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Using Geometric Models to Compose in Virtual Realms

INTRODUCTION

Modern advances in audio recording and engineering allow for the exploration of artificially created auditory realms. One possibility is for a listener wearing headphones to wander through a virtual room using a graphical user interface; the proper auditory sensations are then output to stereo headphones based on the listener's physical coordinates within this virtual room, simulating the direct correlation between sound and surroundings that we intuitively expect from our natural environment. Some composers have sought to take advantage of this new technology, creating interesting sonic collages in which subsequent experiences may vary widely according to each listener's chosen path of movement. Sophisticated musical narratives of a larger temporal scale still remain relatively unexplored, however, and may first require the development of a prescriptive music theory for voice-leading and harmony unique to this virtual environment.

In this paper, I will not delve into technological issues related to programming such a virtual room; my focus will be solely placed upon forming a prescriptive music theory. A practical starting point is the recent scholarship in Neo-Riemannian theory, with its emphasis on diatonic idioms freed from traditional harmonic functions. Jack Douthett's Cube Dance will provide an ideal model for introducing some key concepts. I will explain how the voice-leading properties of the various regions within a virtual room may be defined to reflect those of the

Cube Dance. The Cube Dance and the graph representing the virtual room may then be overlaid and combined, resulting in a multi-dimensional graph which can be used as a compositional space suitable for creating musical narratives within such a virtual room.

CUBE DANCE

The starting point for Douthett's Cube Dance is Richard Cohn's hexatonic systems. A hexatonic system is a cycle of six consonant triads belonging to a hexatonic collection of Forte-class 6-20 (014589), in which adjacent triads share two common tones, with the third tone differing by a semitone.[1] Douthett builds upon these cycles by noting that each consonant triad is also related by the same semitonal voice-leading to one of the two augmented triads belonging to this hexatonic collection, as shown in Figure 1. This combination of consonant and augmented triads may then be depicted as a cube, with each triad representing a single point on the cube, and each edge connecting two points representing parsimonious voice-leading by a semitone between two triads. Figure 2, taken directly from Douthett and Steinbach (1998), shows four such cubes, one for each hexatonic collection, with the hexatonic system from which each cube was derived shown in bold lines. [2]

[1] Richard Cohn, "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," *Music Analysis* 15, no. 1 (March 1996): 18-19. In Cohn's terminology, adjacent triads are IC1-related.
[2] Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (Autumn 1998): 253-54.

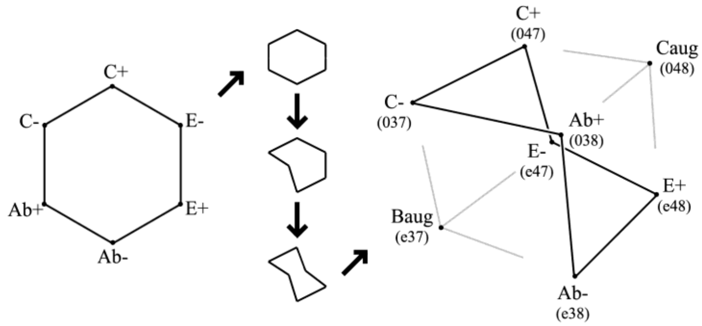

Figure 1: From hexatonic system to hexatonic cube

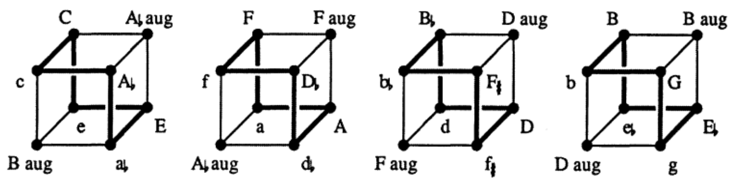

Figure 2: Hexatonic cubes from Douthett and Steinbach (1998)

Each hexatonic cube has an augmented triad in common with two other cubes. The Cube Dance joins all four cubes by their shared augmented triads, forming a single graph as shown in Figure 3, taken from the same paper.[3] This graph displays semitonal voice-leading transformations between all the consonant and augmented triads, and will be used as the compositional space upon which the theory I explain in this paper is based.

[3] Ibid.

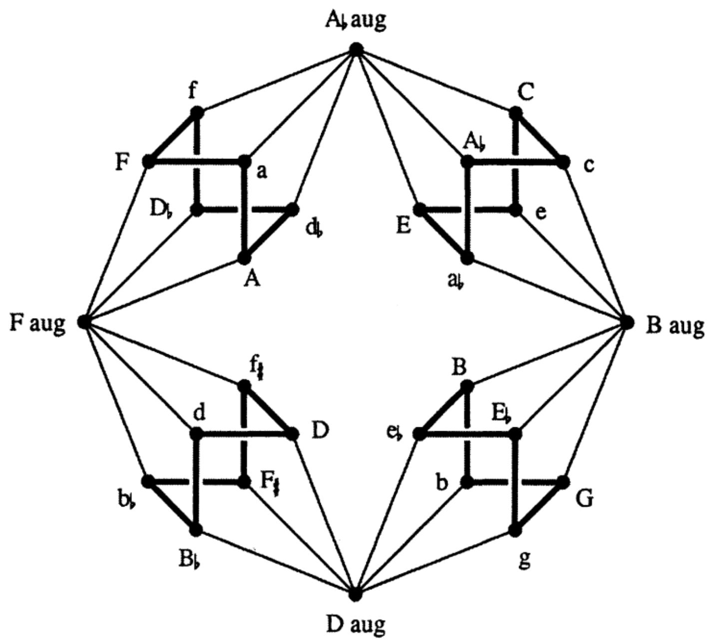

Figure 3: Cube Dance from Douthett and Steinbach (1998)

Compositional Spaces and "Temporal" Compositional Designs

Robert Morris describes the compositional process as a series of stages, beginning with all the ideas and knowledge already possessed by a composer, moving to a draft, then a score, and finally a performance. The realization of a draft may be assisted by two preceding stages: a compositional space, which is an atemporal structure displaying specific musical properties and relations, and a compositional design, which takes certain elements of that compositional space and arranges them in a temporal order.[4] Figure 4 demonstrates a simple compositional design derived from a single hexatonic cube.

[4] Robert Morris, "Compositional Spaces and Other Territories," *Perspectives of New Music* 33, no. 1 (Winter-Summer 1995): 329-30.

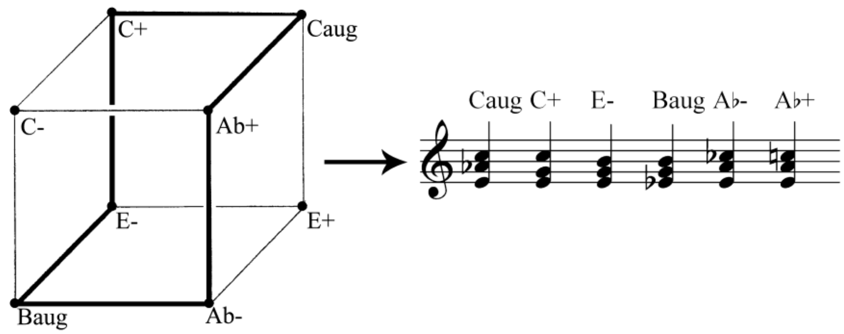

Figure 4: From compositional space to "temporal" compositional design

In addition to a compositional design which arranges musical elements in time, a musical narrative which takes place in a virtual room will also need a manner by which to arrange these elements in a physical dimension. The compositional space, itself a diagram rendered in physical space, is a natural starting point for defining both the physical properties of this virtual room and the manner by which a physical design for each moment of this musical work might be realized.

THE VIRTUAL ROOM

Any virtual auditory realm designed to adhere to a specific music theory must have license to bend or break the natural laws of acoustics when necessary. Understandably, while the creation of a believable free-field environment is certainly a desirable outcome, the ability to create coherent musical narratives for every possible path of movement within this virtual room must remain the first priority. With this freedom in mind, the desired qualities of such a virtual room can be seen as threefold: First, every region within this virtual room must obey the dictates of this music theory. Second, neighboring regions should overlap in ways that allow for smooth musical transitions as the listener wanders from one region to the next. And third, certain regions must remain exclusively separate from each other, for to have it be otherwise would defeat the purpose of realizing a compositional space in a physical dimension.

A simple, two-dimensional physical layout which can meet these requirements is immediately visible within the Cube Dance, in the form of the aforementioned hexatonic system described by Cohn. Neighboring triads of this LP-cycle certainly overlap, since they share two common tones with the third differing only by semitone, while hexatonic poles stay exclusively separate as they do not share any common tones.[5] The hexatonic system thus proves ideal in helping us shape two distinct sets of properties: the physical and acoustic properties of this virtual room as a fixed space, and the harmony and voice-leading properties which allow the Cube Dance to be realized within this space during a single point in time.

Physical and Acoustic Properties

A proposed physical layout for this hexatonic system is a circular room surrounded by six voices equidistantly placed at 60-degree intervals around the room. Perpendicular to the angle of each voice, a line bisecting the circle delineates the limits of that voice's range of projection. At any location in the virtual room inside the near side of this line, the voice is heard at a constant volume, while beyond it, the voice is not heard at all. For the sake of visual clarity, Figure 5 shows a sequential placement of each voice around the virtual room, one by one. In the last diagram, the overlapped regions of audibility for all six voices are displayed, showing that the virtual room is divided into six 60-degree wedges, in each of which three and only three voices are audible.[6]

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[5] Cohn, "Maximally Smooth Cycles," 18.

[6] In Figure 5, each voice's region of audibility actually extends slightly beyond the bisection, creating overlapped areas between the wedges in which the theory will not be applicable. I show that such areas are possible, for three

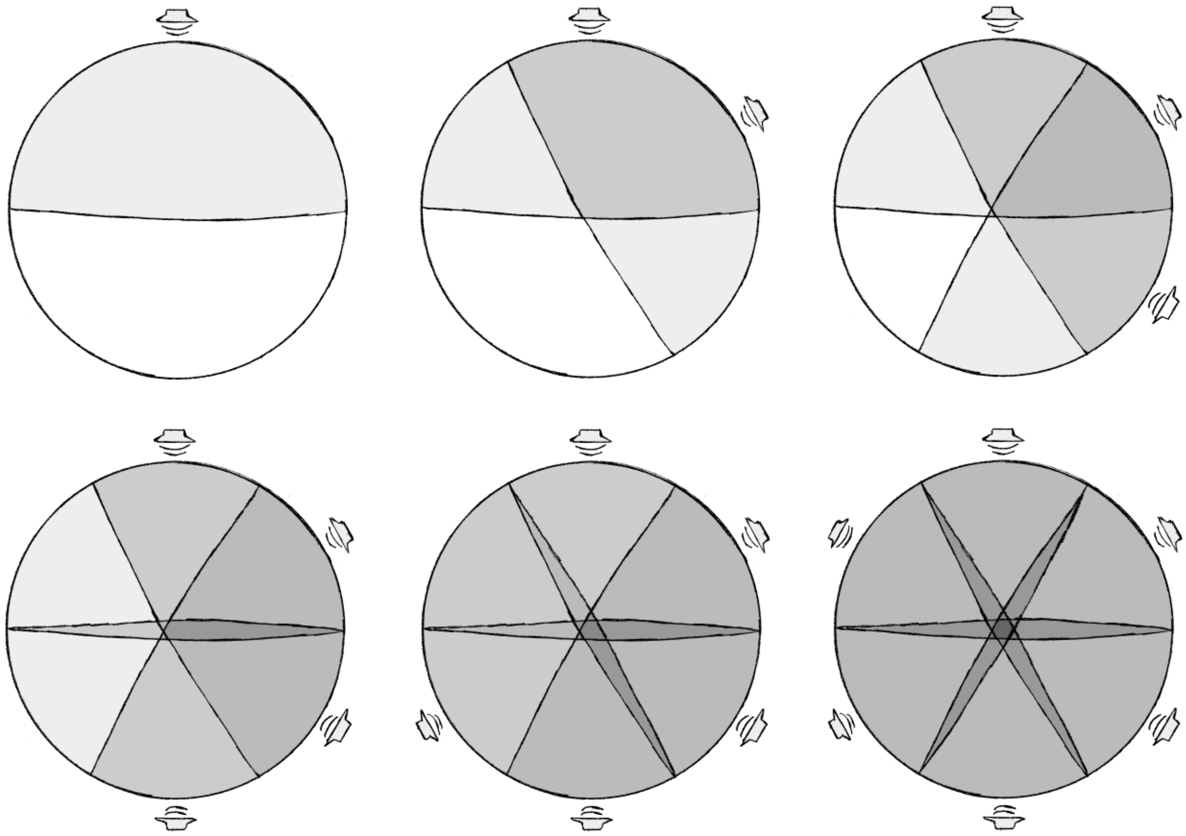

Figure 5: Overlapped regions of audibility in the virtual room

HARMONY AND VOICE-LEADING PROPERTIES

In Figure 6, the six pitch classes of hexatonic system $HEX_{3,4}$ are distributed amongst the six voices in such a way that each wedge of this hexagonal room represents one of the consonant triads. Adjacent wedges are related by parsimonious voice-leading in the same manner as

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reasons: First, they help create a more believable sound environment in which voices momentarily blend together rather than cut off abruptly, and second, they allow the listener to explore artistically interesting possibilities for voices to be combined in ways not necessarily intended by the composer. Thus, in the slivers between two wedges, four voices are heard; in the small triangles where two slivers overlap, five voices; and in the center, all six voices. The third reason is simply that they aid in visualization. These overlapped areas will be ignored for the remainder of this paper.

neighboring triads on the LP-cycle, with two shared common tones and the third differing by a semitone. Although this diagram represents only one possible snapshot in time, it provides a useful template for understanding how other segments of the Cube Dance, including the augmented triads, might also be realized within this virtual room.

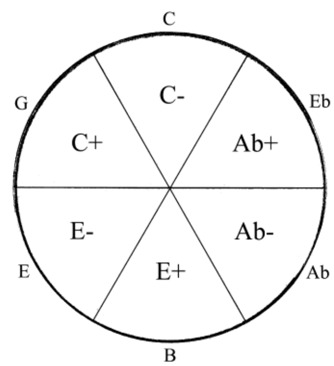


Figure 6: The hexatonic system realized in a virtual room

Because each wedge represents a complete consonant or augmented triad, adjacent voices may only be separated by interval classes 3, 4, and 5, with no two ic3s or two ic5s in a row, and all the intervals must either ascend or descend going clockwise around the room. For example, the LP-cycle in Figure 6 may be labeled as 3-5-3-5-3-5, since the voices alternate between minor thirds and perfect fourths. Additionally, because each cycle returns to the same pitch class after rotating through all six voices, the intervals must add up to 24, the sum of two octaves. There are ten possible permutations which obey these criteria, nine of which may be realized by a single hexatonic cube in the Cube Dance.[7] Examples of each of the nine permutations are shown in Figure 7, alongside their representation within a cube. The top number assigns a label to each

[7] I will overlook permutations containing diminished triads in this paper, as they lie outside the realm of the Cube Dance. However, eight such permutations are possible.

permutation by giving a prime ordering of its interval classes ascending clockwise; the example already given of 3-5-3-5-3-5, which I describe as the "all-consonant hexagon," is one such label.

As we consider these nine permutations, a noticeable pattern emerges: each voice toggles between two pitch classes separated by a semitone, with each pair of pitch classes spaced a major third apart, a property shared by all consonant and augmented triads that belong to a single hexatonic collection.[8] Because a single cycle around the room adds up to a double octave range, polar voices situated 180 degrees apart always toggle within the same pitch-class pair, and therefore either sound out the same pitch class or differ by a semitone. Given that adjacent wedges necessarily share two common tones between them, with the unshared third tone of each wedge facing each other across the room as polar voices, this means that adjacent wedges hear either the same triad or else two triads related by parsimonious voice-leading.

[8] Richard Cohn, "Weitzmann's Regions, My Cycles, and Douthett's Dancing Cubes," *Music Theory Spectrum* 22, no. 1 (Spring 2000): 95. Cohn refers to a musical representation of an LP-cycle as "an endless three-voice canon of interlocked upward and downward togglings."

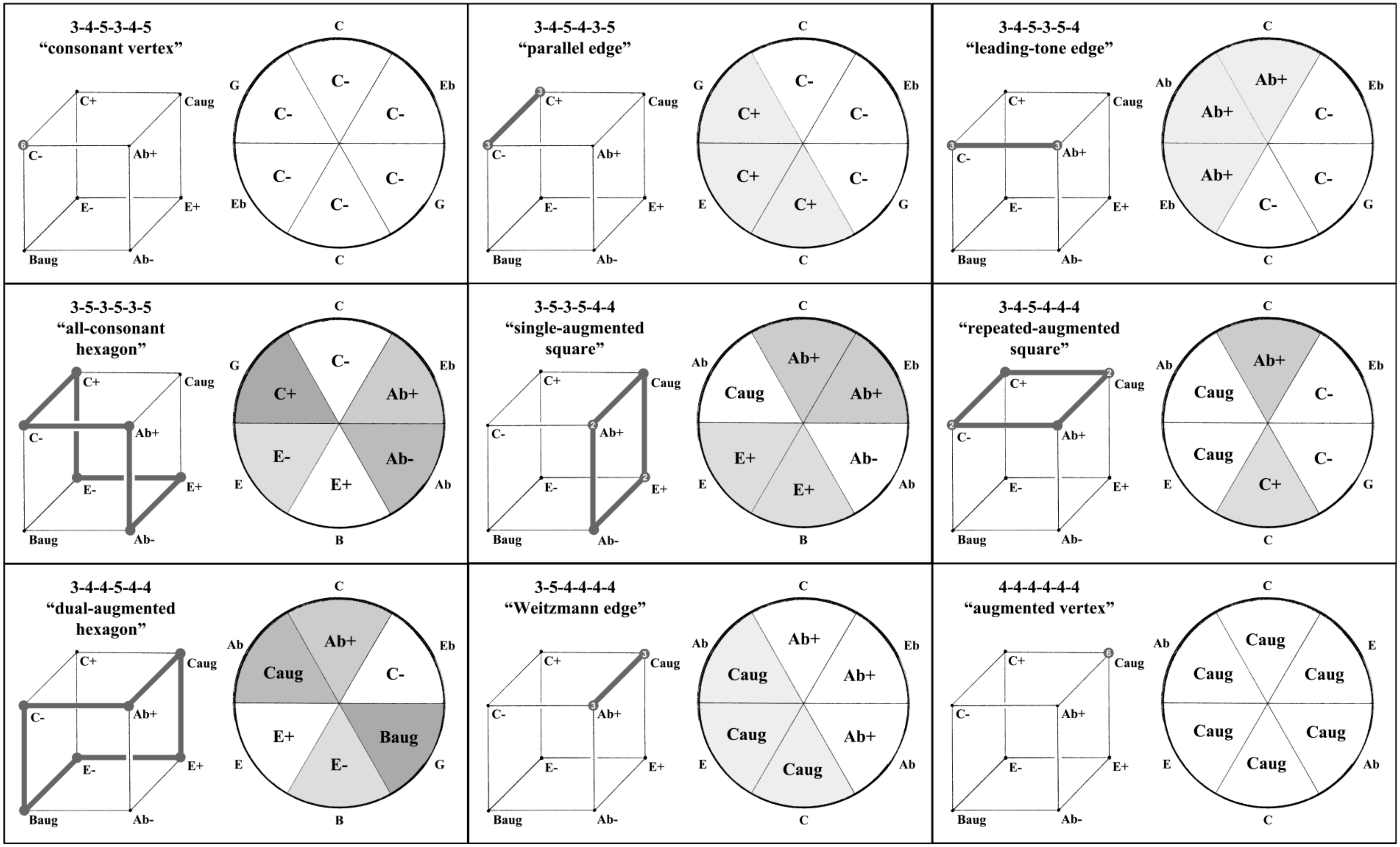


Figure 7: Nine permutations of the virtual room within a hexatonic cube

Thus, if the virtual room is seen as a flexible hexagon, with each of its six wedges represented by a single vertex, we can easily superimpose this hexagon atop the hexatonic cube as a closed loop in bold lines, much like Cohn's hexatonic system in Figures 2 and 3, with one caveat: a single point on the cube may be repeated multiple times by any divisor of six. For example, in the center box of Figure 7 labeled 3-5-3-5-4-4, the square shape of the virtual room's representation on the cube belies its six vertices; two of its points are actually stated twice. An easy way to visualize this is to imagine that the line segment which connects a point to itself is perpendicular to the planar surface of the page. Two other permutations, 3-4-5-3-4-5 and 4-4-4-4-4-4, which represent a single consonant triad and an augmented triad respectively, each represent six instances of a single point on the cube. These may also be visualized in the third dimension of depth, with three line segments rising in succession along a straight line perpendicular to the page, followed by three line segments coming down along the exact same path.

A tenth permutation exists which does not fit into a single hexatonic system and thus does not share all the same properties as the others. This permutation is 3-4-3-5-4-5, which gives us a virtual room evenly divided between relative triads, as shown in Figure 8. Because relative triads differ by a whole tone, this permutation is not easily represented on the Cube Dance. Still, its pattern is clearly identical to those of three other permutations that divide the virtual room evenly between two triads: the "parallel edge" 3-4-5-4-3-5, the "leading-tone edge" 3-4-5-3-5-4, and the "Weitzmann edge" 3-5-4-4-4-4.[9] Naturally, I call this tenth permutation the "relative edge," and happily enough, it allows the virtual room to undergo all three neo-Riemannian

[9] Richard Cohn, "Square Dances with Cubes," *Journal of Music Theory* 42, no. 2 (1998): 290. See also Cohn 2000. Cohn refers to the complete system which connects an augmented triad with three related consonant triads from each of the two hexatonic collections to which it belongs as a Weitzmann region, based on the work of 19th-century theorist Carl Friedrich Weitzmann. "Weitzmann edge" is thus a fitting label, as it represents a virtual room evenly divided between an augmented triad and a related consonant triad.

operations: L, P, and R. Moreover, because the relationship between relative triads is so firmly established in the traditional repertoire, the "relative edge" permutation provides an especially effective and subtle means for modulating from one hexatonic cube to the next.

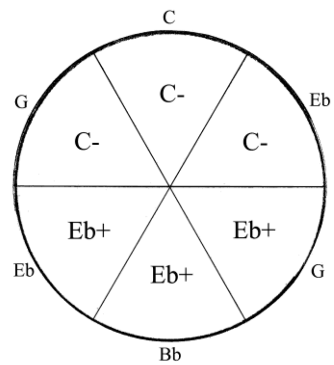


Figure 8: "Relative edge" permutation 3-4-3-5-4-5

A "Physical" Compositional Design and Musical Example

The possibility of deriving a systematic method for creating compositional designs which may be realized as musical narratives in physical space will be the subject of a future paper. In the absence of such a method, the example I show in Figure 9 was arbitrarily chosen and follows no consistent pattern. Figure 10 shows how the instruments for the forthcoming musical example derived from this compositional design are physically situated, with the location of each voice identified by its closest compass direction.

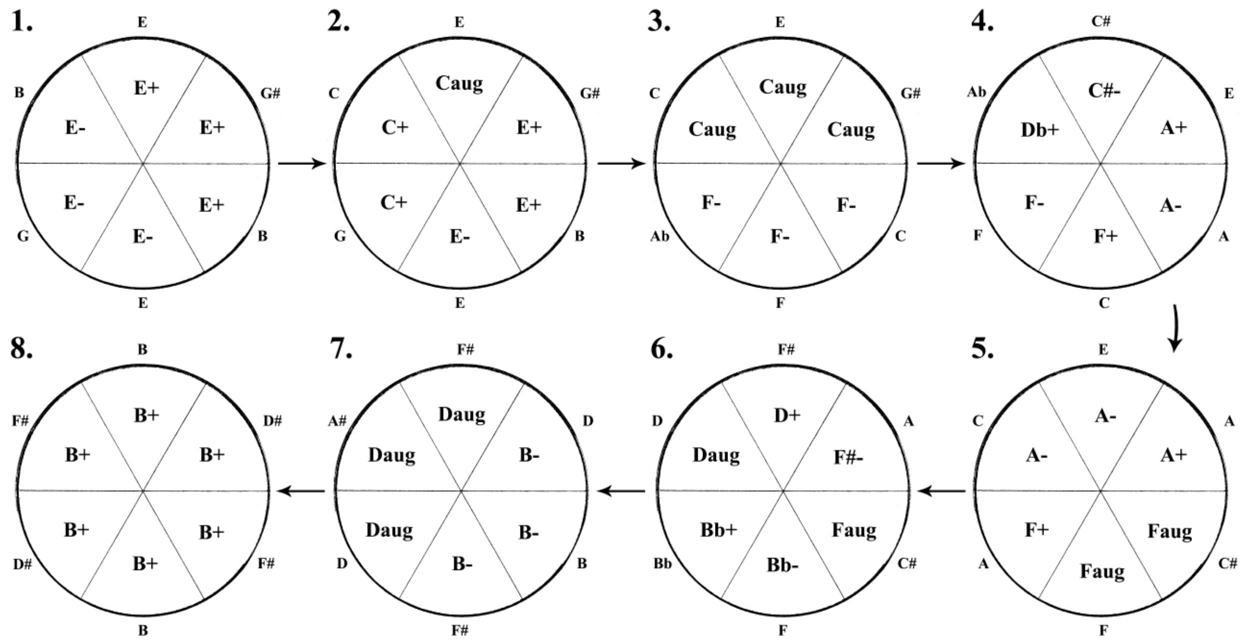


Figure 9: A "physical" compositional design

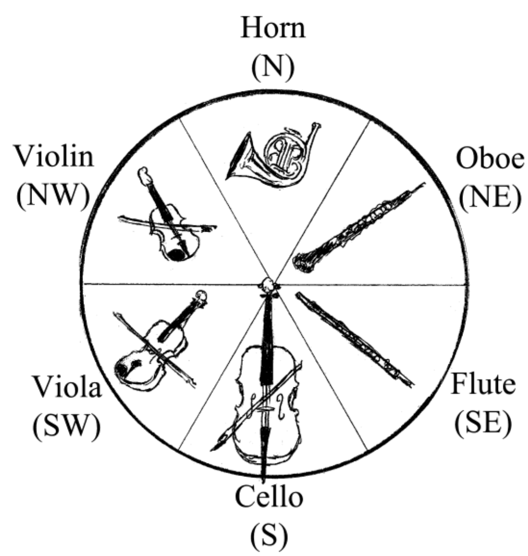


Figure 10: Instrumental layout of the musical example

Example 1 provides the full score for this piece, progressing at the rate of one measure per room snapshot provided by the compositional design. Understandably, our present system of notation does not capture two unique characteristics of this score. First, at any given moment in time, the wandering listener only hears the three instruments closest to the wedge in which he or she is located. For the sake of clarity, the instruments are situated next to each other on the score in the same manner as they are physically arranged around the room. By this reasoning, the second unique characteristic of the score is that the flute and the cello should also be viewed as adjacent neighbors. One can easily visualize rolling the page into a tube in such a way that the flute and the cello are no further apart from each other on the score than the flute from the oboe and the cello from the viola.

Example 1: Full score of the musical example

Figure 11 shows a few possible paths through which a listener may wander during the performance, while Examples 2a through 2d show idealized transcriptions of how each ambulation might sound through the headphones. (In practice, transitions between wedges may not match up so precisely with the barlines or any other temporal markers.)

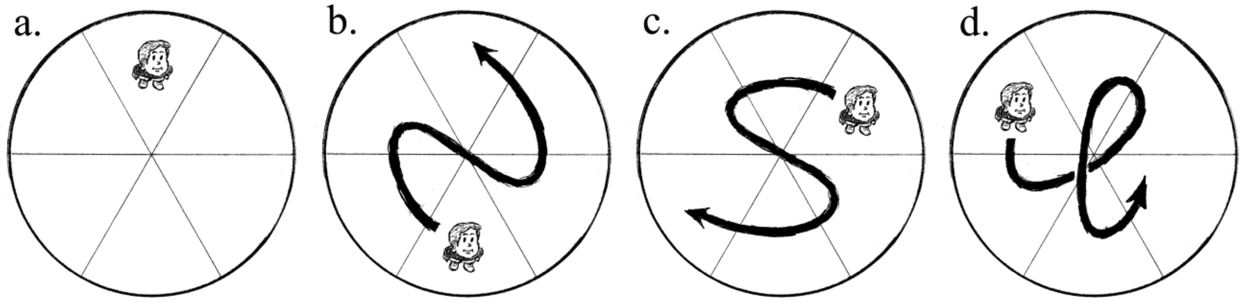
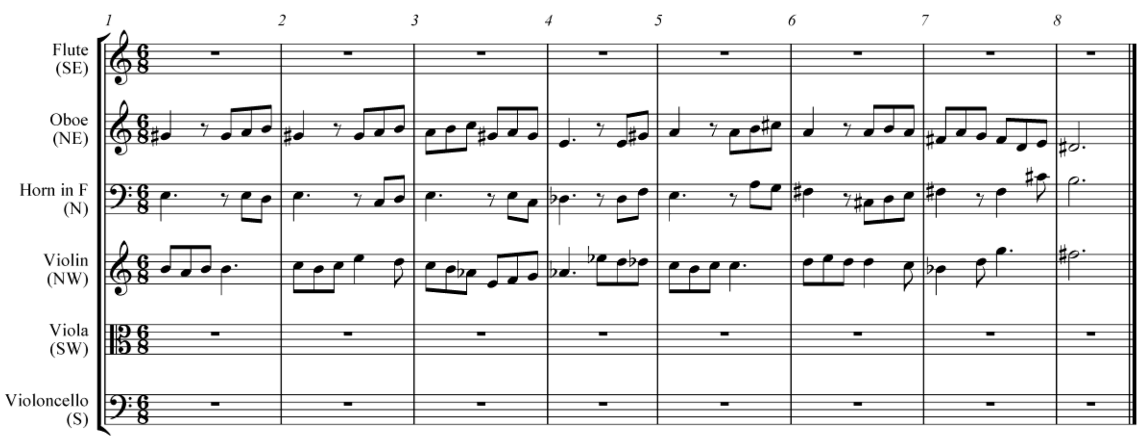


Figure 11: A few possible ambulations



Example 2: Transcriptions of the ambulations

b.

c.

d.

Example 2 (continued)

Future Creative Applications and Research

The interdisciplinary nature of this theory leaves it open-ended in multiple and diverse directions. The following lists a few possibilities for further study.

- A user interface for these virtual rooms has yet to be designed and coded. Wireless tracking may also allow multiple listeners to simulate motion within the virtual room by wandering in an enclosed physical space. Such a setup hints at the possibility for allowing listeners to enjoy the work as a communal experience, or even to interact with each other individually.
- More voices may be added to the virtual room realized by the Cube Dance, logically but perhaps not necessarily in multiples of 3. Each additional set of voices would increase the number of permutations by an exponential degree. A formula for identifying and labeling all possible permutations for any number of voices may be determinable.
- Cohn points out that the consonant triad, the dominant and half-diminished sevenths of Forte-class 4-27, and the Mystic chord of Forte-class 6-34 all share a common characteristic: they are all offset from an equal division of the octave by a single semitone in one voice.[10] Douthett and Adrian Childs have both examined parsimonious voice-leading in 4-27, while Clifton Callender has identified some transformations unique to Scriabin's use of 6-34.[11] Based on their shared voice-leading property with the consonant triad, both set classes may also be well-suited for composition within a virtual room.

[10] Cohn, "Weitzmann's Regions," 100-101.
[11] See Douthett and Steinbach (1998), Adrian P. Childs, "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords," *Journal of Music Theory* 42, no. 2 (Autumn 1998), and Clifton Callender, "Voice-Leading Parsimony in the Music of Alexander Scriabin," *Journal of Music Theory* 42, no. 2 (Autumn 1998).

- Audio engineering techniques such as voice panning and volume attenuation may be refined to simulate the auditory cues given by live sound sources more effectively.

- As previously mentioned, it would be desirable to derive a systematic method for constructing compositional designs which may be used to realize musical narratives in a virtual room. The discovery of such a method, along with other compositional strategies, may be less dependent on a prescriptive approach to theory, however, and more on a descriptive one. As such, the steady practice of composing and executing new pieces in this format will perhaps be the most effective way to solve immediately identifiable problems, while simultaneously revealing new and unforeseen challenges.

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