Estimating Aptera Physical Parameters

October 22, 2023

1 Introduction

This paper describes the methodology used to estimate various physical paramters of an Aptera vehicle, and performs some simple optimality calculations with the derived values.

2 Physical Parameters

To estimate the range of an Aptera production vehicle, we need various physical parameters. Aptera has published the drag coefficient, C_d, as 0.13. However, to use it in the standard drag force equation:

$$F_d = \frac{1}{2}\rho v^2 C_d A \tag{1}$$

we need the frontal area A. Recently, Aptera published on *thingverse* a $\frac{1}{48}$ scale model to allow Aptera enthusiasts to print a 3-d model. The 3-d model was supposedly derived from the engineering models, so simply projecting the model to a plane perpendicular to the front-rear axis should yield a value for A. This is not a novel idea, and a github repository (`https://nathanrooy.github.io/posts/2021-03-26/compute-stl-frontal-area/`) already has code to do this. The computed value is

$$A = 2.21m^2 \tag{2}$$

which compares reasonably well with motormatchup.com's value of $A = 2.1m^2$.

The value of C_{rr} can be estimated if we knew the actual energy used to determine vehicle range at a given speed, by solving for C_{rr} in

$$\begin{aligned} E &= F_d x + F_{\text{rr}} x \\ &= x\left(\frac{1}{2}\rho v^2 C_d A + m g_0 C_{\text{rr}}\right) \end{aligned}$$

but this ignores other energy sinks.

3 Optimal Speed

When an electric vehicle's battery pack is nearly exhausted, it would be nice to know the optimal speed to drive at to maximize the range. Equation 1 gives the drag force, and the force due to rolling resistance is given by:

$$F_{\text{rr}} = C_{\text{rr}} m g_0 \tag{3}$$

where C_{rr} is the coefficient of rolling resistance. Let E represent the total energy available. Using that energy to travel some distance x, we have

$$E = F_{\text{net}} x + P_r t \tag{4}$$

where P_r is the power usage at rest or idle, e.g., energy used by the vehicle's microcontrollers / computers, displays, etc. We derive from equation 4 the following:

$$\begin{aligned} E &= F_{\text{net}} x + P_r \frac{x}{v} \\ &= \left(F_{\text{net}} + \frac{P_r}{v} \right) x \end{aligned}$$

so

$$\begin{aligned} x &= \frac{E}{\frac{1}{2}\rho v^2 C_d A + m g_0 C_{\text{rr}} + \frac{P_r}{v}} \\ \frac{dx}{dv} &= \frac{-E}{\left(\frac{1}{2}\rho v^2 C_d A + m g_0 C_{\text{rr}} + \frac{P_r}{v}\right)^2} \cdot \left(\rho v C_d A - \frac{P_r}{v^2} \right) \end{aligned}$$

and setting $\frac{dx}{dv}$ to zero, we obtain

$$v^3 = \frac{P_r}{\rho C_d A}$$