Lambda^q}{\partial q^s} \tag{154}$$

## A.5 Shrinkflation

## A.6 Motivating example

Suppose that consumers have demand over quality (i.e. quantity of the good within the package) $q$ and price $p$ of the following form:

$$D(q, p) = \exp(m \cdot f(q) - p) \tag{155}$$

Where $f(q)$ is the consumer's dollar valuation for $q$ quantity in a single package; for simplicity, we assume here that $f(q) = \theta q$ measures quality in dollar value. Suppose that a firm in a monopolistically competitive environment faces said demand curve with a quadratic cost of producing quantity $q$: $C(q) = \alpha \cdot q^2$ for $\alpha > 0$. Then we have that the profit function is:

$$\pi = \left( p - \alpha q^2 \right) \exp(m\theta q - p) \tag{156}$$

Their problem is to optimally choose both price and quality. First, price:

$$\frac{d\pi}{dp} = \exp(m\theta q - p) - \left(p - \alpha q^2\right) \exp(m\theta q - p) = 0 \implies \tag{157}$$

$$p^* = 1 + \alpha q^2 \tag{158}$$

Then for quality:

$$\frac{d\pi}{dq} = -2\alpha q \exp(m\theta q - p) + m\theta \left(p - \alpha q^2\right) \exp(m\theta q - p) = 0 \implies \tag{159}$$

$$m\theta \left(p - \alpha q^2\right) - 2\alpha q = 0 \text{ at } p = p^* \implies \tag{160}$$

$$m\theta \left(1 + \alpha q^2 - \alpha q^2\right) - 2\alpha q = 0 \implies \tag{161}$$

$$q^* = \frac{m\theta}{2\alpha} \tag{162}$$

Now plugging $q^*$ into the expression for $p^*$ gives:

$$p^* = 1 + \alpha(q^*)^2 = 1 + \alpha \left(\frac{m\theta}{2\alpha}\right)^2 = 1 + \left(\frac{m\theta}{2\sqrt{\alpha}}\right)^2 \tag{163}$$

So, we can look at two notions of shrinkflation: the ratio of levels of price and quality, or the ratio of pass-through of cost to price and quality.

First looking

Defining $\rho_p = dp^*/d\alpha$ and $\rho_q = dq^*/d\alpha$, we see that:

$$\rho_p = -\left(\frac{m\theta}{2}\right)^2 \frac{1}{\alpha^2} < 0 \tag{164}$$

$$\rho_q = -\frac{m\theta}{2\alpha^2} < 0 \tag{165}$$

This is unsurprising; with higher costs, price goes up and quality goes down. The question is, how does the relative pass-through change over values of $m$? Call $\bar{\rho} \equiv \rho_q/\rho_p > 0$, and then we see that:

$$\frac{d\bar{\rho}}{dm} = \frac{d}{dm}\left[\frac{m\theta}{2\alpha^2} \cdot \frac{4\alpha^2}{(m\theta)^2}\right] = \frac{d}{dm}\left[\frac{2}{m\theta}\right] = -\frac{2}{m^2\theta} < 0 \tag{166}$$

So, as there is higher attention to quality, there is less pass-through to quality (relative to price) when costs are higher.

## A.7 Motivating example: linear demand

Suppose that the monopolist chooses both $p$ and $q$ and faces a linear demand curve and quadratic cost function:

$$\pi(p,q) = \left(p - \alpha q^2\right) \cdot (A - p + \theta q^s) \tag{167}$$

Where $q^s = mq + (1-m)q^d$. Then we have that the optimal price is:

$$\frac{d\pi}{dp} = A - p + \theta q^s - \left(p - \alpha q^2\right) = 0 \implies \tag{168}$$

$$p^* = \frac{A + \theta q^s + \alpha q^2}{2} \tag{169}$$

And we note that $d^2\pi/dp^2 = -2 < 0$. Next, quality:

$$\frac{d\pi}{dq} = -2\alpha q \cdot (A - p + \theta q^s) + \theta m \left(p - \alpha q^2\right) = 0 \implies \tag{170}$$

$$\frac{d\pi}{dq}|_{p=p^*} = -2\alpha q \left(A - \left(\frac{A + \theta q^s + \alpha q^2}{2}\right) + \theta q^s\right) + \theta m \left(\frac{A + \theta q^s + \alpha q^2}{2} - \alpha q^2\right) = 0 \tag{171}$$

$$= -\alpha q \left(2A - A - \theta q^s - \alpha q^2 + 2\theta q^s\right) + \frac{\theta m}{2}\left(A + \theta q^s + \alpha q^2 - 2\alpha q^2\right) = 0 \tag{172}$$

$$= -\alpha q \left(A + \theta q^s - \alpha q^2\right) + \frac{\theta m}{2}\left(A + \theta q^s - \alpha q^2\right) = 0 \implies \tag{173}$$

$$q^* = \frac{m\theta}{2\alpha} \implies \tag{174}$$

$$p^* = \frac{1}{2}\left[A + \theta\left(m\frac{m\theta}{2\alpha} + (1-m)q^d\right) + \alpha\left(\frac{m\theta}{2\alpha}\right)^2\right] \tag{175}$$

$$= \frac{1}{2}\left[A + \theta\left(\frac{m^2\theta}{2\alpha} + (1-m)q^d\right) + \frac{(m\theta)^2}{4\alpha}\right] \tag{176}$$

$$= \frac{1}{2}\left[A + \frac{(m\theta)^2}{2\alpha} + \theta\,(1-m)\,q^d + \frac{(m\theta)^2}{4\alpha}\right] \tag{177}$$

$$= \frac{1}{2}\left[A + \theta\,(1-m)\,q^d + \frac{3\,(m\theta)^2}{4\alpha}\right] \tag{178}$$

This means that the pass-through rates are:

$$\rho_q = \frac{dq^*}{d\alpha} = -\frac{m\theta}{2\alpha^2} \tag{179}$$

$$\rho_p = \frac{dp^*}{d\alpha} = -\frac{3\,(m\theta)^2}{8\alpha^2} \tag{180}$$

$$\overline{\rho} = \frac{m\theta}{2\alpha^2} \cdot \frac{8\alpha^2}{3m\theta} = \frac{4}{3m\theta} \tag{181}$$

Note that when solving for the optimal quality, we assumed that $A + \theta q^s - \alpha q^2 \neq 0$. However, that could be true. The optimal $q$ from that case is:

$$A + \theta q^s - \alpha q^2 = 0 \tag{182}$$

$$A + \theta m q + \theta\,(1-m)\,q^d - \alpha q^2 = 0 \implies \tag{183}$$

$$q^* = \frac{\theta m \pm \sqrt{(\theta m)^2 + 4\alpha \cdot \left(A + \theta\,(1-m)\,q^d\right)}}{2\alpha} \tag{184}$$

The 'minus' solution is not possible because then $q^* < 0$ because $\theta m - \sqrt{(\theta m)^2 + x}$ for $x > 0$ means $q^* < 0$. This leaves the following solution:

$$q^* = \frac{1}{2\alpha}\left(\theta m + \sqrt{(\theta m)^2 + 4\alpha \cdot \left(A + \theta\,(1-m)\,q^d\right)}\right) \tag{185}$$

### A.7.1 Optimal price and quality

We assume that costs $c(q)$ are quadratic, i.e. $c(q) = \alpha q^2$ and valuation for quality $f(q^s) = \theta q^s$ is linear. The demand function is of the form $D(u)$ where $u = f(q^s) - p$; the monopolist's problem is:

$$\max_{p,q} \left(p - \alpha q^2\right) \cdot D(\theta q^s - p) \tag{186}$$

Where $q^s = mq + (1-m)q^d$. Then we have that the optimal choice of price is:

$$\frac{d\pi}{dp} = D(\cdot) - \left(p - \alpha q^2\right) \cdot D'(\cdot) = 0 \implies \tag{187}$$

$$p^* = \alpha q^2 + \frac{D(\cdot)}{D'(\cdot)} \tag{188}$$

And we note that

$$\frac{d^2\pi}{dp^2} = -2D'(\cdot) + \left(p - \alpha q^2\right) D''(\cdot) \tag{189}$$

$$\frac{d^2\pi}{dp^2}|_{p^*} = -2D'(\cdot) + \left(\frac{D(\cdot)}{D'(\cdot)}\right) D''(\cdot) \tag{190}$$

$$= D'(\cdot)\left[\frac{D(\cdot)D''(\cdot)}{(D'(\cdot))^2} - 2\right] \implies \tag{191}$$

$$\frac{d^2\pi}{dp^2}|_{p^*} < 0 \iff \frac{D(\cdot)D''(\cdot)}{(D'(\cdot))^2} - 2 < 0 \iff \tag{192}$$

$$\Lambda < 1 \tag{193}$$

Where $\Lambda = \frac{D(\cdot)D''(\cdot)}{(D'(\cdot)^2}$ is the log-curvature of the demand function w.r.t. $u$. Next, we characterize the optimal choice of quality:

$$\frac{d\pi}{dq} = -2\alpha q \cdot D(\cdot) + \theta m \left(p - \alpha q^2\right) \cdot D'(\cdot) = 0 \tag{194}$$

$$\frac{d\pi}{dq}|_{p=p^*} = -2\alpha q \cdot D(\cdot) + \theta m \left(\frac{D(\cdot)}{D'(\cdot)}\right) \cdot D'(\cdot) = 0 \implies \tag{195}$$

$$D(q^*)\left(\theta m - 2\alpha q^*\right) = 0 \implies \text{(if demand positive)} \tag{196}$$

$$q^* = \frac{\theta m}{2\alpha} \tag{197}$$

This then means that the optimal price is given by:

$$p^* = \alpha\left(\frac{\theta m}{2\alpha}\right)^2 + \frac{D(\cdot)}{D'(\cdot)} \tag{198}$$

$$= \frac{(\theta m)^2}{4\alpha} + \frac{D(\cdot)}{D'(\cdot)} \tag{199}$$

This means that optimal profit is then:

$$\pi^* = \pi(p^*, q^*) = \left(p^* - \alpha\left(q^*\right)^2\right) \cdot D(\cdot) \tag{200}$$

$$= \left(\frac{(\theta m)^2}{4\alpha} + \frac{D(\cdot)}{D'(\cdot)} - \alpha\left(\frac{\theta m}{2\alpha}\right)^2\right) \cdot D(\cdot) \tag{201}$$

$$= \frac{(D(\cdot))^2}{D'(\cdot)} \tag{202}$$

Where we know that only the + solution is possible because otherwise $q < 0$. Further, we note that:

$$\frac{d^2\pi}{dq^2} = -2\alpha D(\cdot) - 2\alpha q\theta m D'(\cdot) - 2\alpha\theta m q \cdot D'(\cdot) + (\theta m)^2 \left(p - \alpha q^2\right) \cdot D''(\cdot) \tag{203}$$

$$\text{At optimum, } p - \alpha q^2 = \frac{D(\cdot)}{D'(\cdot)} \text{ and } q = \frac{m\theta}{2\alpha} \implies \tag{204}$$

$$\frac{d^2\pi}{dq^2}|_{q=q^*} = -2\alpha D(\cdot) - 2\alpha q\theta m D'(\cdot) - 2\alpha\theta m q \cdot D'(\cdot) + (\theta m)^2 \left(\frac{D(\cdot)}{D'(\cdot)}\right) \cdot D''(\cdot) \tag{205}$$

$$= -2\alpha D(\cdot) - 4\alpha\theta m q D'(\cdot) + \theta m \frac{D(\cdot)D''(\cdot)}{D'(\cdot)} 2\alpha q \tag{206}$$

$$= -2\alpha D(\cdot) + 2D'(\cdot)\alpha\theta m q \left[\frac{D(\cdot)D''(\cdot)}{(D'(\cdot))^2} - 2\right] \tag{207}$$

$$= -2\alpha D(\cdot) + 2D'(\cdot)\alpha\theta m q \left[\Lambda - 1\right] \tag{208}$$

$$= -2\alpha D(\cdot) + D'(\cdot)\left(\theta m\right)^2 \left[\Lambda - 1\right] \tag{209}$$

Finally, we note the cross-partial:

$$\frac{d^2\pi}{dqdp} = \frac{d}{dp}\left[-2\alpha q \cdot D(\cdot) + \theta m \left(p - \alpha q^2\right) \cdot D'(\cdot)\right] \tag{210}$$

$$= 2\alpha q \cdot D'(\cdot) + \theta m \cdot D'(\cdot) - \theta m \left(p - \alpha q^2\right) \cdot D''(\cdot) \tag{211}$$

$$\text{At optimum, } p - \alpha q^2 = \frac{D(\cdot)}{D'(\cdot)} \text{ and } q = \frac{m\theta}{2\alpha} \implies \tag{212}$$

$$\frac{d^2\pi}{dqdp}|_{p^*,q^*} = 2\alpha q D'(\cdot) + \theta m \left[D'(\cdot) - \frac{D(\cdot)}{D'(\cdot)} \cdot D''(\cdot)\right] \tag{213}$$

$$= \theta m \cdot D'(\cdot) + \theta m \cdot D'(\cdot) \cdot \left[1 - \frac{D(\cdot)D''(\cdot)}{(D'(\cdot))^2}\right] \tag{214}$$

$$= D'(\cdot)\theta m \cdot \left[1 - \Lambda\right] \tag{215}$$

We already have shown that $d^2\pi/dp^2 < 0$, and so now what remains is to show that $\det(H) > 0$, i.e.:

$$\frac{d^2\pi}{dp^2} \cdot \frac{d^2\pi}{dq^2} - \left(\frac{d^2\pi}{dpdq}\right)^2 > 0 \tag{216}$$

$$D'(\cdot)\,(\Lambda-1)\cdot\left(-2\alpha D(\cdot) + D'(\cdot)\,(\theta m)^2\,[\Lambda-1]\right) > \left(D'(\cdot)\theta m\,(1-\Lambda)\right)^2 \tag{217}$$

$$\left(D'(\cdot)\right)^2(\theta m)^2\,(\Lambda-1)^2 - D'(\cdot)\,(\Lambda-1)\cdot 2\alpha D(\cdot) > \left(D'(\cdot)\theta m\,(1-\Lambda)\right)^2 \tag{218}$$

$$\underbrace{-D'(\cdot)}_{<0}\,\underbrace{(\Lambda-1)}_{<0}\cdot\underbrace{2\alpha D(\cdot)}_{>0} > 0 \tag{219}$$

Thus we have demonstrated that these are the optimal choices of $p$ and $q$.

### A.7.2 Pass-through rates

We next want to find the pass-through rates, i.e. $dp^*/d\alpha$ and $dq^*/d\alpha$. Denote the FOCs as:

$$G_1 \equiv \frac{d\pi}{dp} = D(\cdot) - \left(p - \alpha q^2\right)\cdot D'(\cdot) = 0 \tag{220}$$

$$G_2 \equiv \frac{d\pi}{dq} = -2\alpha q \cdot D(\cdot) + \theta m\left(p - \alpha q^2\right)\cdot D'(\cdot) = 0 \tag{221}$$

The Implicit Function Theorem gives that:

$$\begin{pmatrix} \frac{dp^*}{d\alpha} \\ \frac{dq^*}{d\alpha} \end{pmatrix} = -J^{-1}\begin{pmatrix} \frac{\partial G_1}{\partial\alpha} \\ \frac{\partial G_2}{\partial\alpha} \end{pmatrix} \tag{222}$$

Where the Jacobian is given by:

$$J = \begin{pmatrix} \frac{\partial G_1}{\partial p} & \frac{\partial G_1}{\partial q} \\ \frac{\partial G_2}{\partial p} & \frac{\partial G_2}{\partial q} \end{pmatrix} \tag{223}$$

This is the same matrix as the Hessian, and so we have that $J$ is:

$$J = \begin{pmatrix} D'(\cdot)\,(\Lambda-1) & D'(\cdot)\theta m\,(1-\Lambda) \\ D'(\cdot)\theta m\,(1-\Lambda) & -2\alpha D(\cdot) + D'(\cdot)\,(\theta m)^2\,(\Lambda-1) \end{pmatrix} \implies \tag{224}$$

$$J^{-1} = \frac{1}{\det(J)}\begin{pmatrix} -2\alpha D(\cdot) + D'(\cdot)\,(\theta m)^2\,(\Lambda-1) & D'(\cdot)\theta m\,(\Lambda-1) \\ D'(\cdot)\theta m\,(\Lambda-1) & D'(\cdot)\,(\Lambda-1) \end{pmatrix} \tag{225}$$

Where we know that $\det J = D'(\cdot)\,(1-\Lambda)\,2\alpha D(\cdot) > 0$. Next, we see that

$$\frac{\partial G_1}{\partial \alpha} = q^2 \cdot D'(\cdot) > 0 \tag{226}$$

$$\frac{\partial G_2}{\partial \alpha} = -2qD(\cdot) - \theta m q^2 \cdot D'(\cdot) < 0 \tag{227}$$

Then we have:

$$\begin{pmatrix} \frac{dp^*}{d\alpha} \\ \frac{dq^*}{d\alpha} \end{pmatrix} = -J^{-1} \begin{pmatrix} \frac{\partial G_1}{\partial \alpha} \\ \frac{\partial G_2}{\partial \alpha} \end{pmatrix} \tag{228}$$

We start with $\rho_p = dp^*/d\alpha$:

$$\frac{dp^*}{d\alpha} = -\frac{1}{\det(J)}[\Big(-2\alpha D(\cdot) + D'(\cdot)\,(\theta m)^2\,(\Lambda-1)\Big)\cdot\Big(q^2 \cdot D'(\cdot)\Big) + \tag{229}$$

$$\big(D'(\cdot)\theta m\,(\Lambda-1)\big)\cdot\Big(-2qD(\cdot) - \theta m q^2 \cdot D'(\cdot)\Big)] \tag{230}$$

$$= -\frac{1}{\det(J)}\Big[-2\alpha D(\cdot) q^2 \cdot D'(\cdot) - 2qD(\cdot)D'(\cdot)\theta m\,(\Lambda-1)\Big] \tag{231}$$

$$= \frac{2D(\cdot)D'(\cdot)\,\big[\alpha q^2 + q\theta m\,(\Lambda-1)\big]}{D'(\cdot)\,(1-\Lambda)\,2\alpha D(\cdot)} \text{ at optimum } q = \frac{\theta m}{2\alpha} \implies \tag{232}$$

$$= \frac{\left[\frac{(\theta m)^2}{4\alpha} + \frac{(\theta m)^2}{2\alpha}\,(\Lambda-1)\right]}{\alpha\,(1-\Lambda)} \tag{233}$$

$$= \frac{(\theta m)^2}{4\alpha^2\,(1-\Lambda)}\,[2\Lambda-1] \tag{234}$$

Now for $\rho_q = dq^*/d\alpha$:

$$\frac{dq^*}{d\alpha} = -\frac{1}{\det(J)}\Big[\big(D'(\cdot)\theta m\,(\Lambda-1)\big)\Big(q^2\cdot D'(\cdot)\Big) + \big(D'(\cdot)\,(\Lambda-1)\big)\cdot\Big(-2qD(\cdot) - \theta m q^2 \cdot D'(\cdot)\Big)\Big] \tag{235}$$

$$= \frac{2qD(\cdot)D'(\cdot)\,(\Lambda-1)}{\det(J)} = \frac{2qD(\cdot)D'(\cdot)\,(\Lambda-1)}{D'(\cdot)\,(1-\Lambda)\,2\alpha D(\cdot)} = -\frac{q}{\alpha} = -\frac{\theta m}{2\alpha^2} \tag{236}$$

## A.8 General formula for $d\overline{\rho}/dm$

Given that we found the following general expressions for pass-through are:

$$\rho_q = -\frac{\theta m}{2\alpha^2} \tag{237}$$

$$\rho_p = \left(\frac{\theta m}{2\alpha}\right)^2 \frac{2\Lambda-1}{1-\Lambda} \tag{238}$$

We see that $\rho_q < 0$ always, whereas $\rho_p < 0 \iff \Lambda < 1/2$. First, we note that

$$\frac{d\rho_q}{dm} = -\frac{\theta}{2\alpha^2} < 0 \tag{239}$$

$$\frac{d\rho_p}{dm} = \frac{m\theta^2}{2\alpha^2}\frac{2\Lambda - 1}{1 - \Lambda} \tag{240}$$

And so the $\overline{\rho} \equiv \rho_q/\rho_p$ is:

$$\overline{\rho} = -\frac{\theta m}{2\alpha^2}\left(\frac{4\alpha^2(1-\Lambda)}{(\theta m)^2(2\Lambda - 1)}\right) \tag{241}$$

$$= \frac{2(\Lambda - 1)}{\theta m(2\Lambda - 1)} \tag{242}$$

And for constant $\Lambda$, we have that:

$$\frac{d\overline{\rho}}{dm} = \frac{2(1-\Lambda)}{\theta m^2(2\Lambda - 1)} \tag{243}$$

There are two cases. The first case is that $\Lambda < 1/2$, and so $\rho_q,\ \rho_p < 0$, which means that $\overline{\rho} > 0$. Then, $d\overline{\rho}/dm < 0$ means that the rate of pass-through to quality is getting smaller relative to the pass-through to price. The second case is that $\Lambda > 1/2$, in which case $\rho_q < 0$ and $\rho_p > 0$, meaning that $\overline{\rho} < 0$. Thus $d\overline{\rho}/dm > 0$, i.e. $\overline{\rho}$ is getting closer to zero.

# B Random Utility Models and Perloff-Salop Markups

In the main text, we refer to Perloff-Salop markups as simply markups, but in reference to Perloff and Salop (1985), Gabaix et al. (2016) refer to them using the authors' names.

## B.1 Review of Random Utility Models and Perloff-Salop Markups

The demand function of firm $i$ is the probability that the consumer's surplus at firm $i$, $X_i - p_i$, exceeds the consumer's surplus at all other firms,

$$D(p_1, \ldots, p_n; i) = \mathbb{P}\left(X_i - p_i \geq \max_{j \neq i}\{X_j - p_j\}\right)$$

Using $D(p_i, p; n)$ to denote the demand for good $i$ at price $p_i$ when all other firms set price $p$ and using $D_1(p_i, p; n)$ to denote $\partial D(p_i, p; n) / \partial p_i$, we may calculate

$$D(p_i, p; n) = \int_{w_l}^{w_u} f(x) F^{n-1}(x - p_i + p)\, dx$$
$$D_1(p_i, p; n) = -(n-1) \int_{w_l}^{w_u} f(x) f(x - p_i + p) F^{n-2}(x - p_i + p)\, dx.$$

This then means that for any vector of prices $\overline{p}$, we have the following markups for firm $i$:

$$p_i^* - c = -\frac{\int_{w_l}^{w_u} f(x) F^{n-1}(x - p_i + p)\, dx}{(n-1) \int_{w_l}^{w_u} f(x) f(x - p_i + p) F^{n-2}(x - p_i + p)\, dx} \tag{244}$$

For tractability, we consider only symmetric equilibria; that is, equilibria where $p_i = p$. Then, expressions are simplified:

$$D(p, p; n) = \int_{w_l}^{w_u} f(x) F^{n-1}(x) dx = 1/n,$$
$$D_1(p, p; n) = -(n-1) \int_{w_l}^{w_u} f^2(x) F^{n-2}(x) dx.$$

It follows that the Perloff-Salop markup $\mu_n^{PS}$ is

$$p - c = -\frac{D(p, p; n)}{D_1(p, p; n)} = \frac{1}{n(n-1) \int_{w_l}^{w_u} f^2(x) F^{n-2}(x) dx}.$$

## B.2 Proofs Involving Perloff-Salop Markups

### B.2.1 Behavioral markups: $\mu_s^{PS}$

The formulation of the behavioral markups is simplified because $m$ does not enter into the integral because all prices are the same in the symmetric equilibrium. And because of the linearity of integrals, the approximations of markups in Gabaix et al. (2016) (which are approximations of the integral at the bottom of the equation 27 without the $m$ term), the constant $m$ does not enter the approximation of asymptotic behavior.

### B.2.2 Markup elasticity of number of firms

We seek to calculate the elasticity of $\mu^b$ to $n$. So, note the following:

$$\frac{n}{\mu_n^b} = n \cdot m \cdot n(n-1) \int_{w_l}^{w_u} f(x)^2 f(x)\, F^{n-2}(x)\, dx \tag{245}$$

$$\frac{d\mu^b}{dn} = \frac{d}{dn}\left[\left(m \cdot n \cdot (n-1) \int f^2(x) F^{n-2}(x) dx\right)^{-1}\right] \tag{246}$$

$$= -\left[\frac{m \cdot (2n-1) \int f^2(x) F^{n-2}(x) dx + m \cdot n(n-1) \int f^2(x) F^{n-2}(x) \log F(x) dx}{\left(m \cdot n \cdot (n-1) \int f^2(x) F^{n-2}(x) dx\right)^2}\right] \implies \tag{247}$$

$$\frac{n}{\mu_n^b}\frac{d\mu_n^b}{dn} = -\frac{1}{m}\left[\frac{2n-1}{n-1} + \frac{n \int f^2(x) F^{n-2}(x) \log F(x) dx}{\int f^2(x) F^{n-2}(x) dx}\right] = \frac{1}{m} \cdot \frac{n}{\mu_n^r}\frac{d\mu_n^r}{dn} \tag{248}$$

The result then follows from the invocation of the invocation of Theorem 1,3, and Lemma A1.3 as in the Appendix of Gabaix et al. (2016).

# C Cognitive Cost Pass-Through

## C.1 Monopoly pricing with sparse max consumer

Attention is given by:

$$m^*(p; \kappa, p^d) = \max\left\{0, 1 - \frac{\kappa}{|p - p^d|\, A\alpha}\right\} \tag{249}$$

This formulation of $m^*$ means that there exists a region of prices around the default $p^d$ where attention is zero. This region is where $|p - p^d| \leq \kappa / A\alpha$, i.e. the region where $p \in \left[p^d - \kappa/A\alpha, p^d + \kappa/A\alpha\right]$. In this region, $p^s = p^d$, and so the profit function is:

$$\pi_{m=0} = (p - c)\left(A - \alpha p^d\right) \tag{250}$$

In the region where $m = 0$, demand is unresponsive to changes in price, but revenue grows linearly in price. As a result, the optimum is given by the corner solution at $p^d + \kappa/A\alpha$. In other words, the firm prices at the highest price they can where the consumer will not notice, since for any price $p' < p^d + \kappa/A\alpha$, the firm could price higher without losses in demand, meaning they could make strictly higher profit. We denote the optimal price in the unresponsive region as:

$$p_{\sim}^* = p^d + \frac{\kappa}{A\alpha} \tag{251}$$

However, the firm may also want to price in a region where $m > 0$. Given that the profit function where $m > 0$ is given by:

$$\pi = (p-c)\left(A - \alpha p^s\right) = (p-c)\left(A - \alpha\left[\left(1 - \frac{\kappa}{\left|p - p^d\right| A\alpha}\right) \cdot \left(p - p^d\right) + p^d\right]\right) \tag{252}$$

We see that the profit function in the $m > 0$ region is strictly concave, meaning that there is at most one interior solution. The solution could either be in the region $p < p^d + \kappa / A\alpha = p^*_\sim$ or it could be in the region $p > p^*_\sim$. We can check the optimal prices in each region, and then we can show the conditions under which those prices are indeed the solutions to the first order condition, as a function of $\kappa$.

**Case 1:** $p > p^*_\sim$ Suppose that we search for the optimal price in the region $p > p^*_\sim$; this means that demand is:

$$D(p^s)|_{p > p^*_\sim} = A - \alpha\left[\left(1 - \frac{\kappa}{\left(p - p^d\right) A\alpha}\right) \cdot \left(p - p^d\right) + p^d\right] \tag{253}$$

$$= A - \alpha\left[p - \frac{\kappa}{A\alpha}\right] \tag{254}$$

Then the optimal price is given by:

$$\frac{d\pi}{dp} = \frac{d}{dp}\left[(p-c)\left(A - \alpha p + \frac{\kappa}{A}\right)\right] = 0 \implies \tag{255}$$

$$p^*_> = \frac{A + \alpha c}{2\alpha} + \frac{\kappa}{2A\alpha} \tag{256}$$

Where $p^*_>$ is the optimal price given that the price is higher than the default $p^d$.

**Case 2:** $p < p^*_\sim$ It is easy to see that this is symmetric with Case 1, so $p^*_<$, the optimal price that is below $p^*_\sim$, is:

$$p^*_< = \frac{A + \alpha c}{2\alpha} - \frac{\kappa}{2A\alpha} \tag{257}$$

Now that we have the three candidate prices $p^*_\sim$, $p^*_>$, and $p^*_<$, we need to find out the conditions under which each price will occur. Because the profit function is strictly concave for $m > 0$, we know that only one of $p^*_>$ and $p^*_<$ can satisfy the first order condition. These candidate solutions only occur in their assumed region (either above or below $p^*_\sim$ for the right values of $\kappa$, given fixed values of $A$, $\alpha$, and $c$.

We know that $p^*_> > p^*_\sim$ iff:

$$p^*_> > p^*_\sim \tag{258}$$
$$\frac{A+\alpha c}{2\alpha} + \frac{\kappa}{2A\alpha} > p^d + \frac{\kappa}{A\alpha} \iff \tag{259}$$
$$\frac{A+\alpha c}{2\alpha} - p^d > \frac{\kappa}{2A\alpha} \tag{260}$$

This inequality is easy to interpret. The left side shows the difference between the traditional monopoly price (i.e. monopoly price when $\kappa = 0$ and thus $m = 1\,\forall p$) and the expected price. The right side is the 'behavioral markup', i.e. the markup beyond the traditional monopoly price that stems from positive cognitive costs.

Symmetrically, we know that $p^*_< < p^d - \frac{\kappa}{A\alpha}$ iff:

$$p^*_< < p^d - \frac{\kappa}{A\alpha} \tag{261}$$
$$\frac{A+\alpha c}{2\alpha} - \frac{\kappa}{2A\alpha} < p^d - \frac{\kappa}{A\alpha} \iff \tag{262}$$
$$\frac{\kappa}{2A\alpha} < p^d - \frac{A+\alpha c}{2\alpha} \tag{263}$$

This condition is symmetric to the case for $p^* > p^d + \kappa/A\alpha$. Denoting $\gamma \equiv 2A\alpha \cdot \left[p^d - \frac{A+\alpha c}{2\alpha}\right]$, then we know that:

$$p^*_> > p^d + \frac{\kappa}{A\alpha} \iff \kappa < -\gamma \tag{264}$$
$$p^*_< < p^d - \frac{\kappa}{A\alpha} \iff \kappa < \gamma \tag{265}$$

Since $\kappa > 0$ and $\gamma$ is either negative or positive, then only one of the above conditions can hold. Suppose that $\gamma > 0$; then since $\kappa$ is positive, it is not possible that $\kappa < -\gamma$, and thus it is not possible that $p^*_>$ lies in its assumed region. As a result, it must be true that $p^*_<$ is the interior solution. The case of $\gamma < 0$ holds symmetrically.

It is now clear the conditions under which each price satisfies the first order condition in its region, but it is still not clear the conditions under which that price is actually optimal, i.e. whether or not the prices $p^*_>$ or $p^*_<$ gives higher profit than $p^*_\sim$.

Because of the continuity of the pricing function around $p^d + \kappa/A\alpha$, then we know that there exists some $\kappa$ for which $p^*_> = p^*_\sim$, because of continuity. That value is easy to find, and is given by $\gamma$ (or negative $\gamma$, whatever). The other solution, i.e. the value of $\kappa$ that makes $p^*_\sim$ optimal in comparison to $p^*_<$ is harder to find, because there is no such continuity of the pricing function.

We want to find the solution to $\pi^*_< / \pi^*_\sim = 1$, i.e.:

$$\frac{\pi^*_<}{\pi^*_\sim} = 1 \tag{266}$$

$$\frac{\left(A^2 - A\alpha c - \kappa\right)^2}{4A\left[A\alpha p^d + \kappa - A\alpha c\right] \cdot \left[A - \alpha p^d\right]} = 1 \tag{267}$$

Call the solution(s) to the above equation $\kappa^*_<$, and call $\kappa^*_>$ the solution to $\pi^*_>(\kappa) = \pi^*_\sim(\kappa)$. Then, optimal pricing is given by:

The threshold at which prices switch depends on which interior solution is met. If $\gamma < 0$ and so $p^*_>$ is a local maximum, then define $\zeta = -\gamma$. If $\gamma > 0$ and so $p^*_<$ is the local maximum, then we can implicitly define $\zeta$ as the value of $\kappa$ which satisfies the following equation:

$$\frac{\pi^*_<}{\pi^*_\sim} = \frac{\left(A^2 - A\alpha c - \kappa\right)^2}{4A\left[A\alpha p^d + \kappa - A\alpha c\right] \cdot \left[A - \alpha p^d\right]} = 1 \tag{268}$$

Then the optimal price schedule is given by:

$$\begin{cases} \frac{A+\alpha c}{2\alpha} - \mathrm{sgn}(\gamma) \cdot \frac{\kappa}{2A\alpha} & \text{if } \kappa \leq \zeta \\ p^d + \frac{\kappa}{A\alpha} & \text{if } \kappa > \zeta \end{cases} \tag{269}$$

## C.2 Second Order Conditions

Whereas before we guaranteed quasi-concavity of the profit function by assuming that $\frac{1}{D(p^s)}$ was convex in $p^s$ ($\iff \Lambda < 1$), we now will also have a new second order condition which is needed for the quasi-concavity of profit. Either the function is monotone increasing (in which case it is quasi-concave) or every first order condition is a maximum. This means that we need that:

$$\frac{\partial^2 \pi(p^*)}{\partial p^2} = \frac{\partial D(p^s(p^*))}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s(p^*))}{\partial D(p^s(p^*))/\partial p^s} \cdot \frac{\partial^2 p^s(p^*)/\partial p^2}{\left(\partial p^s(p^*)/\partial p\right)^2}\right] < 0 \tag{270}$$

$$= \frac{\partial D(p^s(p^*))}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s(p^*))}{\partial D(p^s(p^*))/\partial p^s} \cdot \frac{2 \cdot \frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2} \cdot \left(p - p^d\right)}{\left(\partial p^s(p^*)/\partial p\right)^2}\right] < 0 \tag{271}$$

Recall that we want the term in brackets to be positive (since $\frac{\partial D(p^s)}{\partial D/\partial p^s} < 0$ and $\frac{\partial p^s(p^*)}{\partial p^s} > 0$). We can use the old necessary and sufficient condition that $\Lambda < 1$, but we can combine it with a new sufficient (but not necessary) condition, namely that $\frac{\partial^2 p^s(p^*)}{\partial p^2} < 0$ at the optimum. This is not immediately easy to interpret, but adding a stronger set of restrictions makes the situation quite clear. Suppose instead of adopting these sufficient, local conditions, we instead require these restrictions to hold globally. That is, suppose that we require that $p^s$ be strictly increasing and weakly convex for all true prices $p$.

Although the requirement that $\frac{\partial p^s(p^*)}{\partial p} > 0$ is a local and necessary condition, whereas $\frac{\partial^2 p^s(p^*)}{\partial p^2} < 0$ is a local and *sufficient* condition (note that we only need $1 - \Lambda - D(p^s)\frac{\partial^2 p^s}{(\partial p^2/\partial p)^2} < 0$), an intuitive explanation of exactly the formualtion of $p^s$ – and therefore of $m(p, \kappa)$ – that is a *globally* increasing and convex function is easy to interpret. Firstly, consumers should perceive price as increasing when the true price is increasing, but they should be less sensitive to changes at smaller values of price than at larger changes in price. This is somewhat intuitive – their perception of price will not change so much for small prices, but will change a lot for larger values. This coheres with the fact that consumers prefer strictly lower prices, and so will be more sensitive to changes at larger price levels.

## C.3 Deriving $\frac{dp^*}{d\kappa}$

We can also more fully derive the exact expression for cognitive cost pass-through by invoking the Implicit Function Theorem. Thus if we wish to compute $\frac{\partial p^*}{\partial \kappa}$, we can use the implicit function theorem:

$$\frac{\partial p^*}{\partial \kappa} = -\frac{\partial^2 \pi/\partial p \partial \kappa}{\partial^2 \pi/\partial p^2} \tag{272}$$

This requires us to compute both derivatives. Beginning with the denominator:

$$\frac{\partial^2 \pi}{\partial p^2} = \frac{\partial}{\partial p}\left[D(p^s) + (p-c)\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right] \tag{273}$$

$$= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p} + \frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p} + (p-c)\frac{\partial^2 D(p^s)}{\partial p^{s2}} \cdot \left(\frac{\partial p^s}{\partial p}\right)^2 + (p-c)\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial^2 p^s}{\partial p^2} \tag{274}$$

$$= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[2 + (p-c)\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial p^s}{\partial p} + (p-c)\frac{\partial^2 p^s/\partial p^2}{\partial p^s/\partial p}\right] \tag{275}$$

$$\text{At optimum } p^* - c = -D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1} \implies \tag{276}$$

$$\frac{\partial^2 \pi}{\partial p^2} = \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\Bigg[2 - D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1}\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial p^s}{\partial p} - D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1}\frac{\partial^2 p^s/\partial}{\partial p^s/\partial} \tag{277}$$

$$= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[2 - D(p^s)\frac{\partial^2 D(p^s)/\partial p^{s2}}{(\partial D(p^s)/\partial p^s)^2} - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right] \tag{278}$$

$$= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right] \tag{279}$$

Next, we can compute $\frac{\partial^2 \pi}{\partial p \partial \kappa}$:

$$
\begin{aligned}
\frac{\partial^2 \pi}{\partial p \partial \kappa} &= \frac{\partial}{\partial \kappa}\left[D(p^s) + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right] && (280)\\
&= \frac{\partial D(p^s)}{\partial \kappa} + (p-c)\,\frac{\partial^2 D(p^s)}{\partial p^s \partial \kappa}\cdot\frac{\partial p^s}{\partial p} + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa} && (281)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial \kappa} + (p-c)\,\frac{\partial^2 D(p^s)}{\partial p^{s2}}\cdot\frac{\partial p^s}{\partial \kappa}\cdot\frac{\partial p^s}{\partial p} + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa} && (282)\\
&= \frac{\partial D(p^s)}{\partial p^s}\left[\frac{\partial p^s}{\partial \kappa}\left[1 + (p-c)\,\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right] + (p-c)\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa}\right] && (283)\\
&\text{At optimum } p^* - c = -D(p^s)\cdot\left[\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right]^{-1} \implies && (284)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\left[1 - D(p^s)\cdot\frac{\partial^2 D(p^s)/\partial p^{s2}}{(\partial D(p^s)/\partial p^s)^2}\right] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right] && (285)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\,[-\Lambda] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right] && (286)
\end{aligned}
$$

Plugging in these equations for the Implicit Function Theorem gives:

$$
\begin{aligned}
\frac{\partial p^*}{\partial \kappa} &= -\frac{\partial^2 \pi/\partial p \partial \kappa}{\partial^2 \pi/\partial p^2} && (287)\\
&= -\frac{\frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\,[-\Lambda] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right]} && (288)\\
&= \frac{\left[\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right]} && (289)
\end{aligned}
$$

Next, we note the following derivatives:

$$
\begin{aligned}
\frac{\partial p^s}{\partial \kappa} &= \frac{\partial m}{\partial \kappa}\cdot\left(p - p^d\right) && (290)\\
\frac{\partial p^s}{\partial p} &= m(\kappa, p) + \frac{\partial m}{\partial p}\cdot\left(p - p^d\right) && (291)\\
\frac{\partial^2 p^s}{\partial \kappa \partial p} &= \frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(p - p^d\right) + \frac{\partial m}{\partial \kappa} && (292)\\
\frac{\partial^2 p^s}{\partial p^2} &= \frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2}\cdot\left(p - p^d\right) + \frac{\partial m}{\partial p} && (293)\\
&= 2\cdot\frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2}\cdot\left(p - p^d\right) && (294)
\end{aligned}
$$

Plugging these in we get:

$$\frac{\partial p^*}{\partial \kappa} = \frac{\left[\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right]} \tag{295}$$

$$= \frac{\frac{\partial m}{\partial \kappa} \cdot \left(p - p^d\right)\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \left(\frac{\frac{\partial^2 m}{\partial \kappa \partial p} \cdot \left(p - p^d\right) + \frac{\partial m}{\partial \kappa}}{m(\kappa, p) + \frac{\partial m}{\partial p} \cdot \left(p - p^d\right)}\right)}{\left(m(\kappa, p) + \frac{\partial m}{\partial p}\left(p - p^d\right)\right)\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \left(\frac{2 \cdot \frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2} \cdot \left(p - p^d\right)}{\left(m(\kappa, p) + \frac{\partial m}{\partial p} \cdot \left(p - p^d\right)\right)^2}\right)\right]} \tag{296}$$

Given that we know that the denominator of $\frac{\partial p^*}{\partial \kappa}$ is positive, then we can restrict attention to the denominator, noting that:

$$\text{sign}\left(\frac{\partial p^*}{\partial \kappa}\right) = \text{sign}\left(\frac{\partial m}{\partial \kappa} \cdot \left(p - p^d\right)\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \left(\frac{\frac{\partial^2 m}{\partial \kappa \partial p} \cdot \left(p - p^d\right) + \frac{\partial m}{\partial \kappa}}{m(\kappa, p) + \frac{\partial m}{\partial p} \cdot \left(p - p^d\right)}\right)\right) \tag{297}$$

We can break down this equation a bit more. First, let's denote markups as:

$$\mu \equiv -\frac{D(p^s)}{\partial D/\partial p^s} \cdot \left[\frac{\partial p^s}{\partial p}\right]^{-1} \tag{298}$$

Then we can rewrite the numerator of cognitive cost pass-through as:

$$\frac{\partial p^*}{\partial \kappa} = \frac{\partial m}{\partial \kappa} \cdot \left(p^* - p^d\right)\Lambda - \mu\left(\frac{\partial^2 m}{\partial \kappa \partial p} \cdot \left(p^* - p^d\right) + \frac{\partial m}{\partial \kappa}\right) \tag{299}$$

$$= \frac{\partial m}{\partial \kappa} \cdot \left(c + \mu - p^d\right)\Lambda - \mu\left(\frac{\partial^2 m}{\partial \kappa \partial p} \cdot \left(c + \mu - p^d\right) + \frac{\partial m}{\partial \kappa}\right) \tag{300}$$

$$= \frac{\partial m}{\partial \kappa} \cdot \mu\left(\Lambda - 1\right) + \left(c - p^d\right)\left[\Lambda \cdot \frac{\partial m}{\partial \kappa} - \mu\frac{\partial^2 m}{\partial \kappa \partial p}\right] - \mu^2\frac{\partial^2 m}{\partial \kappa \partial p} - \mu\frac{\partial m}{\partial \kappa} \tag{301}$$

Note that the first value is always positive – there is an unambiguous, upward pressure on prices when consumers can see price less (higher $\kappa$). The other terms depend on the functional form of $m(\kappa, p)$ and whether or not expectations are feasible (whether or not $p^d > c$).

## C.4 Example of cognitive cost pass-through

Firstly, we want to compute $\frac{\partial p^s}{\partial \kappa}$, $\Lambda$, $\frac{\partial^2 p^s}{\partial p \partial \kappa}$. Let's begin with the second two, as the first one is visualized in the figure. Recall that we are using the following demand function:

$$D_1 = e^{-\left(m\cdot\left(p-p^d\right)+p^d\right)^{\psi}} \tag{302}$$

And we want to find the value of

$$\Lambda_1 = D_1 \cdot \frac{\partial^2 D_1/\partial p^{s2}}{\left(\partial D_1/\partial p^s\right)^2} - 1 \tag{303}$$

We first note then the partial derivative:

$$\begin{aligned}
\frac{\partial D}{\partial p^s} &= \frac{\partial}{\partial p^s}\left[e^{-(p^s)^{\psi}}\right] && (304)\\
&= -\psi\left(p^s\right)^{\psi-1} e^{-(p^s)^{\psi}} \implies && (305)\\
\frac{\partial^2 D(p^s)}{\partial p^{s2}} &= \frac{\partial}{\partial p^s}\left[-\psi\left(p^s\right)^{\psi-1} e^{-(p^s)^{\psi}}\right] && (306)\\
&= -\psi\left(\psi-1\right)\left(p^s\right)^{\psi-2} e^{-(p^s)^{\psi}} + \left(\psi\left(p^s\right)^{\psi-1}\right)^2 e^{-(p^s)^{\psi}} && (307)\\
&= \psi\left(p^s\right)^{\psi-2} e^{-(p^s)^{\psi}} \cdot \left(\psi\left(\left(p^s\right)^{\psi}-1\right)+1\right) && (308)
\end{aligned}$$

Therefore log-curvature is given by:

$$\begin{aligned}
\Lambda_1 &= e^{-(p^s)^{\psi}} \cdot \frac{\psi\left(p^s\right)^{\psi-2} e^{-(p^s)^{\psi}} \cdot \left(\psi\left(\left(p^s\right)^{\psi}-1\right)+1\right)}{\left(-\psi\left(p^s\right)^{\psi-1} e^{-(p^s)^{\psi}}\right)^2} - 1 && (309)\\
&= \frac{\psi\left(p^s\right)^{\psi-2} e^{-2(p^s)^{\psi}} \cdot \left(\psi\left(\left(p^s\right)^{\psi}-1\right)+1\right)}{\psi^2\left(p^s\right)^{2\psi-2} e^{-2(p^s)^{\psi}}} - 1 && (310)\\
&= \frac{\psi\left(\left(p^s\right)^{\psi}-1\right)+1}{\psi\left(p^s\right)^{\psi}} - 1 && (311)\\
&= \frac{\psi\left(\left(p^s\right)^{\psi}-1\right)+1-\psi\left(p^s\right)^{\psi}}{\psi\left(p^s\right)^{\psi}} && (312)\\
&= \frac{1-\psi}{\psi\left(p^s\right)^{\psi}} < 0\,\forall p^* && (313)
\end{aligned}$$

So we have that this demand function is log-concave for all optimal prices (and for all prices for which demand is positive).

Then, write out $\frac{\partial p^s}{\partial p}$, and then write out $\frac{\partial^2 p^s}{\partial p \partial \kappa}$. Then explain how the main point is that having more cognitive cost is increasing the perception of price as well as the sensitivity to price changes, which forces the monopolist to decrease their price. We'd also like to compute how the perception of price changes with the true price. Given that

$$m_1 = \max\left\{0, 1 - \frac{\kappa}{\left(p - p^d\right)^2}\right\} \tag{314}$$

Then for $m_1$ we have that

$$p_1^s(p, p^d, m_1) = \begin{cases} p^d & \left|p - p^d\right| \leq \sqrt{\kappa} \\ \left(1 - \frac{\kappa}{\left(p-p^d\right)^2}\right)\left(p - p^d\right) + p^d & \text{otherwise} \end{cases} \tag{315}$$

Therefore we see that in the first region, $p^s$ is inelastic to change in price, and so $\frac{\partial p^s}{\partial p} = 0$, whereas in the second region, the change of perception in true price is:

$$\frac{\partial p^s}{\partial p} = \frac{\partial}{\partial p}\left[\left(1 - \frac{\kappa}{\left(p - p^d\right)^2}\right)\left(p - p^d\right) + p^d\right] \tag{316}$$

$$= \frac{2\kappa}{\left(p - p^d\right)^3} \cdot \left(p - p^d\right) + \left(1 - \frac{\kappa}{\left(p - p^d\right)^2}\right) \tag{317}$$

$$= 1 + \frac{\kappa}{\left(p - p^d\right)^2} > 0 \; \forall p \tag{318}$$

Therefore we see that the perception of price is increasing in price always. We can also then immediately see that an increase in cognitive cost amplifies the sensitivity to price changes:

$$\frac{\partial^2 p^s}{\partial p \partial \kappa} = \frac{1}{\left(p - p^d\right)^2} > 0 \tag{319}$$

We can finally see that $\frac{\partial p^s}{\partial \kappa}$ is given by 0 or:

$$\frac{\partial p^s}{\partial \kappa} = \frac{\partial}{\partial \kappa}\left[m(p, \kappa) \cdot \left(p - p^d\right) + p^d\right] \tag{320}$$

$$= \frac{\partial m}{\partial \kappa} \cdot \left(p - p^d\right) \tag{321}$$

$$= -\frac{1}{\left(p - p^d\right)^2} \cdot \left(p - p^d\right) \tag{322}$$

$$= \frac{1}{p^d - p} \tag{323}$$

Which is positive for all $p < p^d$.

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