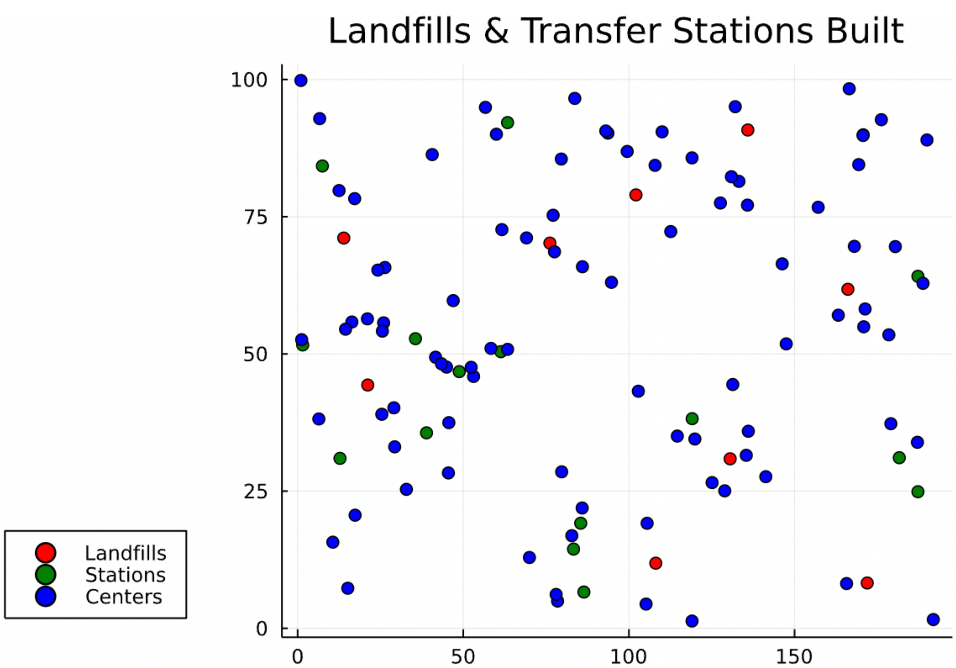
Memorandum: Waste Disposal Plan

To: County Leadership
Written By: Bennett Hellman – Data Scientist

Purpose: The purpose of this memorandum is to outline a plan for waste disposal in the most efficient and cost-effective manner possible.

Recommendation: Based on quantitative evidence, we should pursue a multi-regional waste disposal plan that includes the building of transfer stations and landfills in the following locations:



Evidence: When considering whether to build transfer stations we calculate the estimated cost savings to be $31,844.77. This is because of the reduced cost of compacted waste disposal. Additionally, the total savings of employing a multi-regional disposal plan is $20,046.69. This is because some centers in Region A are closer to transfer stations or landfills in Region B, and vice versa. For example, 5,172.85 units of waste are transported to from a center in Region A to a landfill in Region B. When you impose regional restrictions, you only limit potentially more efficient waste disposal plans. The overall cost of this plan would be $1,865,384.50.

Practical Concerns: The main practical concern is the objective to simply minimize costs does not consider environmental factors. We may want to investigate disposal plans that minimizes the overall carbon footprint. Additionally, there may be some concerns of disposing more waste in one region as opposed to the other. Our current plan does not consider an equitable disposal of waste into both region's landfills.

In [255…
```
using JuMP, Gurobi, LinearAlgebra, CSV, DataFrames, Pkg, Distances, Plots
```

In [262…
```
centers = CSV.read("centers.csv", DataFrame, header=false);
centers2 = CSV.read("centers2.csv", DataFrame, header=false);
landfills = CSV.read("landfills.csv", DataFrame, header=false);
landfills2 = CSV.read("landfills2.csv", DataFrame, header=false);
q = CSV.read("q.csv", DataFrame, header=false);
q2 = CSV.read("q2.csv", DataFrame, header=false);
stations = CSV.read("stations.csv", DataFrame, header=false);
stations2 = CSV.read("stations2.csv", DataFrame, header=false);
```

In [263…
```
first(centers, 5);
```

In [264…
```
first(landfills, 5);
```

In [265…
```
rename!(centers,:Column1 => "xco" );
rename!(centers,:Column2 => "yco" );
rename!(landfills,:Column1 => "xco" );
rename!(landfills,:Column2 => "yco" );
rename!(q,:Column1 => "waste" );
rename!(stations,:Column1 => "xco" );
rename!(stations,:Column2 => "yco" );
```

In [266…
```
dmatx = pairwise(Euclidean(), centers.xco, landfills.xco);
dmaty = pairwise(Euclidean(), centers.yco, landfills.yco);
dmat = sqrt.((dmatx.^2)+(dmaty.^2));
```

A.a

The objective function is simply minimizing the overall distance that all waste is moved. In effect, it attempts to make the waste be disposed in the most efficient

manner. This assumes that all waste transportation is equal and linear. In essence, there is no cost saving for moving waste in bulk.

Constraint 2 ensures that all the waste at centers is transported to a landfill.

Using the big-M method, contraint 3 ensures that waste is only transported to those landfills which are built.

Constraint 4 makes it so that only 5 landfills are built.

Constraint 5 makes it so that waste is a positive value. This assumes waste can only flow in one direction from a center to a landfill.

Constraint 6 makes it so that landfills are either fully built or not. This assumes that all landfills are the same size and serve the same functionality.

A.b

In [267…

```
mod = JuMP.Model(JuMP.optimizer_with_attributes(() -> Gurobi.Optimizer(),"MIPGap" => 0.0001))

set_optimizer_attribute(mod, "OutputFlag", 0)

@variable(mod, z[j=1:15], Bin)
@variable(mod, x[i=1:50,j=1:15]>=0)

@constraint(mod, [i=1:50], sum(x[i,j] for j=1:15)== q[i,1])
@constraint(mod, [i=1:50, j=1:15], x[i,j] <= 1000000*z[j])
@constraint(mod, sum(z[j] for j=1:15)<=5)

@objective(mod, Min, sum(sum(dmat[i,j]*x[i,j] for j=1:15) for i=1:50))

optimize!(mod)
```

```
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```

The total distance traveled by the waste is 840,487.52

In [268…

```
objective_value(mod)
```

Out[268…

```
840487.5240012797
```

In [269…

```
x =  value.(x);
```

In [270…

```
z1=value.(z)
```

Out[270…

```
15-element Vector{Float64}:
 -0.0
  0.0
  1.0
  1.0
 -0.0
 -0.0
  1.0
 -0.0
 -0.0
 -0.0
  0.0
  0.0
  1.0
  1.0
 -0.0
```

In [271…

```
built_ind = [3,4,7,13,14];
```

Landfills are built at the following coordinates:

In [272…

```
lf_b = landfills[built_ind,:]
```

Out[272… 5 rows × 2 columns

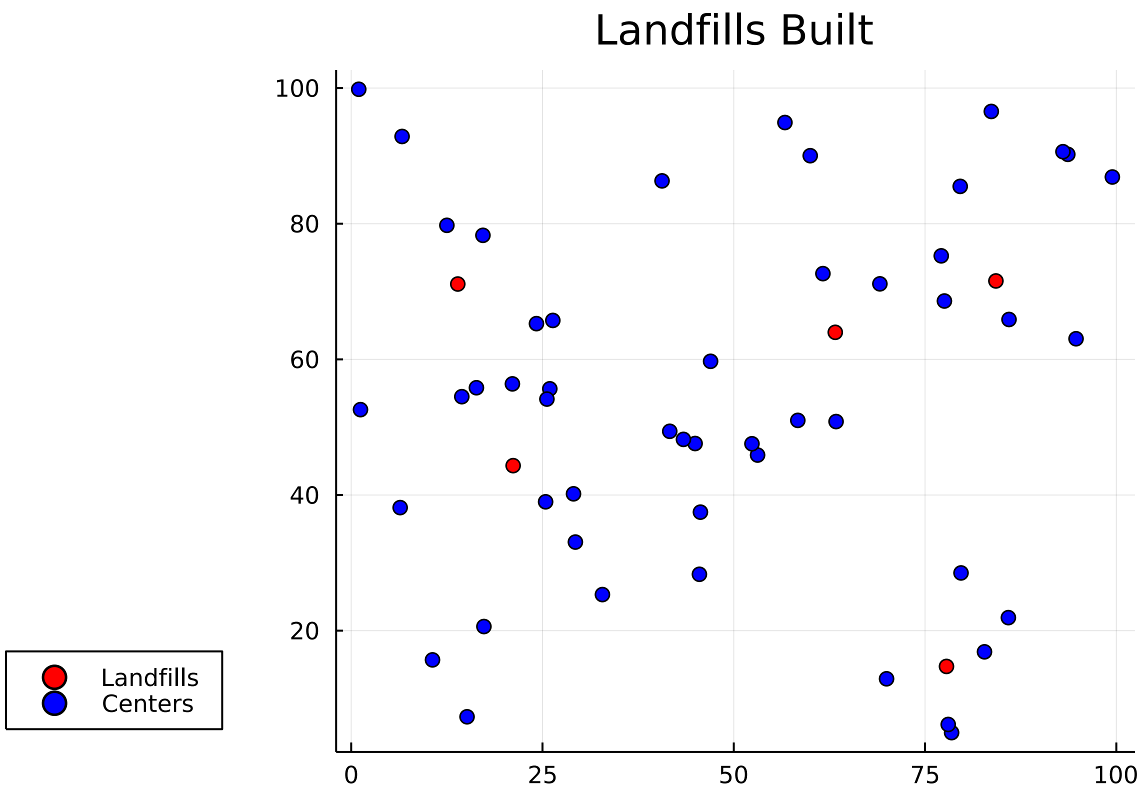
	xco	yco
	Float64	Float64

	xco	yco
	Float64	Float64
1	21.163	44.331
2	63.265	63.989
3	77.801	14.727
4	84.261	71.562
5	13.93	71.108

In [273…

```
plot(lf_b.xco, lf_b.yco, seriestype = :scatter, color = "red", title = "Landfills Built",label = "Landfills", l
plot!(centers.xco, centers.yco, seriestype = :scatter, color = "blue", label = "Centers")
```

Out[273…



B

In [274…

```
dmatx_ct = pairwise(Euclidean(), centers.xco, stations.xco);
dmaty_ct = pairwise(Euclidean(), centers.yco, stations.yco);
dmat_ct = sqrt.((dmatx_ct.^2)+(dmaty_ct.^2));
```

In [275…

```
dmatx_tl = pairwise(Euclidean(), stations.xco, landfills.xco);
dmaty_tl = pairwise(Euclidean(), stations.yco, landfills.yco);
dmat_tl = sqrt.((dmatx_tl.^2)+(dmaty_tl.^2));
```

B.c

Formulation

Sets

$i = 1, \ldots, 50$ centers

$s = 1, \ldots, 50$ potential transfer stations

$j = 1, \ldots, 15$ potential landfills

Parameters

q_i amount of waste generated at center i

d_{ij} (Euclidean) distance between center i and potential landfill j

d_{is} (Euclidean) distance between center i and transfer station s

d_{sj} (Euclidean) distance between transfer station s and landfill j

Variables

$z_j = 1$ if landfill j is built

$t_s = 1$ if transfer station s is built

x_{ij} amount of waste transported from center i to landfill j

a_{is} amount of waste transported from center i to transfer center s

b_{sj} amount of compacted waste transported from transfer center s to landfill j

$$min_{z,x,ta,b} \quad \sum_{i=1}^{50}\sum_{j=1}^{15} d_{ij}x_{ij} + \sum_{i=1}^{50}\sum_{s=1}^{50} d_{is}a_{is} + \sum_{s=1}^{50}\sum_{j=1}^{15} d_{sj}b_{sj} + 1000\sum_{s=1}^{50} t_s$$

$$x_{ij} \leq 1000000 * z_j \quad \forall i, j$$

$$\sum_{j=1}^{15} z_j \leq 5$$

$$\sum_{j=1}^{15} x_{ij} + \sum_{s=1}^{50} a_{is} = q_i \quad \forall i$$

$$\sum_{i=1}^{50} a_{is} = \sum_{j=1}^{15} b_{sj} \quad \forall i, s, j$$

$$b_{sj} \leq 1000000 * z_j \quad \forall s, j$$

$$\sum_{i=1}^{50} a_{is} \leq 2000t_s \quad \forall i, s$$

$$x_{ij}, a_{is}, b_{sj} \leq 0$$

$$z_j, t_s \in \{0, 1\}$$

In [276…

```
mod2 = JuMP.Model(JuMP.optimizer_with_attributes(() -> Gurobi.Optimizer(),"MIPGap" => 0.0001))

set_optimizer_attribute(mod2, "OutputFlag", 0)

#OLD VARIABLES
#if landfill j is built
@variable(mod2, z[j=1:15], Bin)
#amount of waste transported from center i to landfill j
@variable(mod2, x[i=1:50,j=1:15]>=0)

#NEW VARIABLES
#if transfer station s is built
@variable(mod2, t[s=1:50], Bin)
#amount of waste transported from center i to trasnfer station s
@variable(mod2, a[i=1:50,s=1:50]>=0)
#amount of waste transported from center i to trasnfer station s
@variable(mod2, b[s=1:50,j=1:15]>=0)

#OLD CONSTRAINTS
#@constraint(mod2, [i=1:50], sum(x[i,j] for j=1:15)== q[i,1]) modified for reformulation
@constraint(mod2, [i=1:50, j=1:15], x[i,j] <= 1000000*z[j])
@constraint(mod2, sum(z[j] for j=1:15)<=5)

#NEW CONSTRAINTS
#ensure that all waste is taken care of
@constraint(mod2, [i=1:50], sum(x[i,j] for j=1:15) + sum(a[i,s] for s=1:50) == q[i,1])
#conservation of flow for transfer stations
@constraint(mod2, [i=1:50, s=1:50, j=1:15], sum(a[i,s] for i=1:50)==sum(b[s,j] for j=1:15))
#transfer station t cannot serve landfill j unless landfill j is built
@constraint(mod2, [s=1:50, j=1:15], b[s,j] <= 1000000*z[j])
@constraint(mod2, [i=1:50, s=1:50], sum(a[i,s] for i=1:50)<=2000*t[s])
```

```
@objective(mod2, Min, sum(dmat[i,j]*x[i,j] for i = 1:50, j = 1:15)
                +    sum(dmat_ct[i,s]*a[i,s] for i = 1:50, s = 1:50)
                +    sum(dmat_tl[s,j]*b[s,j] for s = 1:50, j = 1:15)/2
                +    sum(10000*t[s] for s=1:50))

optimize!(mod2)
```

```
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```

In [277…

```
objective_value(mod2)
```

Out[277…

```
808642.7541913458
```

In [279…

```
z =  value.(z);
```

In [280…

```
t = value.(t);
```

In [281…

```
built_l_ind2 = [2,3,7,13,14]
lf_b2 = landfills[built_l_ind2,:];
```

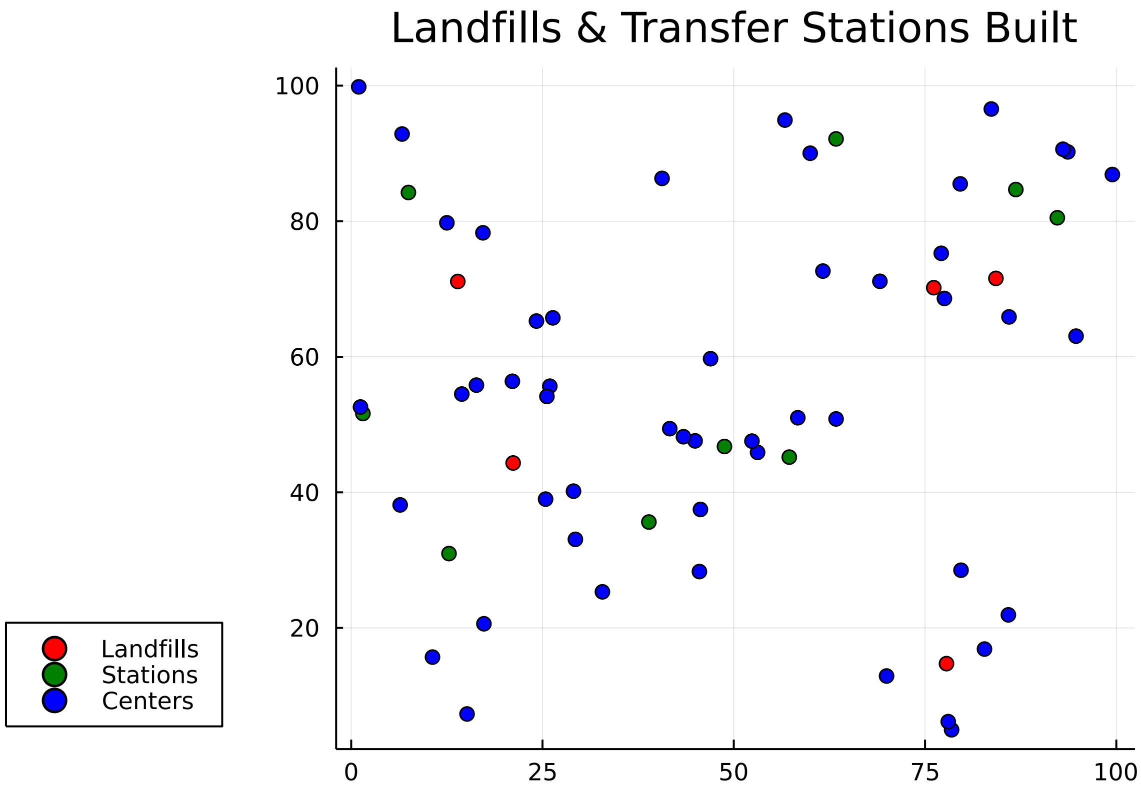
In [282…

```
built_t_ind2 = findall(>(.5), t)
ts_b2 = stations[built_t_ind2,:];
```

In [283…

```
plot(lf_b2.xco, lf_b2.yco, seriestype = :scatter, color = "red", title = "Landfills & Transfer Stations Built",
plot!(ts_b2.xco, ts_b2.yco, seriestype = :scatter, color = "green", label = "Stations")
plot!(centers.xco, centers.yco, seriestype = :scatter, color = "blue", label = "Centers")
```

Out[283…



B.d

9 transfer stations were created.

In [284…

```
round(sum(t))
```

Out[284… 9.0

35.12% of waste in compacted.

In [285…

```
x = value.(x);
```

In [286…

```
sum(x)
```

Out[286…

```
31665.669999999587
```

In [287…

```
a = value.(a);
```

In [288…

```
sum(a)
```

Out[288…

```
17137.500000000487
```

In [289…

```
dist1 = sum(dmat*z1)
dist2 = sum(dmat*z)+sum(dmat_ct*t)+sum(t'*dmat_tl);
```

In the first scenario, the total distance traveled was 11,946.95. In the second scenario the total distance significantly increased to 41,153.68

In [245…

```
print(dist1, "   ")

print(dist2)
```

```
11946.954019037625   41153.67660407018
```

The transportation costs of the first scenario is 840,487.52 and the second scenario is 718,642.75

In [246…

```
print(objective_value(mod), " ")
print(objective_value(mod2)-sum(10000*t))
```

```
840487.5240012797 718642.7541913444
```

The total costs of the first scenario is 840,487.52 and with the

transfer stations in the second scenario is 808,642.75.

In [247…

```
print(objective_value(mod), " ")
print(objective_value(mod2))
```

```
840487.5240012797 808642.7541913458
```

In [290…

```
rename!(centers2,:Column1 => "xco" );
rename!(centers2,:Column2 => "yco" );
rename!(landfills2,:Column1 => "xco" );
rename!(landfills2,:Column2 => "yco" );
rename!(stations2,:Column1 => "xco" );
rename!(stations2,:Column2 => "yco" );
rename!(q2,:Column1 => "waste" );
```

In [291…

```
dmatx = pairwise(Euclidean(), centers2.xco, landfills2.xco);
dmaty = pairwise(Euclidean(), centers2.yco, landfills2.yco);
dmat = sqrt.((dmatx.^2)+(dmaty.^2));
```

C.e

In [292…

```
mod3 = JuMP.Model(JuMP.optimizer_with_attributes(() -> Gurobi.Optimizer(),"MIPGap" => 0.0001))

set_optimizer_attribute(mod2, "OutputFlag", 0)

#OLD VARIABLES
#if landfill j is built
@variable(mod3, z[j=1:15], Bin)
#amount of waste transported from center i to landfill j
@variable(mod3, x[i=1:40,j=1:15]>=0)

#NEW VARIABLES
#if transfer station s is built
@variable(mod3, t[s=1:50], Bin)
#amount of waste transported from center i to trasnfer station s
@variable(mod3, a[i=1:40,s=1:50]>=0)
#amount of waste transported from center i to trasnfer station s
@variable(mod3, b[s=1:50,j=1:15]>=0)
```

```
#OLD CONSTRAINTS
#@constraint(mod3, [i=1:40], sum(x[i,j] for j=1:15)== q[i,1]) modified for reformulation
@constraint(mod3, [i=1:40, j=1:15], x[i,j] <= 1000000*z[j])
@constraint(mod3, sum(z[j] for j=1:15)<=5)

#NEW CONSTRAINTS
#ensure that all waste is taken care of
@constraint(mod3, [i=1:40], sum(x[i,j] for j=1:15) + sum(a[i,s] for s=1:50) == q2[i,1])
#conservation of flow for transfer stations
@constraint(mod3, [i=1:40, s=1:50, j=1:15], sum(a[i,s] for i=1:40)==sum(b[s,j] for j=1:15))
#transfer station t cannot serve landfill j unless landfill j is built
@constraint(mod3, [s=1:50, j=1:15], b[s,j] <= 1000000*z[j])
@constraint(mod3, [i=1:40, s=1:50], sum(a[i,s] for i=1:40)<=2000*t[s])



@objective(mod3, Min, sum(dmat[i,j]*x[i,j] for i = 1:40, j = 1:15)
                +    sum(dmat_ct[i,s]*a[i,s] for i = 1:40, s = 1:50)
                +    sum(dmat_tl[s,j]*b[s,j] for s = 1:50, j = 1:15)/2
                +    sum(10000*t[s] for s=1:50))

optimize!(mod3)
```

```
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Gurobi Optimizer version 9.1.2 build v9.1.2rc0 (mac64)
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 33391 rows, 3415 columns and 1737315 nonzeros
Model fingerprint: 0x0e70716b
Variable types: 3350 continuous, 65 integer (65 binary)
Coefficient statistics:
  Matrix range     [1e+00, 1e+06]
  Objective range  [1e-01, 1e+04]
  Bounds range     [0e+00, 0e+00]
  RHS range        [5e+00, 3e+03]
Presolve removed 31900 rows and 0 columns
Presolve time: 0.82s
Presolved: 1491 rows, 3415 columns, 10115 nonzeros
Variable types: 3350 continuous, 65 integer (65 binary)

Root relaxation: objective 9.744086e+05, 243 iterations, 0.01 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 974408.621    0   25          - 974408.621      -     -    0s
H    0     0                    1454438.7026 974408.621  33.0%     -    0s
H    0     0                    1127090.2538 974408.621  13.5%     -    0s
```

```
     0     0 1023606.60    0   35 1127090.25 1023606.60  9.18%     -    1s
H    0     0                     1096781.9600 1023606.60  6.67%     -    1s
     0     0 1024986.37    0   35 1096781.96 1024986.37  6.55%     -    1s
     0     0 1039198.76    0   36 1096781.96 1039198.76  5.25%     -    1s
     0     0 1040138.98    0   37 1096781.96 1040138.98  5.16%     -    1s
H    0     0                     1096628.8849 1040138.98  5.15%     -    1s
H    0     0                     1085833.6282 1040138.98  4.21%     -    1s
H    0     0                     1077900.7407 1040138.98  3.50%     -    1s
H    0     0                     1076788.4380 1040138.98  3.40%     -    1s
     0     0 1050914.20    0   41 1076788.44 1050914.20  2.40%     -    1s
     0     0 1051161.47    0   40 1076788.44 1051161.47  2.38%     -    1s
     0     0 1051175.88    0   41 1076788.44 1051175.88  2.38%     -    1s
     0     0 1058913.66    0   31 1076788.44 1058913.66  1.66%     -    1s
     0     0 1061498.21    0   29 1076788.44 1061498.21  1.42%     -    1s
     0     0 1061498.21    0   30 1076788.44 1061498.21  1.42%     -    1s

Cutting planes:
  Gomory: 1
  Implied bound: 32
  MIR: 35
  Flow cover: 39

Explored 1 nodes (634 simplex iterations) in 1.23 seconds
Thread count was 8 (of 8 available processors)

Solution count 6: 1.07679e+06 1.08583e+06 1.09663e+06 ... 1.45444e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.076788438001e+06, best bound 1.076788438001e+06, gap 0.0000%

User-callback calls 242, time in user-callback 0.00 sec
```

The daily total cost of region B is $1,076,788.44.

In [293…

```
objective_value(mod3)
```

Out[293…

```
1.0767884380013323e6
```

In [294…

```
t = value.(t);
z = value.(z);
i_t = findall(>(.5), t)
i_z = findall(>(.5), z)
ts_b3 = stations2[i_t,:];
lf_b3 = landfills2[i_z,:];
```

Transfer Stations are built at the following coordinates:

In [296…

```
ts_b3
```

Out[296… 8 rows × 2 columns

	xco	yco
	Float64	Float64
1	175.68	65.393
2	133.12	78.404
3	162.59	70.408
4	172.01	96.693
5	158.35	8.6711
6	187.25	64.139
7	127.51	43.675
8	116.65	48.425

Landfills are built at the following coordinates:

In [297…

```
lf_b3
```

Out[297… 5 rows × 2 columns

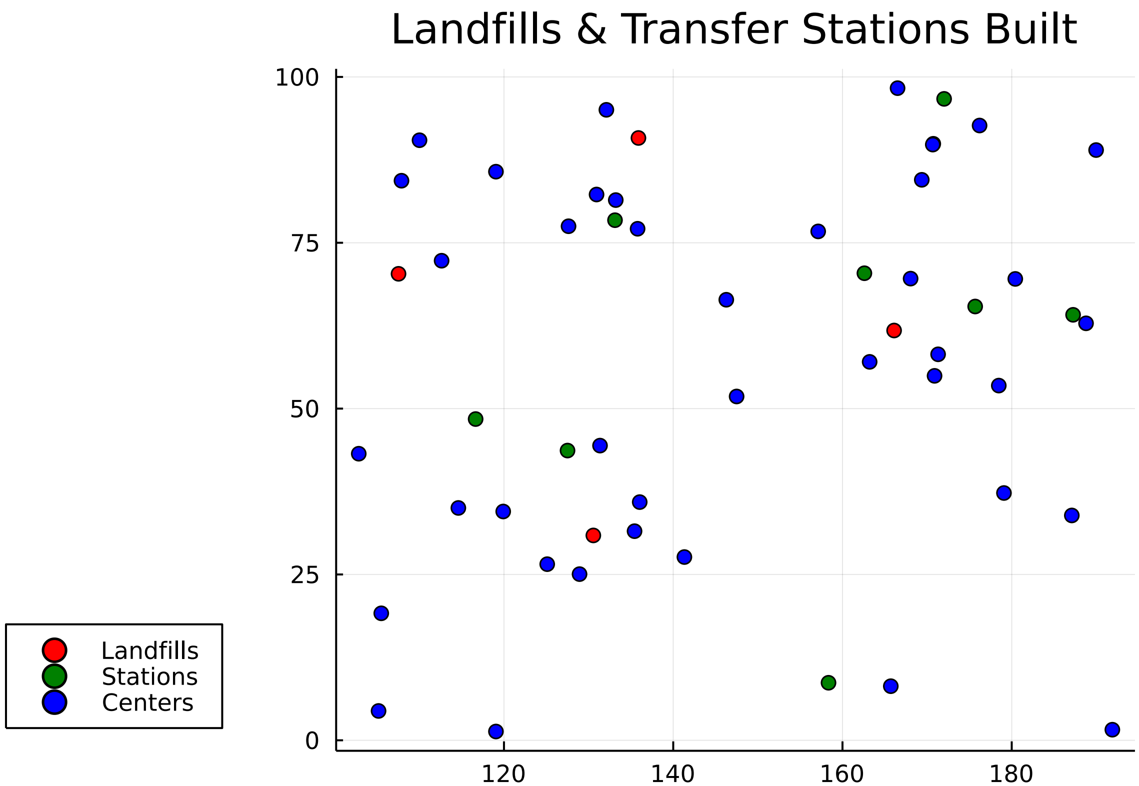
	xco	yco
	Float64	Float64
1	170.73	89.923
2	107.54	70.32
3	130.56	30.882
4	135.89	90.798

	xco	yco
	Float64	Float64
5	166.1	61.775

In [295…

```
plot(lf_b3.xco, lf_b3.yco, seriestype = :scatter, color = "red", title = "Landfills & Transfer Stations Built",
plot!(ts_b3.xco, ts_b3.yco, seriestype = :scatter, color = "green", label = "Stations")
plot!(centers2.xco, centers2.yco, seriestype = :scatter, color = "blue", label = "Centers")
```

Out[295…



C.f

In [298…

```
append!(stations,stations2);
append!(landfills,landfills2);
append!(centers,centers2);
append!(q,q2);
```

In [299…

```
dmatx = pairwise(Euclidean(), centers.xco, landfills.xco);
dmaty = pairwise(Euclidean(), centers.yco, landfills.yco);
dmat = sqrt.((dmatx.^2)+(dmaty.^2));
dmatx_ct = pairwise(Euclidean(), centers.xco, stations.xco);
dmaty_ct = pairwise(Euclidean(), centers.yco, stations.yco);
dmat_ct = sqrt.((dmatx_ct.^2)+(dmaty_ct.^2));
dmatx_tl = pairwise(Euclidean(), stations.xco, landfills.xco);
dmaty_tl = pairwise(Euclidean(), stations.yco, landfills.yco);
dmat_tl = sqrt.((dmatx_tl.^2)+(dmaty_tl.^2));
```

In [300…

```
print(size(stations), size(landfills), size(centers), size(q))
```

```
(100, 2)(30, 2)(90, 2)(90, 1)
```

In [301…

```
mod4 = JuMP.Model(JuMP.optimizer_with_attributes(() -> Gurobi.Optimizer(),"MIPGap" => 0.0001))

set_optimizer_attribute(mod4, "OutputFlag", 0)

#OLD VARIABLES
#if landfill j is built
@variable(mod4, z[j=1:30], Bin)
#amount of waste transported from center i to landfill j
@variable(mod4, x[i=1:90,j=1:30]>=0)

#NEW VARIABLES
#if transfer station s is built
@variable(mod4, t[s=1:100], Bin)
#amount of waste transported from center i to trasnfer station s
@variable(mod4, a[i=1:90,s=1:100]>=0)
#amount of waste transported from center i to trasnfer station s
@variable(mod4, b[s=1:100,j=1:30]>=0)

#OLD CONSTRAINTS
#@constraint(mod4, [i=1:90], sum(x[i,j] for j=1:30)== q[i,1]) modified for reformulation
@constraint(mod4, [i=1:90, j=1:30], x[i,j] <= 1000000*z[j])
@constraint(mod4, sum(z[j] for j=1:30)<=10)

#NEW CONSTRAINTS
#ensure that all waste is taken care of
@constraint(mod4, [i=1:90], sum(x[i,j] for j=1:30) + sum(a[i,s] for s=1:100) == q[i,1])
#conservation of flow for transfer stations
@constraint(mod4, [i=1:90, s=1:100, j=1:30], sum(a[i,s] for i=1:90)==sum(b[s,j] for j=1:30))
```

```
#transfer station t cannot serve landfill j unless landfill j is built
@constraint(mod4, [s=1:100, j=1:30], b[s,j] <= 1000000*z[j])
@constraint(mod4, [i=1:90, s=1:100], sum(a[i,s] for i=1:90)<=2000*t[s])



@objective(mod4, Min, sum(dmat[i,j]*x[i,j] for i = 1:90, j = 1:30)
                +    sum(dmat_ct[i,s]*a[i,s] for i = 1:90, s = 1:100)
                +    sum(dmat_tl[s,j]*b[s,j] for s = 1:100, j = 1:30)/2
                +    sum(10000*t[s] for s=1:100))

optimize!(mod4)
```

```
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```

The total cost of the combined regions is $1,865,384.50

In [302…

```
objective_value(mod4)
```

Out[302…

```
1.8653845034414863e6
```

In [303…

```
t = value.(t);
z = value.(z);
i_t = findall(>(.5), t)
i_z = findall(>(.5), z)
ts_b3 = stations[i_t,:];
lf_b3 = landfills[i_z,:];
```

Transfer Stations are built at the following coordinates:

In [304…

```
ts_b3
```

Out[304… 15 rows × 2 columns

	xco	yco
	Float64	Float64
1	85.45	19.15

	xco	yco
	Float64	Float64
2	63.366	92.143
3	61.369	50.405
4	83.286	14.437
5	38.906	35.617
6	12.781	30.966
7	48.784	46.765
8	1.5279	51.627
9	7.478	84.236
10	86.416	6.6111
11	35.57	52.772
12	119.09	38.195
13	181.62	31.085
14	187.25	24.887
15	187.25	64.139

Landfills are built at the following coordinates:

In [305…

```
lf_b3
```

Out[305…

10 rows × 2 columns

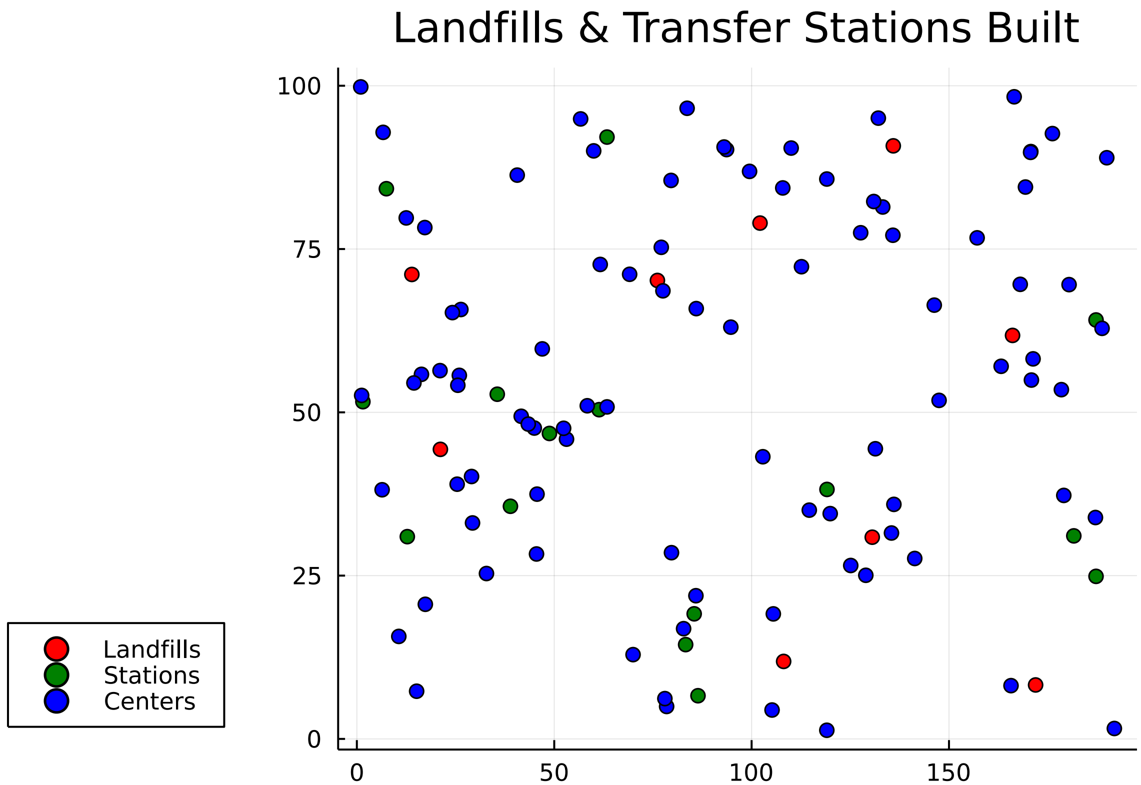
	xco	yco
	Float64	Float64
1	76.138	70.191
2	21.163	44.331
3	13.93	71.108

	xco	yco
	Float64	Float64
4	171.94	8.2604
5	102.13	78.974
6	170.73	89.923
7	108.11	11.864
8	130.56	30.882
9	135.89	90.798
10	166.1	61.775

In [306…

```
plot(lf_b3.xco, lf_b3.yco, seriestype = :scatter, color = "red", title = "Landfills & Transfer Stations Built",
plot!(ts_b3.xco, ts_b3.yco, seriestype = :scatter, color = "green", label = "Stations")
plot!(centers.xco, centers.yco, seriestype = :scatter, color = "blue", label = "Centers")
```

Out[306…



In [307…

```
x=value.(x)
```

Out[307…

```
90×30 Matrix{Float64}:
 0.0    0.0    538.56  0.0  0.0  0.0  …  0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0    910.83  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0    969.95  0.0  0.0  0.0  …  0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0     18.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0  961.35     0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0    955.27  0.0  0.0  0.0  …  0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0    855.61  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 ⋮                               ⋮    ⋱         ⋮
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0     0.0
 0.0    0.0      0.0   0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  2234.3
 0.0    0.0      0.0   0.0  0.0  0.0  …  0.0  0.0  0.0     0.0  0.0     0.0
```

```
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  2057.8  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  2492.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0     0.0  0.0  0.0
```

In [312…

```
cl = sum(sum(x[1:50,15:30],dims = 2))
```

Out[312… 5172.849999999999

In [313…

```
cl = sum(sum(x[50:90,1:15],dims = 2))
```

Out[313… 1033.1

In [ ]: