Inattentive Consumers and Imperfect Competition

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Abstract

Economic intuition suggests that making consumers more rational should make them better off, but is it possible that ignorance is bliss? We construct a framework in which firms set prices while facing consumers who misperceive said prices as a convex combination of the true price and some reference point. The primitives of the model demonstrate how the *inattentive demand function* differs from rational demand and the resultant implications for the level of demand and elasticity of demand. We then show that the relationship between market outcomes like monopoly price or consumer surplus and consumer rationality crucially depends on the curvature of the demand function and on behavioral consumers' reference point. We show that, under plausible conditions, fully rational consumers can face higher monopoly prices than would their behavioral counterparts. Finally, we extend this framework to a setting where firms choose quality rather than price, which then allows for (i) an explanation of why multi-product firms underprovide quality to some goods relative to their flagship product and (ii) a framework to understand 'shrinkflation'. We conclude by explaining how changes in consumer rationality do not necessarily imply a trade-off between producer and consumer welfare, and that changes in attention can benefit both consumers and firms.

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1 Introduction

Economic theory provides a very simple formulation for optimal pricing by firms in the form of the famous inverse elasticity rule, yet the most persistent pattern in pricing is something even simpler: prices ending in .99. Pens are sold for \$4.99 per pack, a bottle of water can be \$1.99, and sometimes a T-shirt can be found for \$19.99. The ubiquity of this pricing pattern suggests that firms choose prices based on the expectation that consumers will perceive a number like \$4.99 as being closer to \$4 than to \$5. They are attempting to exploit what is known in the behavioral economics literature as *left digit bias*, which describes the fact that when reading numbers people place excess weight on leftmost digits in a given number.

Given the ubiquity of this pricing schedule which ostensibly attempts to exploit a behavioral bias, a simple question emerges: would consumers be better off if they were more rational? We answer this question in the case of left digit bias, as well as many more biases: No, not necessarily. Using a canonical framework from Chetty et al. (2009) and Gabaix (2014) that captures behavioral biases like left digit bias, insufficient adjustment from a reference point (like the average or usual price), and more, we demonstrate how everyday intuition fails to capture the complexity of the interaction between behavioral consumers and firms hoping to exploit them.

Demonstrating that more rational consumers can face higher prices requires building a simple framework for demand with behavioral consumers. After constructing and describing the *inattentive demand function*, we then present and explain the counterintuitive result relating consumer rationality and price levels. Next, we turn to understanding how the strength of the competitive forces coming from the entry of firms changes when consumers are partially inattentive to price. We treat this issue in the setting of a symmetric oligopoly with random utility models, and show that behavioral consumers give every firm more market power than they would have in the perfectly rational case, thus dampening the effect of introducing more firms into a market. After this, we define new notions that encode consumer expectations into demand levels and show how making consumers more rational can change not only the monopolist's price, but also change their level of demand given their realization that they misperceived the price.

We then turn to endogenizing the reference price to deal more directly with the case of left digit bias, hidden fees, and more. Further, we demonstrate that even with endogenous models of attention to the true price, the aforementioned relationship between price and consumer rationality is still possible. We then extend the analysis of consumers who misperceive price to instead analyze a market where the monopolist sets a good's vertical quality rather than its price. The observations are essentially analogous to those with price, and so we turn to the final topic: when firms choose both quality and price. Adopting a framework in which the firm sets quality and price when the consumer misperceives quality (e.g. the number of chips in a bag), we theoretically characterize what is colloquially known as 'shrinkflation'. We conclude by rejecting the intuition that changes in consumer attention necessarily benefit only the firm or only the consumer; we demonstrate that both parties can benefit from changes in consumer rationality.

This work sits at the intersection of industrial organization and behavioral economics. This paper takes as a starting point some canonical notions of industrial organization as found in Tirole (1988) and Perloff and Salop (1985) to demonstrate how famous formulae change when consumers are behavioral. Further, we connect insights about the importance of the curvature of demand as found in Bulow and Pfleiderer (1983) and Weyl and Fabinger (2013) to explain how the functional form of demand is a critical ingredient to relating consumer rationality to optimal monopoly prices. Finally, we also rely on theoretical advances in Gabaix et al. (2016) to analyze how competition is dampened by behavioral consumers in the setting of random utility demand oligopoly.

The characterization of consumer rationality comes from canonical expressions for 'anchoring and adjustment' as described in Chetty et al. (2009), Gabaix (2014), and Gabaix (2019). Further, this paper adds to a research agenda that demonstrates how traditional economic theory is augmented when behavioral consumers are introduced; papers like Farhi and Gabaix (2020), Gabaix (2020), and Gabaix (2014) exemplify this literature.

In the last twenty years, an entirely new literature of behavioral industrial organization has emerged, with important theoretical treatments in Gabaix and Laibson (2006) and Spiegler (2011) providing simple and intuitive models of naive consumers. More recently, empirical analyses like Lacetera et al. (2012), List et al. (2023), and structural work in Strulov-Shlain (2022) have focused on the importance of left digit bias in a variety of industries.

Although there are many popular models that include behavioral consumers, such as that found in Gabaix and Laibson (2006), there is not yet a general model which can incorporate a wide range of misperceptions of price into a single, tractable model. This paper seeks to sketch out how fundamental elements of industrial organization – demand elasticity, optimal pricing, competition, etc. – change when consumers are allowed to be imperfectly rational.

The rest of the paper is structured as follows: Section 2 provides definitions for necessary concepts and lemmas that illustrate how inattentive demand differs from rational demand; Section 3 provides the central propositions that describe the relationship between consumer rationality and optimal prices (as well as other market outcomes); Section **??** endogenizes the consumer's reference point and attention to true price; Section 5 extends this framework to a firm choosing quality rather than price; Section **??** models a firm that chooses both quality and price, allowing for a characterization of 'shrinkflation'; Section 7 concludes.

2 Inattentive Demand

Markets operate through the minds of people, and so the extent to which people correctly perceive a given market environment is central to understanding the function of the market itself. Thus, we allow for misperceptions of quantities that are important to economic behavior, like the price of a good or its quality. Further discussion of these issues requires defining a general notion of said behavioral agents' misperceptions of market quantities.

For any variable of interest x – which will be quality or price – we will employ a formulation of imperfect perception from behavioral economics called *anchoring and adjustment*. Starting at some default belief or expectation of the value of x – called the 'reference point' or 'anchor' – people adjust to the true value of x. The extent to which people will shift away from their anchor to the true value can be parametrized by an 'attention parameter', denoted m. We can represent this adjustment as a convex combination of the anchor x^d (d for default) and the true value x. The subjective perception of x, denoted x^s is the result of a convex combination:

$$x^s = mx + (1 - m)\, x^d \tag{1}$$

We assume that $m \in [0, 1]$, where $m = 0$ means no attention to the true value of x, and $m = 1$

means full attention to the true value of x. Thus, note that $x^s = x$ for $m = 1$, and $x^s = x^d$ for $m = 0$.

This representation of x^s is sufficiently general to capture a wide range of misperceptions and economic contexts. Although the attention parameter m can be endogenized – see Gabaix (2014) or Sims (2003) – we mostly abstract away from the process by which attention is formed, and instead focus on the consequences of said partial attention. There are many real-world examples of the phenomena that this paper intends to address; here are a few:

- Consumers underreact to changes in rightmost digits of prices (known as left-digit bias); this leads to .99 cent pricing. Recent work highlights the importance of this; see Strulov-Shlain (2022).

- Consumers face a base fee and an add-on; they may only partially anticipate the add-on fee, such as with banking or concert tickets. See Gabaix and Laibson (2006) or Ellison (2005).

- Consumers underreact to changes in the size of an item in comparison to changes in the price; this allows for firms to change the size of an item rather than raise price in times of rising production costs. This is commonly called 'shrinkflation'. Experimental evidence that consumers do not notice said size changes is provided in Pignatelli and Solano (2020).

- If the acquisition of information is costly, consumers may perceive a price as its average or 'usual' price, implying underreaction to price changes; see Gabaix (2019).

- Consumers may use information about a given brand's flagship good to make an inference about a different good in the product line. For example, prospective master's program enrollees may make (imperfect) inferences about the quality of master's degree program by looking at a university's undergraduate rankings. See Smith-Worthington and Urquiola (2023).

Now turning to the relevant market context, we can suppose that there exists a true price p, and consumers have a default or expected price p^d, which can be microfounded as an average price, the last price they saw, etc. For a given attention parameter $m \in [0, 1]$, we have that the subjective perception of price is $p^s = mp + (1 - m)\, p^d$.

Given this misperception of price, it is necessary to characterize how this misperception coheres with a demand function. For this, we introduce a definition; denote the *rational demand function* as $D^r(p)$ and the *inattentive demand function* as $D^s(p)$. We can relate the two concepts in the following way:[1]

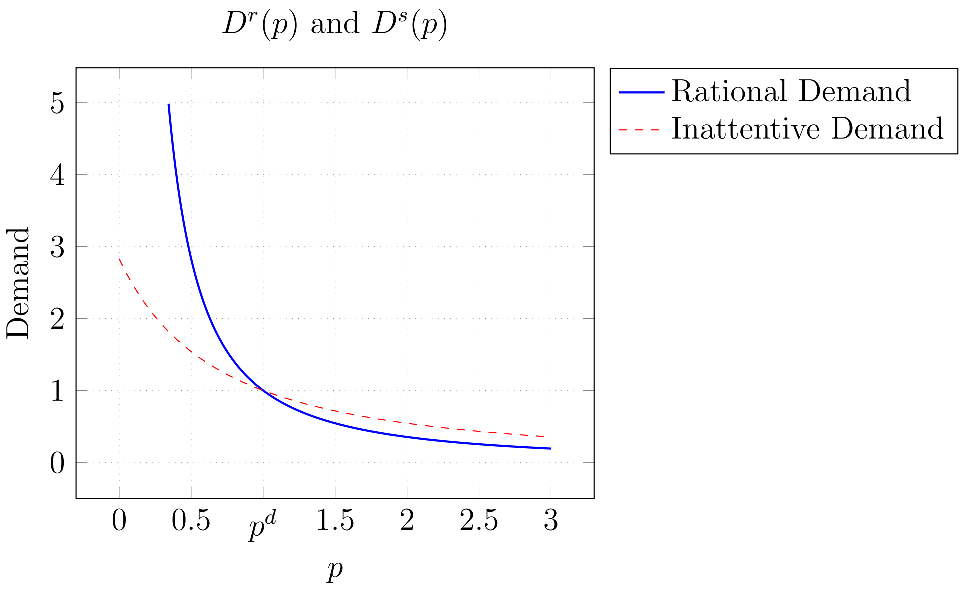
Definition. Inattentive Demand
For a given functional form of demand, $D^r(p)$, the inattentive demand function is the rational demand function evaluated at the subjective perception of price, p^s:

$$D^s(p) = D^r(mp + (1 - m)p^d) \tag{2}$$

For example, if demand is $D^r(p) = \exp(-p)$, then $D^s(p) = \exp\big(-\big(mp + (1 - m)p^d\big)\big)$.

[1]Note that this is similar to the definition of behavioral Marshallian demand as defined in Gabaix (2019), but his application of that definition assumes $p^d = 0$.

Naturally, inattentive demand will be lower than rational demand when $p^s > p$, and it will be higher than when $p^s < p$; this condition reduces to when $p > p^d$.[2] Here, we see that the rational demand is higher below the default price, and inattentive demand is higher above the default price. We can illustrate the inattentive and rational demand functions for $D = p^{-\psi}$:



Parameter values: $m = .5$, $p^d = 1$, $D = p^{-\psi}$, $\psi = 1.5$

As stated, rational demand is lower when the true price is above the default, and rational demand is higher when the true price is below the default. Note that at the default, although the demand functions take on the same value, their slopes differ; thus, it's worth understanding the nature of demand sensitivities too. The derivative of inattentive demand with respect to price is given by the following equation:

$$\frac{\partial D^s}{\partial p} = \frac{\partial D^r}{\partial p^s}\frac{\partial p^s}{\partial p} = \frac{\partial D^r}{\partial p^s} \cdot m \tag{3}$$

Note that if $m = 1$, meaning that there is full attention and therefore $p^s = p$, then we would have right hand side be $\frac{\partial D^r}{\partial p}$; if there is full attention, the behavioral and rational demand functions exhibit identical behavior. However, suppose that $m < 1$; in this case, things are not so simple:

Lemma 1. ***Inattentive Demand Sensitivity to Price***
Inattentive demand sensitivity to price is given by:

$$\frac{\partial D(p^s)}{\partial p}\Big|_{m<1} = \frac{\partial D(mp + (1-m)p^d)}{\partial p^s} \cdot m \tag{4}$$

And it is not globally larger or smaller (in magnitude) than the rational demand sensitivity.

[2]We can easily demonstrate this fact, and will do so for the case where $p^d > p$. Suppose the true price is lower than the default; we wish to show that the rational demand will be higher than the inattentive demand: $p^d > p \implies (1-m)\,p^d > (1-m)\,p \implies mp + (1-m)\,p^d > (1-m)\,p + mp \implies p^s > p \implies D^r(p^s) < D^r(p) \iff D^s(p) < D^r(p)$ Where the last line follows from the fact that D^r is decreasing in its argument.

There are two forces that make the inattentive demand sensitivity distinct from the rational demand sensitivity. Firstly, the argument in D is $p^s = mp + (1-m)p^d \neq p \ \forall p \neq p^d$, so the derivatives are evaluated at different points. Secondly, we see that the inattentive demand function has sensitivity to price equal to the rational demand sensitivity evaluated at $p^s \neq p$ and is dampened by inattention via the term $m < 1$. Note further that we arrive at the representation of behavioral Marshallian demand that is given in Gabaix (2019) if we take the special case where the behavioral consumers' default is the true price ($p = p^d$):

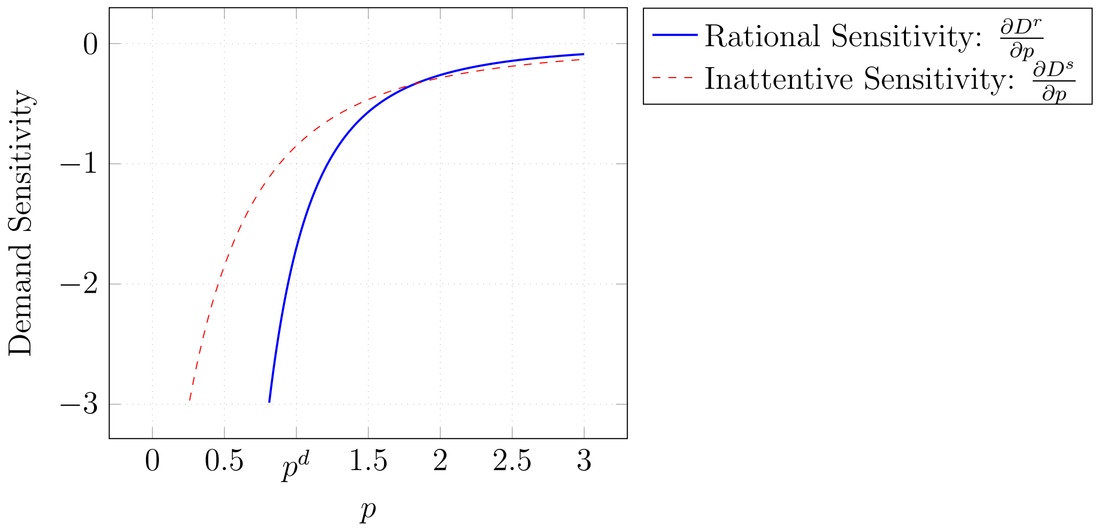
$$\frac{\partial D(p^s)}{\partial p}|_{m<1,p^d=p} = \frac{\partial D(p)}{\partial p} \cdot m \tag{5}$$

All of this can be made more explicit with an example. Suppose that $D^r = p^{-\psi}$ where $\psi > 1$ is price elasticity; then, it follows that

$$\frac{\partial D^r}{\partial p} = -\psi p^{-\psi-1} \tag{6}$$

$$\frac{\partial D^s}{\partial p} = -\psi \left(mp + (1-m)\,p^d\right)^{-\psi-1} \cdot m \tag{7}$$

We can see this represented in the following figure:



It is clear that at $p = p^d$, the behavioral sensitivity is dampened (closer to zero) by a factor of $m = .5$. However, the inattentive demand function having lower sensitivity to changes in the price is not true along the entire domain of prices. It is clear that there is an entire range of prices over which the rational consumers are less sensitive than the behavioral consumers; the point at which the two functions are equal is given by p':

$$p' = \frac{(1-m)\,p^d}{m^{\frac{1}{\psi+1}} - m} \approx 1.83 \tag{8}$$

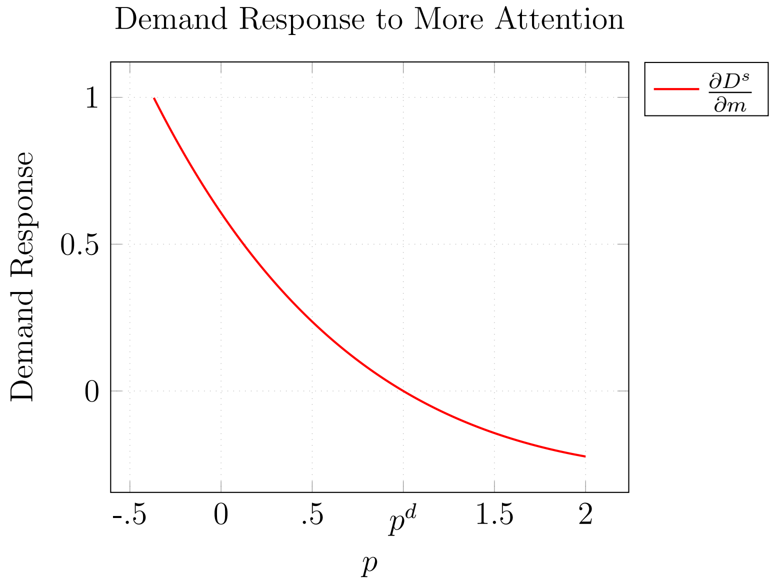
This is a simple example to demonstrate that the inattentive demand function is not simply a less sensitive version of the rational demand function, although it does behave that way around the default p^d. This will have important implications later in the analysis.

So far, we have only discussed the relationship of demand to price and changes in price. However, changes in attention itself can generate changes in demand as well, and we demonstrate that the relationship between demand and attention is highly intuitive:

Lemma 2. ***Inattentive Demand and Changes in Attention***
Holding price fixed, inattentive demand's response to a change in attention is given by the following:

$$\frac{\partial D^s}{\partial m} = \frac{\partial D^r(p^s)}{\partial p^s} \cdot \left(p - p^d\right) \tag{9}$$

Since $\frac{\partial D}{\partial p^s} < 0$, then it is clear that the effect of attention on demand depends entirely on whether or not the true price is above or below the default. This is natural – if consumers' default is that an apple's price is $p^d = 1$, but they begin to notice that actually $p = 1.99$, then this should lead to a drop in demand, since $p^s \to p > p^d$ as $m \to 1$. Using $D^r = e^{-p}$ as an example, we can see how demand changes with attention, noting that the change is zero at the default $p^d = 1$:



So, if the true price is higher than the reference price, we would expect that since higher attention means the perceived price gets larger, there would be decrease in demand. Indeed, this is what we see above: the line is above 0 for $p < p^d$ (more attention means lower perceived price, so higher demand), and the line is below 0 for $p > p^d$ (more attention means higher perceived price, so lower demand).

Now that we have defined the primitives of the inattentive demand function, we can investigation how traditional notions in industrial organization change with behavioral consumers.

3 Optimal Pricing with Consumer Inattention

3.1 Prices and Attention

Suppose a monopolist faces consumers whose perception of price is $p^s = mp + (1 - m)\, p^d$; how does this monopolist price optimally? If the profit function is of the following form:

$$\pi = (p - c)\, D(mp + (1 - m)p^d) \tag{10}$$

where the demand function has the property that $1/D(p^s)$ is convex in p^s,[3] then we have the following lemma:

Lemma 3. ***Behavioral Inverse Elasticity Rule***
For a demand function D that satisfies the property that $1/D(p)$ is convex, then the optimal price is given by the following equation:

$$p^* = c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m} \tag{11}$$

So, holding all else fixed, we arrive at the traditional inverse elasticity rule as $m \to 1$, and markups get larger as $m \to 0$. Importantly, however, changes in attention m enter this equation in multiple ways, and the relationship of attention to optimal prices is not straightforward. We will investigate how attention changes optimal price, called here *attentional pass-through*.

It is clear from Equation 11 that changes in attention will affect markups through three channels: 1. Changes in demand D, 2. Changes in demand sensitivity $\partial D/\partial p^s$, and 3. Direct dampening of the price through $\frac{1}{m}$. Evaluating the derivative and rearranging terms (proof can be found in Appendix Appendix A.1), we come to the following proposition:

Proposition 1. ***More consumer rationality does not always imply lower prices***
Defining $\Lambda \equiv \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1$, where $\Lambda < 0$ means D is log-concave, we can write the expression for attentional pass-through as:

$$\frac{\partial p^*}{\partial m} = \frac{\Lambda}{\Lambda - 1}\left(\frac{p^d - p^{pc}}{m}\right) + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{12}$$

Where p^{pc} is the perfectly competitive price. Thus, there are three forces that determine attentional pass-through:

- *The log-concavity of $D(p^s)$*
- *Whether p^d is above the perfectly competitive price p^{pc}*
- *An unambiguously downward pressure on prices from the dampening of markups*

We can summarize the relation of these three conditions in the following table, which states how log-curvature of demand interacts with the reference price to cause the relevant change in optimal price:

Table 1: Relationship between consumer rationality and price

	$\Lambda < 0$	$0 < \Lambda < 1$
$p^d > p^{pc}$	Ambiguous	$\frac{\partial p^*}{\partial m} < 0$
$p^d < p^{pc}$	$\frac{\partial p^*}{\partial m} < 0$	Ambiguous

[3]Note that this weak condition implies quasi-concavity of the profit function; see Chapter 6 of Anderson et al. (1992)

This result requires some explanation. Note that attentional pass-through is given by:

$$\frac{\partial p^*}{\partial m} = \frac{\partial}{\partial m}\left[-\frac{D(p^s)}{\partial D/\partial p^s}\right] \cdot \frac{1}{m} + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{13}$$

When attention changes, it effects the optimal price p^* in two ways. Firstly, it has a *direct effect* by increasing the size of m which dampens prices by decreasing $1/m$. It also has an indirect effect on markups by changing the size of $-\frac{D(p^s)}{\partial D/\partial p^s}$. This is because increasing attention shifts the demand curve away from the default and towards the true value of price, meaning that the demand function now takes on a different value for every true price p; in addition, increasing m also increases the first derivative of the demand function. So, the question is whether or not the change in m constitutes a change in the demand such that the quantity $-D(p^s)/\partial D/\partial p^s$ increases or decreases.

Thus, one needs to analyze the indirect effect to understand how p^* will change. We see that the log-curvature of the demand function plays a central role:

$$\frac{\partial}{\partial m}\left(-\frac{D(p^s)}{\partial D/\partial p^s}\right) = \frac{\Lambda}{\Lambda - 1} \cdot \left(p^d - c\right) \tag{14}$$

Where D is log-concave $\iff \Lambda < 0$. So, suppose that we take $p^d > c$ and log-concave demand ($\Lambda < 0$); then this means that $\frac{\Lambda}{\Lambda-1} \cdot \left(p^d - c\right) > 0$. That is, changing m is increasing the ratio of demand levels to demand sensitivities. And since the behavioral inverse elasticity rule tells us that

$$p^* \propto -\frac{D(p^s(p^*))}{\partial D(p^s(p^*))/\partial p^s} \tag{15}$$

Then this means that if higher attention implies higher $-\frac{D(p^s)}{\partial D/\partial p^s}$, then price may go up. It is important to note that this is only a necessary, but not a sufficient condition; because the direct impact always exerts a downward pressure on prices, we typically need a somewhat large gap $p^d - c$ for $\frac{\partial p^*}{\partial m} > 0$.

Thus, if consumers' reference p^d is greater than the perfectly competitive price – which is reasonable if consumers form default prices by observation of prices in markets with similar costs, where prices would typically not be lower than p^{pc} – then this proposition shows that consumers with log-concave demand may be made worse off if made more attentive to price. However, there may be many real-world cases in which consumers' reference price could be lower than cost; for example, if consumers take a reference price from a market where firms are subsidized by the government, then they may be seeing much lower prices than would be reasonable for an unsubsidized good.

Another way of understanding the intuition is available if we imagined ourselves wanting to know the answer to a simple question: if consumers are made to be more attentive, will they face a lower price? It turns out that there are two key mechanisms that constitute the answer to this question. Before proceeding, we first define the *markup function* as the following:

$$\mu(p^s(p^*(m))) \equiv -\frac{D(p^s)}{\partial D/\partial p^s} \tag{16}$$

Note that importantly, the subjective perception of price $p^s = mp^* + (1 - m)p^d$ will depend on m directly, but also indirectly via $p^*(m)$. Why does this matter? It matters because the combination of the direct and indirect dependence implies that we cannot actually know immediately whether or not the subjective perception of price is increasing or decreasing with attention, as both attention m itself and $p^*(m)$ will move:

$$\frac{\partial p^s(p^*(m))}{\partial m} = p^* - p^d + m \cdot \frac{\partial p^*}{\partial m} \tag{17}$$

Further, it is not a priori obvious whether or not the markup function μ is increasing or decreasing in p^s. The derivative of μ with respect to the subjective perception of price p^s is given by its log-curvature:

$$\begin{aligned} \frac{\partial \mu(p^s)}{\partial p^s} &= \frac{\partial}{\partial p^s}\left[-\frac{D(p^s)}{\partial D/\partial p^s}\right] && (18)\\ &= -\frac{\partial D/\partial p^s}{\partial D/\partial p^s} + D(p^s) \cdot \frac{\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} && (19)\\ &= \Lambda && (20) \end{aligned}$$

We care about these two mechanisms because they will determine whether or not it is possible that more attention can imply higher price (the counterintuitive result). This becomes clear immediately if we return to how optimal price varies with attention:

$$\begin{aligned} \frac{\partial p^*}{\partial m} &= \frac{\partial}{\partial m}\left[-\frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}\right] && (21)\\ &= \frac{\partial}{\partial m}\left[\frac{\mu}{m}\right] && (22)\\ &= \frac{\partial \mu}{\partial m} \cdot \frac{1}{m} - \frac{\mu}{m^2} && (23)\\ &= \frac{\partial \mu}{\partial p^s}\frac{\partial p^s}{\partial m}\frac{1}{m} - \frac{\mu}{m^2} && (24)\\ &= \frac{\Lambda}{m} \cdot \frac{\partial p^s}{\partial m} - \frac{\mu}{m^2} && (25) \end{aligned}$$

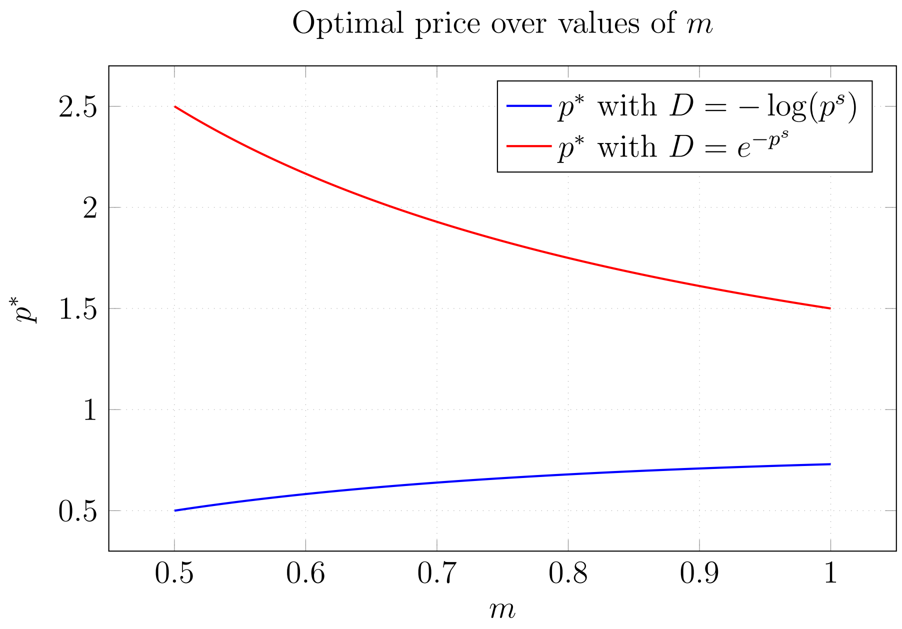
Thus the change in optimal price with a movement in m depends on whether or not $p^s(m, p^*(m))$ is increasing or decreasing, and whether or not higher p^s implies a higher or lower markup. Thus, the heart of the question about optimal price and attention is whether or not consumers' perception of price is getting higher or lower with more attention; in a sense, we want to know if they are getting more pessimistic as they get more attentive.

Next, we can answer the question of whether or not monopoly prices with behavioral agents are higher than the prices with fully rational agents more concretely using the results from Proposition 1 by looking at small deviations in attention away from the fully rational case. That is, we wish to look at decreases in attention when $m = 1$ to see if it is possible that consumers would be better off – that is, if that prices would be lower:

Corollary 1. ***When a little inattention helps consumers***
Slightly inattentive agents would face lower prices than fully rational agents if the following expression is negative:

$$\frac{\partial p^*}{\partial(-m)}|_{m=1} = \frac{\Lambda}{1-\Lambda}\left(p^d - p^c\right) - \frac{D(p)}{\partial D/\partial p} \tag{26}$$

Because the conditions under which $\partial p^*/\partial m > 0$ are not particularly intuitive, it may be helpful to plot two polar examples of pricing behavior over values of m with different demand. We plot a demand function that is not $1/D(p^s)$ convex, namely $D = -\log(p^s)$, along with $D = e^{-p^s}$, which has strictly negative attentional pass-through.[4]



Parameter values: $c = .5$, $p^d = 1.5$ for $D_1 = -\log(p^s)$ and $D_2 = e^{-p^s}$

If we make more assumptions about the value of the default price p^d, we can generate stronger results that eschew restrictions on the functional form of demand. Suppose that consumers overestimate how competitive a market is, and indeed assume that the market is perfectly competitive. Then, since $p^c \equiv c = p^d$, we have the following corollary:

Corollary 2. ***Attentional pass-through with a perfect-competition benchmark*** *If consumers have a default price as $p^d = c \equiv p^{pc}$ where p^{pc} is the perfectly competitive price, then we have the following equation for attentional pass-through:*

$$\frac{\partial p^*}{\partial m}\Big|_{p^d=c} = \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m^2} < 0 \tag{27}$$

Thus, when consumers assume that a market is perfectly competitive when it is not, then improving their perception of prices will unequivocally decrease prices. Since $p^d < p^*$ because of the consumers' reference point is perfect competition, we already know the sign of the first term in equation 12. However, we can eschew any restrictions on the demand curve; in fact, we get the simpler expression where the first two effects to the demand and demand sensitivity are no longer relevant – only the direct dampening remains.

[4]Note that with the log demand function, we do not have a closed form solution for p^*, so solutions are numerically solved.

3.2 Consumer Inattention and Market Competition

Although it is not trivial to determine whether or not markups are higher with inattention because of possibly conflicting effects from the log-curvature of demand and direct dampening of the markup, we can make a conclusive statement about the behavior of markups as the number of firms increases. For tractability, we consider random utility demand in a symmetric oligopoly setting as originally outlined in Perloff and Salop (1985), as the formulation for markups and their asympotitic behavior as given by Gabaix et al. (2016) are particularly straightforward. We give a review of the canonical random utility formulation and the resultant markups from Gabaix et al. (2016) in Appendix Appendix C.1. Because we focus mostly on asymptotics here, we use 'prices' and 'markups' interchangeably.

We can make an easy extension from the results in Gabaix et al. (2016) to demonstrate how behavioral markups are always higher than rational markups in a symmetric equilibrium. We will see that the factors that lead to uncertainty about whether or not the behavioral markup is larger than the rational markup disappears in the symmetric equilibrium, and instead we find that each firm is given unequivocally more market power by the consumer's dampened response to price.

Reformulating the discussion around random utility models with behavioral perceptions of price, we make the simplifying assumption that consumers' reference price is the same for every firm; this means that they take the reference price to be some attribute of the market, and not of any single firm. This is a good approximation when consumers are interested in deviations from a market average or some other market-wide quantity, but not if their default is different for every firm. Given this modification, we can restate the demand function in the following way:

$$D_i\left(p_1, \ldots, p_n; p^d, m\right) = \mathbb{P}\left(X_i - mp_i + (1-m)p^d \geq \max_{j \neq i}\left\{X_j - mp_j + (1-m)p^d\right\}\right) \tag{28}$$

$$= \mathbb{P}\left(X_i + m\left(p_j - p_i\right) \geq \max_{j \neq i}\left\{X_j\right\}\right) \tag{29}$$

Now looking at the case where firm i posts price p_i and all other firms post price p, we get the following equations for demand and demand sensitivity:

$$D\left(p_i, p; n, m, p^d\right) = \int_{w_l}^{w_u} f(x) F^{n-1}\left(x + m\left(p - p_i\right)\right) dx \tag{30}$$

$$\frac{\partial D\left(p_i, p; n, m, p^d\right)}{\partial p_i} = -m(n-1)\int_{w_l}^{w_u} f(x)^2 f\left(x + m\left(p - p_i\right)\right) F^{n-2}\left(x + m\left(p - p_i\right)\right) dx \tag{31}$$

Now restricting attention to a symmetric equilibrium ($p_i = p$) for tractability (as in Gabaix et al. (2016) and Anderson et al. (1992)), the demand expression simply becomes $1/n$. Therefore we can express markups, denoted as μ^b where the b denotes the markups are with the behavioral modification:

$$\mu^b = \frac{1}{m \cdot n(n-1)\int_{w_l}^{w_u} f(x)^2 f\left(x\right) F^{n-2}\left(x\right) dx} = \frac{\mu^r}{m} \tag{32}$$

Therefore markups are stated to be a function of i) the attentional parameter m ii) the distribution of taste shocks $F(x)$ and iii) the number of firms. So, for a fixed distribution of taste shocks, these markups are some function of m and n. Further, the fact that inattention enters the markup only multiplicatively means that we can easily extend this formulation to relate it to the asymptotic behavior of markups with respect to the number of firms:

Proposition 2. ***Inattention limits the reach of competition***
In a symmetric equilibrium with random utility demand, behavioral markups are strictly larger than rational markups, and behavioral asymptotic markups are higher than traditional asymptotic markups. For a given distribution of random taste shocks, if

$$\frac{D^r(p,n)}{\partial D^r/\partial p} = \mu_n^r \implies \frac{D^s(p,n)}{\partial D^s/\partial p} = \frac{\mu_n^r}{m} \tag{33}$$

And if

$$\lim_{n\to\infty} \mu_n^r \equiv \mu_\infty \implies \lim_{n\to\infty} \mu_n^b = \frac{\mu_\infty}{m} \tag{34}$$

Thus, behavioral markups are always higher than rational markups. Further, the markup elasticity of number of firms is γ/m, where γ is the tail index of the distribution of preferences.

The proofs are a basic extension of Gabaix et al. (2016) and can be found in Appendix Appendix C.2.[5] The reason that we can make such a strong statement about the size of behavioral markups relative to the size of fully rational markups is the fact that we are in the context of a symmetric equilibrium with a random utility model, so price levels do not determine the size of the markups – only the distribution of random taste shocks determines markups. Because prices themselves cancel out, all that remains is markups as a function of the number of firms and the factor $\frac{1}{m}$ which unambiguously amplifies prices.

Taking the example where demand is given by the random utility model with Gumbel noise, we get the following rational markups which are then contrasted with demand when price is misperceived:

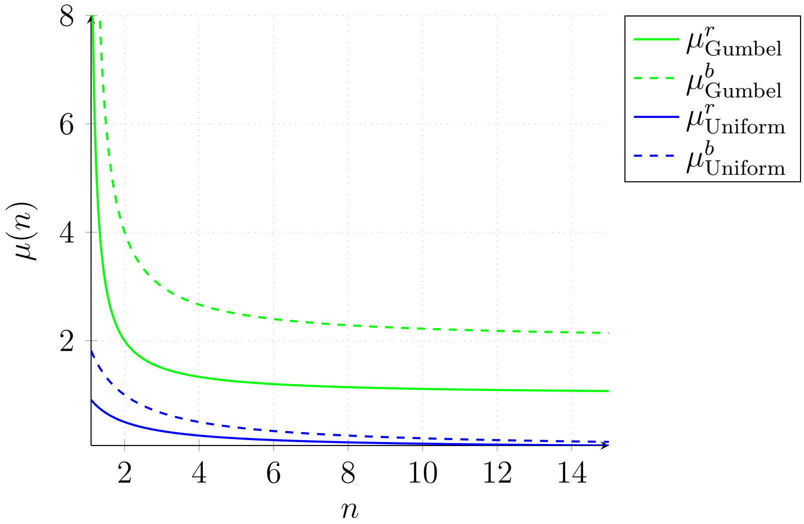
$$\mu^r_{\text{Gumbel}} = \frac{n}{n-1} \to 1 \tag{35}$$

$$\mu^b_{\text{Gumbel}} = \frac{n}{m\,(n-1)} \to \frac{1}{m} \tag{36}$$

If we repeat the above exercise with markups resulting from uniform noise, we can plot the difference between the μ^r and μ^b under each distribution to make the point clear:

[5] Technically, this statement is only true for distributions for which the tail index γ is negative. As noted in Gabaix et al. (2016), distributions with positive tail indices have markups that *grow* with the number of firms in the market. Somewhat like upward sloping demand, we ignore this case.

Rational and Behavioral Markups and the Number of Firms



We have thus far explained how changes in attention affect price and how the presence of consumer inattention dampens the competitive forces stemming from the introduction of firms into a market. But it is also useful to investigate more precisely how we can relate changes in attention to changes in consumer, rather than firm, behavior.

3.3 Consumer Welfare: Perceived Surplus, Consumer Surplus, and their Relationships to Inattention

To better understand the way that consumer welfare and inattention relate to each other, we return to the context of monopoly pricing. Denoting the reservation price as $\overline{p}$, we introduce the following definition:

Definition. Perceived Surplus
The consumers' perceived surplus is the difference between how much they think they will pay and how much they would be willing to pay:

$$\text{PS} \equiv \int_{p^*(m)}^{\overline{p}} D(p^s(p))dp \tag{37}$$

This definition captures what consumers believe their surplus will be, given their misperception of price; it is a similar formulation to consumer surplus, but instead uses the inattentive demand function $D(p^s)$ rather than the rational demand function. The use of this definition is that it encodes two distinct and equally important effects of changes in attention and allows us to state the following:

Proposition 3. ***More rationality does not necessarily imply higher perceived surplus, even if prices decrease***
A change in attention changes the optimal price of the monopolist, but it also generates an

independent change in demand:

$$\frac{\partial PS}{\partial m} = \underbrace{-D(p^s(p^*))\frac{\partial p^*}{\partial m}}_{\textit{Change in price}} + \underbrace{\int\limits_{p^*(m)}^{\overline{p}} \frac{\partial D}{\partial p^s}\left(p - p^d\right) dp}_{\textit{Change in perception}} \tag{38}$$

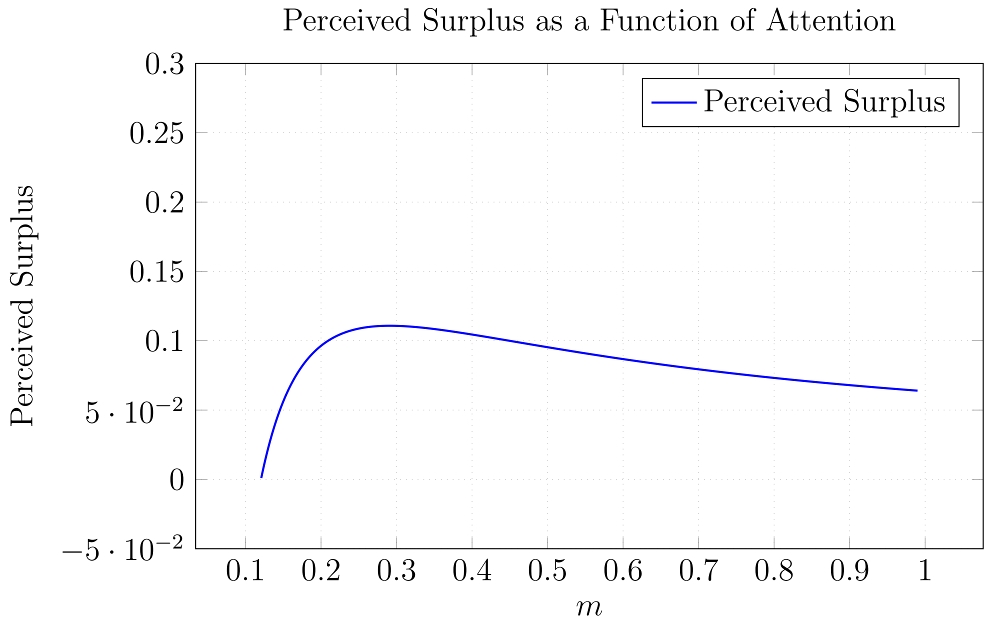
The first term captures the change in perceived surplus that comes from the change in the monopolist's optimal price due to the shift in attention. The second term captures the change in demand that occurs from the consumer's change in perceived price. The change in the optimal price has been discussed at length; the more important contribution of this definition and proposition is that it also contains the change in demand from the consumer described in the rightmost term. Suppose that you are deciding whether or not to buy an item, and your friend points out to you its true price. Depending on if you are disappointed or excited by your realization, you will be more or less likely to buy; this effect is captured by the second term.

A possibly surprising result, is that negative attentional pass-through ($\frac{\partial p^*}{\partial m} < 0$) is not sufficient for perceived surplus to be increasing in consumer attention. This highlights the role that the secondary shift in demand (given by the integral) generated by a change in attention has – even if the firm's optimal price goes down, consumers may react so negatively to the realization that they are paying more than their default that the decrease in demand outweighs the increase in surplus from the downward shift in p^*.

We can demonstrate this phenomenon with an example, using $D = (p^s)^{-\psi}$, which yields the following expression for the relationship between rationality and consumer surplus:

$$\frac{\partial \text{PS}}{\partial m} = (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{\psi(p^{pc} - p^d)}{m\,(\psi - 1)}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{39}$$

The derivation of this expression can be found in Appendix Appendix A.3.2. Recall that with this demand function, prices are strictly decreasing in attention. Nonetheless, putting a sign on this expression is not trivial because of the complexity of the relationship of $\overline{p}$, p^d, and p^{pc}. So, we illustrate perceived surplus below, demonstrating that for a reasonable set of parameters and a wide range of values of m, perceived surplus is actually decreasing in attention:



Parameter values: $\psi = 3$, $\overline{p} = 10$, $c = 1.8$, $p^d = 2$

Although price is decreasing, the demand curve undergoes a downward shift because consumers realize that the price is higher than they had expected. Note that for these parameter values, $p^* \in (2.71, \overline{p}) \implies p^* \geq p^d$, and so consumers are buying less because they are realizing (when they pay more attention) that the real price is actually higher than they had thought. This causes a downward shift in the demand curve. All of this means that even if price decreases, consumers won't necessarily think that they're getting a better deal.

However, even if consumers do not believe they are getting a better deal before buying, they can still experience higher levels of utility after their purchase given the lower price. In other words, there is a difference between perceived utility (before buying) and experienced utility (after buying); the difference is discussed in Farhi and Gabaix (2020). To discuss the second point – the utility that people actually experience, rather than just their perceptions – we can revisit the usual formulation of consumer surplus:

$$\text{CS} = \int_{p^*(m)}^{\overline{p}} D^r(p)dp \tag{40}$$

It is immediately obvious, then that the relationship between attention and consumer (experienced) surplus depends entirely on the sign of attentional pass-through.

Proposition 4. ***More rationality does not necessarily imply higher consumer surplus***
The relationship between consumer surplus and attention is characterized entirely by attentional pass-through:

$$\frac{\partial CS}{\partial m} = -D^r(p) \cdot \frac{\partial p^*}{\partial m} \tag{41}$$

Therefore the conclusions drawn from Proposition 1 allows us to immediately infer the relationship of consumer surplus to attention, but not the relationship of perceived surplus to attention.

4 Endogenous Attention

4.1 Endogenous Attention and Cognitive Cost Pass-Through: A Motivating Example

In the previous treatment, we assumed that m was simply a parameter. However, a more realistic expression of boundedly rational behavior in a market would be a case where attention is reflective of the market's conditions. To fix ideas, we will begin with an example and study the following question: will increasing the cognitive cost κ of observing price necessarily increase price? Intuitively, it should – since price is harder to see with higher κ, then increasing κ is like decreasing m. We call the rate of change of the monopolist's optimal price w.r.t. the consumer's cognitive cost $\frac{dp^*}{d\kappa}$ *cognitive cost pass-through*, and illustrate it in the following example.

Suppose that attention is given by:

$$m_1 = \max\left\{0, 1 - \frac{\kappa}{(p - p^d)^2}\right\} \tag{42}$$

And that demand is given by (for $\psi > 1$):

$$D_1 = e^{-(p^s)^\psi} \tag{43}$$

Neither function exhibits exotic behavior – attention is increasing in price's distance from the default and decreasing in cognitive cost. Demand is decreasing in perceived price.

We have some intuiton from our treatment of dp^*/dm that what matters is the log-curvature of demand and whether or not p^s is increasing in attention. Naturally, if p^s is a function of attention m, where $m(p, \kappa)$ is a function of price and cognitive cost κ, then we should expect to have to investigate how p^s varies with p and κ (whereas before we only had to concern ourselves with how p^s varied with m). We see that (all derivations can be found in Appendix Appendix D.3):

$$\Lambda_1 = \frac{1 - \psi}{\psi\,(p^s)^\psi} < 0 \tag{44}$$

$$\frac{\partial p^s}{\partial p} = 1 + \frac{\kappa}{(p - p^d)^2} > 0\ \forall\ |p - p^d| > \kappa \text{ and } 0 \text{ otherwise} \tag{45}$$

$$\frac{\partial p^s}{\partial \kappa} = \frac{1}{p^d - p} \tag{46}$$

It turns out that we will need one more ingredient, which reflects the new interaction between changes in price and cognitive cost; we also need to study how the responsiveness of p^s to p changes with higher κ:

$$\frac{\partial^2 p^s}{\partial p \partial \kappa} = \frac{1}{(p - p^d) > 2} > 0\ \forall\ |p - p^d| > \kappa \text{ and } 0 \text{ otherwise} \tag{47}$$

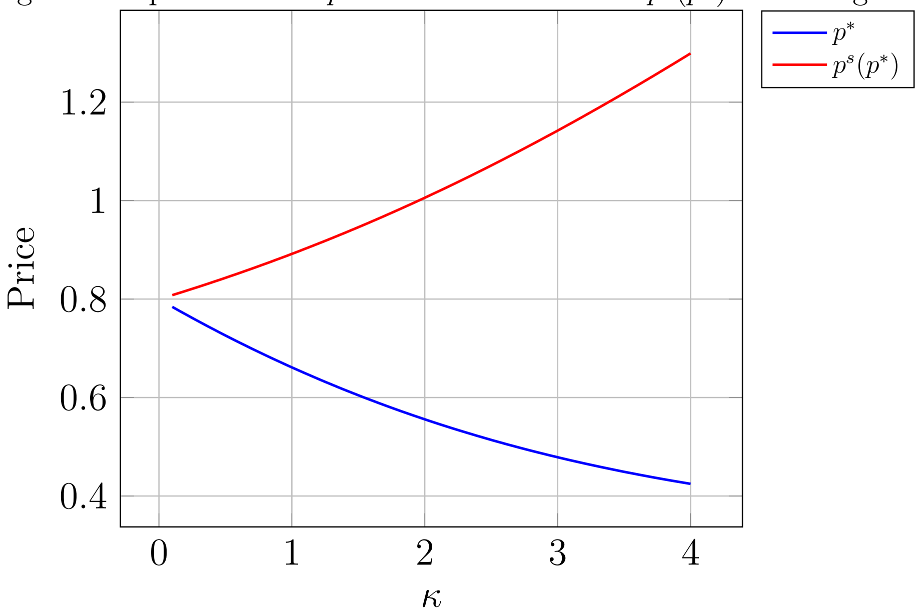
So, we have derived a simple situation where we have log-concave demand, and the attention function is such that:

1. Consumers perceive price as increasing if price is increasing outside of an 'unresponsive' region

2. Consumers perceive price as increasing if cognitive cost κ is increasing if $p^d > p$[6]
3. Consumers' perception of price is more sensitive to changes in price when cognitive cost κ is higher

We know intuitively that the firm will never price within an attentionally 'unresponsive' region because they could price higher without the consumer noticing (we will be able to derive this in a general setting soon). Thus we know that $\frac{\partial p^s(p^*)}{\partial p} > 0$ at any optimum. We also see that the conditions above mean that the consumer will perceive higher price levels and be more sensitive to price changes as κ rises if the default price exceeds the true value. This means that the consumer is perceives price to be increasing, and his perception of price is more sensitive to price changes; thus, we should expect that an increase in cognitive cost should a) increase their perception of price and b) in turn (because $\Lambda_1 < 0$) decrease prices. Indeed, this is what we see:

Figure 1: Optimal Price p^* and Perceived Price $p^s(p^*)$ Over Cognitive Costs κ



Parameter values: $p^d = 5$, $c = .3$, $\psi = 2.3$, $D = e^{-(p^s)^\psi}$, $m = \max\left\{0, 1 - \frac{\kappa}{(p-p^d)^2}\right\}$

We have worked out an example for intuition, but all of this can be derived more generally. We can do so by first revisiting sensible properties of the attention function, then observing the behavior of $p^s(m(p, p^d, \kappa))$, and then solving the monopolist's optimization problem (while deducing necessary and sufficient conditions for optimality), and the analyzing cognitive cost pass-through $\frac{dp^*}{d\kappa}$.

[6]This is sensible – if consumers are facing higher cognitive costs, they rely on their default more. So, if they rely on their default more and that default value is higher than the true value, then they will perceive price as increasing.

4.2 Fundamentals with Endogenous Attention

To achieve a microfoundation of attention, we will instead treat $m \in [0,1]$ as a function of one variable and two parameters: price p and parameters cognitive cost κ and default p^d:

$$m \equiv f(p, p^d, \kappa) \text{ such that} \tag{48}$$

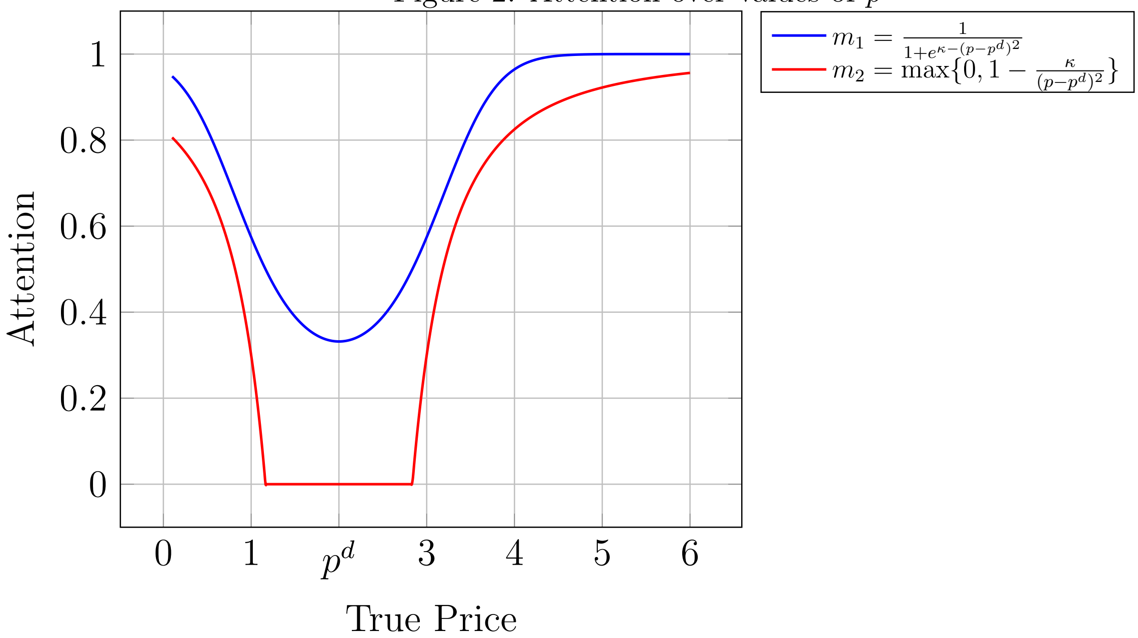
$$\frac{\partial f}{\partial \left|p - p^d\right|} \geq 0 \text{ and } \frac{\partial f}{\partial \kappa} \leq 0 \tag{49}$$

For example, we can consider the attentional function m_1 as discussed in the previous section, or we could consider:

$$m_2 = \frac{1}{1 + e^{\kappa - \left(p - p^d\right)^2}} \tag{50}$$

Both of which are bounded between 0 and 1. For $p \approx p^d$, consumers pay no attention to price, and for large deviations from their expectation p^d they pay maximal attention. We can see that here:

Figure 2: Attention over values of p



Parameter values: $p^d = 2$, $\kappa = .7$

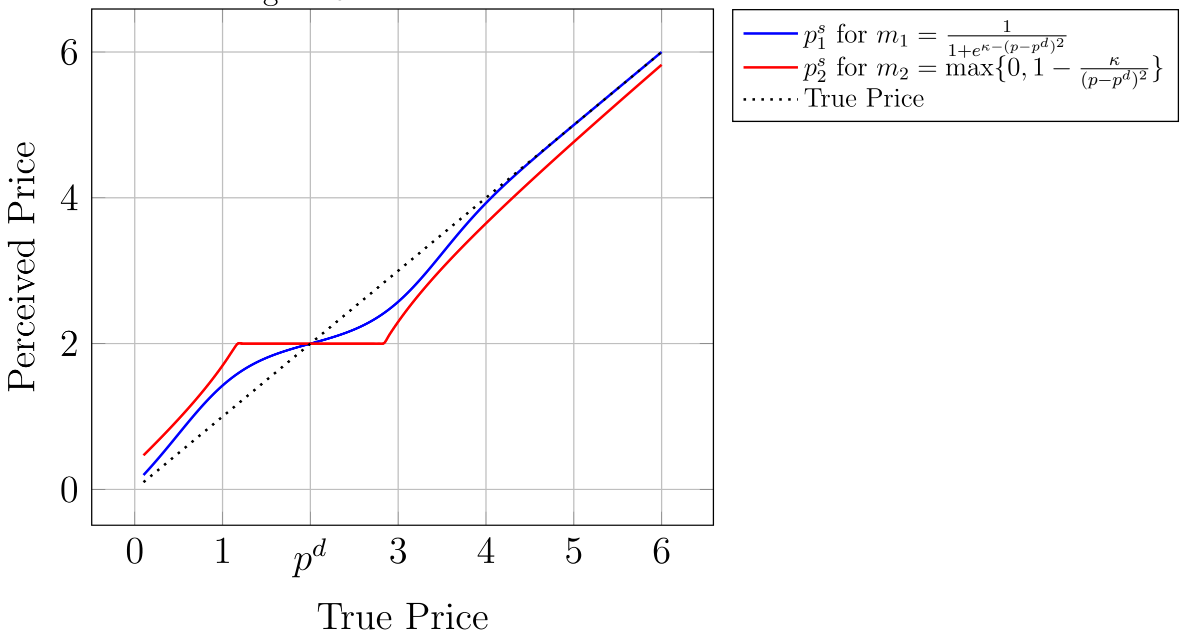
For values sufficiently close to the default, attention is very low, and for values very far from the default, attention is near or equal to one.

We can see that the perception of price will still be of the form

$$p^s(m(\kappa, p), p, p^d) = m(p, \kappa)\left(p - p^d\right) + p^d \tag{51}$$

And we can again visualize how the perception of price changes over true values of price for the above attentional functions:

Figure 3: Perceived Price for Different Attention Functions



Parameter values: $p^d = 2$, $\kappa = .7$

What can we learn from these visuals? The first point is that it is clear that any optimal price p' could not be in a region where $\frac{\partial p^s}{\partial p} \leq 0$. Take for example the region around p^d with m_2. In this region, the perception of price is inelastic to true price, and so the firm could price strictly higher than p^d, have no change in demand, and have strictly higher revenue. This means that the entire range of prices $p \in \left(p^d - \kappa, p^d + \kappa\right)$ could not be optimal. We will see later how this same intuitive condition arises naturally from the firm's optimization problem.

Given that $m(p)$, it is now necessary to rederive the monopolist's optimization problem. Facing a profit function of the form:

$$\pi(p; p^d, m(\kappa, p)) = (p - c)\, D(m(\kappa, p)\left(p - p^d\right) + p^d) \tag{52}$$

Then we have that first order condition is given by:

$$p^* = c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \left[\frac{\partial p^s}{\partial p}\right]^{-1} \tag{53}$$

$$= c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \left[m(p,\kappa) + \frac{\partial m}{\partial p} \cdot \left(p^* - p^d\right)\right]^{-1} \tag{54}$$

Where if m is *not* a function of p, then we simply have the usual $p^* = c - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{1}{m}$.

Since $p^* - c \geq 0$ at optimum and $-\frac{\partial D(p^s)}{\partial D(p^s)/\partial p^s} > 0$, we now have a new condition: the rightmost term in brackets must be positive. This is entirely sensible – the firm should not price in a region where consumers don't respond to price increases. If at a candidate solution p' the perception of price is going *down*, then why not increase the price more? Consumers would see a lower price (so more demand) and the firm would make more revenue per sale. Therefore, a solution is only optimal if the perception of price is increasing at that point. This derivation gives the general form of the intuition that we saw in the previous section about not pricing in the 'unresponsive region'.

We deal with second order conditions in Appendix Appendix D.1 and demonstrate that a sufficient condition for optimality of price is that p^s be convex in p, but note that this is not necessary.

Ultimately, we are interested in understanding how optimal price of a monopolist varies over values of cognitive cost κ; this is the endogenous version of the main proposition that explained *attentional pass-through*.

4.3 Cognitive Cost Pass-Through

We can demonstrate how the perception of price changes with cognitive cost when we treat $p^*(\kappa)$; this is where the complexity of endogenous attention enters:

$$\frac{\partial p^s}{\partial \kappa} = \frac{\partial}{\partial \kappa}\left[m(\kappa, p^*(\kappa)\left(p^*(\kappa) - p^d\right) + p^d\right] \tag{55}$$

$$= \left[\frac{\partial f}{\partial \kappa} + \frac{\partial f}{\partial p}\frac{\partial p^*}{\partial \kappa}\right]\left(p^* - p^d\right) + m \cdot \frac{\partial p^*}{\partial \kappa} \tag{56}$$

It is immediately unclear whether or not an increase in cognitive cost will decrease or increase attention m. Although an increase in κ has the direct effect of decreasing attention via $\frac{\partial f}{\partial \kappa} < 0$, a change in cognitive cost also changes price, thus changing attention. If $\frac{\partial p^*}{\partial \kappa} > 0$ – optimal price increases with cognitive cost – then the sign of $\frac{\partial m}{\partial \kappa}$ (given by the term in brackets) is unclear, as the consumer faces higher cognitive cost (making it harder to see the world) and the impact of higher prices on attention (which is unclear since we don't know where the default is). It is this tension between the direct impact of cognitive cost and the indirect impact of cognitive cost via changes in optimal price that complicates the analysis of how price relates to changes in κ with endogenous attention.

However, by using the Implicit Function Theorem we can get easily observe and evaluate the forces that govern how optimal price changes with more cognitive cost.

Proposition 5. ***Cognitive Cost Pass-Through***
The change in a monopolist's optimal price when consumers have higher cost κ of observing price p^* is given by:

$$\frac{\partial p^*}{\partial \kappa} = \frac{\left[\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right]} \tag{57}$$

Because of the second order condition at the optimum, the denominator is positive. Therefore, the sign of cognitive cost pass-through is given by the numerator. We see then that the sign of the cognitive cost pass-through is given by:

$$\text{sign}\left(\frac{dp^*}{d\kappa}\right) = \text{sign}\left(\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right) \tag{58}$$

Recall that $\frac{D(p^s)}{\partial D(p^s)/\partial p^s} < 0$ and $\frac{\partial p^s(p^*)}{\partial p} > 0$. The forces at play are exactly the same as those examined in the example section. If demand is log-convex ($\Lambda > 0$) and prices are perceived as decreasing ($\frac{\partial p^s}{\partial \kappa}$ and sensitivity to price changes decrease, then we have that cognitive cost pass-through is negative. We can list all possibilities in this table:

	2 $\Lambda < 0$	$\Lambda < 0$	$0 < \Lambda < 1$	$0 < \Lambda < 1$
	$\frac{\partial p^s}{\partial \kappa} < 0$	$\frac{\partial p^s}{\partial \kappa} > 0$	$\frac{\partial p^s}{\partial \kappa} < 0$	$\frac{\partial p^s}{\partial \kappa} > 0$
$\frac{\partial p^s}{\partial p \partial \kappa}$	$\frac{dp^*}{d\kappa} > 0$	Ambiguous	Ambiguous	$\frac{dp^*}{d\kappa} > 0$
$\frac{\partial^2 p^s}{\partial p \partial \kappa}$	Ambiguous	$\frac{dp^*}{d\kappa} < 0$	$\frac{dp^*}{d\kappa} < 0$	Ambiguous

Table 2: Cognitive Cost Pass-Through

Revisiting the example from the previous section, we see that for $p^d = 1.5$ and D_1, m_1:

$$\text{sign}\left(\frac{dp^*}{d\kappa}\right) = \underbrace{\frac{\partial p^s}{\partial \kappa}}_{>0} \overbrace{\Lambda}^{<0} + \underbrace{\frac{D(p^s)}{\partial D(p^s)/\partial p^s}}_{<0} \cdot \underbrace{\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}}_{>0} < 0 \tag{59}$$

So, the **Markup effect** appears again via the log-curvature of the demand function.[7] The **Behavioral Effect** concerns whether or not higher cognitive cost implies higher perception of attention, $\frac{\partial p^s}{\partial \kappa}$, as well as whether or not the consumer is more/less sensitive to price changes with higher cognitive cost: $\frac{\partial^2 p^s}{\partial p \partial \kappa}$.

Key Takeaways

1. We derive the optimal behavior of a monopolist facing consumers with an endogenous attention function $m(p, p^d, \kappa)$ and derive optimality conditions for the pricing problem.

2. We demonstrate how changes in cognitive cost κ affect perception of price levels, but also sensitivity of p^s to price changes, and show how these are the two key mechanisms in determining the sign of cognitive cost pass-through $\frac{dp^*}{d\kappa}$.

5 Optimal Quality Choice with Inattention

5.1 Inattentive Demand for Quality

We now extend the previous framework to deal with when firms choose the quality of a good rather than the price. The results are symmetric and demonstrate how quality markdowns, rather than price markups, will behave under different levels of attention. Suppose the monopolist faces a demand function where the consumer misperceives quality as $q^s = mq + (1-m)\, q^d$. Then, if they have a linear cost function $C(q) = b$, we have a profit function of the form:

$$\pi = (p - bq)\, D(mq + (1-m)q^d) \tag{60}$$

[7] Recall that this effect deals with whether or not higher perceived prices imply a higher markup or not. That is, is $\frac{\partial \mu}{\partial p^s} > 0$.

Facing this profit function, the optimal quality is given by:

$$q^* = \frac{p}{b} - D\left[\frac{\partial D}{\partial q^s}\right]^{-1} \cdot \frac{1}{m} \tag{61}$$

Where again convexity of $1/D(q^s)$ is sufficient for quasi-concavity of the profit function, as demonstrated in Appendix Appendix A.4.1. Thus we can revisit the question of attentional pass-through but now in reference to quality:

Proposition 6. ***Attentional pass-through for quality***

$$\frac{\partial q^*}{\partial m} = \left[\frac{\Lambda^q}{1-\Lambda^q}\left(q^{pc} - q^d\right) + \frac{D(q^s)}{\partial D/\partial q^s}\right] \cdot \frac{1}{m} \tag{62}$$

Which is analogous to that which we found for price, as in equation 12. Now that we have established that quality choice with inattention is analogous to the case of price, then we can move to a more applicable question: how do firms that offer multiple goods of varying quality optimize when some goods are more prominent than others?

5.2 Flagship and Peripheral Goods

Suppose a firm produces two goods, where one is the central or flagship product, whereas the second good is a peripheral, or auxiliary good. For example, a vehicle manufacturer may have a model of car that is their most well-known and celebrated, while also producing lesser known, less popular models. Denote the flagship good as having characteristics $\boldsymbol{x}^d = (p^d, q^d)$ for quality and price, while the peripheral good has quality q and price p.

Importantly, however, consumers see the two goods as asymmetrically related: they infer something about the quality of the peripheral good from the flagship product, but not the other way around. So, the perceived quality of the central good, for some adjustment parameter m, is given by the following equation:

$$q^s = mq + (1-m)\, q^d \tag{63}$$

So when $m = 0$, people assume that the peripheral good's quality is the same as the flagship product, while when $m = 1$, the quality of the central good is irrelevant. However, we can assume that usually $m \in (0, 1)$, meaning that consumers think that the peripheral good has quality that is somewhat similar to the flagship product. Importantly, there is no misperception of the central good's quality – consumers know the true quality of the central good, but not of the peripheral one. From this asymmetry, the rest will follow.

The monopolist makes profit from each good:

$$\pi(\boldsymbol{x^d}, \boldsymbol{x}) = (p - bq)D(mq + (1-m)\, q^d, p) + (p^d - bq^d)D^d(q^d, p^d) \tag{64}$$

So, note that the demand for the peripheral good (left) has the convex combination between peripheral and central quality, whereas the demand for the central good (right) contains no such confusion. The optimal choice of quality for the peripheral good is:

$$q^* = \underbrace{\frac{p}{b}}_{\text{Competitive quality}} - \underbrace{\frac{D}{m}\left[\frac{\partial D}{\partial q^s}\right]^{-1}}_{\text{Quality Markdown}} \tag{65}$$

In contrast, the optimal choice of quality for the central good contains an extra term:

$$q^d = \underbrace{\frac{p^d}{b}}_{\text{Competitive quality}} + \underbrace{\left(\frac{p - bq}{b}\right)\left(\frac{\partial D/\partial q^s}{\partial D/\partial q^d}\right) \cdot (1-m)}_{\text{Centrality Markup}} - \underbrace{D^d \left[\frac{\partial D^d}{\partial q^d}\right]^{-1}}_{\text{Quality Markdown}} \tag{66}$$

These very simple derivations can be found in Appendix Appendix B.

We have just demonstrated how a firm will endogenously supply more quality to the central good and less to the peripheral good due to the asymmetry in demand dependencies, and that the extent of this depends on the attentional parameter m.

Next, we wish to demonstrate how the optimal choice of the peripheral good changes when firms are endowed with different central good qualities. That is, suppose that different companies have different 'histories' or reputations. Consumers believe that when they observe some new good's quality q_i from firm i, that this good has a quality level related to that of firm i's reputation, q_i^d. So, do firms with better reputations have higher or lower peripheral quality?

It immediately follows (see Appendix Appendix B.2) that *reputational pass-through* is given by the following:

Proposition 7. ***Reputational Pass-Through***
As a firm's reputation q^d increases, the firm changes their peripheral good's quality following this expression:

$$\frac{\partial q^*}{\partial q^d} = \frac{1-m}{m}\Lambda^q \tag{67}$$

Where $\Lambda^q < 0$ if $D(q^s)$ is log-concave.

It is immediately clear that for any log-concave demand function, optimal quality is decreasing in default quality. So, if Mercedes faces a log-concave demand function (multinomial logit, for example) when offering their cars, then as the reputation of their most famous car goes up, they have an incentive to undercut the quality of their auxiliary vehicles.

Now that we have drawn an analogy of attentional pass-through from price to quality and to the multiproduct monopolist, we can now analyze the situation in which the firm chooses both quality and price.

6 Rationality and the Resolution to Consumer-Firm Tensions

The analysis in Section 3 suggests that there may be some tension between the interests of the firm and the benefits to the consumer. Whereas the previous discussion focused on the behavior of optimal price and attention, we instead focus here on the behavior of *profit* and attention. In a sense, price levels do not matter to the monopolist – prices are only a step in the path to profit. So, although we may have implicitly assumed that a lower price resulting from more consumer attention hurts the monopolist, there is of course no reason to believe that this is true, as it amounts to an assumption about the value function π^*, which depends on the functional form of $D(p)$. As a result, we can present the conditions under which firms and consumers alike benefit from more rationality.

To do so, we first make a very weak assumption: m is not 0. If someone is agreeing to buy a product, it is not plausible that they pay literally no attention to price,[8] so it may be of more use to think that $m \in [\underline{m}, 1]$ for some $\underline{m} > 0$, like $m = .05$ or $m = .1$. The reason that this matters is that at $m = 0$, the monopolist can price at infinity; but once we exclude unrealistically low levels of attention, then an unintuitive result becomes clear: more consumer attention is often more profitable for the firm.

It is not easy to give a global characterization of the optimal profit function π^* as a function of m for a general demand function $D(p)$, but individual examples can return simple conclusions. To demonstrate this, we derive the relationship between π^* and m for $D = p^{-\psi}$ in Appendix Appendix E.1 and show that the relationship is determined by a simple ratio.

Due to the difficulty involved in developing a global characterization of the relation of π^* to consumer attention m, we instead ask a simpler question: under what conditions is a slight deviation in consumer attention away from full rationality profitable for the firm? That is, we can evaluate $-\frac{\partial \pi^*}{\partial m}|_{m=1}$.

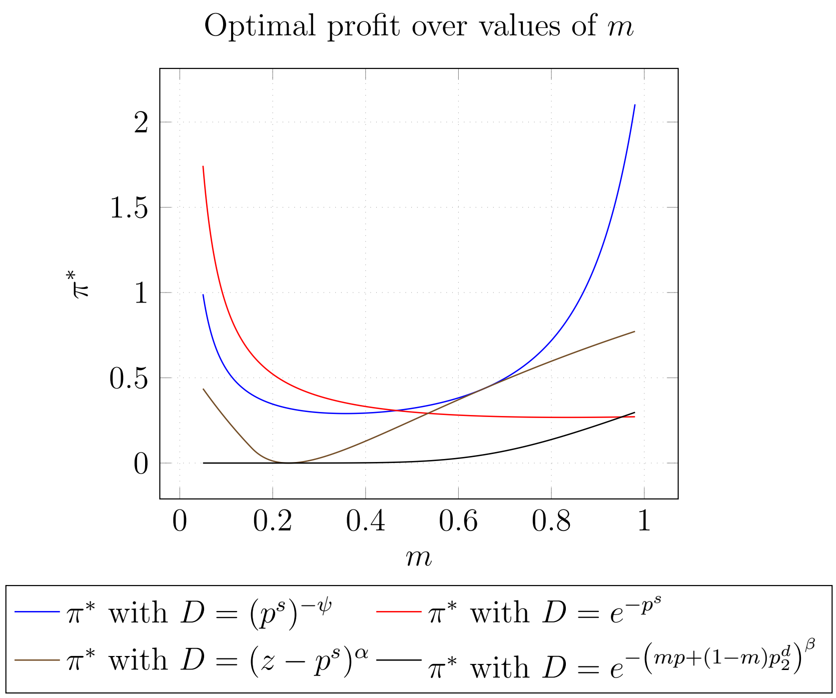
Proposition 8. ***When firms benefit from more consumer attention***
Defining $\pi^* \equiv \pi(p^*(m))$ and p_r^* as monopoly price with fully rational consumers, then firms would experience a decrease in profits if perfectly rational consumers became slightly inattentive if the following inequality holds:

$$p^d > p_r^* \tag{68}$$

The proof can be found in Appendix Appendix E.2, but the result follows simply from the Envelope Theorem. This result means that profits will decrease if consumers' reference point for price is higher than that which they would be charged if they were fully rational. Whereas Proposition 1 requires a comparison of the true price to the perfectly competitive price, this result instead depends on a comparison of the expected price and perfectly rational price. This suggests, then, that a different (but related form) of consumer expectations about price is central to understanding the relationship of profit and attention.

Finally, we illustrate optimal profit over values of m for a variety of demand functions to emphasize the complexity and diversity of relationships between profit and attention:

[8] The case of $m = 0$ also makes the problem trivial and uninteresting in other ways, and so it is a polar case best left ignored.



Parameter values: $c = .3$, $z = 2$, $\alpha = 1.3$, $\beta = 2.3$, $\psi = 3.5$, $p^d = 1.5$, $p_2^d = 2.3, \underline{m} = .05$

This means that for demand functions where optimal price is decreasing in m and profit is increasing in m, then full rationality ($m = 1$) can be characterized as the value of m that maximizes consumer surplus and producer surplus separately, and as a result maximizes total surplus. Our intuition about attention, consumers, and firms may make us think that more attention implies a trade-off for consumer surplus vs. profit, but these preliminary investigations instead demonstrate that full rationality often involves no trade-offs and makes everyone better off.

So, if consumers and firms could vote on the value of $m \in [\underline{m}, 1]$, then it seems that in many cases, they would converge on $m = 1$. Therefore, depending on the demand specification, there are some markets for which there is a free-market solution to the problem of consumer inattention – firms and consumers both will want to maximize attention. For other markets, there will be a genuine trade-off between consumer and producer surplus, and so government intervention may be needed to balance the scales appropriately. Characterizing the demand functions that lead to one outcome or the other is the next step in this research agenda.

7 Conclusion

We have demonstrated that for a very general formulation in which consumers misperceive prices or qualities, making them less behavioral does not necessarily make them better off. Their welfare does not necessarily increase, whether this is understood in terms of prices, consumer surplus, or perceived surplus. Having applied a similar logic to the choice of quality, we then attempt a preliminary investigation of shrinkflation, and demonstrate that its occurrence relies on a new third order effect, here defined 'markup concavity'. Finally, we demonstrate that in some markets, it may be beneficial for both firms and consumers to have fully rational consumers, and that the assumption that more or less attention implies a trade-off between consumer and producer welfare is misguided.

Appendix A Monopoly with Inattention

Appendix A.1 Attentional Pass-Through

First, we must note that the key derivatives $\frac{\partial D(p^s(p^*(m)))}{\partial m}$ and $\frac{\partial^2 D(p^s(p^*(m)))}{\partial p^s \partial m}$ must be rederived because the optimal price $p^*(m)$ is a function of attention, so the previous equations are no longer complete. So, we can reexpress the derivatives as follows:

$$\frac{\partial D}{\partial m} = \frac{\partial D}{\partial p^s} \cdot \left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right) \tag{69}$$

$$\frac{\partial^2 D}{\partial p^s \partial m} = \frac{\partial^2 D}{\partial p^{s2}} \cdot \left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right) \tag{70}$$

Now deriving attentional pass-through will immediately yield the above two derivatives:

$$\frac{\partial p^*}{\partial m} = \frac{\partial}{\partial m}\left[c - \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m}\right] \tag{71}$$

$$= -\frac{\partial D/\partial m}{\partial D/\partial p^s} \cdot \frac{1}{m} + D \cdot \frac{\partial^2 D/\partial p^s \partial m}{(\partial D/\partial p^s)^2} \cdot \frac{1}{m} + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{72}$$

$$= -\frac{\frac{\partial D}{\partial p^s} \cdot \left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)}{\partial D/\partial p^s} \cdot \frac{1}{m} + D \cdot \frac{\frac{\partial^2 D}{\partial p^{s2}} \cdot \left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)}{(\partial D/\partial p^s)^2} \cdot \frac{1}{m} + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{73}$$

$$= -\left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right) \cdot \frac{1}{m} + \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} \cdot \frac{\left(p^* - p^d\right)}{m} + \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} \cdot \frac{\partial p^*}{\partial m} + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{74}$$

$$2\frac{\partial p^*}{\partial m} = -\frac{p^* - p^d}{m} + \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{\partial p^*}{\partial m}\right) + \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{p^* - p^d}{m}\right) + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{75}$$

$$2\frac{\partial p^*}{\partial m} - \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{\partial p^*}{\partial m}\right) = -\frac{p^* - p^d}{m} + \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}\left(\frac{p^* - p^d}{m}\right) + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \tag{76}$$

$$\frac{\partial p^*}{\partial m}[2 - \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}] = \left(\frac{p^* - p^d}{m}\right)\left(\frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2} \implies \tag{77}$$

$$\frac{\partial p^*}{\partial m} = \frac{\left(\frac{p^* - p^d}{m}\right)\left(\frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s} \cdot \frac{1}{m^2}}{2 - \frac{D \cdot \partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}} = \tag{78}$$

$$= \frac{\left(p^* - p^d\right)\Lambda + \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}}{1 - \Lambda} \cdot \frac{1}{m} \tag{79}$$

However, we can see that this formulation does not get at the primitives of the model – we can push further. Now plugging in optimal price $p^* = c - \frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}$ gives:

$$\frac{\partial p^*}{\partial m} = \frac{\left(c - \frac{D(p^s)}{\partial D/\partial p^s} - p^d\right)\left(\frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1\right) + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m}}{2 - \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2}} \cdot \frac{1}{m} \tag{80}$$

$$\text{Now defining } \Lambda \equiv \frac{D\cdot\partial^2 D/\partial p^{s2}}{(\partial D/\partial p^s)^2} - 1 \implies \tag{81}$$

$$\frac{\partial p^*}{\partial m} = \frac{\left(c - \frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m} - p^d\right)\Lambda + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m}}{1-\Lambda} \cdot \frac{1}{m} \tag{82}$$

$$= \frac{\left(c - p^d\right)\Lambda - \left(\frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m}\right)\Lambda + \frac{D}{\partial D/\partial p^s}\cdot\frac{1}{m}}{1-\Lambda} \cdot \frac{1}{m} \tag{83}$$

$$= \frac{\left(c - p^d\right)\Lambda + \frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m}(1-\Lambda)}{1-\Lambda} \cdot \frac{1}{m} \tag{84}$$

$$= \left[\frac{\Lambda}{1-\Lambda}\left(c - p^d\right) + \frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m}\right] \cdot \frac{1}{m} \tag{85}$$

Now if we call the perfectly competitive price $p^{pc} = c$, then we have that the equation is:

$$\frac{\partial p^*}{\partial m} = \left[\frac{\Lambda}{\Lambda-1}\left(p^d - p^{pc}\right) + \frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m}\right] \cdot \frac{1}{m} \tag{86}$$

We can verify the above equation for attentional pass-through by comparing it to the direct derivation of $\frac{\partial p^*}{\partial m}$ when $D = \left(mp + (1-m)p^d\right)^{-\psi}$; see Appendix 93.

Appendix A.2 Optimal price with $D^s = \left(mp + (1-m)\,p^d\right)^{-\psi}$

The derivations for a monopolist facing inattention (whose process of formation is unspecificied) will price when facing $D^s = \left(mp + (1-m)\,p^d\right)^{-\psi}$. The profit function is of the form $\pi = (p-c)\left(mp + (1-m)\,p^d\right)^{-\psi}$, and so the optimal price is given by:

$$\frac{\partial \pi}{\partial p} = \left(mp + (1-m)\,p^d\right)^{-\psi} - m\psi\,(p-c)\left(mp + (1-m)\,p^d\right)^{-\psi-1} = 0 \tag{87}$$

$$= 1 - \frac{m\psi\,(p-c)}{mp + (1-m)\,p^d} = 0 \tag{88}$$

$$= mp + (1-m)\,p^d - m\psi\,(p-c) = 0 \tag{89}$$

$$= mp - m\psi p = -\left(\psi c m + (1-m)\,p^d\right) \implies \tag{90}$$

$$p^* = \frac{\psi c m + (1-m)\,p^d}{m\,(\psi-1)} \tag{91}$$

$$= \frac{\psi c}{\psi - 1} + \frac{(1-m)\,p^d}{m\,(\psi-1)} \tag{92}$$

Next, it is trivial to check the derivative of the optimal price with respect to m:

$$\frac{\partial p^*}{\partial m} = \frac{-p^d}{m(\psi-1)} - \frac{(1-m)p^d}{m^2(\psi-1)} < 0 \tag{93}$$

Appendix A.3 Changes in Perceived Surplus with More Rationality

These calculations are done in order to understand the quantity $\frac{\partial \mathrm{PS}}{\partial m}$: do consumers think they are made better off when they are more rational? First, we derive the simple general equation for any demand function, and then we compute some examples.

Appendix A.3.1 General expression

We have the following expression for perceived surplus, which is denoted PS:

$$\mathrm{PS} = \int_{p^*(m)}^{\overline{p}} D(p^s(p))dp \tag{94}$$

Then, using Leibniz's rule for differentiation under the integral sign, we can find the effect of changes in attention on perceived surplus:

$$\frac{\partial \mathrm{PS}}{\partial m} = D(p^s(\overline{p})) \cdot \frac{\partial \overline{p}}{\partial m} - D(p^s(p^*))\frac{\partial p^*}{\partial m} + \int_{p^*}^{\overline{p}} \frac{\partial D(p^s(p, m, p^d))}{\partial m} dp \tag{95}$$

$$= -D(p^s(p^*))\frac{\partial p^*}{\partial m} + \int_{p^*}^{\overline{p}} \frac{\partial D}{\partial p^s} \left(p - p^d\right) dp \tag{96}$$

Appendix A.3.2 Derivation of examples

First, suppose that $D = (p^s)^{-\psi}$. Then, we have that perceived surplus is:

$$\mathrm{PS} = \int_{p^*(m)}^{\overline{p}} (p^s)^{-\psi} dp = \left[\frac{(p^s)^{1-\psi}}{m(1-\psi)}\right]_{p^*}^{\overline{p}} \tag{97}$$

$$= \frac{(m\overline{p} + (1-m)p^d)^{1-\psi} - (mp^* + (1-m)p^d)^{1-\psi}}{m(1-\psi)} \implies \tag{98}$$

$$\frac{\partial \mathrm{PS}}{\partial m} = \frac{(1-\psi)\left(p^s(\overline{p})\right)^{-\psi}\left(\overline{p} - p^d\right) - (1-\psi)\left(p^s(p^*(m))\right)^{-\psi}\left(p^* - p^d + \frac{\partial p^*}{\partial m} \cdot m\right)}{m\,(1-\psi)} \tag{99}$$

$$- \frac{\left[(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}\right](1-\psi)}{m^2(1-\psi)^2} \tag{100}$$

$$= (p^s(\overline{p})^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - (p^s(p^*))^{-\psi}\left(\frac{p^* - p^d}{m} + \frac{\partial p^*}{\partial m}\right) - \frac{(p^s(\overline{p}))^{1-\psi} - (p^s(p^*))^{1-\psi}}{m^2(1-\psi)} \tag{101}$$

We want to check that equation 101 is the same as when we just plug in $D = (p^s)^{-\psi}$ to the general expression given by equation 96. Doing so, we get:

$$
\begin{aligned}
\frac{\partial \mathrm{PS}}{\partial m} &= -\left(p^s\right)^{-\psi} \cdot \frac{\partial p^*}{\partial m} - \int_{p^*}^{\overline{p}} \psi \left(p^s(p)\right)^{-\psi-1} \left(p - p^d\right) dp && (102)\\
&= -\left(p^s\right)^{-\psi} \cdot \frac{\partial p^*}{\partial m} + \left[\frac{p - p^d}{m\left(p^s(p)\right)^{\psi}} - \frac{\left(p^s(p)\right)^{1-\psi}}{m^2(1-\psi)}\right]_{p^*}^{\overline{p}} && (103)\\
&= -\left(p^s\right)^{-\psi} \cdot \frac{\partial p^*}{\partial m} + \left(\frac{\overline{p} - p^d}{m}\right)\left(p^s(\overline{p})\right)^{-\psi} - \frac{\left(p^s(\overline{p})\right)^{1-\psi}}{m^2(1-\psi)} - \left(\frac{p^* - p^d}{m}\right)\left(p^s(p^*)\right)^{-\psi} + \frac{\left(p^s(p^*)\right)^{1-\psi}}{m^2(1-\psi)} && (104)\\
&= \left(\frac{\overline{p} - p^d}{m}\right)\left(p^s(\overline{p})\right)^{-\psi} - \left(p^s(p^*)\right)^{-\psi}\left(\frac{p^* - p^d}{m} + \frac{\partial p^*}{\partial m}\right) + \frac{\left(p^s(p^*)\right)^{1-\psi} - \left(p^s(\overline{p})\right)^{1-\psi}}{m^2(1-\psi)} && (105)
\end{aligned}
$$

Thus the general expression is correct. We can further simplify terms.

Using the fact that $p^* = \frac{\psi cm + (1-m)p^d}{m(\psi-1)}$ (equation 92) and $\frac{\partial p^*}{\partial m} = \frac{-p^d}{m^2(\psi-1)}$ (equation 93), we have the following:

$$
\begin{aligned}
\frac{\partial \mathrm{PS}}{\partial m} &= \left(p^s(\overline{p})\right)^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - \left(p^s(p^*)\right)^{-\psi}\left(\frac{\frac{\psi cm + (1-m)p^d}{m(\psi-1)} - p^d}{m} + \frac{-p^d}{m^2(\psi-1)}\right) - \frac{\left(p^s(\overline{p})\right)^{1-\psi} - \left(p^s(p^*)\right)^{1-\psi}}{m^2(1-\psi)} && (106)\\
&= \left(p^s(\overline{p})\right)^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - \left(p^s(p^*)\right)^{-\psi}\left(\frac{\psi cm + (1-m)\,p^d - m(\psi-1)p^d - p^d}{m^2\left(\psi - 1\right)}\right) - \frac{\left(p^s(\overline{p})\right)^{1-\psi} - \left(p^s(p^*)\right.}{m^2(1-\psi)} && (107)\\
&= \left(p^s(\overline{p})\right)^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - \left(p^s(p^*)\right)^{-\psi}\left(\frac{\psi cm + -m\psi p^d}{m^2\left(\psi - 1\right)}\right) - \frac{\left(p^s(\overline{p})\right)^{1-\psi} - \left(p^s(p^*)\right)^{1-\psi}}{m^2(1-\psi)} && (108)\\
&= \left(p^s(\overline{p})\right)^{-\psi}\left(\frac{\overline{p} - p^d}{m}\right) - \left(p^s(p^*)\right)^{-\psi}\left(\frac{\psi(c - p^d)}{m\left(\psi - 1\right)}\right) - \frac{\left(p^s(\overline{p})\right)^{1-\psi} - \left(p^s(p^*)\right)^{1-\psi}}{m^2(1-\psi)} && (109)
\end{aligned}
$$

The above expression is not easy to sign, and so instead we draw it for a set of reasonable parameter values in the main text.

Appendix A.4 Optimal quality

Appendix A.4.1 Convexity of $1/D(q^s)$ and quasi-concavity of profit

We demonstrate here that, just as for price, the quasi-concavity of the profit function is ensured with the convexity of $1/D(q^s)$; note that this argument is symmetric to that for price given in Anderson et al. (1992) 6.3.1. Given the profit function

$$\pi(q) = (p - bq)\, D(mq + (1-m)q^d) \tag{110}$$

Then we wish to show that convexity of $1/D(q^s)$ is sufficient to ensure quasi-concavity of the profit function; that is, we want to show that any value of q that satisfies the first order condition, we also have $\partial^2\pi/\partial q^2 < 0$. So, we have that at the FOC:

$$\frac{\partial \pi}{\partial q} = -bD(q^s) + (p - bq)\,\frac{\partial D(q^s)}{\partial q^s} \cdot m = 0 \tag{111}$$

And further we have that the second order condition is:

$$\frac{\partial^2 \pi}{\partial q^2} = -b\frac{\partial D(q^s)}{\partial q^s} \cdot m - b\frac{\partial D(q^s)}{\partial q^s} \cdot m + (p - bq)\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m^2 \tag{112}$$

$$= \frac{\partial^2 \pi}{\partial q^2} = -2b\frac{\partial D(q^s)}{\partial q^s} \cdot m + (p - bq)\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m^2 \tag{113}$$

Next, evaluating the second order condition when the first order condition is zero ($p - bq = \frac{bD(q^s)}{\partial D(q^s)/\partial q^s} \cdot \frac{1}{m}$) gives:

$$\frac{\partial^2 \pi}{\partial q^2} = -2b\frac{\partial D(q^s)}{\partial q^s} \cdot m + \frac{bD(q^s)}{\partial D(q^s)/\partial q^s}\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot \frac{m^2}{m} \tag{114}$$

We want to see what condition under which the above expression is negative, so we will write it as less than zero, and then divide both sides by $\frac{\partial D}{\partial q^s} > 0$:

$$\frac{\partial^2 \pi}{\partial q^2} = -2b \cdot m + \frac{bD(q^s)}{(\partial D(q^s)/\partial q^s)^2}\frac{\partial^2 D(q^s)}{\partial q^{s2}} \cdot m < 0 \tag{115}$$

$$= -bm\left[2 - D(q^s) \cdot \frac{\partial^2 D(q^s)/\partial q^{s2}}{(\partial D(q^s)/\partial q^s)^2}\right] < 0 \tag{116}$$

Thus it is clear that the quantity above is negative if the value in the square brackets is positive. This condition is satisfied if $1/D(q^s)$ is convex.

Appendix A.4.2 Attentional pass-through with quality

Recall that optimal quality choice is given by:

$$q^* = \frac{p}{b} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} \tag{117}$$

Then looking at how this changes with respect to m gives:

$$\frac{\partial q^*}{\partial m} = -\frac{\partial D/\partial m}{\partial D/\partial q^s} \cdot \frac{1}{m} + D \cdot \frac{\partial^2 D/\partial q^s \partial m}{(\partial D/\partial q^s)^2} \cdot \frac{1}{m} + \frac{D}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \tag{118}$$

$$= -\frac{\partial D/\partial q^s}{\partial D/\partial q^s} \cdot \frac{q^* - q^d + m \cdot \frac{\partial q^*}{\partial m}}{m} + D \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} \frac{q^* - q^d + m \cdot \frac{\partial q^*}{\partial m}}{m} + \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \tag{119}$$

$$= \left(\frac{q^* - q^d}{m}\right) \left[D \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} - 1\right] + \frac{\partial q^*}{\partial m} \left[D \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} - 1\right] + \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \tag{120}$$

$$\frac{\partial q^*}{\partial m} [1 - \Lambda^q] = \left(\frac{q^* - q^d}{m}\right) \Lambda^q + \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \tag{121}$$

$$\frac{\partial q^*}{\partial m} = \left[\frac{\left(q^* - q^d\right) \Lambda^q + \frac{D}{\partial D/\partial q^s} \cdot \frac{1}{m}}{1 - \Lambda^q}\right] \cdot \frac{1}{m} \tag{122}$$

$$= \left[\frac{\left(\frac{p}{b} - \frac{D}{\partial D/\partial q^s} \cdot \frac{1}{m} - q^d\right) \Lambda^q + \frac{D}{\partial D/\partial q^s} \cdot \frac{1}{m}}{1 - \Lambda^q}\right] \cdot \frac{1}{m} \tag{123}$$

$$= \left[\frac{\left(q^{pc} - q^d\right) \Lambda^q + \frac{D}{\partial D/\partial q^s} (1 - \Lambda^q) \cdot \frac{1}{m}}{1 - \Lambda^q}\right] \cdot \frac{1}{m} \tag{124}$$

$$= \left[\frac{\Lambda^q}{1 - \Lambda^q} \left(q^{pc} - q^d\right) + \frac{D}{\partial D/\partial q^s} \cdot \frac{1}{m}\right] \cdot \frac{1}{m} \tag{125}$$

Appendix A.4.3 Markdown concavity with quality

We can easily see that quality markdown concavity, which is $\frac{\partial \Lambda^q}{\partial q^s} < 0$ and is analagous to price markup concavity is given by:

$$\frac{\partial \Lambda^q}{\partial q} = \frac{\partial D}{\partial q^s} \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} + D(q^s) \cdot \frac{\partial^3 D/\partial q^{s3}}{(\partial D/\partial q^s)^2} - 2D(q^s) \cdot \frac{(\partial^2 D/\partial q^{s2})^2}{(\partial D/\partial q^s)^3} \tag{126}$$

Further, notice that:

$$\frac{\partial \Lambda^q(q^s)}{\partial m} = \left(m\frac{\partial q^*}{\partial m} + q^* - q^d\right) \frac{\partial \Lambda^q}{\partial q^s} \tag{127}$$

$$= \left(m \cdot \left(\left[\frac{\Lambda^q}{1 - \Lambda^q} \left(q^{pc} - q^d\right) + \frac{D(q^s)}{\partial D/\partial q^s} \frac{1}{m}\right] \cdot \frac{1}{m}\right) + \frac{p}{b} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} - q^d\right) \frac{\partial \Lambda^q}{\partial q^s} \tag{128}$$

$$= \left(\frac{\Lambda^q}{1 - \Lambda^q} \left(q^{pc} - q^d\right) + \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} + q^{pc} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} - q^d\right) \frac{\partial \Lambda^q}{\partial q^s} \tag{129}$$

$$= \left(\frac{q^{pc} - q^d}{1 - \Lambda^q}\right) \frac{\partial \Lambda^q}{\partial q^s} \tag{130}$$

So, to get $\frac{\partial \rho^q}{\partial m}$, then we need to evaluate the following:

$$\frac{\partial \rho^q(q^s)}{\partial m} = \frac{\partial}{\partial m}\left[\frac{-q^{pc}}{b\,(1-\Lambda^q)}\right] \tag{131}$$

$$= -\frac{q^{pc}}{b}\frac{\partial \Lambda^q(q^s)/\partial m}{(1-\Lambda^q)^2} \tag{132}$$

$$= -\frac{q^{pc}}{b(1-\Lambda^q)^2}\left(\frac{q^{pc}-q^d}{1-\Lambda^q}\right)\frac{\partial \Lambda^q}{\partial q^s} \tag{133}$$

$$= \frac{q^{pc}\left(q^d - q^{pc}\right)}{b\,(1-\Lambda^q)^3}\frac{\partial \Lambda^q}{\partial q^s} \tag{134}$$

Appendix B Concentric Goods

Appendix B.1 Optimal choice of peripheral and central quality

Given the profit function $\pi = (p - bq)\,D(mq + (1-m)\,q^d, p) + (p^d - bq^d)D^d(q^d, p^d)$, we will show the optimal choices of central quality q^d and peripheral quality q.

First, the optimal choice of peripheral quality:

$$\frac{\partial \pi}{\partial q} = -bD(mq + (1-m)\,q^d, p) + (p - bq)\frac{\partial D}{\partial q^s}\cdot m = 0 \implies \tag{135}$$

$$q^* = \frac{p}{b} - \frac{D}{m}\left[\frac{\partial D}{\partial q^s}\right]^{-1} \tag{136}$$

Next, the optimal choice of central quality:

$$\frac{\partial \pi}{\partial q^d} = (p - bq)\,\frac{\partial D}{\partial q^s}\cdot(1-m) - bD^d + \left(p^d - bq^d\right)\frac{\partial D^d}{\partial q^d} = 0 \implies \tag{137}$$

$$q^d = \frac{p^d}{b} + \left(\frac{p - bq}{b}\right)\left(\frac{\partial D/\partial q^s}{\partial D^d/\partial q^d}\right)\cdot(1-m) - D^d\left[\frac{\partial D^d}{\partial q^d}\right]^{-1} \tag{138}$$

Now evaluating q^{d*} with q^* yields:

$$q^{d*} = \left(\frac{p}{b} - \left(\frac{p}{b} - \frac{D}{m}\left[\frac{\partial D}{\partial q^s}\right]^{-1}\right)\right)\left(\frac{\partial D/\partial q^s}{\partial D^d/\partial q^d}\right)\cdot(1-m) + \frac{p^d}{b} - D^d\left[\frac{\partial D^d}{\partial q^d}\right]^{-1} \tag{139}$$

$$= \left(\frac{D}{m}\left[\frac{\partial D}{\partial q^s}\right]^{-1}\right)\left(\frac{\partial D/\partial q^s}{\partial D^d/\partial q^d}\right)\cdot(1-m) + \frac{p^d}{b} - D^d\left[\frac{\partial D^d}{\partial q^d}\right]^{-1} \tag{140}$$

$$= \left(\frac{D}{\partial D^d/\partial q^d}\right)\cdot\left(\frac{1-m}{m}\right) + \frac{p^d}{b} - D^d\left[\frac{\partial D^d}{\partial q^d}\right]^{-1} \tag{141}$$

Next, we can define the quality markups for each good as:

$$\mu^{q^d} \equiv q^{d*} - p^d \tag{142}$$

$$\mu^q \equiv q - p \tag{143}$$

To avoid reliance on price differences, we can show that the markups in quality relative to own price are higher for the central good than for the peripheral good. We can do this using an example; suppose that demand is given by $D = \exp(q-p)$ where q can be whatever relevant quality and price is whatever relevant price.

To simplify calculation, set $b = 1$, and we can find the following optimal qualities. Given that $\pi = (p-q)\, e^{mq+(1-m)q^d-p} + (p^d - q^d)e^{q^d-p^d}$, then we can calculate the optimal choice of peripheral quality q:

$$\frac{\partial \pi}{\partial q} = -e^{mq+(1-m)q^d-p} + m\,(p-q)\, e^{mq+(1-m)q^d-p} = 0 \implies \tag{144}$$

$$= -1 + m\,(p-q) = 0 \implies \tag{145}$$

$$q^* = p - \frac{1}{m} \tag{146}$$

This illustrates the intuition laid out earlier – as consumers get less attentive to true quality ($m \to 0$), quality gets lower and lower than its competitive quality.

Next, we can find optimal central quality q^{d*}:

$$\frac{\partial \pi}{\partial q^d} = (p-q^*)\, e^{mq^*+(1-m)q^d-p} \cdot (1-m) - e^{q^d-p^d} + \left(p^d - q^d\right) e^{q^d-p^d} = 0 \implies \tag{147}$$

$$= (p-q^*)\, e^{mq^*+(1-m)q^d-p-(q^d-p^d)}\,(1-m) - 1 + (p^d - q^d) = 0 \implies \tag{148}$$

$$\text{Since } q^* = p - \frac{1}{m} \implies \tag{149}$$

$$q^{d*} = e^{mp-1-mq^d+p^d-p}\left(\frac{1-m}{m}\right) - 1 + p^d \tag{150}$$

So, our goal is to show that the quality for the central good is higher than the value of the peripheral good, given their prices; that is, we wish to show that $\mu^{q^d} > \mu^q$:

$$e^{mp-1-mq^d+p^d-p} > -1 \text{ Multiply both sides by } \frac{1-m}{m} \tag{151}$$

$$e^{mp-1-mq^d+p^d-p}\left(\frac{1-m}{m}\right) > \frac{m-1}{m} \tag{152}$$

$$e^{mp-1-mq^d+p^d-p}\left(\frac{1-m}{m}\right) > 1 - \frac{1}{m} \tag{153}$$

$$e^{mp-1-mq^d+p^d-p}\left(\frac{1-m}{m}\right) - 1 > -\frac{1}{m} \iff \tag{154}$$

$$\mu^{q^d} > \mu^q \tag{155}$$

So, the intuition of greater quality for the central good given by the 'centrality term' holds literally when looking at quality exceeding prices.

Appendix B.2 Optimal choice q^* when q^d is exogenous

Above we have demonstrated how since $q^s = mq + (1-m)q^d$ introduces asymmetric demand dependence between q and q^d, then firms will endogenously choose to supply (relative to price) more quality to q^d. Suppose now that we are interested in the short-term decisions of a company that already has a reputation, given by some exogenous q^d. Suppose that we want to understand the differences in choices of q as a result of differences in companies' reputations. Thus, we have that

$$\pi(q^s(q, q^d, m)) = (p - bq)D(mq + (1 - m)q^d) \tag{156}$$

Then the optimal choice of quality is given by:

$$q^* = \frac{p}{b} - \frac{D(q^s)}{\partial D/\partial q^s} \cdot \frac{1}{m} \tag{157}$$

And our central comparative static is how the optimal choice of peripheral quality changes with a change in flagship product quality ('reputation'). Then we observe:

$$\begin{aligned}
\frac{\partial q^*}{\partial q^d} &= -\frac{\partial D/\partial q^d}{\partial D/\partial q^s} \cdot \frac{1}{m} + D(q^s) \cdot \frac{\partial^2 D/\partial q^s \partial q^d}{(\partial D/\partial q^s)^2} \cdot \frac{1}{m} && (158)\\
&= -\frac{\partial D/\partial q^s}{\partial D/\partial q^s} \cdot \frac{1-m}{m} + D(q^s) \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} \cdot \frac{1-m}{m} && (159)\\
&= \frac{1-m}{m}\left(D(q^s) \cdot \frac{\partial^2 D/\partial q^{s2}}{(\partial D/\partial q^s)^2} - 1\right) && (160)
\end{aligned}$$

And so if $D(q^s)$ is log-concave, then we have $\frac{\partial q^*}{\partial q^d} < 0$.

Appendix B.3 Shrinkflation

Appendix B.3.1 Jacobian proof

We need to demonstrate that the determinant of the Jacobian at the candidate solution p^* and q^* (from equations **??** and **??**) is nonzero. Rewriting the first order conditions:

$$G_1 \equiv p^* - bq^* + \frac{D(q^s - p)}{\partial D/\partial p} = 0 \tag{161}$$

$$G_2 \equiv q^* - \frac{p^*}{b} + \frac{D(q^s - p)}{\partial D/\partial q^s} \cdot \frac{1}{m} = 0 \tag{162}$$

Recall that the Jacobian is given by:

$$\mathcal{J} = \begin{bmatrix} \frac{\partial G_1}{\partial p} & \frac{\partial G_1}{\partial q} \\ \frac{\partial G_2}{\partial p} & \frac{\partial G_2}{\partial q} \end{bmatrix} \tag{163}$$

Our goal is to show that $\det \mathcal{J} \neq 0$. We will proceed by calculating the elements of $\mathcal{J}$ and then simplify them.

$$\frac{\partial G_1}{\partial p} = 1 + \frac{\partial D/\partial p}{\partial D/\partial p} - D() \cdot \frac{\partial^2 D/\partial p^2}{\left(\partial D/\partial p\right)^2} \tag{164}$$

$$= 1 - \Lambda \tag{165}$$

$$\frac{\partial G_2}{\partial p} = -\frac{1}{b} + \frac{\partial D/\partial p}{\partial D/\partial q^s} \cdot \frac{1}{m} - D() \cdot \frac{\partial^2 D/\partial q^s \partial p}{\left(\partial D/\partial q^s\right)^2} \cdot \frac{1}{m} \tag{166}$$

$$= -\frac{1}{b} - \frac{\partial D/\partial p}{\partial D/\partial p} \cdot \frac{1}{m} + D() \cdot \frac{\partial^2 D/\partial p^2}{\left(\partial D/\partial p\right)^2} \cdot \frac{1}{m} \tag{167}$$

$$= -\frac{1}{b} + \frac{\Lambda}{m} = \frac{\Lambda b - m}{mb} \tag{168}$$

$$\frac{\partial G_1}{\partial q} = -b + \frac{\partial D/\partial q}{\partial D/\partial p} - D() \frac{\partial^2 D/\partial p \partial q}{\left(\partial D/\partial p\right)^2} \tag{169}$$

$$= -b + \frac{\partial D/\partial q^s}{\partial D/\partial p} \cdot m - D() \frac{\partial^2 D/\partial p \partial q^s}{\left(\partial D/\partial p\right)^2} \cdot m \tag{170}$$

$$= -b - \frac{\partial D/\partial p}{\partial D/\partial p} \cdot m + D() \frac{\partial^2 D/\partial p^2}{\left(\partial D/\partial p\right)^2} \cdot m \tag{171}$$

$$= -b + \Lambda m \tag{172}$$

$$\frac{\partial G_2}{\partial q} = 1 + \frac{\partial D/\partial q}{\partial D/\partial q^s} \cdot \frac{1}{m} - D() \frac{\partial^2 D/\partial q^s \partial q}{\left(\partial D/\partial q^s\right)^2} \cdot \frac{1}{m} \tag{173}$$

$$= 1 + \frac{\partial D/\partial q^s}{\partial D/\partial q^s} \cdot \frac{m}{m} - D() \frac{\partial^2 D/\partial q^{s2}}{\left(\partial D/\partial q^s\right)^2} \cdot \frac{m}{m} \tag{174}$$

$$= 1 - \Lambda \tag{175}$$

Now, we can write $\mathcal{J}$:

$$\mathcal{J} = \begin{bmatrix} 1 - \Lambda & -b + \Lambda m \\ \frac{\Lambda b - m}{mb} & 1 - \Lambda \end{bmatrix} \tag{176}$$

And so we can demonstrate that $\det \mathcal{J} \neq 0$:

$$\det \mathcal{J} = \left(1 - \Lambda\right)^2 - \left(\frac{\Lambda b - m}{bm}\right)\left(\Lambda m - b\right) \tag{177}$$

$$= \frac{bm\left(1 - \Lambda\right)^2 - \left(\Lambda b - m\right)\left(\Lambda m - b\right)}{bm} \tag{178}$$

$$= \frac{bm\left(1 + \Lambda^2 - 2\Lambda\right) - \left(\Lambda^2 mb - \Lambda b^2 - m^2 \Lambda + bm\right)}{bm} \tag{179}$$

$$= \frac{bm + bm\Lambda^2 - 2\Lambda bm - \Lambda^2 mb + \Lambda b^2 + m^2 \Lambda - bm}{bm} \tag{180}$$

$$= \frac{\Lambda\left(b^2 + m^2 - 2bm\right)}{bm} \tag{181}$$

$$= \frac{\Lambda\left(b - m\right)^2}{bm} < 0 \tag{182}$$

So long as $\Lambda \neq 0$, which we have assumed via strict log-concavity (Λ¡0), we have that $\det \mathcal{J} < 0$. Note also that this result makes use of the assumption that $b > 1$ and the restriction that $m \in [0, 1]$.

Appendix B.3.2 Demand Derivatives

We will consider demand function that take as an argument some increasing function of consumer utility; that is, suppose that $D(f(q^s - p))$ where $f'(u) > 0$. Then we can write the following relations, which will be used to simplify the above expressions:

$$\frac{\partial D}{\partial q^s} = -\frac{\partial D}{\partial p} \tag{183}$$

$$\frac{\partial D}{\partial m} = \frac{\partial D}{\partial q^s} \cdot \left(q - q^d\right) \tag{184}$$

$$\frac{\partial^2 D}{\partial p^2} = \frac{\partial^2 D}{\partial q^{s2}} \tag{185}$$

$$\frac{\partial^2 D}{\partial p \partial q} = -m \cdot \frac{\partial^2 D}{\partial p^2} \tag{186}$$

$$\frac{\partial^2 D}{\partial q^s \partial p} = -\frac{\partial^2 D}{\partial p^2} \tag{187}$$

$$\frac{\partial^2 D}{\partial q^s \partial q} = \frac{\partial^2 D}{\partial q^{s2}} \cdot m \tag{188}$$

Finally, we can write the log-concavity of the demand function D in an argument x as the following:

$$\Lambda^x = D \cdot \frac{\partial^2 D / \partial x^2}{\left(\partial D / \partial x\right)^2} - 1 \tag{189}$$

So, Λ is the second derivative of the log of demand in price, or Λ^{q^s} for perceived quality. Given the way they enter the demand function, these curvatures will be equal, so we will simply denote $\Lambda^p = \Lambda^{q^s} \equiv \Lambda$. Then, we know from the Implicit Function Theorem that:

$$\begin{pmatrix} \frac{dp^*}{dm} \\ \frac{dq^*}{dm} \end{pmatrix} = -\mathcal{J}^{-1} \begin{pmatrix} \frac{\partial G_1}{\partial m} \\ \frac{\partial G_2}{\partial m} \end{pmatrix} \tag{190}$$

Appendix B.3.3 Finding $\frac{dq^*}{dm}$

Recall that the Jacobian is given by:

$$\mathcal{J} = \begin{bmatrix} \frac{\partial G_1}{\partial p} & \frac{\partial G_1}{\partial q} \\ \frac{\partial G_2}{\partial p} & \frac{\partial G_2}{\partial q} \end{bmatrix} = \begin{bmatrix} 1 - \Lambda & -b + m\Lambda \\ \frac{\Lambda b - m}{mb} & 1 - \Lambda \end{bmatrix} \tag{191}$$

And note that $\mathcal{J}$ is invertible under if $\det \mathcal{J} = \frac{\Lambda (b-m)^2}{bm} < 0 \neq 0$, which is guranteed under the assumption that $b > 1$, as $m \in (0, 1)$. Further, we know that the inverse of the Jacobian is:

$$\mathcal{J}^{-1} = \frac{1}{\det \mathcal{J}} \begin{pmatrix} 1 - \Lambda & b - m\Lambda \\ \frac{-\Lambda b + m}{mb} & 1 - \Lambda \end{pmatrix} \tag{192}$$

$$= \frac{bm}{\Lambda \left(b - m\right)^2} \begin{pmatrix} 1 - \Lambda & b - m\Lambda \\ \frac{-\Lambda b + m}{mb} & 1 - \Lambda \end{pmatrix} \tag{193}$$

Further, we have that:

$$\frac{\partial G_1}{\partial m} = \left(q - q^d\right)\Lambda \tag{194}$$

$$\frac{\partial G_2}{\partial m} = \left(q - q^d\right)(-\Lambda) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \tag{195}$$

So, we can set up the IFT as:

$$\begin{pmatrix} \frac{dp^*}{dm} \\ \frac{dq^*}{dm} \end{pmatrix} = -\mathcal{J}^{-1} \begin{pmatrix} \frac{\partial G_1}{\partial m} \\ \frac{\partial G_2}{\partial m} \end{pmatrix} \tag{196}$$

$$= -\frac{bm}{\Lambda (b-m)^2} \begin{pmatrix} 1-\Lambda & b - m\Lambda \\ \frac{-\Lambda b + m}{mb} & 1-\Lambda \end{pmatrix} \begin{pmatrix} \left(q - q^d\right)\Lambda \\ \left(q - q^d\right)(-\Lambda) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1}{m^2} \end{pmatrix} \tag{197}$$

$$= -\frac{bm}{\Lambda (b-m)^2} \begin{pmatrix} (1-\Lambda)\Lambda\left(q - q^d\right) + (b - m\Lambda)\left[-\Lambda\left(q - q^d\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1}{m^2}\right] \\ \frac{-\Lambda b + mb}{mb} \cdot \left(q - q^d\right)\Lambda + (1-\Lambda)\left[\left(q - q^d\right)(-\Lambda) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1}{m^2}\right] \end{pmatrix} \tag{198}$$

Now, let's restrict attention to $\frac{dq^*}{dm}$ and denote $\gamma(\Lambda) \equiv -\frac{bm}{\Lambda(b-m)^2}$ and note that $\gamma(\Lambda) > 0 \iff \Lambda < 0$:

$$\frac{dq^*}{dm} = -\frac{bm}{\Lambda (b-m)^2} \left[\frac{-\Lambda b + mb}{mb} \cdot \left(q - q^d\right)\Lambda + (1-\Lambda)\left[\left(q - q^d\right)(-\Lambda) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1}{m^2}\right]\right] \tag{199}$$

$$= \gamma(\Lambda)\left[\left(q - q^d\right)\Lambda\left(\frac{-\Lambda b + mb}{mb} - (1-\Lambda)\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1-\Lambda}{m^2}\right] \tag{200}$$

$$= \gamma(\Lambda)\left[\left(q - q^d\right)\Lambda\left(\frac{-\Lambda b + mb - mb(1-\Lambda)}{mb}\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1-\Lambda}{m^2}\right] \tag{201}$$

$$= \gamma(\Lambda)\left[\left(q - q^d\right)\Lambda\left(\frac{-\Lambda b + \Lambda mb}{mb}\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1-\Lambda}{m^2}\right] \tag{202}$$

$$= \gamma(\Lambda)\left[\left(q - q^d\right)\Lambda^2\left(\frac{mb - b}{mb}\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1-\Lambda}{m^2}\right] \tag{203}$$

$$= \gamma(\Lambda)\left[\left(q^* - q^d\right)\Lambda^2\left(\frac{b(m-1)}{mb}\right) - \frac{D()}{\partial D/\partial q^s} \cdot \frac{1-\Lambda}{m^2}\right] \text{ at optimum} \tag{204}$$

Appendix C Random Utility Models and Perloff-Salop Markups

In the main text, we refer to Perloff-Salop markups as simply markups, but in reference to Perloff and Salop (1985), Gabaix et al. (2016) refer to them using the authors' names.

Appendix C.1 Review of Random Utility Models and Perloff-Salop Markups

The demand function of firm i is the probability that the consumer's surplus at firm $i, X_i - p_i$, exceeds the consumer's surplus at all other firms,

$$D\left(p_1, \ldots, p_n; i\right) = \mathbb{P}\left(X_i - p_i \geq \max_{j \neq i}\left\{X_j - p_j\right\}\right)$$

Using $D\left(p_i, p; n\right)$ to denote the demand for good i at price p_i when all other firms set price p and using $D_1\left(p_i, p; n\right)$ to denote $\partial D\left(p_i, p; n\right) / \partial p_i$, we may calculate

$$\begin{aligned}
D\left(p_i, p; n\right) &= \int_{w_l}^{w_u} f(x) F^{n-1}\left(x - p_i + p\right) dx \\
D_1\left(p_i, p; n\right) &= -(n-1) \int_{w_l}^{w_u} f(x) f\left(x - p_i + p\right) F^{n-2}\left(x - p_i + p\right) dx.
\end{aligned}$$

This then means that for any vector of prices $\overline{p}$, we have the following markups for firm i:

$$p_i^* - c = -\frac{\int_{w_l}^{w_u} f(x) F^{n-1}\left(x - p_i + p\right) dx}{(n-1) \int_{w_l}^{w_u} f(x) f\left(x - p_i + p\right) F^{n-2}\left(x - p_i + p\right) dx} \tag{205}$$

For tractability, we consider only symmetric equilibria; that is, equilibria where $p_i = p$. Then, expressions are simplified:

$$\begin{aligned}
D(p, p; n) &= \int_{w_l}^{w_u} f(x) F^{n-1}(x) dx = 1/n, \\
D_1(p, p; n) &= -(n-1) \int_{w_l}^{w_u} f^2(x) F^{n-2}(x) dx.
\end{aligned}$$

It follows that the Perloff-Salop markup μ_n^{PS} is

$$p - c = -\frac{D(p, p; n)}{D_1(p, p; n)} = \frac{1}{n(n-1) \int_{w_l}^{w_u} f^2(x) F^{n-2}(x) dx}.$$

Appendix C.2 Proofs Involving Perloff-Salop Markups

Appendix C.2.1 Behavioral markups: μ_s^{PS}

The formulation of the behavioral markups is simplified because m does not enter into the integral because all prices are the same in the symmetric equilibrium. And because of the linearity of integrals, the approximations of markups in Gabaix et al. (2016) (which are approximations of the integral at the bottom of the equation 32 without the m term), the constant m does not enter the approximation of asymptotic behavior.

Appendix C.2.2 Markup elasticity of number of firms

We seek to calculate the elasticity of μ^b to n. So, note the following:

$$\frac{n}{\mu_n^b} = n \cdot m \cdot n(n-1) \int_{w_l}^{w_u} f(x)^2 f(x)\, F^{n-2}(x)\, dx \tag{206}$$

$$\frac{d\mu^b}{dn} = \frac{d}{dn}\left[\left(m \cdot n \cdot (n-1) \int f^2(x) F^{n-2}(x) dx\right)^{-1}\right] \tag{207}$$

$$= -\left[\frac{m \cdot (2n-1) \int f^2(x) F^{n-2}(x) dx + m \cdot n(n-1) \int f^2(x) F^{n-2}(x) \log F(x) dx}{\left(m \cdot n \cdot (n-1) \int f^2(x) F^{n-2}(x) dx\right)^2}\right] \implies \tag{208}$$

$$\frac{n}{\mu_n^b}\frac{d\mu_n^b}{dn} = -\frac{1}{m}\left[\frac{2n-1}{n-1} + \frac{n \int f^2(x) F^{n-2}(x) \log F(x) dx}{\int f^2(x) F^{n-2}(x) dx}\right] = \frac{1}{m} \cdot \frac{n}{\mu_n^r}\frac{d\mu_n^r}{dn} \tag{209}$$

The result then follows from the invocation of the invocation of Theorem 1,3, and Lemma A1.3 as in the Appendix of Gabaix et al. (2016).

Appendix D Cognitive Cost Pass-Through

Appendix D.1 Second Order Conditions

Whereas before we guaranteed quasi-concavity of the profit function by assuming that $\frac{1}{D(p^s)}$ was convex in p^s ($\iff \Lambda < 1$), we now will also have a new second order condition which is needed for the quasi-concavity of profit. Either the function is monotone increasing (in which case it is quasi-concave) or every first order condition is a maximum. This means that we need that:

$$\frac{\partial^2 \pi(p^*)}{\partial p^2} = \frac{\partial D(p^s(p^*))}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s(p^*))}{\partial D(p^s(p^*))/\partial p^s} \cdot \frac{\partial^2 p^s(p^*)/\partial p^2}{\left(\partial p^s(p^*)/\partial p\right)^2}\right] < 0 \tag{210}$$

$$= \frac{\partial D(p^s(p^*))}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s(p^*))}{\partial D(p^s(p^*))/\partial p^s} \cdot \frac{2 \cdot \frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2} \cdot \left(p - p^d\right)}{\left(\partial p^s(p^*)/\partial p\right)^2}\right] < 0 \tag{211}$$

Recall that we want the term in brackets to be positive (since $\frac{\partial D(p^s)}{\partial D/\partial p^s} < 0$ and $\frac{\partial p^s(p^*)}{\partial p^s} > 0$). We can use the old necessary and sufficient condition that $\Lambda < 1$, but we can combine it with a new sufficient (but not necessary) condition, namely that $\frac{\partial^2 p^s(p^*)}{\partial p^2} < 0$ at the optimum. This is not immediately easy to interpret, but adding a stronger set of restrictions makes the situation quite clear. Suppose instead of adopting these sufficient, local conditions, we instead require these restrictions to hold globally. That is, suppose that we require that p^s be strictly increasing and weakly convex for all true prices p.

Although the requirement that $\frac{\partial p^s(p^*)}{\partial p} > 0$ is a local and necessary condition, whereas $\frac{\partial^2 p^s(p^*)}{\partial p^2} < 0$ is a local and *sufficient* condition (note that we only need $1 - \Lambda - D(p^s)\frac{\partial^2 p^s}{(\partial p^2/\partial p)^2} < 0$), an intuitive explanation of exactly the formualtion of p^s – and therefore of $m(p, \kappa)$ – that is a *globally* increasing and convex function is easy to interpret. Firstly, consumers should perceive price as increasing when the true price is increasing, but they should be less sensitive to changes at smaller values of price than at larger changes in price. This is somewhat intuitive – their perception of price will not change so much for small prices, but will change a lot for larger values. This coheres with the fact that consumers prefer strictly lower prices, and so will be more sensitive to changes at larger price levels.

Appendix D.2 Deriving $\frac{dp^*}{d\kappa}$

We can also more fully derive the exact expression for cognitive cost pass-through by invoking the Implicit Function Theorem. Thus if we wish to compute $\frac{\partial p^*}{\partial \kappa}$, we can use the implicit function theorem:

$$\frac{\partial p^*}{\partial \kappa} = -\frac{\partial^2 \pi / \partial p \partial \kappa}{\partial^2 \pi / \partial p^2} \tag{212}$$

This requires us to compute both derivatives. Beginning with the denominator:

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial p^2} &= \frac{\partial}{\partial p}\left[D(p^s) + (p-c)\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right] && (213)\\
&= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p} + \frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p} + (p-c)\frac{\partial^2 D(p^s)}{\partial p^{s2}} \cdot \left(\frac{\partial p^s}{\partial p}\right)^2 + (p-c)\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial^2 p^s}{\partial p^2} && (214)\\
&= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[2 + (p-c)\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial p^s}{\partial p} + (p-c)\frac{\partial^2 p^s/\partial p^2}{\partial p^s/\partial p}\right] && (215)\\
&\text{At optimum } p^* - c = -D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1} \implies && (216)\\
\frac{\partial^2 \pi}{\partial p^2} &= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[2 - D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1}\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial p^s}{\partial p} - D(p^s)\left[\frac{\partial D(p^s)}{\partial p^s} \cdot \frac{\partial p^s}{\partial p}\right]^{-1}\frac{\partial^2 p^s/\partial p^2}{\partial p^s/\partial p}\right] && (217)\\
&= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[2 - D(p^s)\frac{\partial^2 D(p^s)/\partial p^{s2}}{\left(\partial D(p^s)/\partial p^s\right)^2} - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{\left(\partial p^s/\partial p\right)^2}\right] && (218)\\
&= \frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s} \cdot \frac{\partial^2 p^s/\partial p^2}{\left(\partial p^s/\partial p\right)^2}\right] && (219)
\end{aligned}$$

Next, we can compute $\frac{\partial^2 \pi}{\partial p \partial \kappa}$:

$$
\begin{aligned}
\frac{\partial^2 \pi}{\partial p \partial \kappa} &= \frac{\partial}{\partial \kappa}\left[D(p^s) + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right] && (220)\\
&= \frac{\partial D(p^s)}{\partial \kappa} + (p-c)\,\frac{\partial^2 D(p^s)}{\partial p^s \partial \kappa}\cdot\frac{\partial p^s}{\partial p} + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa} && (221)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial \kappa} + (p-c)\,\frac{\partial^2 D(p^s)}{\partial p^{s2}}\cdot\frac{\partial p^s}{\partial \kappa}\cdot\frac{\partial p^s}{\partial p} + (p-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa} && (222)\\
&= \frac{\partial D(p^s)}{\partial p^s}\left[\frac{\partial p^s}{\partial \kappa}\left[1 + (p-c)\,\frac{\partial^2 D(p^s)/\partial p^{s2}}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right] + (p-c)\cdot\frac{\partial^2 p^s}{\partial p \partial \kappa}\right] && (223)\\
&\text{At optimum } p^* - c = -D(p^s)\cdot\left[\frac{\partial D(p^s)}{\partial p^s}\cdot\frac{\partial p^s}{\partial p}\right]^{-1} \implies && (224)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\left[1 - D(p^s)\cdot\frac{\partial^2 D(p^s)/\partial p^{s2}}{\left(\partial D(p^s)/\partial p^s\right)^2}\right] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right] && (225)\\
&= \frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\left[-\Lambda\right] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right] && (226)
\end{aligned}
$$

Plugging in these equations for the Implicit Function Theorem gives:

$$
\begin{aligned}
\frac{\partial p^*}{\partial \kappa} &= -\frac{\partial^2 \pi/\partial p \partial \kappa}{\partial^2 \pi/\partial p^2} && (227)\\
&= -\frac{\frac{\partial D(p^s)}{\partial p^s}\cdot\left[\frac{\partial p^s}{\partial \kappa}\left[-\Lambda\right] - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial D(p^s)}{\partial p^s}\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p^2}{\left(\partial p^s/\partial p\right)^2}\right]} && (228)\\
&= \frac{\left[\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial p^s}{\partial p}\left[1 - \Lambda - \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p^2}{\left(\partial p^s/\partial p\right)^2}\right]} && (229)
\end{aligned}
$$

Next, we note the following derivatives:

$$
\begin{aligned}
\frac{\partial p^s}{\partial \kappa} &= \frac{\partial m}{\partial \kappa}\cdot\left(p - p^d\right) && (230)\\
\frac{\partial p^s}{\partial p} &= m(\kappa, p) + \frac{\partial m}{\partial p}\cdot\left(p - p^d\right) && (231)\\
\frac{\partial^2 p^s}{\partial \kappa \partial p} &= \frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(p - p^d\right) + \frac{\partial m}{\partial \kappa} && (232)\\
\frac{\partial^2 p^s}{\partial p^2} &= \frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2}\cdot\left(p - p^d\right) + \frac{\partial m}{\partial p} && (233)\\
&= 2\cdot\frac{\partial m}{\partial p} + \frac{\partial^2 m}{\partial p^2}\cdot\left(p - p^d\right) && (234)
\end{aligned}
$$

Plugging these in we get:

$$\frac{\partial p^*}{\partial \kappa} = \frac{\left[\frac{\partial p^s}{\partial \kappa}\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p \partial \kappa}{\partial p^s/\partial p}\right]}{\frac{\partial p^s}{\partial p}\left[1-\Lambda-\frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\frac{\partial^2 p^s/\partial p^2}{(\partial p^s/\partial p)^2}\right]} \tag{235}$$

$$= \frac{\frac{\partial m}{\partial \kappa}\cdot\left(p-p^d\right)\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\left(\frac{\frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(p-p^d\right)+\frac{\partial m}{\partial \kappa}}{m(\kappa,p)+\frac{\partial m}{\partial p}\cdot\left(p-p^d\right)}\right)}{\left(m(\kappa,p)+\frac{\partial m}{\partial p}\left(p-p^d\right)\right)\left[1-\Lambda-\frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\left(\frac{2\cdot\frac{\partial m}{\partial p}+\frac{\partial^2 m}{\partial p^2}\cdot\left(p-p^d\right)}{\left(m(\kappa,p)+\frac{\partial m}{\partial p}\cdot\left(p-p^d\right)\right)^2}\right)\right]} \tag{236}$$

Given that we know that the denominator of $\frac{\partial p^*}{\partial \kappa}$ is positive, then we can restrict attention to the denominator, noting that:

$$\text{sign}\left(\frac{\partial p^*}{\partial \kappa}\right) = \text{sign}\left(\frac{\partial m}{\partial \kappa}\cdot\left(p-p^d\right)\Lambda + \frac{D(p^s)}{\partial D(p^s)/\partial p^s}\cdot\left(\frac{\frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(p-p^d\right)+\frac{\partial m}{\partial \kappa}}{m(\kappa,p)+\frac{\partial m}{\partial p}\cdot(p-p^d)}\right)\right) \tag{237}$$

We can break down this equation a bit more. First, let's denote markups as:

$$\mu \equiv -\frac{D(p^s)}{\partial D/\partial p^s}\cdot\left[\frac{\partial p^s}{\partial p}\right]^{-1} \tag{238}$$

Then we can rewrite the numerator of cognitive cost pass-through as:

$$\frac{\partial p^*}{\partial \kappa} = \frac{\partial m}{\partial \kappa}\cdot\left(p^*-p^d\right)\Lambda - \mu\left(\frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(p^*-p^d\right)+\frac{\partial m}{\partial \kappa}\right) \tag{239}$$

$$= \frac{\partial m}{\partial \kappa}\cdot\left(c+\mu-p^d\right)\Lambda - \mu\left(\frac{\partial^2 m}{\partial \kappa \partial p}\cdot\left(c+\mu-p^d\right)+\frac{\partial m}{\partial \kappa}\right) \tag{240}$$

$$= \frac{\partial m}{\partial \kappa}\cdot\mu\left(\Lambda-1\right) + \left(c-p^d\right)\left[\Lambda\cdot\frac{\partial m}{\partial \kappa} - \mu\frac{\partial^2 m}{\partial \kappa \partial p}\right] - \mu^2\frac{\partial^2 m}{\partial \kappa \partial p} - \mu\frac{\partial m}{\partial \kappa} \tag{241}$$

Note that the first value is always positive – there is an unambiguous, upward pressure on prices when consumers can see price less (higher κ). The other terms depend on the functional form of $m(\kappa, p)$ and whether or not expectations are feasible (whether or not $p^d > c$).

Appendix D.3 Example of cognitive cost pass-through

Firstly, we want to compute $\frac{\partial p^s}{\partial \kappa}$, Λ, $\frac{\partial^2 p^s}{\partial p \partial \kappa}$. Let's begin with the second two, as the first one is visualized in the figure. Recall that we are using the following demand function:

$$D_1 = e^{-\left(m\cdot\left(p-p^d\right)+p^d\right)^{\psi}} \tag{242}$$

And we want to find the value of

$$\Lambda_1 = D_1\cdot\frac{\partial^2 D_1/\partial p^{s2}}{\left(\partial D_1/\partial p^s\right)^2} - 1 \tag{243}$$

We first note then the partial derivative:

$$\frac{\partial D}{\partial p^s} = \frac{\partial}{\partial p^s}\left[e^{-(p^s)^\psi}\right] \tag{244}$$

$$= -\psi\,(p^s)^{\psi-1}\,e^{-(p^s)^\psi} \implies \tag{245}$$

$$\frac{\partial^2 D(p^s)}{\partial p^{s2}} = \frac{\partial}{\partial p^s}\left[-\psi\,(p^s)^{\psi-1}\,e^{-(p^s)^\psi}\right] \tag{246}$$

$$= -\psi\,(\psi-1)\,(p^s)^{\psi-2}\,e^{-(p^s)^\psi} + \left(\psi\,(p^s)^{\psi-1}\right)^2 e^{-(p^s)^\psi} \tag{247}$$

$$= \psi\,(p^s)^{\psi-2}\,e^{-(p^s)^\psi}\cdot\left(\psi\left((p^s)^\psi-1\right)+1\right) \tag{248}$$

Therefore log-curvature is given by:

$$\Lambda_1 = e^{-(p^s)^\psi}\cdot\frac{\psi\,(p^s)^{\psi-2}\,e^{-(p^s)^\psi}\cdot\left(\psi\left((p^s)^\psi-1\right)+1\right)}{\left(-\psi\,(p^s)^{\psi-1}\,e^{-(p^s)^\psi}\right)^2} - 1 \tag{249}$$

$$= \frac{\psi\,(p^s)^{\psi-2}\,e^{-2(p^s)^\psi}\cdot\left(\psi\left((p^s)^\psi-1\right)+1\right)}{\psi^2\,(p^s)^{2\psi-2}\,e^{-2(p^s)^\psi}} - 1 \tag{250}$$

$$= \frac{\psi\left((p^s)^\psi-1\right)+1}{\psi\,(p^s)^\psi} - 1 \tag{251}$$

$$= \frac{\psi\left((p^s)^\psi-1\right)+1-\psi\,(p^s)^\psi}{\psi\,(p^s)^\psi} \tag{252}$$

$$= \frac{1-\psi}{\psi\,(p^s)^\psi} < 0\ \forall p^* \tag{253}$$

So we have that this demand function is log-concave for all optimal prices (and for all prices for which demand is positive).

Then, write out $\frac{\partial p^s}{\partial p}$, and then write out $\frac{\partial^2 p^s}{\partial p \partial \kappa}$. Then explain how the main point is that having more cognitive cost is increasing the perception of price as well as the sensitivity to price changes, which forces the monopolist to decrease their price. We'd also like to compute how the perception of price changes with the true price. Given that

$$m_1 = \max\left\{0, 1-\frac{\kappa}{(p-p^d)^2}\right\} \tag{254}$$

Then for m_1 we have that

$$p_1^s(p,p^d,m_1) = \begin{cases} p^d & \left|p-p^d\right| \le \sqrt{\kappa} \\ \left(1-\frac{\kappa}{(p-p^d)^2}\right)\left(p-p^d\right)+p^d & \text{otherwise} \end{cases} \tag{255}$$

Therefore we see that in the first region, p^s is inelastic to change in price, and so $\frac{\partial p^s}{\partial p} = 0$, whereas in the second region, the change of perception in true price is:

$$\frac{\partial p^s}{\partial p} = \frac{\partial}{\partial p}\left[\left(1-\frac{\kappa}{\left(p-p^d\right)^2}\right)\left(p-p^d\right)+p^d\right] \tag{256}$$

$$= \frac{2\kappa}{\left(p-p^d\right)^3}\cdot\left(p-p^d\right)+\left(1-\frac{\kappa}{\left(p-p^d\right)^2}\right) \tag{257}$$

$$= 1+\frac{\kappa}{\left(p-p^d\right)^2} > 0\ \forall p \tag{258}$$

Therefore we see that the perception of price is increasing in price always. We can also then immediately see that an increase in cognitive cost amplifies the sensitivity to price changes:

$$\frac{\partial^2 p^s}{\partial p \partial \kappa} = \frac{1}{\left(p-p^d\right)^2} > 0 \tag{259}$$

We can finally see that $\frac{\partial p^s}{\partial \kappa}$ is given by 0 or:

$$\frac{\partial p^s}{\partial \kappa} = \frac{\partial}{\partial \kappa}\left[m(p,\kappa)\cdot\left(p-p^d\right)+p^d\right] \tag{260}$$

$$= \frac{\partial m}{\partial \kappa}\cdot\left(p-p^d\right) \tag{261}$$

$$= -\frac{1}{\left(p-p^d\right)^2}\cdot\left(p-p^d\right) \tag{262}$$

$$= \frac{1}{p^d-p} \tag{263}$$

Which is positive for all $p < p^d$.

Appendix E Profit and Attention

Appendix E.1 $\frac{\partial \pi^*}{\partial m}$ for $D = \left(p^s\right)^{-\psi}$

$$\pi^* = \left(p^*-c\right)\left(mp^*+(1-m)p^d\right)^{-\psi} \implies \tag{264}$$

$$\frac{\partial \pi^*}{\partial m} = \frac{\partial p^*}{\partial m}\cdot\left(mp^*+(1-m)p^d\right)^{-\psi}-\psi\left(p^*-c\right)\left(mp^*+(1-m)p^d\right)^{-\psi-1}\cdot\left(m\frac{\partial p^*}{\partial m}+p^*-p^d\right) \tag{265}$$

$$= \frac{1}{\left(p^s\right)^{\psi+1}}\left[\frac{\partial p^*}{\partial m}\cdot\left(mp^*+(1-m)p^d\right)-\psi\left(p^*-c\right)\cdot\left(m\frac{\partial p^*}{\partial m}+p^*-p^d\right)\right] \tag{266}$$

Next, recall the following facts for $D = \left(p^s\right)^{-\psi}$:

$$p^* = \frac{\psi cm+(1-m)\,p^d}{m\left(\psi-1\right)} \tag{267}$$

$$\frac{\partial p^*}{\partial m} = \frac{-p^d}{m^2\left(\psi-1\right)} \tag{268}$$

The expressions will be too long, so let us first deal with the first term, $\frac{\partial p^*}{\partial m} \cdot \left(mp^* + (1-m)p^d\right)$. Plugging in these two values to the first term gives:

$$\frac{-p^d}{m^2\left(\psi-1\right)} \cdot \left(m\left(\frac{\psi cm+\left(1-m\right)p^d}{m\left(\psi-1\right)}\right)+(1-m)p^d\right) = \tag{269}$$

$$\frac{-p^d}{m^2\left(\psi-1\right)} \cdot \left(\frac{\psi cm+\left(1-m\right)p^d+\left(\psi-1\right)\left(1-m\right)p^d}{\left(\psi-1\right)}\right) = \tag{270}$$

$$\frac{-p^d}{m^2\left(\psi-1\right)} \cdot \left(\frac{\psi cm+p^d-mp^d+\psi p^d-\psi mp^d-p^d+mp^d}{\left(\psi-1\right)}\right) = \tag{271}$$

$$\frac{-p^d}{m^2\left(\psi-1\right)} \cdot \left(\frac{\psi cm+\psi p^d-\psi mp^d}{\left(\psi-1\right)}\right) = \tag{272}$$

$$\frac{-p^d}{m^2\left(\psi-1\right)} \cdot \left(\frac{\psi\left(m\left(c-p^d\right)+p^d\right)}{\left(\psi-1\right)}\right) = \tag{273}$$

$$\left(\frac{\psi p^d\left(m\left(p^d-c\right)-p^d\right)}{\left(m\left(\psi-1\right)\right)^2}\right) \tag{274}$$

Now we can deal with the second term of the original equation, $-\psi\left(p^*-c\right) \cdot \left(m\frac{\partial p^*}{\partial m}+p^*-p^d\right)$

$$-\psi\left(\frac{\psi cm+\left(1-m\right)p^d}{m\left(\psi-1\right)}-c\right) \cdot \left(m \cdot \left(\frac{-p^d}{m^2\left(\psi-1\right)}\right)+\frac{\psi cm+\left(1-m\right)p^d}{m\left(\psi-1\right)}-p^d\right) = \tag{275}$$

$$-\psi\left(\frac{\psi cm+\left(1-m\right)p^d-cm\left(\psi-1\right)}{m\left(\psi-1\right)}\right) \cdot \left(\frac{-p^d}{m\left(\psi-1\right)}+\frac{\psi cm+\left(1-m\right)p^d}{m\left(\psi-1\right)}-p^d\right) = \tag{276}$$

$$-\psi\left(\frac{\left(1-m\right)p^d+cm}{m\left(\psi-1\right)}\right) \cdot \left(\frac{-p^d}{m\left(\psi-1\right)}+\frac{\psi cm+\left(1-m\right)p^d}{m\left(\psi-1\right)}-p^d\right) = \tag{277}$$

$$-\psi\left(\frac{\left(1-m\right)p^d+cm}{m\left(\psi-1\right)}\right) \cdot \left(\frac{-p^d+\psi cm+\left(1-m\right)p^d-mp^d\left(\psi-1\right)}{m\left(\psi-1\right)}\right) = \tag{278}$$

$$-\psi\left(\frac{\left(1-m\right)p^d+cm}{m\left(\psi-1\right)}\right) \cdot \left(\frac{\psi cm-m\psi p^d}{m\left(\psi-1\right)}\right) = \tag{279}$$

$$-\psi^2\left(\frac{\left(1-m\right)p^d+cm}{m\left(\psi-1\right)}\right) \cdot \left(\frac{m\left(c-p^d\right)}{m\left(\psi-1\right)}\right) = \tag{280}$$

$$-\psi^2\left(\frac{\left(\left(1-m\right)p^d+cm\right) \cdot \left(m\left(c-p^d\right)\right)}{\left(m\left(\psi-1\right)\right)^2}\right) \tag{281}$$

Now we can combine the left and right terms to get $\frac{\partial \pi^*}{\partial m}$:

$$\frac{\partial \pi^*}{\partial m} = (p^s)^{-\psi-1}\left[\left(\frac{\psi p^d\left(m\left(p^d-c\right)-p^d\right)}{\left(m\left(\psi-1\right)\right)^2}\right)-\psi^2\left(\frac{\left(\left(1-m\right)p^d+cm\right)\cdot\left(m\left(c-p^d\right)\right)}{\left(m\left(\psi-1\right)\right)^2}\right)\right] \tag{282}$$

$$= \frac{(p^s)^{-\psi-1}}{\left(m\left(\psi-1\right)\right)^2}\left[\psi p^d\left(m\left(p^d-c\right)-p^d\right)-\psi^2\left(\left(1-m\right)p^d+cm\right)\cdot\left(m\left(c-p^d\right)\right)\right] \tag{283}$$

$$= \frac{\psi\,(p^s)^{-\psi-1}}{\left(m\left(\psi-1\right)\right)^2}\left[p^d\left(m\left(p^d-c\right)-p^d\right)-\psi\left(\left(1-m\right)p^d+cm\right)\cdot\left(m\left(c-p^d\right)\right)\right] \tag{284}$$

$$= -\frac{\psi\,(p^s)^{-\psi-1}}{\left(m\left(\psi-1\right)\right)^2}\left[p^d\left(m\left(c-p^d\right)+p^d\right)+.\psi\left(\left(1-m\right)p^d+cm\right)\cdot\left(m\left(c-p^d\right)\right)\right] \tag{285}$$

$$= \frac{\psi\,(p^s)^{-\psi-1}}{\left(m\left(\psi-1\right)\right)^2}\left[-\left(cm+p^d-mp^d\right)\left(p^d+cm\psi-mp^d\psi\right)\right] \tag{286}$$

$$= \underbrace{-\frac{\psi\,(p^s)^{-\psi-1}}{\left(m\left(\psi-1\right)\right)^2}}_{<0}\left[\underbrace{\left(cm+\left(1-m\right)p^d\right)}_{>0}\left(p^d+m\psi\left(c-p^d\right)\right)\right] \tag{287}$$

So, we arrive at the following: the sign of $\frac{\partial \pi^*}{\partial m}$ depends entirely on the sign of $p^d + m\psi(c - p^d)$. Therefore, we can represent the behavior of the value function in m over the values of m:

$$\begin{cases} \frac{\partial \pi^*}{\partial m} > 0, & \text{if } m > \frac{p^d}{\psi\left(p^d-c\right)} \\ \frac{\partial \pi^*}{\partial m} < 0, & \text{if } m < \frac{p^d}{\psi\left(p^d-c\right)} \end{cases} \tag{288}$$

Appendix E.2 When a little inattention helps the firm

We can look at local deviations to inattention from full rationality to observe the impact on optimal profits.

$$\pi^* = (p^* - c)\, D(mp^* + (1-m)p^d) \implies \tag{289}$$

$$-\frac{\partial \pi^*}{\partial m} = -\left[\frac{\partial p^*}{\partial m}D(mp^* + (1-m)p^d) + (p^*-c)\,\frac{\partial D(p^s)}{\partial p^s}\cdot\left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)\right] \tag{290}$$

Note that at the optimal price, the following relation holds:

$$p^* - c = -\frac{D(p^s)}{\partial D/\partial p^s}\cdot\frac{1}{m} \tag{291}$$

And so plugging this in to $-\frac{\partial \pi^*}{\partial m}$ gives:

$$-\frac{\partial \pi^*}{\partial m} = -\left[\frac{\partial p^*}{\partial m} D(mp^* + (1-m)p^d) + \left(-\frac{D(p^s)}{\partial D/\partial p^s} \cdot \frac{1}{m}\right) \frac{\partial D(p^s)}{\partial p^s} \cdot \left(p^* - p^d + m\frac{\partial p^*}{\partial m}\right)\right] \tag{292}$$

$$= -\left[\frac{\partial p^*}{\partial m} \cdot D(p^s) - D(p^s) \cdot \left(\frac{p^* - p^d}{m} + \frac{\partial p^*}{\partial m}\right)\right] \tag{293}$$

$$= -D(p^s)\left[\frac{\partial p^*}{\partial m} - \frac{p^* - p^d}{m} - \frac{\partial p^*}{\partial m}\right] \text{Now evaluating at } m = 1 \tag{294}$$

$$= D(p^s) \cdot \left[p_r^* - p^d\right] \tag{295}$$

Appendix F Monopoly with Sparse Max

Appendix F.1 Calculating $\frac{\partial Q^*}{\partial p}$ and u_{QQ}

We need to calculate these two objects so that we can compute m^*. Recall from before that $Q^* = p^{-\psi}$, so this means that $\frac{\partial Q^*}{\partial p} = -\psi\,(p)^{-\psi-1}$. Next, we wish to evaluate $u_{QQ}|_{Q^*}$.

Next, we want to compute $u_{QQ}|_{Q^*} = \frac{\partial^2 u}{\partial Q^2}|_{Q^*}$:

$$\frac{\partial}{\partial Q}\left[y + \frac{Q^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - pQ\right] = Q^{-\frac{1}{\psi}} - p \implies \tag{296}$$

$$\frac{\partial^2 u}{\partial Q^2} = -\frac{1}{\psi} Q^{-\frac{1}{\psi}-1} \text{ now evaluating at } Q^* = p^{-\psi}\text{:} \tag{297}$$

$$\frac{\partial^2 u}{\partial Q^2}|_{Q^*} = -\frac{1}{\psi}\left(p^{-\psi}\right)^{-\frac{1}{\psi}-1} \tag{298}$$

$$= -\frac{1}{\psi}\left(p^{-\psi}\right)^{-\left(\frac{1+\psi}{\psi}\right)} \tag{299}$$

$$= -\frac{p^{1+\psi}}{\psi} \tag{300}$$

Appendix F.2 Calculating m^*

Now that we have the necessary quantities calculated, we can recall that optimal attention comes from the following equation:

$$m^* = \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{p^2 \left| \left(\frac{\partial Q^*}{\partial p} \right)^2 u_{QQ} \right|}{\kappa} \right) + m \tag{301}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{p^2 \left(-\psi(p)^{-(1+\psi)} \right)^2 \left(\frac{p^{1+\psi}}{\psi} \right)}{\kappa} \right) + m \tag{302}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{p^2 \psi^2 (p)^{-2(1+\psi)} \left(\frac{p^{1+\psi}}{\psi} \right)}{\kappa} \right) + m \tag{303}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{\psi}{\kappa} \right) \left(p^2 p^{-2(1+\psi)} p^{1+\psi} \right) + m \tag{304}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{\psi}{\kappa} \right) \left(p^{2-2(1+\psi)+1+\psi} \right) + m \tag{305}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{\psi}{\kappa} \right) \left(p^{2-2(1+\psi)+1+\psi} \right) + m \tag{306}$$

$$= \underset{m}{\text{argmin}} \, \frac{1}{2}(m-1)^2 \left(\frac{\psi}{\kappa} \right) \left(p^{1-\psi} \right) + m \implies \tag{307}$$

$$= (m-1) \left(\frac{\psi \left(p^{1-\psi} \right)}{\kappa} \right) + 1 = 0 \implies \tag{308}$$

$$m^* = 1 - \frac{\kappa}{\psi p^{1-\psi}} = 1 - \frac{\kappa p^{\psi - 1}}{\psi} \tag{309}$$

The fact that we have attention decreasing in price is a huge problem. This must mean that there is an error somewhere. It doesn't look like any of the calculations themselves are wrong (and the relationship with ψ and κ looks right), so something must be wrong in the set up?. Further, the problem can't be a negative sign missing (I don't think); I think the issue is that there is some issue with the exponent $p^{\psi-1}$.

$$= (1-m) \left(\frac{\psi \left(p^{1-\psi} \right)}{\kappa} \right) + 1 = 0 \implies \tag{310}$$

$$(1-m) + \frac{\kappa}{\psi \left(p^{1-\psi} \right)} = 0 \implies \tag{311}$$

$$m^* = 1 + \frac{\kappa p^{\psi-1}}{\psi} \tag{312}$$

Which is always greater than one, which is inadmissable. **Btw, he doesn't derive anything that looks like penalized regression. He just randomly chose that cost function!!! We can choose our own cost functions and do it ourselves. This could open the door for something like a cost function that is related to the 'salience' of the default. i.e.**

	$\Lambda < 0$	$0 < \Lambda < 1$
$p^d > p^{pc}$	Ambiguous	$\frac{\partial p^*}{\partial m} < 0$
$p^d < p^{pc}$	$\frac{\partial p^*}{\partial m} < 0$	Ambiguous

suppose that you know the price of $n-1$ goods, and you know what you think p_n is, i.e. they have some p_n^d. Based on the salience of p_n^d in comparison to the other $p_1, ..., p_{n-1}$, then maybe we make the cost lower? The 'lower' cost could be the fixed cost, but then allow the marginal cost to be independent of that salience? Seems reasonable. We can use this to try to explain pop-ups; you immediately notice it, but also very quickly realize it's useless. Whereas there could be a quantity, say, the price of a car you want, and just imagine that it's really hard to find that number, but once you find the number it's very easy to then optimize. Could the fixed vs marginal cost deal with the stuff in that BU guy's 2023 QJE about how to *acquire* information vs. make use of information

Appendix G Miscellaneous

Appendix G.1 Code for table of attentional pass-through

Recall that we inlcude the *image* of the table rather than the table itself for formatting reasons. So, here's the code for the table:

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