15.093/6.215 Optimization Methods

Problem Set 3

Due: November 4, 2021

Problem 1: Solving a network simplex problem. (10 points) Bertsimas & Tsitsiklis, Exercise 7.9.

Problem 2: Turning robust optimization constraint into tractable linear constraints. (10 points) In this problem, we consider the robust optimization constraint of the form:

$$\left(\bar{a} + \overline{P}z\right)^{\top} x \leq b, \quad \forall z \in U, \tag{ROC}$$

where $\bar{a} \in \mathbf{R}^n, \overline{P} \in \mathbf{R}^{n \times p}, b \in \mathbf{R}$ are known problem data, $x \in \mathbf{R}^n$ is the decision variable, but the vector $z \in \mathbf{R}^p$ is uncertain and lies in some uncertainty set U. For the following uncertainty sets, convert (ROC) into a set containing linear constraints. *Hint: Duality Trick.*

(a) Box uncertainty set: $U = \left\{z \in \mathbf{R}^p \mid \|z\|_\infty = \max_{i \in \{1,\ldots,p\}} |z_i| \leq \rho\right\}$, where $\rho > 0$ is a known constant. (3 points)

(b) Polyhedral uncertainty set: $U = \{z \in \mathbf{R}^p \mid Dz \leq d\}$, where $D \in \mathbf{R}^{m \times p}, d \in \mathbf{R}^m$ are known. The set U is known to be nonempty and bounded. (4 points)

(c) l_1-uncertainty set: $U = \{z \in \mathbf{R}^p \mid \|z\|_1 = \sum_{i=1}^p |z_i| \leq \rho\}$, where $\rho > 0$ is a known constant. (3 points)

Problem 3: Moving problems. (10 points) Suppose you are planning to move to your new house. You have n items of size a_j, $j = 1, \ldots, n$, that need to be moved. You have rented a truck that has size Q and you have bought m boxes. Box i has size b_i, $i = 1, \ldots, m$. Formulate an integer programming problem in order to decide whether the move is possible in only one travel. Specify what each integer variable models.

Problem 4: Relax or not? (10 points) Give an MIP formulation of each of the following problems. For each problem, can the integer constraints be relaxed in your formulation? Either prove it or give an example where the optimal solution of your relaxed problem is not integer (you can chose specific numerical values of the problem's parameters).

1. (6 points) We have 3 different coin denominations with values $v_1 = 1$, $v_2 = 2$ and $v_3 = 5$. The objective is to provide exact change for some amount $C \in \mathbb{N}$ using as few coins as possible.
2. (4 points) Let $n > 2$. Suppose there are n people to be assigned to n tasks. Every task has to be completed and each task has to be handled by only one person. Here $c_{ij} > 0$ measures the benefits gained by assigning the person i to the task j. The objective here is to maximize the overall benefits by devising the optimal assignment pattern.

Problem 5: The tournament problem. (10 points) Bertsimas & Tsitsiklis, Exercise 7.3.