PSET 1

Based on Lecture 1 and Lecture 2 Solve for Questions 4 and 5

Import the necessary dependencies

In [26]:

```
# Ignore the pip dependecy error

import cv2
import math
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import convolve2d as conv2d
import scipy.sparse as sps
from PIL import Image
from sys import platform
import sparseqr
```

In [27]:

```
! wget http://6.869.csail.mit.edu/fa19/psets19/pset1/img1.png
```

```
--2022-02-11 14:40:13--  http://6.869.csail.mit.edu/fa19/psets19/pset1/img1.png
Resolving 6.869.csail.mit.edu... 128.30.100.221
Connecting to 6.869.csail.mit.edu|128.30.100.221|:80... connected.
HTTP request sent, awaiting response... 200 OK
Length: 96264 (94K) [image/png]
Saving to: 'img1.png.3'

img1.png.3          100%[===================>]  94.01K  --.-KB/s    in 0.01s

2022-02-11 14:40:13 (6.83 MB/s) - 'img1.png.3' saved [96264/96264]
```

1. Define the Sparse Matrix

In [36]:

```
def sparseMatrix(i, j, Aij, imsize):
    """ Build a sparse matrix containing 2D linear neighborhood operators
    Input:
        Aij = [ni, nj, nc] nc: number of neighborhoods with contraints
        i: row index
        j: column index
        imsize: [nrows ncols]
    Returns:
        A: a sparse matrix. Each row contains one 2D linear operator
    """
    ni, nj, nc = Aij.shape
    nij = ni*nj

    a = np.zeros((nc*nij))
    m = np.zeros((nc*nij))
    n = np.zeros((nc*nij))
    grid_range = np.arange(-(ni-1)/2, 1+(ni-1)/2)
    jj, ii = np.meshgrid(grid_range, grid_range)
    ii = ii.reshape(-1,order='F')
```

```
    jj = jj.reshape(-1,order='F')


    k = 0
    for c in range(nc):
        # Get matrix index
        x = (i[c]+ii) + (j[c]+jj)*nrows
        a[k:k+nij] = Aij[:,:,c].reshape(-1,order='F')
        m[k:k+nij] = c
        n[k:k+nij] = x

        k += nij

    m = m.astype(np.int32)
    n = n.astype(np.int32)
    A = sps.csr_matrix((a, (m,  n)))

    return A
```

In [53]:

```
print(np.cos(alpha))
```

```
0.8191520442889918
```

2. Define world parameters and plot the edges

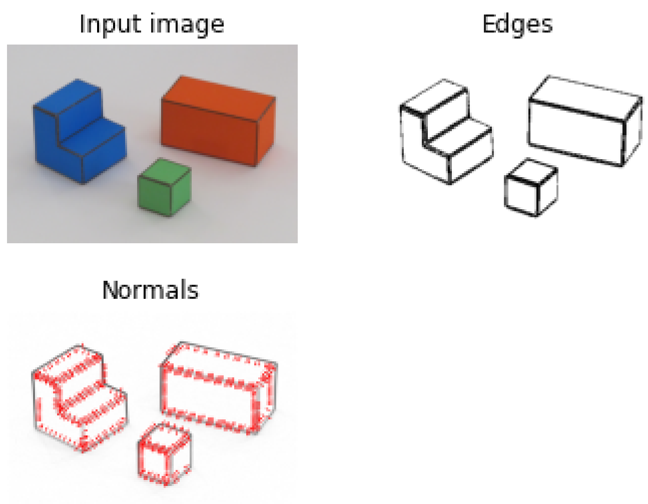
In [37]:

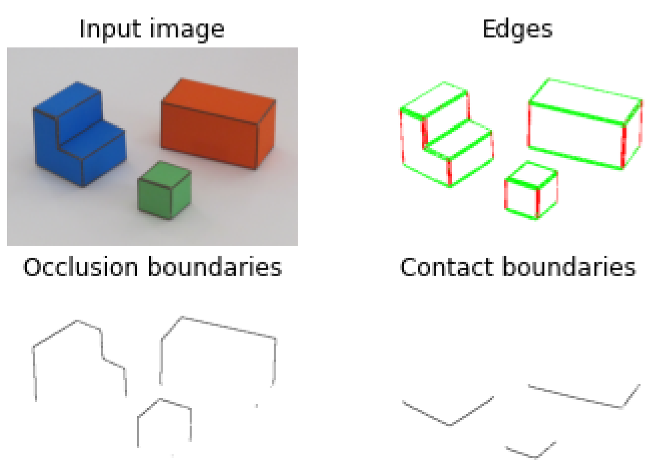
```
# World parameters
alpha = 35*math.pi/180;

img = cv2.imread('img1.png')
img = img[:, :, ::-1].astype(np.float32)

nrows, ncols, colors = img.shape
ground = (np.min(img, axis=2) > 110).astype(np.float32)
print('ground', ground.shape, ground)
foreground = (ground == 0).astype(np.float32)

m = np.mean(img, 2)
kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float32)
dmdx = conv2d(m, kern, 'same')
dmdy = conv2d(m, kern.transpose(), 'same')

mag = np.sqrt(dmdx**2 + dmdy**2)
mag[0, :] = 0
mag[-1, :] = 0
mag[:, 0] = 0
mag[:, -1] = 0

theta = np.arctan2(dmdx, dmdy)
edges = mag >= 30
edges = edges * foreground

## Occlusion and contact edges
pi = math.pi
vertical_edges = edges*((theta<115*pi/180)*(theta>65*pi/180)+(theta<-65*pi/180)*
horizontal_edges = edges * (1-vertical_edges)

kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float32)
```

```
horizontal_ground_to_foreground_edges = (conv2d(ground, kern, 'same'))>0;
horizontal_foreground_to_ground_edges = (conv2d(foreground, kern, 'same'))>0;
vertical_ground_to_foreground_edges = vertical_edges*np.abs(conv2d(ground, kern.


occlusion_edges = edges*(vertical_ground_to_foreground_edges + horizontal_ground
contact_edges   = horizontal_edges*(horizontal_foreground_to_ground_edges);


E = np.concatenate([vertical_edges[:,:,None],
                    horizontal_edges[:,:,None],
                    np.zeros(occlusion_edges.shape)[:,:,None]], 2)


# Plot
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')
plt.subplot(2,2,2)
plt.imshow(edges == 0, cmap='gray')
plt.axis('off')
plt.title('Edges')

# Normals
K = 3
ey, ex = np.where(edges[::K, ::K])
ex *= K
ey *= K
plt.figure()
plt.subplot(2,2,3)
plt.imshow(np.max(mag)-mag, cmap='gray')
dxe = dmdx[::K, ::K][edges[::K, ::K] > 0]
dye = dmdy[::K, ::K][edges[::K, ::K] > 0]
n = np.sqrt(dxe**2 + dye**2)
dxe = dxe/n
dye = dye/n
plt.quiver(ex, ey, dxe, -dye, color='r')
plt.axis('off')
plt.title('Normals')
plt.show()



# Edges and boundaries
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')


plt.subplot(2,2,2)
plt.imshow(E+(edges == 0)[:, :, None])
plt.axis('off')
plt.title('Edges')


plt.subplot(2,2,3)
```

```
plt.imshow(1-(occlusion_edges>0), cmap='gray')
plt.axis('off')
plt.title('Occlusion boundaries')

plt.subplot(2,2,4)
plt.imshow(1-contact_edges, cmap='gray')
plt.axis('off')
plt.title('Contact boundaries');
```

```
ground (256, 373) [[1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 ...
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]]
```





3. TODO: Populate edge variables

Corresponds to Question 4 in the problem set

In [69]:

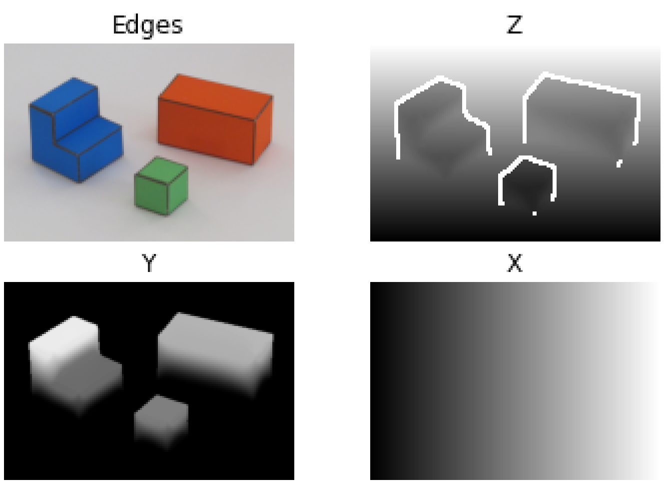
```
Nconstraints = nrows*ncols*20
Aij = np.zeros((3, 3, Nconstraints))
b = np.zeros((Nconstraints, 1))

#Indices and counters
ii = np.zeros((Nconstraints, 1))
```

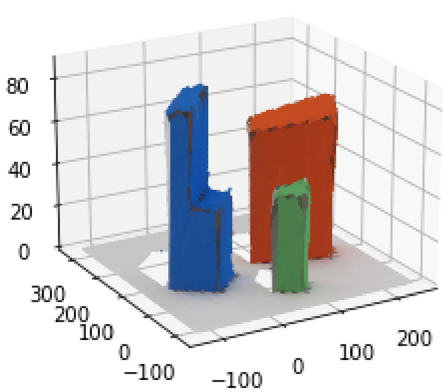
```
jj = np.zeros((Nconstraints, 1))
global c
c = 0

# These will always be updated with the current indices
def update_indices():
  global c
  ii[c] = i
  jj[c] = j
  c += 1

# Create linear contraints
for i in range(1, nrows-1):
  for j in range(1, ncols-1):
    # Y = 0
    if ground[i,j]:
      Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]])
      b[c]       = 0
      update_indices()
    else:
      # Check if current neighborhood touches an edge
      edgesum = np.sum(edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches ground pixels
      groundsum = np.sum(ground[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches vertical pixels
      verticalsum = np.sum(vertical_edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches horizontal pixels
      horizontalsum = np.sum(horizontal_edges[i-1:i+2,j-1:j+2])

      # TODO: edge orientation (average of edge pixels in current neighborhood)
      # Populate Aij, ii, jj, b, and c using alpha, theta, and
      # the constraint/transform matrices you derived in the written segment

      ###############################################################
      ### COPY YOUR CODE BELOW UNTIL THE LOOP INTO YOUR REPORT ###
      ###############################################################

      # Contact edge: dY/dy = ?
      # Requires: a transform matrix
      if contact_edges[i, j]:
        Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]], dtype=np.float3
        b[c]       = 0
        update_indices()

      # Vertical edge: dY/dy = 1/cos(theta)
      # Requires: a transform matrix, alpha

      if verticalsum > 0 and groundsum == 0:
        Aij[:,:,c] = 0.125*np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=
        b[c]       = 1/np.cos(alpha)
        update_indices()


      # dY/dt = 0 (you'll have to express t using other variables)
      # Requires: a transform matrix, i, j, theta
      if horizontalsum > 0 and groundsum == 0 and verticalsum == 0:
        g_norm = np.sqrt(dmdx[i,j]**2 + dmdy[i,j]**2)
        dmdx_norm = dmdx[i,j]/g_norm
        dmdy_norm = dmdy[i,j]/g_norm
        dx_kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float3
```

```
        dy_kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float3
        Aij[:,:,c] = -dx_kern*dmdy_norm +  dy_kern*dmdx_norm
        b[c]       = 0
        update_indices()

      # laplacian = 0 (weighted by 0.1 to reduce constraint strength)
      # Requires: multiple transform matrices
      if groundsum == 0:
        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()

        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()

        Aij[:,:,c] = 0.1*np.array([[1, 0, -1], [0, 0, 0], [-1, 0, 1]], dtype=np.
        b[c]       = 0
        update_indices()

# Splve for constraints
ii = ii[:c]
jj = jj[:c]
Aij = Aij[:,:,:c]
b = b[:c]
A = sparseMatrix(ii, jj, Aij, nrows)
Y = sparseqr.solve( A, b)

# Transform vector into image
Y = np.reshape(Y, [nrows, ncols], order='F')

# Recover 3D world coordinates
x, y = np.meshgrid(np.arange(ncols), np.arange(nrows))
x = x.astype(np.float32)
y = y.astype(np.float32)
x -= nrows/2
y -= ncols/2

# Final coordinates
X = x
Z = Y*np.cos(alpha)/np.sin(alpha) - y/np.sin(alpha)
Y = -Y
Y = np.maximum(Y, 0);

E = occlusion_edges.astype(np.float32);
E[E > 0] = np.nan;
Z = Z+E; #  remove occluded edges

# Visualize solution
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img[1:-1, 1:-1].astype(np.uint8))
plt.axis('off')
plt.title('Edges')

plt.subplot(2,2,2)
plt.imshow(Z[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Z')
```

```
plt.subplot(2,2,3)
plt.imshow(Y[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Y')

plt.subplot(2,2,4)
plt.imshow(X[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('X')

# 3D projection
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')


# TODO (p5/6) Rerun the script with a different image and set a differnt
# view angle. Note that we expect results to be quite brittle -- in
# answering q6, think about the strong assumptions that this approach makes
# we'll see more robust methods for similar problems later in the course

# Specify here the angle you want to see
ax.view_init(20, -120)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[69]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8eb71e9100>





In [ ]:

4. TODO: Run the code, show new view points for the images included with the code

Corresponds to Question 5 in the Problem Set.

In [77]:

```
### TODO: Your runs and parameters here
! wget http://6.869.csail.mit.edu/fa19/psets19/pset1/img2.png
img = cv2.imread('img2.png')
img = img[:, :, ::-1].astype(np.float32)
```

```
--2022-02-13 16:42:30--  http://6.869.csail.mit.edu/fa19/psets19/pset1/img2.png
Resolving 6.869.csail.mit.edu... 128.30.100.221
Connecting to 6.869.csail.mit.edu|128.30.100.221|:80... connected.
HTTP request sent, awaiting response... 200 OK
Length: 76847 (75K) [image/png]
Saving to: 'img2.png.3'

img2.png.3          100%[===================>]  75.05K  --.-KB/s    in 0.03s

2022-02-13 16:42:30 (2.55 MB/s) - 'img2.png.3' saved [76847/76847]
```

In [79]:

```
# World parameters
alpha = 35*math.pi/180;

img = cv2.imread('img2.png')
img = img[:, :, ::-1].astype(np.float32)

nrows, ncols, colors = img.shape
ground = (np.min(img, axis=2) > 110).astype(np.float32)
print('ground', ground.shape, ground)
foreground = (ground == 0).astype(np.float32)

m = np.mean(img, 2)
kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float32)
dmdx = conv2d(m, kern, 'same')
dmdy = conv2d(m, kern.transpose(), 'same')

mag = np.sqrt(dmdx**2 + dmdy**2)
mag[0, :] = 0
mag[-1, :] = 0
mag[:, 0] = 0
mag[:, -1] = 0

theta = np.arctan2(dmdx, dmdy)
edges = mag >= 30
edges = edges * foreground

## Occlusion and contact edges
pi = math.pi
vertical_edges = edges*((theta<115*pi/180)*(theta>65*pi/180)+(theta<-65*pi/180)*
horizontal_edges = edges * (1-vertical_edges)

kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float32)
horizontal_ground_to_foreground_edges = (conv2d(ground, kern, 'same'))>0;
```

```
horizontal_foreground_to_ground_edges = (conv2d(foreground, kern, 'same'))>0;
vertical_ground_to_foreground_edges = vertical_edges*np.abs(conv2d(ground, kern.


occlusion_edges = edges*(vertical_ground_to_foreground_edges + horizontal_ground
contact_edges   = horizontal_edges*(horizontal_foreground_to_ground_edges);


E = np.concatenate([vertical_edges[:,:,None],
                    horizontal_edges[:,:,None],
                    np.zeros(occlusion_edges.shape)[:,:,None]], 2)


# Plot
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')
plt.subplot(2,2,2)
plt.imshow(edges == 0, cmap='gray')
plt.axis('off')
plt.title('Edges')

# Normals
K = 3
ey, ex = np.where(edges[::K, ::K])
ex *= K
ey *= K
plt.figure()
plt.subplot(2,2,3)
plt.imshow(np.max(mag)-mag, cmap='gray')
dxe = dmdx[::K, ::K][edges[::K, ::K] > 0]
dye = dmdy[::K, ::K][edges[::K, ::K] > 0]
n = np.sqrt(dxe**2 + dye**2)
dxe = dxe/n
dye = dye/n
plt.quiver(ex, ey, dxe, -dye, color='r')
plt.axis('off')
plt.title('Normals')
plt.show()



# Edges and boundaries
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')


plt.subplot(2,2,2)
plt.imshow(E+(edges == 0)[:, :, None])
plt.axis('off')
plt.title('Edges')


plt.subplot(2,2,3)
plt.imshow(1-(occlusion_edges>0), cmap='gray')
```

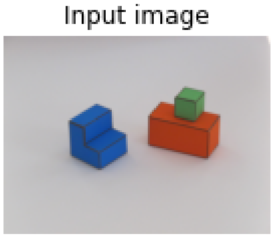
```
plt.axis('off')
plt.title('Occlusion boundaries')

plt.subplot(2,2,4)
plt.imshow(1-contact_edges, cmap='gray')
plt.axis('off')
plt.title('Contact boundaries');


Nconstraints = nrows*ncols*20
Aij = np.zeros((3, 3, Nconstraints))
b = np.zeros((Nconstraints, 1))

#Indices and counters
ii = np.zeros((Nconstraints, 1))
jj = np.zeros((Nconstraints, 1))
global c
c = 0

# These will always be updated with the current indices
def update_indices():
  global c
  ii[c] = i
  jj[c] = j
  c += 1

# Create linear contraints
for i in range(1, nrows-1):
  for j in range(1, ncols-1):
    # Y = 0
    if ground[i,j]:
      Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]])
      b[c]       = 0
      update_indices()
    else:
      # Check if current neighborhood touches an edge
      edgesum = np.sum(edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches ground pixels
      groundsum = np.sum(ground[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches vertical pixels
      verticalsum = np.sum(vertical_edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches horizontal pixels
      horizontalsum = np.sum(horizontal_edges[i-1:i+2,j-1:j+2])

      # TODO: edge orientation (average of edge pixels in current neighborhood)
      # Populate Aij, ii, jj, b, and c using alpha, theta, and
      # the constraint/transform matrices you derived in the written segment

      ###############################################################
      ### COPY YOUR CODE BELOW UNTIL THE LOOP INTO YOUR REPORT ###
      ###############################################################

      # Contact edge: dY/dy = ?
      # Requires: a transform matrix
      if contact_edges[i, j]:
        Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]], dtype=np.float3
        b[c]       = 0
        update_indices()

      # Vertical edge: dY/dy = 1/cos(theta)
```

```
      # Requires: a transform matrix, alpha

      if verticalsum > 0 and groundsum == 0:
        Aij[:,:,c] = 0.125*np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=
        b[c]       = 1/np.cos(alpha)
        update_indices()


      # dY/dt = 0 (you'll have to express t using other variables)
      # Requires: a transform matrix, i, j, theta
      if horizontalsum > 0 and groundsum == 0 and verticalsum == 0:
        g_norm = np.sqrt(dmdx[i,j]**2 + dmdy[i,j]**2)
        dmdx_norm = dmdx[i,j]/g_norm
        dmdy_norm = dmdy[i,j]/g_norm
        dx_kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float3
        dy_kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float3
        Aij[:,:,c] = -dx_kern*dmdy_norm +  dy_kern*dmdx_norm
        b[c]       = 0
        update_indices()

      # laplacian = 0 (weighted by 0.1 to reduce constraint strength)
      # Requires: multiple transform matrices
      if groundsum == 0:
        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()

        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()

        Aij[:,:,c] = 0.1*np.array([[1, 0, -1], [0, 0, 0], [-1, 0, 1]], dtype=np.
        b[c]       = 0
        update_indices()

# Splve for constraints
ii = ii[:c]
jj = jj[:c]
Aij = Aij[:,:,:c]
b = b[:c]
A = sparseMatrix(ii, jj, Aij, nrows)
Y = sparseqr.solve( A, b)

# Transform vector into image
Y = np.reshape(Y, [nrows, ncols], order='F')

# Recover 3D world coordinates
x, y = np.meshgrid(np.arange(ncols), np.arange(nrows))
x = x.astype(np.float32)
y = y.astype(np.float32)
x -= nrows/2
y -= ncols/2

# Final coordinates
X = x
Z = Y*np.cos(alpha)/np.sin(alpha) - y/np.sin(alpha)
Y = -Y
Y = np.maximum(Y, 0);

E = occlusion_edges.astype(np.float32);
```

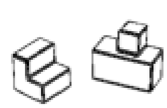
```python
E[E > 0] = np.nan;
Z = Z+E; #  remove occluded edges

# Visualize solution
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img[1:-1, 1:-1].astype(np.uint8))
plt.axis('off')
plt.title('Edges')

plt.subplot(2,2,2)
plt.imshow(Z[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Z')


plt.subplot(2,2,3)
plt.imshow(Y[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Y')

plt.subplot(2,2,4)
plt.imshow(X[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('X')

# 3D projection
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')


# TODO (p5/6) Rerun the script with a different image and set a differnt
# view angle. Note that we expect results to be quite brittle -- in
# answering q6, think about the strong assumptions that this approach makes
# we'll see more robust methods for similar problems later in the course

# Specify here the angle you want to see
ax.view_init(20, -120)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

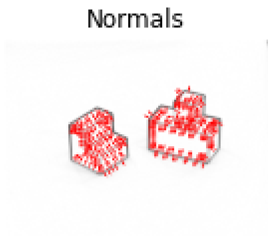
```
ground (256, 342) [[1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 ...
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]]
```

Input image

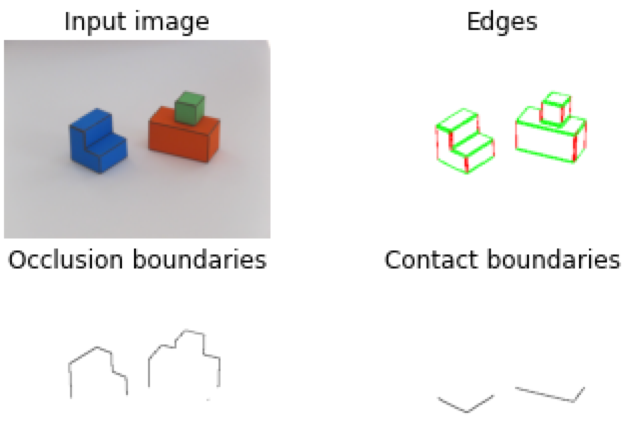


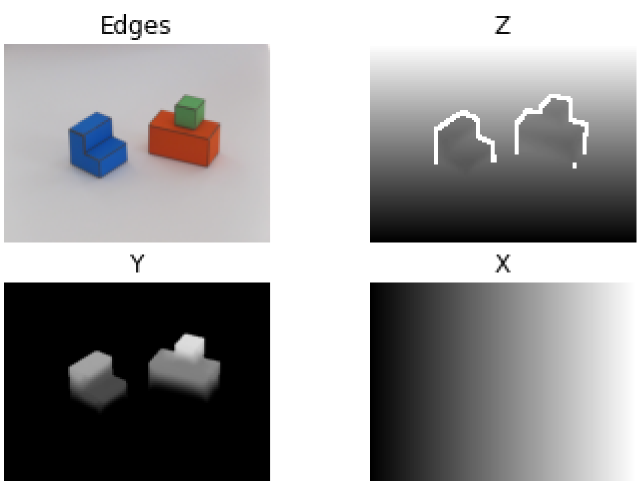
Edges

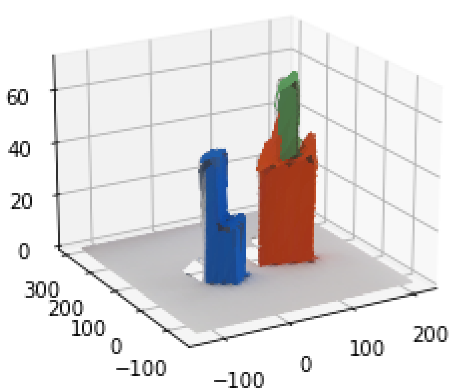




Out[79]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8e215b6cd0>





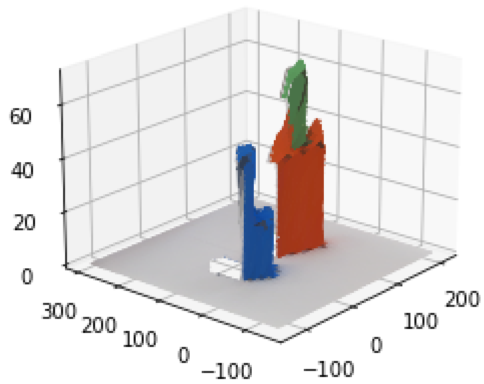


In [88]:

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

```
ax.view_init(20, -140)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

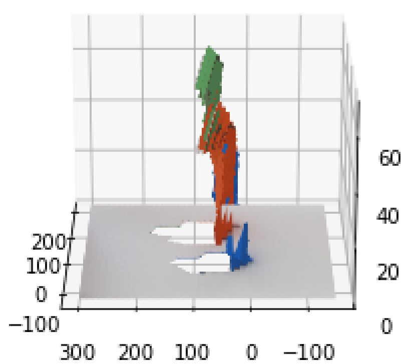
Out[88]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea3e34760>



In [89]:

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(20, -180)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

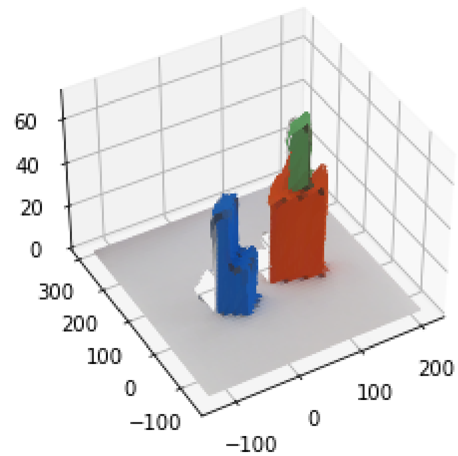
Out[89]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8e22fda6a0>



In [90]:

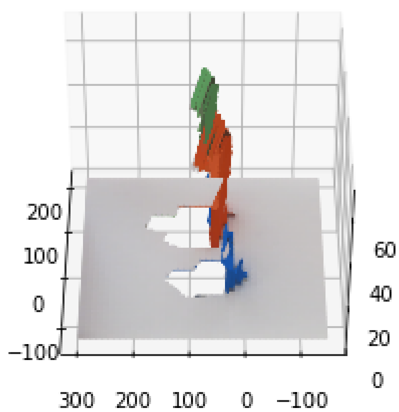
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(40, -120)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[90]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8e226e1e80>



In [96]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(40, -180)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[96]: `<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea55bcb80>`



In [97]:
```
! wget http://6.869.csail.mit.edu/fa19/psets19/pset1/img3.png
img = cv2.imread('img3.png')
img = img[:, :, ::-1].astype(np.float32)
```

```
--2022-02-13 16:54:05--  http://6.869.csail.mit.edu/fa19/psets19/pset1/img3.png
Resolving 6.869.csail.mit.edu... 128.30.100.221
Connecting to 6.869.csail.mit.edu|128.30.100.221|:80... connected.
HTTP request sent, awaiting response... 200 OK
Length: 76434 (75K) [image/png]
Saving to: 'img3.png.2'

img3.png.2          100%[===================>]  74.64K  --.-KB/s    in 0.03s

2022-02-13 16:54:05 (2.25 MB/s) - 'img3.png.2' saved [76434/76434]
```

In [98]:
```
# World parameters
alpha = 35*math.pi/180;

img = cv2.imread('img3.png')
```

```python
img = img[:, :, ::-1].astype(np.float32)

nrows, ncols, colors = img.shape
ground = (np.min(img, axis=2) > 110).astype(np.float32)
print('ground', ground.shape, ground)
foreground = (ground == 0).astype(np.float32)

m = np.mean(img, 2)
kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float32)
dmdx = conv2d(m, kern, 'same')
dmdy = conv2d(m, kern.transpose(), 'same')

mag = np.sqrt(dmdx**2 + dmdy**2)
mag[0, :] = 0
mag[-1, :] = 0
mag[:, 0] = 0
mag[:, -1] = 0

theta = np.arctan2(dmdx, dmdy)
edges = mag >= 30
edges = edges * foreground

## Occlusion and contact edges
pi = math.pi
vertical_edges = edges*((theta<115*pi/180)*(theta>65*pi/180)+(theta<-65*pi/180)*
horizontal_edges = edges * (1-vertical_edges)

kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float32)
horizontal_ground_to_foreground_edges = (conv2d(ground, kern, 'same'))>0;
horizontal_foreground_to_ground_edges = (conv2d(foreground, kern, 'same'))>0;
vertical_ground_to_foreground_edges = vertical_edges*np.abs(conv2d(ground, kern.


occlusion_edges = edges*(vertical_ground_to_foreground_edges + horizontal_ground
contact_edges   = horizontal_edges*(horizontal_foreground_to_ground_edges);


E = np.concatenate([vertical_edges[:,:,None],
                    horizontal_edges[:,:,None],
                    np.zeros(occlusion_edges.shape)[:,:,None]], 2)



# Plot
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')
plt.subplot(2,2,2)
plt.imshow(edges == 0, cmap='gray')
plt.axis('off')
plt.title('Edges')

# Normals
K = 3
ey, ex = np.where(edges[::K, ::K])
ex *= K
ey *= K
plt.figure()
plt.subplot(2,2,3)
```

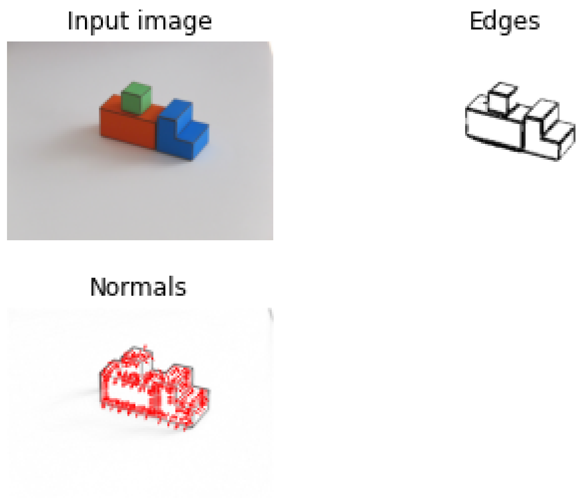
```
plt.imshow(np.max(mag)-mag, cmap='gray')
dxe = dmdx[::K, ::K][edges[::K, ::K] > 0]
dye = dmdy[::K, ::K][edges[::K, ::K] > 0]
n = np.sqrt(dxe**2 + dye**2)
dxe = dxe/n
dye = dye/n
plt.quiver(ex, ey, dxe, -dye, color='r')
plt.axis('off')
plt.title('Normals')
plt.show()



# Edges and boundaries
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')


plt.subplot(2,2,2)
plt.imshow(E+(edges == 0)[:, :, None])
plt.axis('off')
plt.title('Edges')


plt.subplot(2,2,3)
plt.imshow(1-(occlusion_edges>0), cmap='gray')
plt.axis('off')
plt.title('Occlusion boundaries')

plt.subplot(2,2,4)
plt.imshow(1-contact_edges, cmap='gray')
plt.axis('off')
plt.title('Contact boundaries');


Nconstraints = nrows*ncols*20
Aij = np.zeros((3, 3, Nconstraints))
b = np.zeros((Nconstraints, 1))

#Indices and counters
ii = np.zeros((Nconstraints, 1))
jj = np.zeros((Nconstraints, 1))
global c
c = 0

# These will always be updated with the current indices
def update_indices():
  global c
  ii[c] = i
  jj[c] = j
  c += 1

# Create linear contraints
for i in range(1, nrows-1):
  for j in range(1, ncols-1):
    # Y = 0
    if ground[i,j]:
```

```
      Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]])
      b[c]       = 0
      update_indices()
    else:
      # Check if current neighborhood touches an edge
      edgesum = np.sum(edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches ground pixels
      groundsum = np.sum(ground[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches vertical pixels
      verticalsum = np.sum(vertical_edges[i-1:i+2,j-1:j+2])
      # Check if current neirborhood touches horizontal pixels
      horizontalsum = np.sum(horizontal_edges[i-1:i+2,j-1:j+2])

      # TODO: edge orientation (average of edge pixels in current neighborhood)
      # Populate Aij, ii, jj, b, and c using alpha, theta, and
      # the constraint/transform matrices you derived in the written segment

      ###############################################################
      ### COPY YOUR CODE BELOW UNTIL THE LOOP INTO YOUR REPORT ###
      ###############################################################

      # Contact edge: dY/dy = ?
      # Requires: a transform matrix
      if contact_edges[i, j]:
        Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]], dtype=np.float3
        b[c]       = 0
        update_indices()

      # Vertical edge: dY/dy = 1/cos(theta)
      # Requires: a transform matrix, alpha

      if verticalsum > 0 and groundsum == 0:
        Aij[:,:,c] = 0.125*np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=
        b[c]       = 1/np.cos(alpha)
        update_indices()


      # dY/dt = 0 (you'll have to express t using other variables)
      # Requires: a transform matrix, i, j, theta
      if horizontalsum > 0 and groundsum == 0 and verticalsum == 0:
        g_norm = np.sqrt(dmdx[i,j]**2 + dmdy[i,j]**2)
        dmdx_norm = dmdx[i,j]/g_norm
        dmdy_norm = dmdy[i,j]/g_norm
        dx_kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float3
        dy_kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float3
        Aij[:,:,c] = -dx_kern*dmdy_norm +  dy_kern*dmdx_norm
        b[c]       = 0
        update_indices()

      # laplacian = 0 (weighted by 0.1 to reduce constraint strength)
      # Requires: multiple transform matrices
      if groundsum == 0:
        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()

        Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
        b[c]       = 0
        update_indices()
```

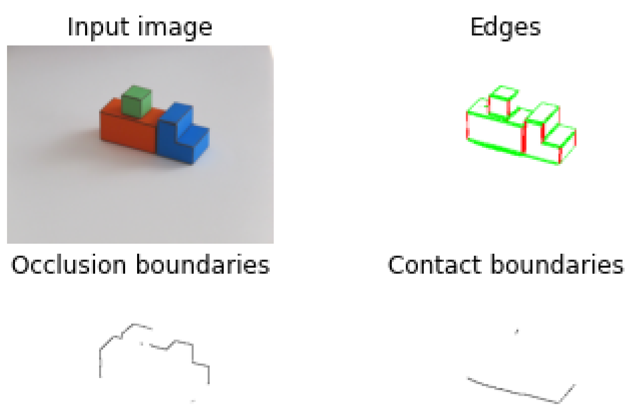
```
        Aij[:,:,c] = 0.1*np.array([[1, 0, -1], [0, 0, 0], [-1, 0, 1]], dtype=np.
        b[c]       = 0
        update_indices()

# Splve for constraints
ii = ii[:c]
jj = jj[:c]
Aij = Aij[:,:,:c]
b = b[:c]
A = sparseMatrix(ii, jj, Aij, nrows)
Y = sparseqr.solve( A, b)

# Transform vector into image
Y = np.reshape(Y, [nrows, ncols], order='F')

# Recover 3D world coordinates
x, y = np.meshgrid(np.arange(ncols), np.arange(nrows))
x = x.astype(np.float32)
y = y.astype(np.float32)
x -= nrows/2
y -= ncols/2

# Final coordinates
X = x
Z = Y*np.cos(alpha)/np.sin(alpha) - y/np.sin(alpha)
Y = -Y
Y = np.maximum(Y, 0);

E = occlusion_edges.astype(np.float32);
E[E > 0] = np.nan;
Z = Z+E; #  remove occluded edges

# Visualize solution
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img[1:-1, 1:-1].astype(np.uint8))
plt.axis('off')
plt.title('Edges')

plt.subplot(2,2,2)
plt.imshow(Z[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Z')


plt.subplot(2,2,3)
plt.imshow(Y[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Y')

plt.subplot(2,2,4)
plt.imshow(X[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('X')

# 3D projection
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

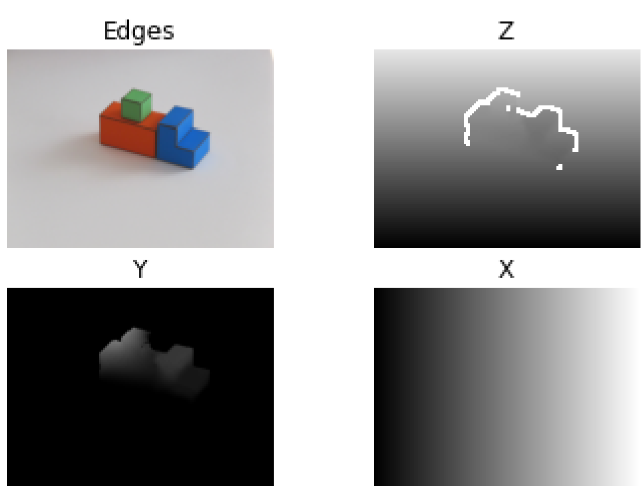
```
ax.view_init(20, -120)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

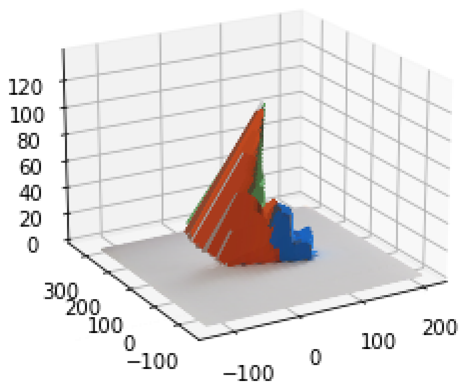
```
ground (256, 342) [[1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 ...
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]]
```



Out[98]: `<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea6319340>`

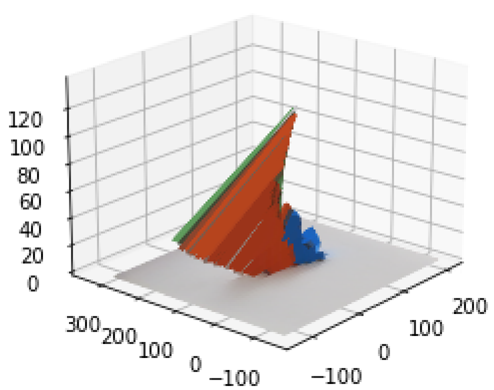






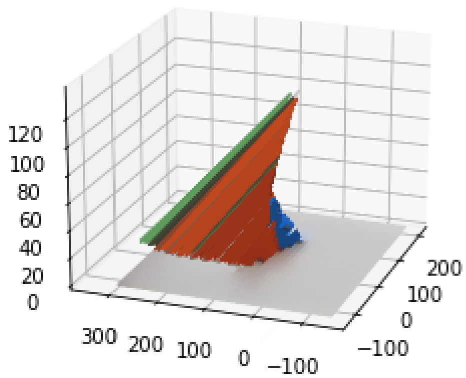
In [99]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(20, -140)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[99]: <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea5105eb0>



In [100…
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')


# TODO (p5/6) Rerun the script with a different image and set a differnt
# view angle. Note that we expect results to be quite brittle -- in
# answering q6, think about the strong assumptions that this approach makes
# we'll see more robust methods for similar problems later in the course

# Specify here the angle you want to see
ax.view_init(20, -160)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

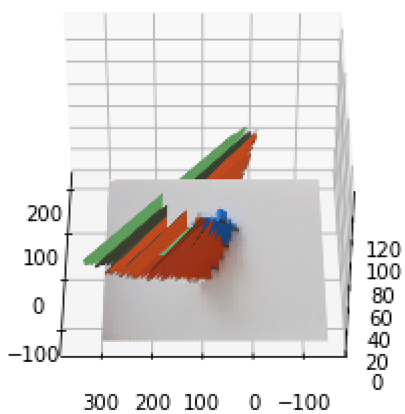
Out[100… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8eb2140eb0>



In [101…

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(40, -180)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[101… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea4a6ad90>



In [102…

```
! wget http://6.869.csail.mit.edu/fa19/psets19/pset1/img4.png
img = cv2.imread('img4.png')
img = img[:, :, ::-1].astype(np.float32)
```

```
--2022-02-13 16:55:20--  http://6.869.csail.mit.edu/fa19/psets19/pset1/img4.png
Resolving 6.869.csail.mit.edu... 128.30.100.221
Connecting to 6.869.csail.mit.edu|128.30.100.221|:80... connected.
HTTP request sent, awaiting response... 200 OK
Length: 78473 (77K) [image/png]
Saving to: 'img4.png.1'

img4.png.1          100%[===================>]  76.63K  --.-KB/s    in 0.04s

2022-02-13 16:55:20 (2.11 MB/s) - 'img4.png.1' saved [78473/78473]
```

In [103…

```
# World parameters
alpha = 35*math.pi/180;

img = cv2.imread('img4.png')
img = img[:, :, ::-1].astype(np.float32)
```

```
nrows, ncols, colors = img.shape
ground = (np.min(img, axis=2) > 110).astype(np.float32)
print('ground', ground.shape, ground)
foreground = (ground == 0).astype(np.float32)

m = np.mean(img, 2)
kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float32)
dmdx = conv2d(m, kern, 'same')
dmdy = conv2d(m, kern.transpose(), 'same')

mag = np.sqrt(dmdx**2 + dmdy**2)
mag[0, :] = 0
mag[-1, :] = 0
mag[:, 0] = 0
mag[:, -1] = 0

theta = np.arctan2(dmdx, dmdy)
edges = mag >= 30
edges = edges * foreground

## Occlusion and contact edges
pi = math.pi
vertical_edges = edges*((theta<115*pi/180)*(theta>65*pi/180)+(theta<-65*pi/180)*
horizontal_edges = edges * (1-vertical_edges)

kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float32)
horizontal_ground_to_foreground_edges = (conv2d(ground, kern, 'same'))>0;
horizontal_foreground_to_ground_edges = (conv2d(foreground, kern, 'same'))>0;
vertical_ground_to_foreground_edges = vertical_edges*np.abs(conv2d(ground, kern.


occlusion_edges = edges*(vertical_ground_to_foreground_edges + horizontal_ground
contact_edges   = horizontal_edges*(horizontal_foreground_to_ground_edges);


E = np.concatenate([vertical_edges[:,:,None],
                    horizontal_edges[:,:,None],
                    np.zeros(occlusion_edges.shape)[:,:,None]], 2)


# Plot
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')
plt.subplot(2,2,2)
plt.imshow(edges == 0, cmap='gray')
plt.axis('off')
plt.title('Edges')

# Normals
K = 3
ey, ex = np.where(edges[::K, ::K])
ex *= K
ey *= K
plt.figure()
plt.subplot(2,2,3)
plt.imshow(np.max(mag)-mag, cmap='gray')
```

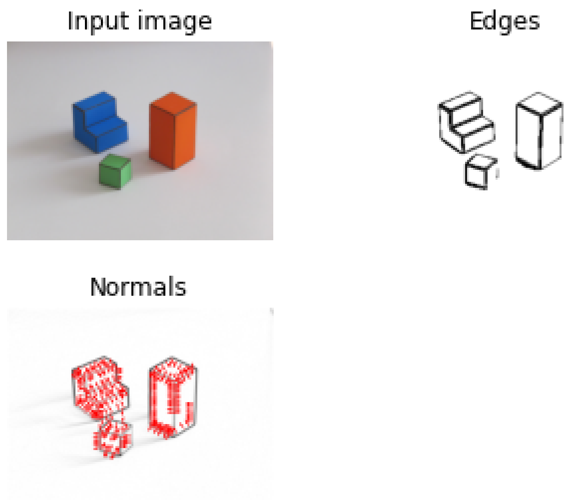
```
dxe = dmdx[::K, ::K][edges[::K, ::K] > 0]
dye = dmdy[::K, ::K][edges[::K, ::K] > 0]
n = np.sqrt(dxe**2 + dye**2)
dxe = dxe/n
dye = dye/n
plt.quiver(ex, ey, dxe, -dye, color='r')
plt.axis('off')
plt.title('Normals')
plt.show()



# Edges and boundaries
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img.astype(np.uint8))
plt.axis('off')
plt.title('Input image')


plt.subplot(2,2,2)
plt.imshow(E+(edges == 0)[:, :, None])
plt.axis('off')
plt.title('Edges')


plt.subplot(2,2,3)
plt.imshow(1-(occlusion_edges>0), cmap='gray')
plt.axis('off')
plt.title('Occlusion boundaries')

plt.subplot(2,2,4)
plt.imshow(1-contact_edges, cmap='gray')
plt.axis('off')
plt.title('Contact boundaries');


Nconstraints = nrows*ncols*20
Aij = np.zeros((3, 3, Nconstraints))
b = np.zeros((Nconstraints, 1))

#Indices and counters
ii = np.zeros((Nconstraints, 1))
jj = np.zeros((Nconstraints, 1))
global c
c = 0

# These will always be updated with the current indices
def update_indices():
  global c
  ii[c] = i
  jj[c] = j
  c += 1

# Create linear contraints
for i in range(1, nrows-1):
  for j in range(1, ncols-1):
    # Y = 0
    if ground[i,j]:
      Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]])
```

```
          b[c]       = 0
          update_indices()
        else:
          # Check if current neighborhood touches an edge
          edgesum = np.sum(edges[i-1:i+2,j-1:j+2])
          # Check if current neirborhood touches ground pixels
          groundsum = np.sum(ground[i-1:i+2,j-1:j+2])
          # Check if current neirborhood touches vertical pixels
          verticalsum = np.sum(vertical_edges[i-1:i+2,j-1:j+2])
          # Check if current neirborhood touches horizontal pixels
          horizontalsum = np.sum(horizontal_edges[i-1:i+2,j-1:j+2])

          # TODO: edge orientation (average of edge pixels in current neighborhood)
          # Populate Aij, ii, jj, b, and c using alpha, theta, and
          # the constraint/transform matrices you derived in the written segment

          ###############################################################
          ### COPY YOUR CODE BELOW UNTIL THE LOOP INTO YOUR REPORT ###
          ###############################################################

          # Contact edge: dY/dy = ?
          # Requires: a transform matrix
          if contact_edges[i, j]:
            Aij[:,:,c] = np.array([[0, 0, 0], [0, 1, 0], [0, 0, 0]], dtype=np.float3
            b[c]       = 0
            update_indices()

          # Vertical edge: dY/dy = 1/cos(theta)
          # Requires: a transform matrix, alpha

          if verticalsum > 0 and groundsum == 0:
            Aij[:,:,c] = 0.125*np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=
            b[c]       = 1/np.cos(alpha)
            update_indices()


          # dY/dt = 0 (you'll have to express t using other variables)
          # Requires: a transform matrix, i, j, theta
          if horizontalsum > 0 and groundsum == 0 and verticalsum == 0:
            g_norm = np.sqrt(dmdx[i,j]**2 + dmdy[i,j]**2)
            dmdx_norm = dmdx[i,j]/g_norm
            dmdy_norm = dmdy[i,j]/g_norm
            dx_kern = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]], dtype=np.float3
            dy_kern = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]], dtype=np.float3
            Aij[:,:,c] = -dx_kern*dmdy_norm +  dy_kern*dmdx_norm
            b[c]       = 0
            update_indices()

          # laplacian = 0 (weighted by 0.1 to reduce constraint strength)
          # Requires: multiple transform matrices
          if groundsum == 0:
            Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
            b[c]       = 0
            update_indices()

            Aij[:,:,c] = 0.1*np.array([[1, -2, 1], [2, -4, 2], [1, -2, 1]], dtype=np
            b[c]       = 0
            update_indices()

            Aij[:,:,c] = 0.1*np.array([[1, 0, -1], [0, 0, 0], [-1, 0, 1]], dtype=np.
```

```
        b[c]        = 0
        update_indices()

# Splve for constraints
ii = ii[:c]
jj = jj[:c]
Aij = Aij[:,:,:c]
b = b[:c]
A = sparseMatrix(ii, jj, Aij, nrows)
Y = sparseqr.solve( A, b)

# Transform vector into image
Y = np.reshape(Y, [nrows, ncols], order='F')

# Recover 3D world coordinates
x, y = np.meshgrid(np.arange(ncols), np.arange(nrows))
x = x.astype(np.float32)
y = y.astype(np.float32)
x -= nrows/2
y -= ncols/2

# Final coordinates
X = x
Z = Y*np.cos(alpha)/np.sin(alpha) - y/np.sin(alpha)
Y = -Y
Y = np.maximum(Y, 0);

E = occlusion_edges.astype(np.float32);
E[E > 0] = np.nan;
Z = Z+E; #  remove occluded edges

# Visualize solution
plt.figure()
plt.subplot(2,2,1)
plt.imshow(img[1:-1, 1:-1].astype(np.uint8))
plt.axis('off')
plt.title('Edges')

plt.subplot(2,2,2)
plt.imshow(Z[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Z')


plt.subplot(2,2,3)
plt.imshow(Y[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('Y')

plt.subplot(2,2,4)
plt.imshow(X[1:-1, 1:-1], cmap='gray')
plt.axis('off')
plt.title('X')

# 3D projection
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

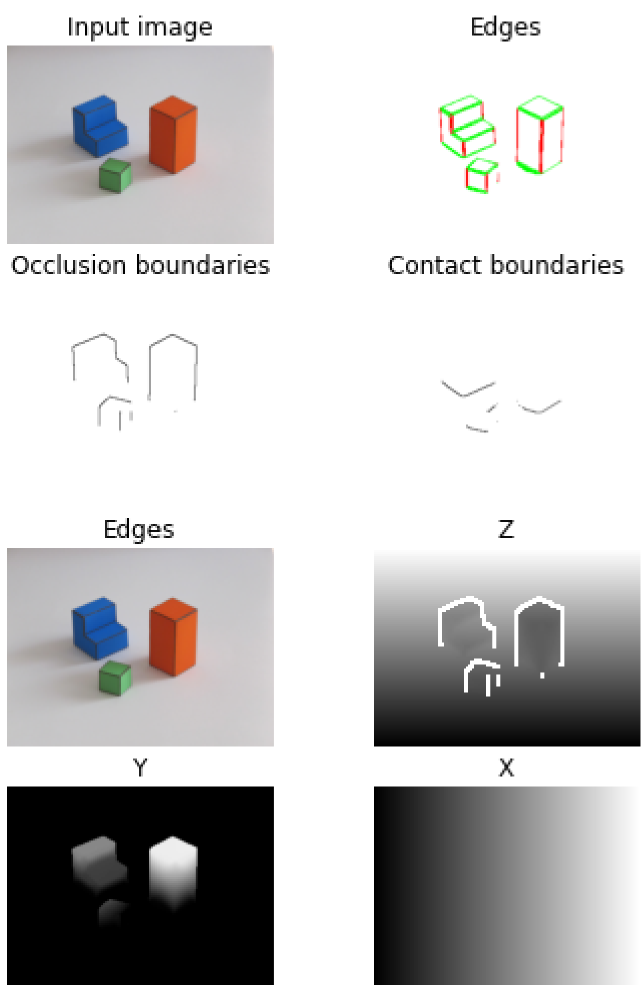
```
ax.view_init(20, -120)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

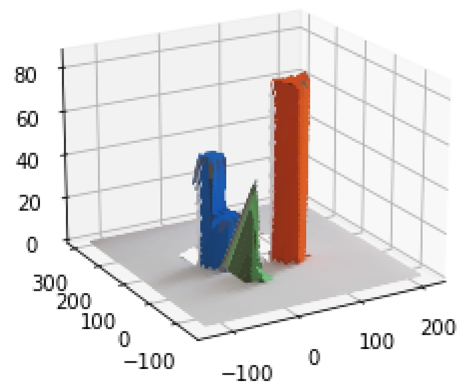
```
ground (256, 342) [[1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 ...
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]
 [1. 1. 1. ... 1. 1. 1.]]
```



Out[103…

```
<mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8e2432d9d0>
```

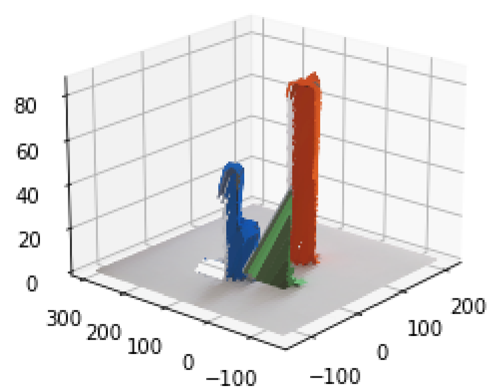




In [104…

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(20, -140)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

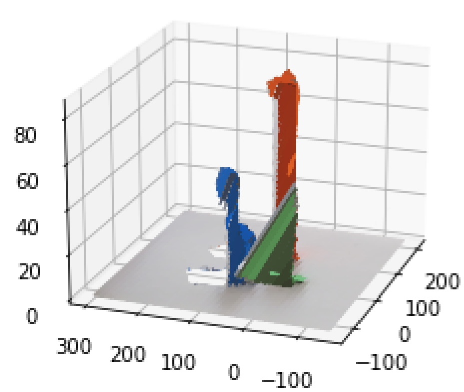
Out[104… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea631f820>



In [105…

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(20, -160)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

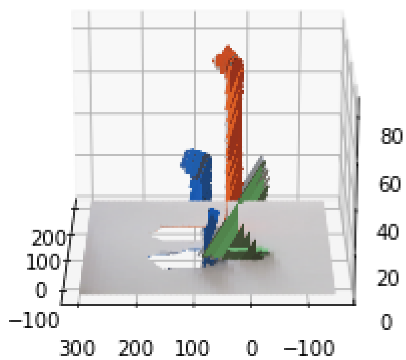
Out[105… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8e227f7190>



In [106…

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(20, -180)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

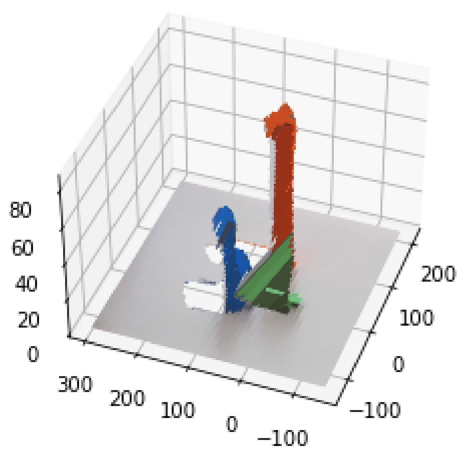
Out[106… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea53581c0>



In [107…

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.view_init(40, -160)
ax.plot_surface(X,Z,Y, facecolors=img/255., shade=False)
```

Out[107… <mpl_toolkits.mplot3d.art3d.Poly3DCollection at 0x7f8ea54f0f70>



In [ ]: