

Agent Based Discrete Event Simulations of Markets under Different Market Trends

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Abstract

The goal of modern financial markets is to ensure pricing efficiency remains strong even in the presence of a discrepancy amongst the number of buyers and sellers. In this study, we showcase the efficiency under varying market conditions. This is accomplished by utilizing a discrete event-based simulation (DES) with agents representing buyers and sellers who arrive according to stochastic interarrival processes and place limit orders with price beliefs generated as random deviations away from the prevailing market midpoint. The manipulation of interarrival rates for buyers and sellers allows for the simulation of different market trends while showcasing their effectiveness in pricing an asset. The architecture Limit-Order Book (LOB) tracks key market statistics such as bid-ask spreads, queue sizes, waiting times and order filling data which will be key in quantitatively contrasting market efficiency. Multiple independent simulation runs are performed across markets to reduce noise and construct confidence intervals for market metrics.

1 Introduction

This study uses 'zero-intelligence' agents who interact in a market to trade a singular asset. Zero intelligence agents generate random bids/asks dependent on the most recent price of the asset, we assume no memory, and no individual characteristics. [1] The investors are simply generating variations away from the price which allows for trading to occur in a market place. Agents are split into buyers and sellers, where buyers are looking to purchase the asset and sellers are looking to offload their owned asset. Ultimately the presence of buyers and sellers with their own price beliefs dictate the price of the asset due to interacting laws of supply and demand. Many different market environments can be simulated by the manipulation of these presences of buyers and sellers to understand how pricing efficiency changes.

The buy and sell orders sent by investors will be placed in a double sided queue. A priority system based on both price and time of order will be implemented in order to match buy and sell orders. An order is associated with an investor, and therefore an order being in the queue can be likened to the investor waiting in the queue. Exiting the queue, or equivalently, having an order filled, can be likened to receiving service. Due to the instantaneous nature of an order being filled, there is no service time in this simulation.

The emulation of the bid-ask spread will be crucial to understanding pricing effectiveness. The bid-ask spread of an asset is the difference between the highest purchasing price sent to the market by the buyer and lowest selling price sent to the market by a seller. The orders to buy the assets are the "bids" and the orders to sell the assets are the "asks". The bid-ask spread provides insight into the liquidity of the market for the asset. The liquidity of an asset is how easily and frequently the asset is traded, it is one of the industry standard ways of quantifying market efficiency. A highly liquid market experiences frequent and consistent orders from both buyers and sellers resulting in a high number of trades. A non-liquid asset is one that is not often traded, leading to higher friction in attempting to complete a transaction, which usually results in one side of the market having to alter their price to get a trade to take place. As an example, an asset like Apple Inc. stock has many people willing to buy or trade shares, so completing a successful buy or sell order near the bid-ask spread has less friction. Conversely, an asset that not many people know of will not have as many orders in a given period of time, causing the process of finding an investor to accept a given order more difficult and slow. An asset that has high liquidity is likely to have a smaller bid-ask spread, due to the higher number of individuals trading the stock which increases competition for an order to be traded. An illiquid asset will have a higher bid ask spread as there is less competition which makes the pricing belief between the buyers and sellers larger. By altering the prevalence of buyers and sellers in the market, it is hypothesized that the liquidity of the asset being traded will change. This is to be measured using the bid-ask spread alongside other market metrics.

The use of a Limit-Order Book to track all transactions and their necessary details will be the centerpiece of the simulation, and the main way in which statistics will be calculated and presented. The Order Book will track metrics such as the list of bids and asks, a history of the orders and trades, the best bid or ask at any time, and the fundamental price of the asset before simulation. Orders themselves will have identifying information such as the investor ID associated with the order, price of the order, as the times the order was placed and filled (if it ever was).

Varying market trends will be simulated by manipulating the underlying frequency of buyers and sellers coming to the market, allowing for bull, bear, and neutral markets to be emulated. A bull market is categorized as a market in which investors are optimistic about the asset and will be wanting to place buy orders more frequently than sell orders. Prices are often rising in bull markets. A bear market shows the opposite, a market of low morale and pessimism about an asset reduces the frequency of buy orders with more asset holders wanting to sell their asset. A neutral market tends to have prices staying within a certain range and a somewhat equal proportion of buyers and sellers. The liquidity of the simulations will be studied in order to understand if the simulation architecture lends itself to replicating many aspects of a market trend, not just the movement of the price.

2 Methods

2.1 Investors

In order to establish a market, there must be two types of investors; buyers and sellers. Object-Oriented Programming is used to create these investors. The buyers and sellers are two child classes of the parent class Investor. The buyers are only able to buy the asset, whereas sellers are only able to sell. Investors have two defining features: their interarrival rate and their price belief.

Interarrival Rate: The key feature that impacts whether an investor is optimistic, pessimistic, or neutral about the asset at its current price is the distribution that defines the investor's interarrival times, which is set as an exponential distribution. The cumulative distribution function (CDF) of an exponential random variable is given below:

$$P(X \leq x) = 1 - e^{-\lambda x} \quad (1)$$

[2] The buyers have different exponential distributions assigned to them than the sellers. By varying the parameter of the distribution that generates interarrival times of the sellers or the buyers, it changes the mean of the rate of arrival. Therefore, by shifting the parameters of the investors, the "sentiment" of the market can be impacted by creating more demand (buying) than supply (selling), or the opposite.

Price Belief: The prices at which investors can place orders is determined by a uniform distribution and the most recent price that is defined with a lower bound that is 2% below the initial asset price, and an upper bound that is 2% above the initial asset price. This is a key assumption of our simulation, as we assume that the noise distribution is dependent on the noise around the initial price rather than the most recent price. This simulation assumes a starting price of \$100.

$$\tilde{P}_t = P_t(1 + U_t), \quad U_t \sim \text{Uniform}(-0.02, 0.02) \quad (2)$$

As this price shifts, the midpoint of the interval changes to the current asset price, but the length of the interval remains the same. That is, if the asset price decreases to \$99 in the simulation, then at that time the new distribution in which prices are generated is $U(97, 101)$. This noise around the most recent midpoint takes into account the two key components of an investors price belief; the market price and their price belief.

2.2 Market Trends/Conditions

Utilizing the above architecture for the agents in the market, the difference in the number of buyers and sellers can be initialized and tested. This is how a bull market can be simulated and analyzed. Conversely, a market where sellers are more active than buyers can be used to simulate a bear market. Finally, the simplest way to simulate a neutral market is to allow the mean arrival rate to be the same for the buyers and sellers. Under these various market conditions, it is hypothesized that the frequency of orders placed will have a large effect on how the market behaves and which transactions are carried out.

In a bull market, it is suspected that more buy orders will go unfilled, as the demand will be much larger than the supply. As a result of this, it is suspected that the sell orders that are placed will be filled quicker than under any other market trend, due to there being such high demand present. As a result of the above conditions, we expect the price of the asset to be driven upwards towards a new equilibrium and the overall liquidity to suffer, due to the imbalance between buy and sell orders

In a bear market, the exact opposite of what is experienced in the bull market is hypothesized to occur. More supply than demand is expected, leading to suffering liquidity and more sell orders going unfilled, thereby remaining in the queue. The asset price will likely be driven downwards to a new equilibrium price.

2.3 Limit-Order Book

The Limit-Order Book (LOB) is the core concept of the simulation and governs all price formation and trade execution. It maintains the pending bid and ask priority queues with their associated prices, volumes, and time of order. These queues enable the price-time priority mechanism that underlies the order matching algorithm to take place. The bid and ask queues make use of a heap-based priority queue data structure which abides by a priority system. Orders are prioritized first by price and then by time of submission commonly referred to as a FIFO queue. In the bid queue the higher the price the higher priority and in the ask queue the lower the price the higher the priority. This structure ensures a hierarchical system so that orders closest to the current asset price are matched first compared to those further away.

A transaction occurs when an incoming order crosses the opposing price in the book. For example, a buy order is a successful transaction if a buy order is placed above the minimum sell price, then the order is matched to the minimum sell price and those orders become a transaction. The same occurs when a seller's price is below the maximum bid, that maximum bid price becomes the transaction price and the trade is executed with both participants being removed from their priority queues in the LOB. This naturally arises to the idea of the bid-ask spread which is defined as the distance between the top of the bid queue and at the top of the ask queue. This difference tells us how much a buyer/seller must be willing to trade away from their current best price in order to have their transaction take place, which serves as a way of measuring liquidity and pricing efficiency. As trades are executed over the course of the DES, the state of the bid and ask queues evolve which ultimately changes the bid-ask spread, consequently the asset price as well. The asset price is the midpoint of the bid-ask spread and is updated with each trade that takes place. This asset price serves as the market price and is the reference for new incoming orders. This information is stored in the LOB and allows for Figures 1 through 9 in the Appendix to give the snapshot of the LOB throughout the market trading day.

2.4 Tracking Statistics

In order to conduct output analysis on the simulation and find the statistics on interest, functions are defined that automatically calculate the statistics from the record of the simulation in the Limit-Order Book. Statistics and quantities that are calculated include the best bid price, best ask price, the midpoint, the average size of the spread, the average wait time for a filled order, the average wait time for an unfilled order, the average time for all orders, the average bid queue size, the average ask queue size, and the percent of orders filled. The simulations for each market trend are run multiple times, and from this the variance of the statistics and quantities are calculated, which allows for confidence intervals to be calculated.

2.5 Simulation Architecture

Initialization: The simulation is conducted using Simpy, a Python simulation library. Parameters such as the initial price of the asset, the number of investors, the distributions that generate arrival times and prices, and how long the simulation is run from are given to the simulation program. These parameters are set intentionally by the user.

Inverse Transform Method: The random variates for interarrival and price for an investor are determined using the inverse-transform method. This method takes the uniformly random generated variates and uses the inverse of the desired distribution, in this case the exponential distribution, in order to generate realizations from that distribution. In scenarios where uniform random variates are needed, NumPy is utilized.

Common Random Number (CRN) Generation: A key aspect of this simulation structure is the variance reduction technique that is employed to limit variation between simulation run statistics. A method named Common Random Numbers (CRN) is utilized in order to get more accurate comparisons between the runs. This method involves using the same string of random numbers for different simulations to see how the simulation performs under the same experimental conditions [2]. This is done easily using "seeds" of random number generation with numpy. Providing a seed to numpy means that if the same seed is provided more than once, the same random number sequence will be realized in generation. To ensure consistency, the number stream used to generate the interarrival times is separate from the number stream used to generate the prices. This ensures that the same random numbers are used for the same purpose in each simulation.

An additional concern when using CRN is that we should also ensure that the same stream of random numbers is used in all three market situations for all the times it is run. In other words, since the simulation is run multiple times under each condition (bull, bear, neutral), CRN must be implemented in such a way that the first run of each condition should use the same stream of random numbers, then second run of each condition should share another stream of random numbers, and so on. This can be done in the following way: say each market condition is run ten times. Then ten seeds should be generated, and the first seed used in the first run of the bull, bear, and neutral simulations. The same is true for the second through tenth run. Implementing CRN is a powerful tool that will allow for great results, but it must be done diligently and carefully. The important outcome of using CRN across multiple simulations runs is the consequence of Independent Identically Distributed (IID) Output variates. This inhibits statistical analysis to explore the variance, sample mean, and ultimately the confidence interval of our output analytics which ensures more robust and reliable results. Without such a method the impact of a certain random process may outweigh the underlying mechanisms which are being invested which would skew results.

3 Results

Each market has been initialized under their distinct presumption of market behavior. In each simulation, the initial price of the asset was set to 100, the simulation was run for 6 hours, and the number of times the market condition was simulated was 30. With 30 simulations it can be appropriately assumed that the distribution of the results converges to the normal distribution in each case, which means the sample variance and sample mean hold significance in computing confidence in the results.

3.1 Neutral Market

In order to create a neutral market, the arrival rate for buyers and sellers were set to be equal. Each was set to 1, therefore lambda in the exponential distribution is equal to 1. A summary of the 95% confidence intervals under this setup follows.

Table 1: Neutral Market Statistics (95% Confidence Intervals)

Statistic	Value
best_bids	[99.734, 100.016]
best_asks	[100.142, 100.423]
midpoints	[99.938, 100.22]
spreads	[0.405, 0.421]
completed_wait_times	[5.653, 6.102]
ongoing_wait_times	[72.486, 74.893]
total_wait_times	[20.049, 21.258]
bid_queue_size	[38.2, 41.555]
ask_queue_size	[36.851, 40.167]
pct_filled	[0.79, 0.799]

Under the neutral market, the 95% confidence interval of the midpoint of the bid-ask spread, which determines the price of the asset in this setup, still contains the original asset price of one hundred. Therefore, at the 0.05 significance level, it cannot be concluded that the price of the asset changes under this setup. This is strong evidence for the simulation of the neutral market being valid, since under this

market it is expected to have an asset price staying near its initial level. The confidence intervals of the best bids and best asks also support this conclusion, as these intervals both contain one hundred as well, and they overlap. This shows that the equilibrium price on average stays near the initial price, even after six hours of simulation time.

Figure 1 in the appendix has a visualization of the bid-ask spread over time for one of the simulation runs of the neutral market. This graph shows how the spread does shift as transactions occur, but ultimately the midpoint stays very close to the original price despite the fluctuations. Additionally, in this figure it is shown that the limit-order book at the termination of the simulation has almost no orders remaining near the initial price, but many orders unfilled below and above the price. This is further evidence supporting the claim that the neutral market encourages the price to stay near the initial value throughout the simulation.

Shifting focus to the liquidity of the market, there is strong evidence that the neutral market has high levels of liquidity, as evidenced by the confidence intervals for the percentage of orders filled, the queue sizes, the total delay (wait time), and the average spread. The average percentage of orders filled over the thirty simulations is between 78.45% and 79.88% at the 0.05 significance level. This is relatively high and shows that the demand matches the supply. The confidence intervals for the average bid queue size and the average ask queue size overlap from 37.64 to 40.96. This demonstrates that the architecture that defines the buyers' demand and the sellers' supply does work. It is also evidence for the neutral market having high liquidity under this simulation architecture.

3.2 Bull Market

In order to create a bull market, the arrival rate for buyers was set to 2, and the arrival rate for sellers was held at 1. A summary of the 95% confidence intervals under this setup follows.

Table 2: Bull Market Statistics (95% Confidence Intervals)

Statistic	Value
best_bids	[101.610, 101.917]
best_asks	[101.527, 102.918]
midpoints	[102.130, 102.520]
spreads	[0.785, 0.824]
completed_wait_times	[3.539, 4.761]
ongoing_wait_times	[81.024, 85.320]
total_wait_times	[29.024, 31.381]
bid_queue_size	[169.408, 182.806]
ask_queue_size	[1.132, 1.509]
pct_filled	[0.662, 0.682]

Under the bull market, the 95% confidence interval for the midpoint of the bid-ask spread did not contain the initial price of one hundred. It spanned from 102.13 to 102.52, giving strong indication that the bull market increased the equilibrium price of the asset as was intended. The confidence intervals also show a higher level of best bids and best asks. Taking these factors into consideration, it seems reasonable to conclude that under the architecture of the bull market where demand outpaces supply, the equilibrium price rises on average. Figure 2 in the appendix shows how for one of the simulation runs, the bid-ask spread tends to rise over time under this architecture.

Analyzing the liquidity of the bull market in this simulation leads to a few interesting findings. It seems that the orders that are filled tend to be filled quicker, but the overall wait time for an order to be filled does increase. This is due to the fact that since there was much more demand than supply, the sell orders that were placed were filled very quickly as there was a high likelihood that there was a buy order to match with it. This is confirmed by noticing that the average delay in the ask queue (sellers) drops significantly but the average delay in the bid queue (buyers) rises significantly. Additionally, figure 2 shows that the orders remaining in the queue after the simulation terminates are mostly buy orders in one simulation, providing even more evidence for this assertion. Therefore, orders that were filled were filled quicker on average than a neutral market. However, the overall wait time likely increases due to there being a large number of demand being unfilled, leading to many orders staying in the queue for a long time. This causes the overall average waiting time to increase when compared to the neutral

market. This also leads to the percentage of orders filled being less than the neutral market, with a 95% confidence interval of 66.24% to 68.18%. Therefore, it does appear that the overall liquidity of the market does suffer under a bull market, but when looking at only filled orders, the delay drops.

3.3 Bear Market

In order to create a bear market, the arrival rate for buyers was set back to 1, and the arrival rate for sellers was increased to 2. A summary of the 95% confidence intervals under this setup follows.

Table 3: Bear Market Statistics (95% Confidence Intervals)

Statistic	Value
best_bids	[97.065, 97.443]
best_asks	[98.075, 98.333]
midpoints	[97.448, 97.821]
spreads	[0.756, 0.803]
completed_wait_times	[4.037, 5.170]
ongoing_wait_times	[80.436, 85.327]
total_wait_times	[29.379, 31.840]
bid_queue_size	[1.285, 1.775]
ask_queue_size	[170.819, 183.596]
pct_filled	[0.662, 0.681]

Under the bear market, the 95% confidence interval for the best bid, best ask, and the midpoint all drop from the initial price of one hundred. It seems that higher supply than demand does indeed force the price downward, similarly to what was seen in the bull market when the price was pushed upwards. Figure 3 shows a simulation run where the bid-ask spread drifts downwards over time. It appears that in all three tested market conditions, the intuition about how the price would shift was consistent with the results.

Shifting the focus to liquidity, there are many similar findings to the bull market with the key difference being the supply now outpaces the demand. Therefore, we see the total wait time increase due to the unfilled order wait times being higher than the neutral market, and the wait times for the completed orders being lower than the neutral market. This is the same result seen in the bull market but for a different reason. Both the bull and bear markets show suffering liquidity due to mismatches in demand and supply, but for different reasons. In the bear market case, the average bid queue size is very small but the average ask queue size is exceedingly large, which is directly opposite to the bull market. This is predictable, since the bear market has an abundance of supply but not much demand, which is the opposite of the bull market. Similarly to the bull market, the bear market experiences the average percent of orders filled being lower than the neutral market, solidifying the finding of the bear market experiencing less liquidity than the neutral market. Figure 3 in the appendix shows how the unfilled orders at termination are mostly sell orders that never match with any buy orders.

3.4 Numerical Comparison of Prevalence of Buyers and Sellers

From the preceding results it is clear that there are changes in the price due to market conditions however the overall market efficiency difference is not as clear. Taking a numerical approach to contrast the discrepancy in number of buyers and the number of sellers, the following is an analysis on the ratio between buyers and sellers in the market. The ratio is determined by the numbers of buyers relative to the number of sellers. More precisely, where number of sellers is kept constant and the number of buyers is a list of [0.5, 0.55,..., 1.95, 2.0] are created showcasing a ratio of 0.5 to 2.0 A ratio of 2.0 suggests for every 1 seller there are 2.0 buyers, whereas a ratio of 0.5 suggests there is 1 buyer for every 2 sellers. These bounds are chosen for realism as beyond these ratio bounds the fundamental match making algorithm begins to fall apart due to the extent of the difference in buyers and sellers in the market.

The spread has been a priority in understanding the efficiency of the market in pricing and liquidity. As seen in Figure 4 the minimal spread is between 0.8 and 1.2 and as the ratio moves further away from equilibrium the spread grows. In addition Figure 5 showcases the optimal ratio to be between a similar range of 0.8 and 1.2 as the percentage of orders filled find their maximum in this range and fall steeply

outside of this region. These two visualizations showcase the initial findings of the bull and bear markets which suggest that orders are more likely to be filled when the ratio between sellers and buyers are equal as trades take place around a fundamental price which as a result reduces the bid-ask spread. Figure 6 furthers this idea with the wait times being lowest in a region between 0.8 and 1.2 between buyers and sellers.

Figure 7 further showcases how pricing efficiency changes with the number of buyers and sellers. When there are more buyers than sellers there is an upward force on the price as the best ask continues to be pushed up and the same applies in the opposite direction. However once we get to values outside of 0.8 and 1.2 the price change levels off, this suggests a serious issue in the efficiency of the market. There is not enough buyers or sellers to allow the price to be pushed in a certain direction and the number of trades that take place significantly reduced as seen by Figure 6 which shows the wait times increasing. This is shown in Figures 8 and 9 as the sizes of queues change drastically based on the discrepancy in buyers and sellers.

4 Conclusions and Future Work

Statistics and quantities supporting various claims were found by simulating the different types of market conditions. In a bull market it was found that the price is driven upwards by an abundance of demand for purchasing the asset that is greater than the supply by those selling it. The liquidity thereby suffered overall, but was much greater for those looking to sell the asset, as there were many buyers looking to match their orders. Under the bear market, the price was driven down due to the supply from those selling being much larger than the demand from those buying. The liquidity also suffered overall in this scenario, with the proportion of orders that are filled decreasing and wait time increasing overall, but the opposite was found for the investors who were buying the asset. The neutral market provided a good benchmark to base the results of the other markets on. In this market demand roughly matches supply, leading to a price that was relatively stationary and high liquidity.

The method in which this simulation was carried out lent itself to making calculations of statistics and confidence intervals simple. The output analysis conducted on these simulations demonstrated was extremely useful in understanding how the structure of our simulation affected the statistics and quantities of interest. By calculating and comparing these statistics, we were able to understand how the market we simulated behaved under varying conditions. Confidence intervals allowed for a quantitative measure on how certain we could be in the estimates we received. Techniques such as running multiple simulations of the same condition and using the Common Random Numbers approach allowed for more confidence in the results that were received.

The simulation that was run here had many simplifications when compared to a real market to allow for completion in the time allotted and for results that would be interpretable. There are many aspects of the model that can be changed or added upon and tested to see if they add more realism or any kind of insight that was not received from this model. An example of an extension to this model would be adding a third type of investor, the market maker. A market maker is a trader or institution that seeks to connect buyers and sellers by trading rapidly around the bid-ask spread and profiting from the difference. For example, if a market maker can quickly find a buy order that is higher than a sell order, they can buy the asset at the low selling price and sell it at the high buying price, pocketing the difference. This increases the liquidity of the market. Market makers could be added to this simulation in some capacity and the liquidity in the various scenarios could be examined. Another example for extension could be the inclusion of large market events such as a price shock or political event. These types of events often push prices up or down very quickly and their effects could be quick or long lasting. Simulating events such as these could also be attempted. Additionally, one could add large institutional investors such as hedge funds or banks. Hedge funds or banks have large amounts of capital to trade with and if an order that is very large in quantity is placed and filled, the price could be impacted drastically.

References

- [1] Gode, D. K., & Sunder, S. (1993). Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of Political Economy*, 101(1), 119–137. <https://faculty.sites.iastate.edu/tesfatsi/archive/tesfatsi/ZITraders.GodeSunder.JPE1993.pdf>
- [2] Law, A. M. (2015). *Simulation modeling and analysis* (5th ed.). McGraw-Hill Education.

A Appendix

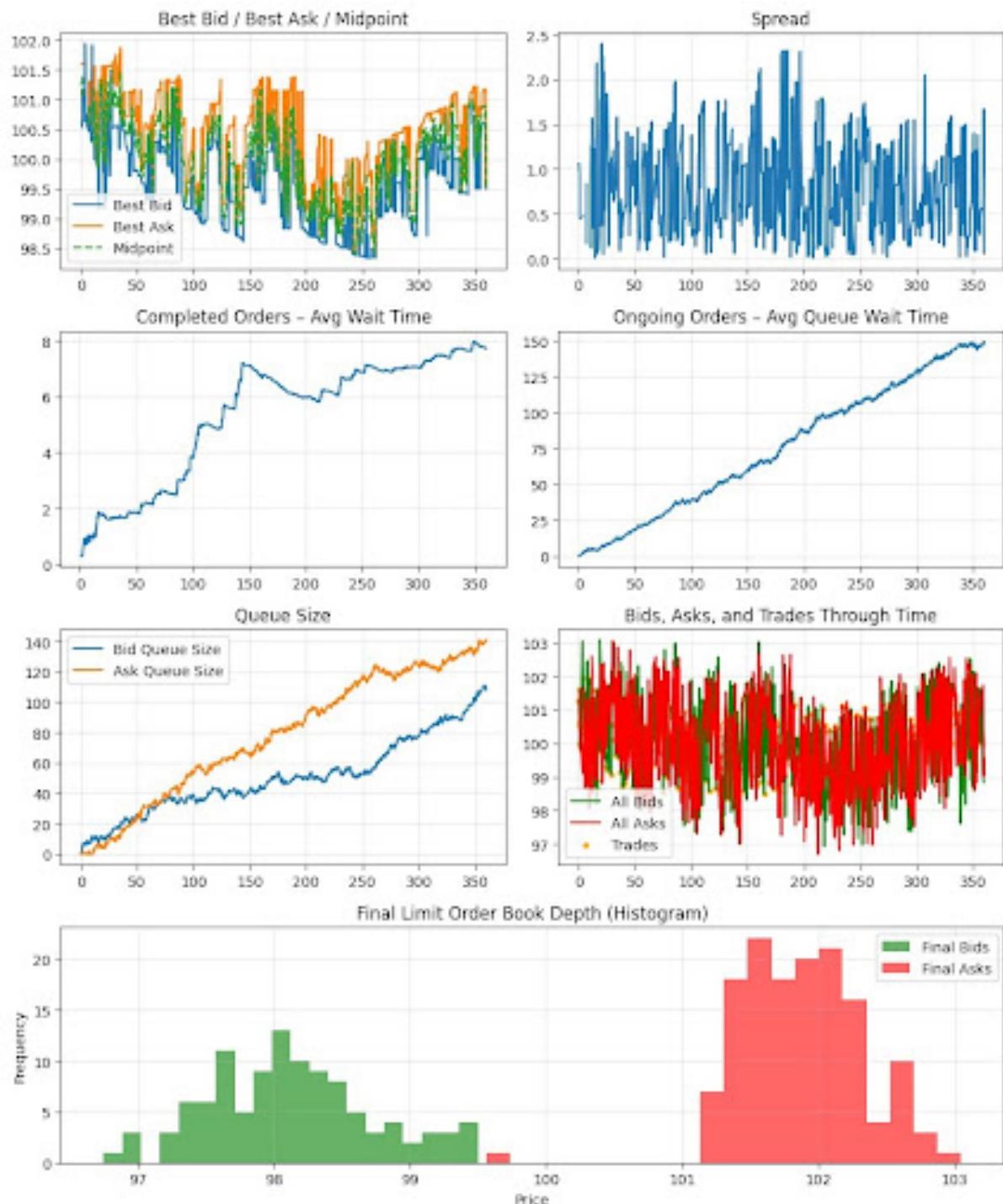


Figure 1: Various Visualizations of Market Information under the Neutral Market

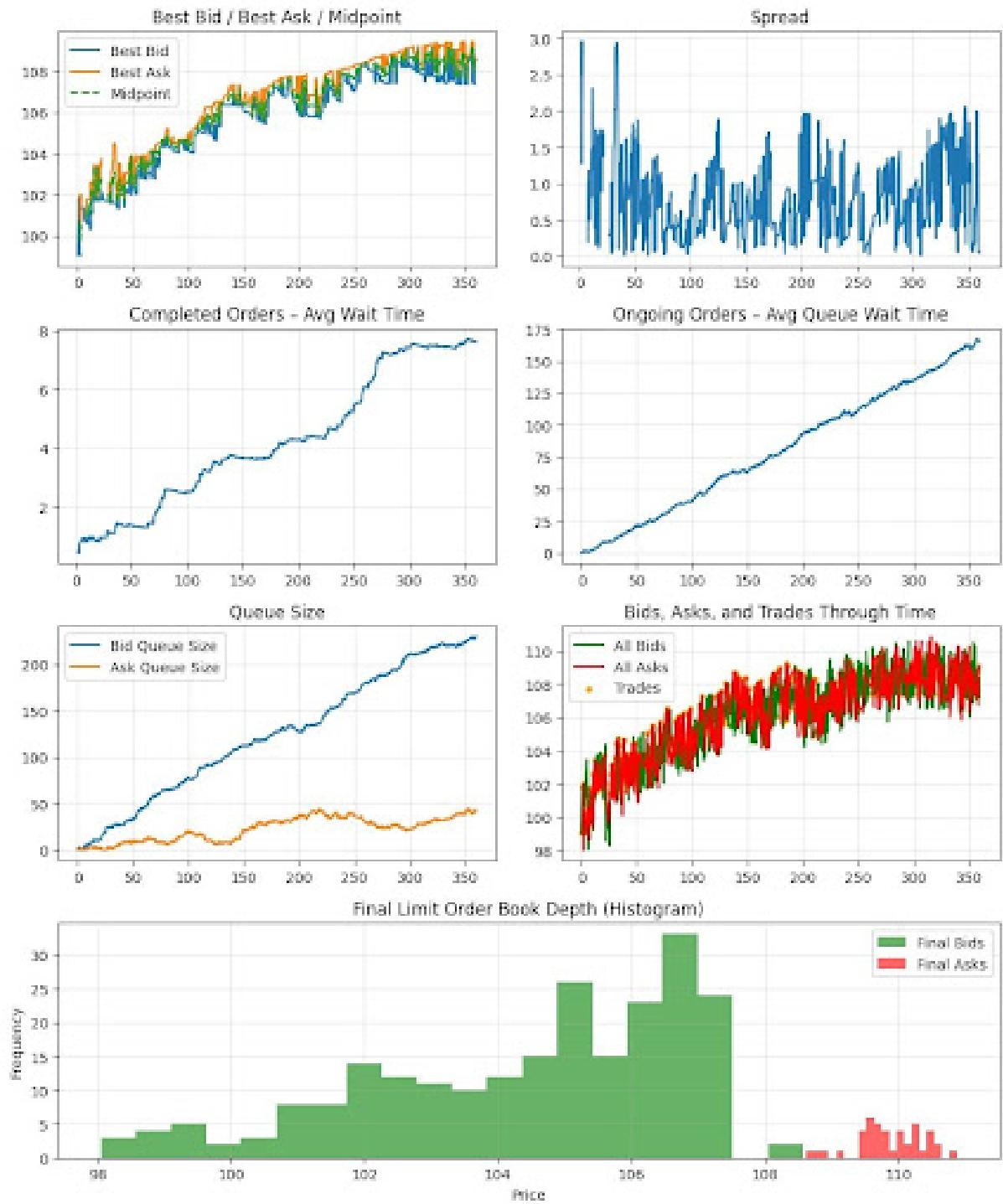


Figure 2: Various Visualizations of Market Information under the Bull Market

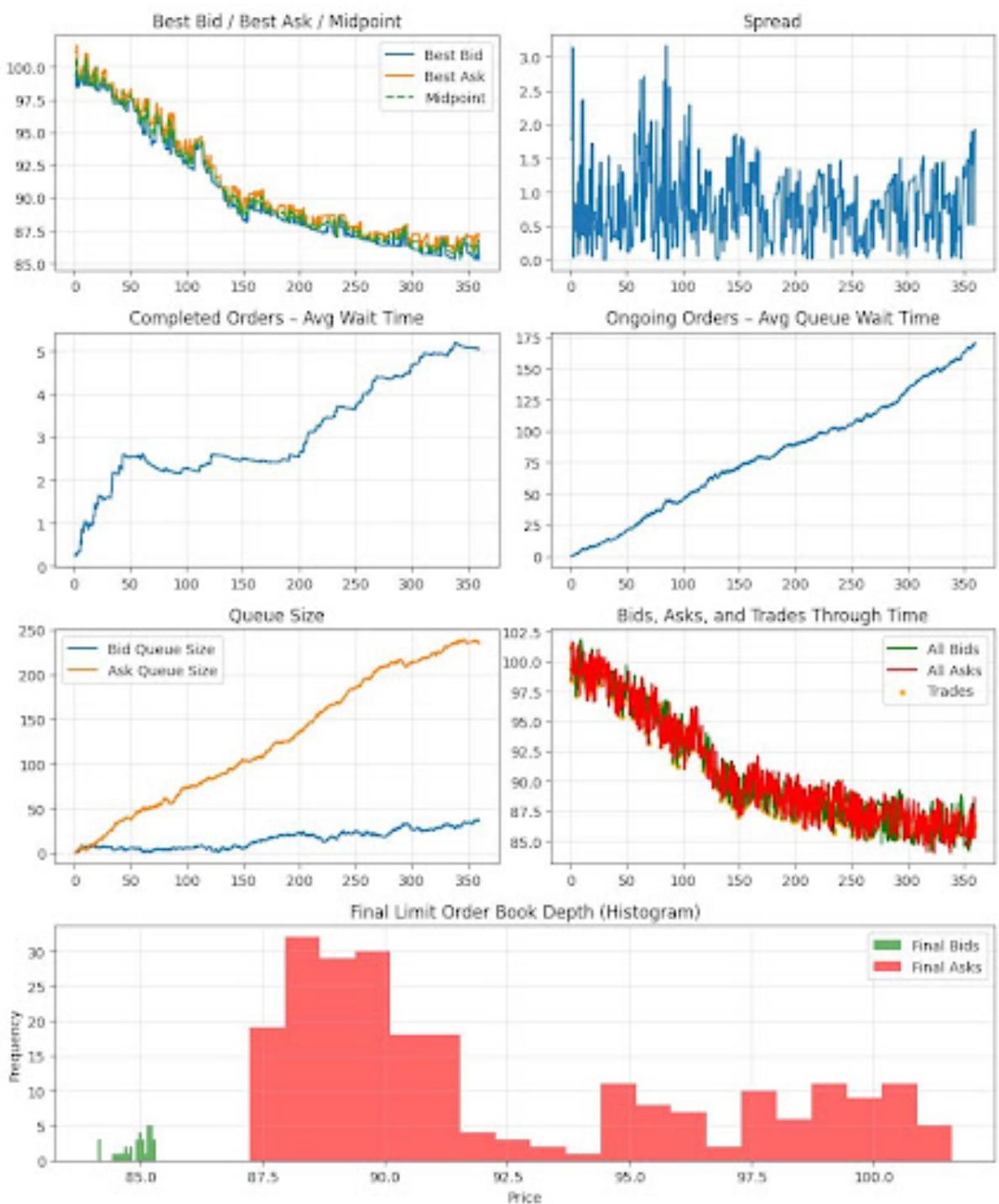


Figure 3: Various Visualizations of Market Information under the Bear Market



Figure 4: Average Spreads across Ratios of Buyers to Sellers

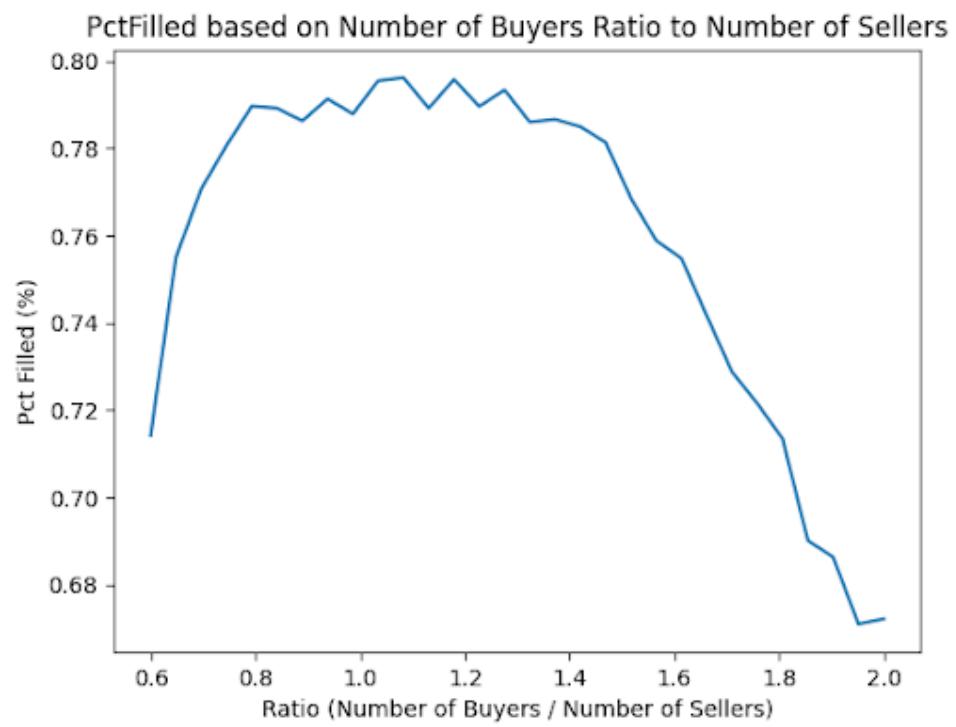


Figure 5: Average Percent of Orders Filled across Ratios of Buyers to Sellers

Total Wait Times based on Number of Buyers Ratio to Number of Sellers

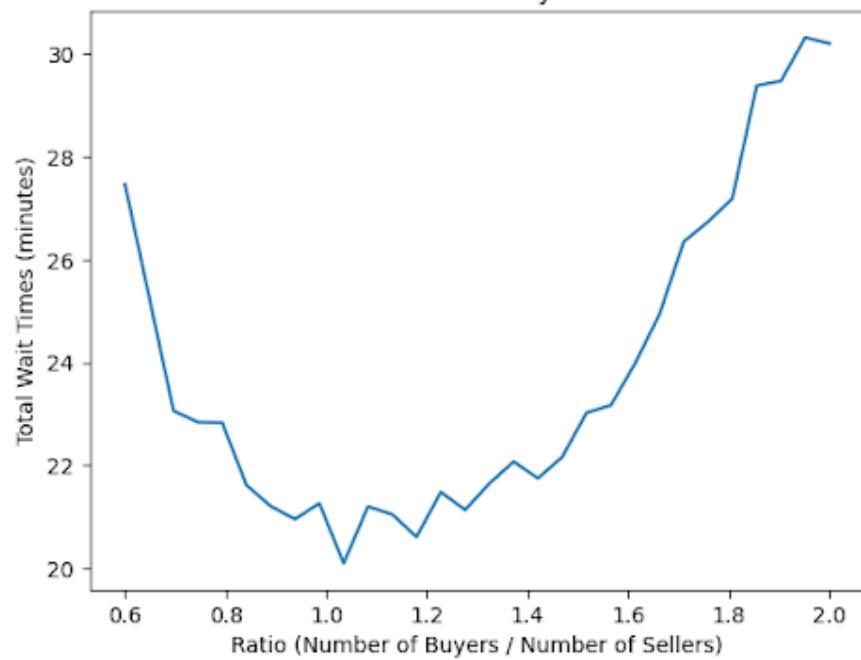


Figure 6: Average Total Wait Time across Ratios of Buyers to Sellers

Fundamental Price based on Number of Buyers Ratio to Number of Sellers

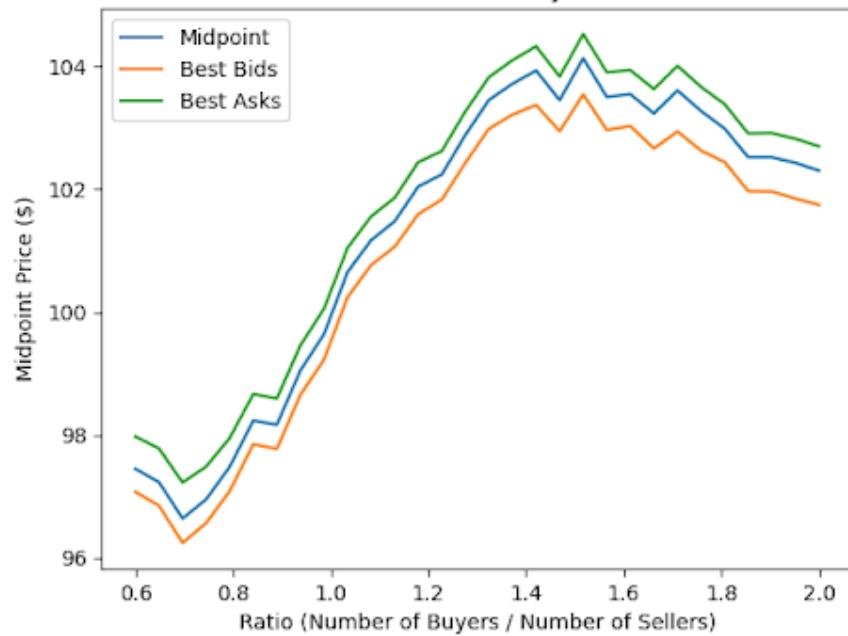


Figure 7: Fundamental Price across Ratios of Buyers to Sellers

Ask Queue Size based on Number of Buyers Ratio to Number of Sellers

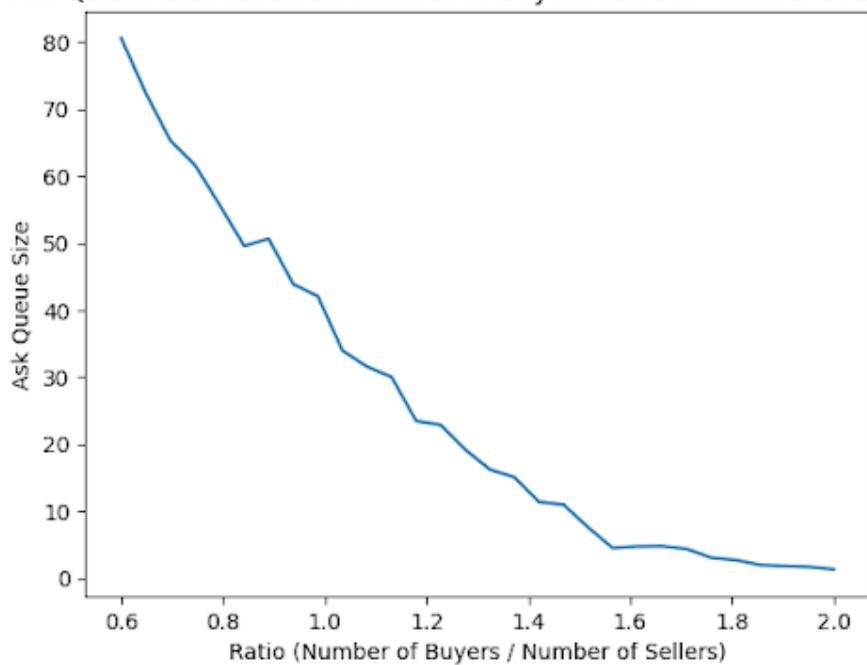


Figure 8: Ask Queue Size across Ratios of Buyers to Sellers

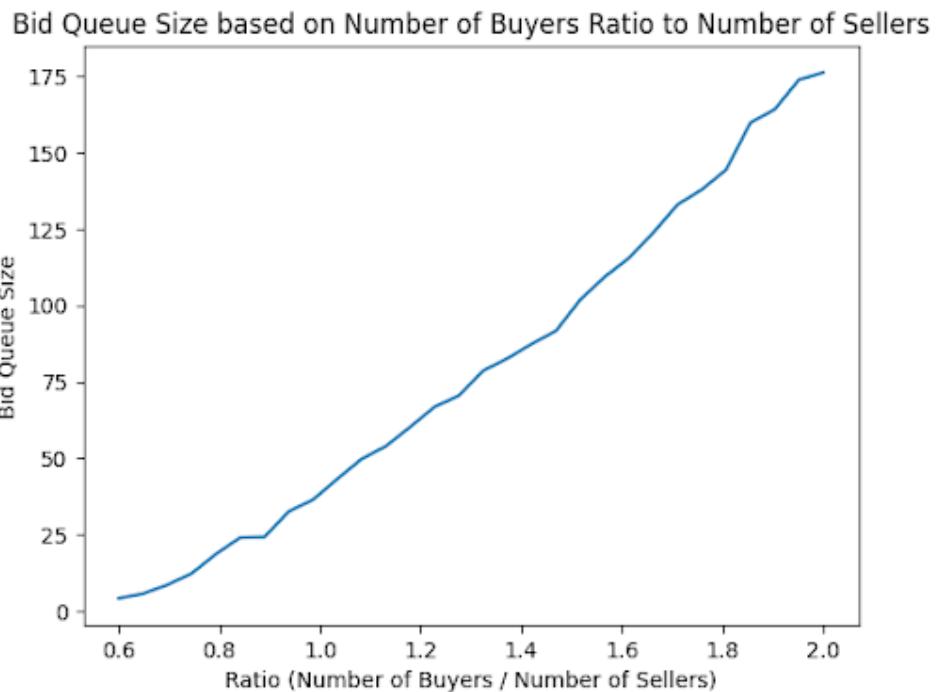


Figure 9: Bid Queue Size across Ratios of Buyers to Sellers

To view the code for this study, visit: <https://github.com/bennicholson2/LOB-Microstructure-AMS-553-Project>