### 15.072: Advanced Analytics Edge Fall 2021 Homework 4: From Predictions to Prescriptions

a.i)

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)
             2.312e+00 3.317e-02
                                  69.712
                                            <2e-16 ***
linc
            -2.351e-01 3.147e-03 -74.711
                                            <2e-16 ***
Week_Num
            4.771e-03 2.832e-05 168.495
                                            <2e-16 ***
bseason2
             2.167e-03 3.757e-03
                                   0.577
                                            0.564
             2.182e-03 3.762e-03
                                   0.580
                                            0.562
bseason3
bseason4
            1.750e-03 3.770e-03
                                   0.464
                                            0.643
            1.315e-03 3.782e-03
                                   0.348
                                            0.728
bseason5
                                  13.336
                                            <2e-16 ***
bseason6
             5.064e-02 3.797e-03
             5.318e-02 3.816e-03
                                  13.936
                                            <2e-16 ***
bseason7
                                            <2e-16 ***
bseason8
            4.891e-02 3.838e-03
                                  12.744
                                            <2e-16 ***
bseason9
            5.253e-02 3.863e-03
                                  13.600
bseason10
             5.409e-02 3.891e-03
                                  13.903
                                            <2e-16 ***
             2.517e-03 3.922e-03
                                   0.642
bseason11
                                            0.521
bseason12
             1.423e-01 3.956e-03
                                  35.974
                                            <2e-16 ***
             1.410e-01 3.993e-03 35.317
bseason13
                                            <2e-16 ***
```

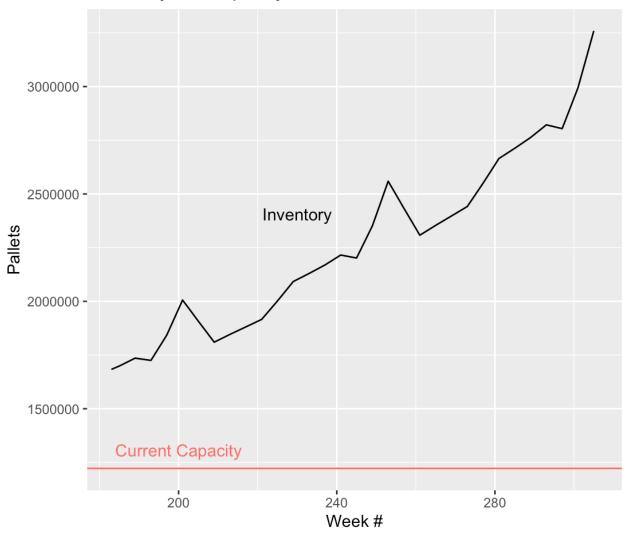
```
his<-his%>%mutate(lsale = log(Sales/Population),saleper = Sales/Population,
linc = log(Income), bseason = as.factor(Season))
train <- his%>%filter(Year <=2013)
test<-his%>%filter(Year ==2014)
mod <- lm(lsale~linc+Week_Num+bseason, data = train)
summary(mod)
```

a.ii)

It is important to note that none of these estimates are causal and can only be interpreted as correlation. A one percent increase in income per capita is associated with a 0.235% decrease in sales per capita. Another week away from the beginning of 2012 is associated with a .477% increase in sales per capita. Using season 2 as an example, being in season 2 is associated with a .217% increase in sales per capita. All other seasons can be interpreted the same was a 100\*coefficient increase in sales per capita.

a.iii) Out-of-sample performance:  $R^2 = 0.777$ , MAE = 0.163, RMSE = 0.209

b.i) Inventory vs. Capacity Over Time



### b.ii) **Shortage = 166.674%**

 $percent\_cap\_short <- (inv\_df[123,3]-current\_cap)/(current\_cap)*100$ 

c.i)

**Decision Variables:** 

 $b_i = 1 if DC_i$  is chosen, 0 otherwise

 $u_i = 1$  if  $DC_i$  is serves  $County_j$ , 0 otherwise

 $c_i$  = the capacity utilized of  $DC_i$ 

#### Inputs:

 $FixedCost_i = fixed\ cost\ of\ constructing\ DC_i$   $VariableCost_i = variable\ cost\ of\ constucting\ DC_i\ per\ sqft$   $d_{ij} = distance\ from\ DC_i\ to\ county_j$   $CapacityDC_i = maximum\ capacity\ of\ DC_i$   $x_{it} = predicted\ demand\ of\ county\ j\ in\ week\ t$ 

$$\begin{aligned} & \text{Min}_{\text{b,u,c}} \sum_{\mathbf{i}} FixedCost_{i}b_{i} + \sum_{\mathbf{i}} VariableCost_{i}c_{i} + \sum_{\mathbf{i}} \sum_{\mathbf{j}} \sum_{\mathbf{t}} \left(\frac{1.55}{20}\right) d_{ij}x_{jt}u_{ij} \\ & b_{i} = 1 \ \forall \ i = 1,2,3 \\ & c_{i} = 1,200,000 \ \forall \ i = 1,2 \\ & c_{i} = 900,000 \ \forall \ i = 3 \end{aligned} \\ & \sum_{\mathbf{j}} u_{ij} = 1 \ \forall \ j \\ & u_{ij} \leq b_{i} \ \forall \ i,j \\ & c_{i} = 1,200,000 \ \forall \ i = 1,2 \\ & 0 \leq c_{i} \leq CapacityDC_{i}b_{i} \end{aligned} \\ & \sum_{\mathbf{j}} u_{ij} \leq CapacityDC_{i} \left(\frac{5}{13.5}\right) \ \forall \ i \end{aligned}$$

$$c_{i} \leq 1,200,000b_{i} \ \forall \ i \\ & u_{ij}, b_{i} \in \{0,1\} \end{aligned}$$

model = Model(with optimizer(Gurobi.Optimizer, Gurobi.Env()))

set\_optimizer\_attribute(model, "OutputFlag", 0)

 $n = size(county\_tot\_d,1)$ 

- @variable(model, b[i=1:20], Bin)
- @variable(model, u[i=1:20,j=1:n], Bin)
- @variable(model, c[i=1:20] >= 0)
- @constraint(model, [i=1:3], b[i]==1)
- @constraint(model, [i=1:2], c[i]==1200000)
- @constraint(model, [i=3], c[i]==900000)
- @constraint(model, [j=1:n], sum(u[:,j])== 1)

```
(a) constraint(model, [i=1:20, j=1:n], u[i,j] \le b[i])
       @constraint(model, [i=1:20], c[i]*(5/13.5) >= sum(df[j]*u[i,j] for j=1:n))
       @constraint(model, [i=1:20], c[i] <= b[i] * 1200000)
       @objective(model, Min, sum(variable cost[i]*c[i] for i=1:20) + sum(fixed cost[i]*b[i]
       for i=1:20) +
         sum(sum((1.55/20)*d mat[i,j]*county tot d[j,:d pallets sum]*u[i,j] for j=1:n) for
       i=1:20)
       optimize!(model)
d.i) In addition to the 3 original DCs, there will also be DCs at Kalamazo, Lancaster,
Scranton, Syracuse, Toledo.
     b=value.(b)
Providence = 57
Richmond = 122
Youngstown = 120
Kalamazo = 222
Lancaster = 41
Scranton = 27
Syracuse = 60
Toledo = 116
       sum(u, dims=2)
Construction Cost = $556,050,072.64
Transportation Cost = $197,324,788.82
       construction cost = sum(variable cost[i]*c[i] for i=1:20) + sum(fixed cost[i]*b[i] for
       i=1:20)
       transpo cost = sum(sum((1.55/20)*d mat[i,j]*county tot d[j,:d pallets sum]*u[i,j] for
      i=1:n) for i=1:20)
Cheapest DC's w/ capacity (in sqft)
   Providence = 1,200,000
   Richmond = 1,200,000
   Youngstown = 900,000
   Bangor = 1,200,000
   Burlington = 1.200.000
   Dover = 1,200,000
```

d.ii)

d.iii)

e.i)

```
Kalamazoo = 700237.0249
   Worcester = 1,200,000
Construction Costs = $517,077,966.49
Transportation Costs = $505,919,172.89
       mod1 = Model(with optimizer(Gurobi.Optimizer, Gurobi.Env()))
       set optimizer attribute(mod1, "OutputFlag", 0)
       n = size(county tot d,1)
       @variable(mod1, c[i=1:20] \ge 0)
       @variable(mod1, b[i=1:20], Bin)
       (a) constraint(mod1, [i=1:2], c[i]==1200000)
       (a) constraint(mod1, [i=3], c[i]==900000)
       @constraint(mod1, [i=1:3], b[i]==1)
       @constraint(mod1, [i=1:20], c[i]<=b[i]*1200000)
       @constraint(mod1, sum(c[i]*(5/13.5) for i=1:20) >= sum(df[i] for i=1:n))
       @objective(mod1, Min, sum(fixed cost[i]*b[i] for i=1:20) + sum(variable cost[i]*c[i]
       for i=1:20)
       optimize!(mod1)
       mod2 = Model(with optimizer(Gurobi.Optimizer, Gurobi.Env()))
       set optimizer attribute(mod2, "OutputFlag", 0)
       n = size(county tot d,1)
       @variable(mod2,u[i=1:20,j=1:n], Bin)
       b = value.(b)
       c = value.(c)
       (a) constraint(mod2, [i=1:n], sum(u[:,i])== 1)
       @constraint(mod2, [i=1:20, j=1:n], sum(df[j]*u[i,j] for j=1:n) \leq 1.001*c[i]*5/13.5)
       @objective(mod2, Min,
       sum(sum(1.55/20*d mat[i,j]*county tot d[i,:d pallets sum]*u[i,j] for j=1:n) for
       i=1:20)
       optimize!(mod2)
```

e.ii) In the baseline solution, which first optimizes for construction cost and then generates transportation costs, the solution reflects the priorities. Construction cost is lower in the

second model because it is explicitly minimizes it first. However, this comes at the cost of higher transportation costs. In part d, we were only concerned with the overall cost and thus that was decreased, and transportation costs were significantly less.

#### **R-Code Appendix**

```
library(tidyverse)
library(Metrics)
library(zoo)
his = read.csv("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute of
Technology/AE/HWs/HW4/Dartboard historical-1.csv");
fut = read.csv("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute of
Technology/AE/HWs/HW4/Dartboard future.csv.crdownload");
dc = read.csv("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute of
Technology/AE/HWs/HW4/Dartboard dcs.csv");
#dc<-dc%>%mutate(County.Name = Location)
#mdf = merge(his, dc, by.x = "State.Name", by.y = "Location", all.x = TRUE)
his<-his%>%mutate(lsale = log(Sales/Population), saleper = Sales/Population,
          linc = log(Income), bseason = as.factor(Season))
train <- his%>%filter(Year <=2013)
test<-his%>%filter(Year == 2014)
mod <- lm(lsale~linc+Week Num+bseason, data = train)
summary(mod)
summary(mod)$r.squared
pred <- predict(mod, newdata=test)</pre>
#OSR^2
1 - sum((pred - test$lsale)^2) / sum((mean(train$lsale) - test$lsale)^2)
#OMAE
mae(test$lsale, pred)
#ORMSE
rmse(test$lsale, pred)
fut<-fut%>%mutate(linc = log(Income), bseason = as.factor(Season))
fut$forecast <- exp(predict(mod, newdata=fut))*fut$Population</pre>
#b
inv df <- fut %>% group by(Week Num) %>% summarize(tot d= sum(forecast)) %>%
 mutate(inv = (rollapply(data = tot d, FUN=sum, width=8, align="left", fill=NA)/1000))
#Claire Sailard helped me with this function
current cap<- dc%>%summarise(tot pallets cap = sum(5*Current Size/(13.5)))
ggplot(main = "Inventory vs. Capacity Over Time")+geom line(aes(x = Week Num, y = inv),
data = inv df) + geom hline(aes(vintercept=1222222, color = "red")) +
 geom text(aes(200,1222222,label = "Current Capacity", vjust = -1, color = "red")) +
geom text(aes(230,2322222,label = "Inventory", vjust = -1))+
```

```
labs(x ="Week #", y="Pallets", title="Inventory vs. Capacity Over Time")+
theme(legend.position = "none")+xlim(183,306)

#bi
percent_cap_short <- (inv_df[123,3]-current_cap)/(current_cap)*100
percent_cap_short

#exportation
fut$d_pallets = fut$forecast/1000
write.csv(fut,'/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute of Technology/AE/HWs/HW4/pred_d.csv')
```

# Julia Code Appendix

```
In [1]:
    using JuMP, Gurobi, LinearAlgebra, CSV, DataFrames, Pkg, Distances

In [2]:
    his = CSV.read("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute
    fut = CSV.read("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute
    dc = CSV.read("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute
    pred_d = CSV.read("/Users/bennetthellman/Desktop/OneDrive - Massachusetts Institute
    pred_d
In [3]:

pred_d
```

Out[3]: 99,450 rows × 16 columns (omitted printing of 8 columns)

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
1	1	9001	Connecticut	Fairfield County	41.244	-73.363	2015	27
2	2	9001	Connecticut	Fairfield County	41.244	-73.363	2015	28
3	3	9001	Connecticut	Fairfield County	41.244	-73.363	2015	29
4	4	9001	Connecticut	Fairfield County	41.244	-73.363	2015	30
5	5	9001	Connecticut	Fairfield County	41.244	-73.363	2015	31
6	6	9001	Connecticut	Fairfield County	41.244	-73.363	2015	32
7	7	9001	Connecticut	Fairfield County	41.244	-73.363	2015	33
8	8	9001	Connecticut	Fairfield County	41.244	-73.363	2015	34
9	9	9001	Connecticut	Fairfield County	41.244	-73.363	2015	35
10	10	9001	Connecticut	Fairfield County	41.244	-73.363	2015	36
11	11	9001	Connecticut	Fairfield County	41.244	-73.363	2015	37
12	12	9001	Connecticut	Fairfield County	41.244	-73.363	2015	38
13	13	9001	Connecticut	Fairfield County	41.244	-73.363	2015	39
14	14	9001	Connecticut	Fairfield County	41.244	-73.363	2015	40
15	15	9001	Connecticut	Fairfield County	41.244	-73.363	2015	41
16	16	9001	Connecticut	Fairfield County	41.244	-73.363	2015	42
17	17	9001	Connecticut	Fairfield County	41.244	-73.363	2015	43
18	18	9001	Connecticut	Fairfield County	41.244	-73.363	2015	44
19	19	9001	Connecticut	Fairfield County	41.244	-73.363	2015	45
20	20	9001	Connecticut	Fairfield County	41.244	-73.363	2015	46
21	21	9001	Connecticut	Fairfield County	41.244	-73.363	2015	47
22	22	9001	Connecticut	Fairfield County	41.244	-73.363	2015	48
23	23	9001	Connecticut	Fairfield County	41.244	-73.363	2015	49
24	24	9001	Connecticut	Fairfield County	41.244	-73.363	2015	50

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
25	25	9001	Connecticut	Fairfield County	41.244	-73.363	2015	51
26	26	9001	Connecticut	Fairfield County	41.244	-73.363	2015	52
27	27	9003	Connecticut	Hartford County	41.82	-72.718	2015	27
28	28	9003	Connecticut	Hartford County	41.82	-72.718	2015	28
29	29	9003	Connecticut	Hartford County	41.82	-72.718	2015	29
30	30	9003	Connecticut	Hartford County	41.82	-72.718	2015	30
:	:	:	:	:	:	:	÷	÷

In [4]: agg\_county = groupby(pred\_d, :FIPS\_Code)

 $_{\texttt{Out}\,[\,4\,]\,:}$  GroupedDataFrame with 765 groups based on key: FIPS\_Code

First Group (130 rows): FIPS\_Code = 9001

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
1	1	9001	Connecticut	Fairfield County	41.244	-73.363	2015	27
2	2	9001	Connecticut	Fairfield County	41.244	-73.363	2015	28
3	3	9001	Connecticut	Fairfield County	41.244	-73.363	2015	29
4	4	9001	Connecticut	Fairfield County	41.244	-73.363	2015	30
5	5	9001	Connecticut	Fairfield County	41.244	-73.363	2015	31
6	6	9001	Connecticut	Fairfield County	41.244	-73.363	2015	32
7	7	9001	Connecticut	Fairfield County	41.244	-73.363	2015	33
8	8	9001	Connecticut	Fairfield County	41.244	-73.363	2015	34
9	9	9001	Connecticut	Fairfield County	41.244	-73.363	2015	35
10	10	9001	Connecticut	Fairfield County	41.244	-73.363	2015	36
11	11	9001	Connecticut	Fairfield County	41.244	-73.363	2015	37
12	12	9001	Connecticut	Fairfield County	41.244	-73.363	2015	38
13	13	9001	Connecticut	Fairfield County	41.244	-73.363	2015	39
14	14	9001	Connecticut	Fairfield County	41.244	-73.363	2015	40
15	15	9001	Connecticut	Fairfield County	41.244	-73.363	2015	41
16	16	9001	Connecticut	Fairfield County	41.244	-73.363	2015	42
17	17	9001	Connecticut	Fairfield County	41.244	-73.363	2015	43
18	18	9001	Connecticut	Fairfield County	41.244	-73.363	2015	44
19	19	9001	Connecticut	Fairfield County	41.244	-73.363	2015	45
20	20	9001	Connecticut	Fairfield County	41.244	-73.363	2015	46

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
21	21	9001	Connecticut	Fairfield County	41.244	-73.363	2015	47
22	22	9001	Connecticut	Fairfield County	41.244	-73.363	2015	48
23	23	9001	Connecticut	Fairfield County	41.244	-73.363	2015	49
24	24	9001	Connecticut	Fairfield County	41.244	-73.363	2015	50
25	25	9001	Connecticut	Fairfield County	41.244	-73.363	2015	51
26	26	9001	Connecticut	Fairfield County	41.244	-73.363	2015	52
27	19891	9001	Connecticut	Fairfield County	41.244	-73.363	2016	1
28	19892	9001	Connecticut	Fairfield County	41.244	-73.363	2016	2
29	19893	9001	Connecticut	Fairfield County	41.244	-73.363	2016	3
30	19894	9001	Connecticut	Fairfield County	41.244	-73.363	2016	4
:	÷	÷	:	:	:	÷	:	:
:								

Last Group (130 rows): FIPS\_Code = 54109

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
1	19865	54109	West Virginia	Wyoming County	37.634	-81.539	2015	27
2	19866	54109	West Virginia	Wyoming County	37.634	-81.539	2015	28
3	19867	54109	West Virginia	Wyoming County	37.634	-81.539	2015	29
4	19868	54109	West Virginia	Wyoming County	37.634	-81.539	2015	30
5	19869	54109	West Virginia	Wyoming County	37.634	-81.539	2015	31
6	19870	54109	West Virginia	Wyoming County	37.634	-81.539	2015	32
7	19871	54109	West Virginia	Wyoming County	37.634	-81.539	2015	33
8	19872	54109	West Virginia	Wyoming County	37.634	-81.539	2015	34
9	19873	54109	West Virginia	Wyoming County	37.634	-81.539	2015	35
10	19874	54109	West Virginia	Wyoming County	37.634	-81.539	2015	36
11	19875	54109	West Virginia	Wyoming County	37.634	-81.539	2015	37
12	19876	54109	West Virginia	Wyoming County	37.634	-81.539	2015	38
13	19877	54109	West Virginia	Wyoming County	37.634	-81.539	2015	39
14	19878	54109	West Virginia	Wyoming County	37.634	-81.539	2015	40
15	19879	54109	West Virginia	Wyoming County	37.634	-81.539	2015	41
16	19880	54109	West Virginia	Wyoming County	37.634	-81.539	2015	42
17	19881	54109	West Virginia	Wyoming County	37.634	-81.539	2015	43
18	19882	54109	West Virginia	Wyoming County	37.634	-81.539	2015	44

	Column1	FIPS_Code	State.Name	County.Name	Latitude	Longitude	Year	Week
	Int64	Int64	String	String	Float64	Float64	Int64	Int64
19	19883	54109	West Virginia	Wyoming County	37.634	-81.539	2015	45
20	19884	54109	West Virginia	Wyoming County	37.634	-81.539	2015	46
21	19885	54109	West Virginia	Wyoming County	37.634	-81.539	2015	47
22	19886	54109	West Virginia	Wyoming County	37.634	-81.539	2015	48
23	19887	54109	West Virginia	Wyoming County	37.634	-81.539	2015	49
24	19888	54109	West Virginia	Wyoming County	37.634	-81.539	2015	50
25	19889	54109	West Virginia	Wyoming County	37.634	-81.539	2015	51
26	19890	54109	West Virginia	Wyoming County	37.634	-81.539	2015	52
27	59619	54109	West Virginia	Wyoming County	37.634	-81.539	2016	1
28	59620	54109	West Virginia	Wyoming County	37.634	-81.539	2016	2
29	59621	54109	West Virginia	Wyoming County	37.634	-81.539	2016	3
30	59622	54109	West Virginia	Wyoming County	37.634	-81.539	2016	4
:	:	÷	:	:	:	:	:	:

## Part D

```
In [5]:
          county_tot_d = combine(agg_county, :d_pallets => sum);
 In [6]:
          df = []
          for i in 1:size(agg_county,1)
              row num = size(agg county[i])[1]
              x = sum(agg_county[i][row_num-7:row_num,:d_pallets])
              append!(df, x)
          end
 In [7]:
          variable cost = Vector(dc[:,:Variable Cost]);
          fixed cost = Vector(dc[:,:Fixed Cost]);
 In [8]:
          d mat = zeros((size(fixed cost,1), size(agg county,1)))
          for i in 1:size(fixed_cost,1)
              dc lat = dc[i,:Latitude]
              dc long = dc[i,:Longitude]
              for j in 1:size(agg county,1)
                  county_lat = agg_county[j][1,:Latitude]
                  county_long = agg_county[j][1,:Longitude]
                  d mat[i,j] = haversine((dc lat,dc long),(county lat,county long), 3958.8)
              end
          end
In [44]:
          model = Model(with_optimizer(Gurobi.Optimizer, Gurobi.Env()))
```

set optimizer attribute(model, "OutputFlag", 0)

```
n = size(county_tot_d,1)
          @variable(model, b[i=1:20], Bin)
          @variable(model, u[i=1:20,j=1:n], Bin)
          @variable(model, c[i=1:20]>=0)
          @constraint(model, [i=1:3], b[i]==1)
          @constraint(model, [i=1:2], c[i]==1200000)
          @constraint(model, [i=3], c[i]==900000)
          @constraint(model, [j=1:n], sum(u[:,j])== 1)
          @constraint(model, [i=1:20, j=1:n], u[i,j] \le b[i])
          @constraint(model, [i=1:20], c[i]*(5/13.5) >= sum(df[j]*u[i,j] for j=1:n))
          @constraint(model, [i=1:20], c[i]<=b[i]*1200000)</pre>
          @objective(model, Min, sum(variable cost[i]*c[i] for i=1:20) + sum(fixed cost[i]
              sum(sum((1.55/20)*d_mat[i,j]*county_tot_d[j,:d_pallets_sum]*u[i,j] for j=1:n
          optimize!(model)
         Academic license - for non-commercial use only - expires 2022-08-19
In [45]:
          objective_value(model)
Out[45]: 7.533748614634609e8
In [46]:
          b=value.(b)
Out[46]: 20-element Vector{Float64}:
          1.0
          1.0
          1.0
          0.0
          0.0
          0.0
          0.0
          0.0
          0.0
          0.0
          1.0
          0.0
          0.0
          1.0
          0.0
          0.0
          1.0
          1.0
          1.0
          0.0
In [47]:
          u=value.(u)
Out[47]: 20×765 Matrix{Float64}:
          0.0 1.0 0.0 1.0 1.0
                                  1.0
                                         1.0 1.0
                                                      0.0
                                                           0.0
                                                               0.0
                                                                     0.0
                                                                         0.0
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In [48]:
            c=value.(c)
Out[48]: 20-element Vector{Float64}:
                  1.2e6
                  1.2e6
            900000.0
                  0.0
                  0.0
                  0.0
                  0.0
                  0.0
                  0.0
                  0.0
                  1.1999247319021097e6
                  0.0
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            701461.9812655906
                  0.0
                  0.0
                  1.199839817498397e6
                  1.1992867442985747e6
                  1.1999464575535741e6
                  0.0
In [49]:
            sum(u, dims=2)
Out[49]: 20×1 Matrix{Float64}:
             57.0
            122.0
            120.0
              0.0
              0.0
              0.0
              0.0
              0.0
              0.0
              0.0
            222.0
              0.0
              0.0
             41.0
              0.0
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             27.0
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10.0
116.0
116.0
0.0

In [50]: construction_cost = sum(variable_cost[i]*c[i] for i=1:20) + sum(fixed_cost[i]*b[
Out[50]: 5.560500726415393e8

In [51]: transpo_cost = sum(sum((1.55/20)*d_mat[i,j]*county_tot_d[j,:d_pallets_sum]*u[i,j]
Out[51]: 1.9732478882192114e8
```

## Part E

```
In [64]:
          mod1 = Model(with optimizer(Gurobi.Optimizer, Gurobi.Env()))
          set_optimizer_attribute(mod1, "OutputFlag", 0)
          n = size(county_tot_d,1)
          @variable(mod1, c[i=1:20]>=0)
          @variable(mod1, b[i=1:20], Bin)
          @constraint(mod1, [i=1:2], c[i]==1200000)
          @constraint(mod1, [i=3], c[i]==900000)
          @constraint(mod1, [i=1:3], b[i]==1)
          @constraint(mod1, [i=1:20], c[i]<=b[i]*1200000)</pre>
          @constraint(mod1, sum(c[i]*(5/13.5) for i=1:20) >= sum(df[j] for j=1:n))
          @objective(mod1, Min, sum(fixed cost[i]*b[i] for i=1:20) + sum(variable cost[i]*
          optimize! (mod1)
         Academic license - for non-commercial use only - expires 2022-08-19
In [65]:
          objective value(mod1)
Out[65]: 5.1707796649073195e8
In [66]:
          b=value.(b)
Out[66]: 20-element Vector{Float64}:
            1.0
            1.0
           1.0
           -0.0
           -0.0
           1.0
           -0.0
           -0.0
           1.0
           -0.0
           1.0
           1.0
           -0.0
```

```
-0.0
           -0.0
           -0.0
           -0.0
           -0.0
           1.0
           -0.0
In [67]:
          c=value.(c)
Out[67]: 20-element Vector{Float64}:
                1.2e6
                1.2e6
           900000.0
                0.0
                0.0
                1.2e6
                0.0
                0.0
                1.2e6
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                1.2e6
           700237.024943693
                0.0
                0.0
                0.0
                0.0
                0.0
                0.0
                1.2e6
                0.0
In [68]:
          mod2 = Model(with optimizer(Gurobi.Optimizer, Gurobi.Env()))
          set optimizer attribute(mod2, "OutputFlag", 0)
          n = size(county tot d, 1)
          @variable(mod2,u[i=1:20,j=1:n], Bin)
          b = value.(b)
          c = value.(c)
          @constraint(mod2, [j=1:n], sum(u[:,j])== 1)
          \texttt{@constraint(mod2, [i=1:20, j=1:n], sum(df[j]*u[i,j] for j=1:n) <= 1.001*c[i]*5/1}
          @objective(mod2, Min, sum(sum(1.55/20*d_mat[i,j]*county_tot_d[j,:d_pallets_sum]*
          optimize!(mod2)
         Academic license - for non-commercial use only - expires 2022-08-19
In [69]:
          objective value(mod2)
Out[69]: 5.059191728940411e8
```