6.215/6.255J/15.093J/IDS.200J: Optimization Methods

Problem Set 2

Due: October 12, 2021 1:00 PM

Several of the problems are from the course textbook, *Introduction to linear optimization*, by D. Bertsimas and J. Tsitsiklis.

Problem 1: (10 points) Bertsimas & Tsitsiklis, Exercise 2.4

Problem 2: (10 points) Bertsimas & Tsitsiklis, Exercise 4.1

Problem 3: (15 points) Solving primal and dual problems in Julia.

(a) Find the dual problem of the following LP:

$$\begin{array}{ll} \text{minimize} & c'x + d'y \\ \text{subject to} & Ax + By = b \\ & Dx \leq f \\ & Gy \leq g \end{array}$$

where $x \in \mathbf{R}^n, y \in \mathbf{R}^m$ are the decision variables, and everything else is the problem data. We have $A \in \mathbf{R}^{p_1 \times n}, B \in \mathbf{R}^{p_1 \times m}, D \in \mathbf{R}^{p_2 \times n}$, and $G \in \mathbf{R}^{p_3 \times m}$.

(b) For the given data in `data.jld2` (i.e., instances of c, d, A, B, b, D, f, G, g), solve both the primal problem and the dual problem and show that both have the same optimal solution. In this data file, $p_1 = 25, p_2 = 100, p_3 = 200, n = 50, m = 100$. Load the data into your Julia code by running the following commands:

```
using Pkg

Pkg.add("JLD2")

using JLD2

@load "data.jld2" c d A B b D f G g
```

Problem 4: (20 points) In order to implement complicated nonlinear functions in a computer, sometimes polynomial approximations are used. In this exercise, we explore one way of computing these using linear programming.

Consider a scalar function $f(x)$, which we are trying to approximate over the interval $[a, b]$ with a polynomial $p(x)$ of degree d. As a measure of how well the polynomial approximates the function, we can use the norm

$$||f - p||_\infty := \sup_{x \in [a,b]} |f(x) - p(x)|.$$

The *minimax* or *Chebyshev* polynomial approximation of degree d of $f(x)$ is then defined as

$$\min_{p \in P_d} ||f(x) - p(x)||_\infty,$$

where P_d is the set of polynomials of degree less than or equal to d. Because the true $||\cdot||_\infty$ norm can sometimes be troublesome to compute, throughout this exercise we will use instead a discrete approximation given by:

$$||g||_{\infty,N} := \max_{x_i \in [a,b]} |g(x_i)|,$$

where the x_i is a set of N points equispaced on the interval.

(a) Give a linear programming formulation of the Chebyshev approximation problem in the $||\cdot||_{\infty,N}$ norm.

(b) Write the corresponding dual problem.

(c) Give an interpretation of the complementary slackness conditions.

(d) Bonus: Using the finite approximation described, compute with Julia the minimax polynomial approximant of the function $f(x) = e^x$ in the interval $[-1, 1]$, for $d = 0, 1, 2, 3, 4$ and $N = 100$. Plot the resulting approximating polynomials, as well as the approximation errors.
You do not need to provide your code for this question. You are only required to write the resulting errors and show the plots.

(e) Bonus: Graphically compare the results with the dth order Taylor expansion around the origin $\sum_{i=0}^{d} x^i/(i!)$. What do you observe?

Problem 5: (15 points) Bertsimas & Tsitsiklis, Exercise 4.28