Lecture 18: The Method of Images, Introduction to Magnetism

ECE221: Electric and Magnetic Fields

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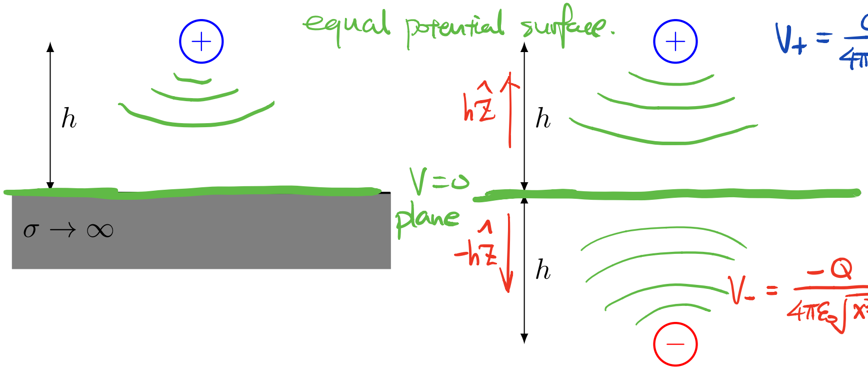
Outline

Method of Images

Given a charge configuration above an infinite grounded PEC plane may be replaced by the charge configuration itself, its image, and an equipotential surface in place of the conducting plane.

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$



$$V_+ = \frac{Q}{4\pi\varepsilon_0 |\vec{r} - h\hat{z}|} = \frac{Q}{4\pi\varepsilon_0 \sqrt{x^2+y^2+(z-h)^2}}$$

$$V_- = \frac{-Q}{4\pi\varepsilon_0 \sqrt{x^2+y^2+(z+h)^2}}$$

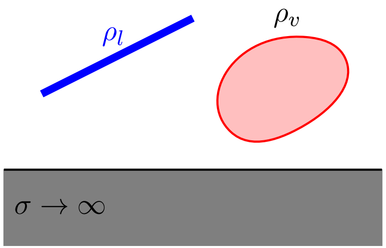
$$V = V_+ + V_- = \frac{Q}{4\pi\varepsilon_0 \sqrt{x^2+y^2+(z-h)^2}} - \frac{Q}{4\pi\varepsilon_0 \sqrt{x^2+y^2+(z+h)^2}}$$

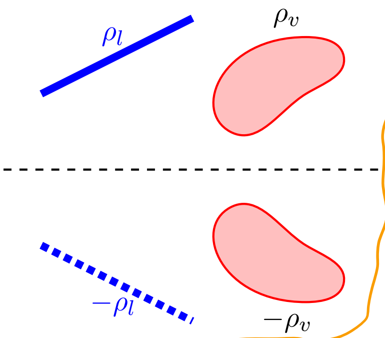
Method of Images

In electric field: $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{x\hat{x}+y\hat{y}+\hat{z}(z-h)}{(x^2+y^2+(z-h)^2)^{3/2}}\right]$$

$$- \frac{Q}{4\pi\varepsilon_0}\left[\frac{x\hat{x}+y\hat{y}+\hat{z}(h+z)}{(x^2+y^2+(z-h)^2)^{3/2}}\right]$$





$\vec{E}(z=0)$

$$= \frac{-Q}{4\pi\varepsilon_0}\,\frac{2h\hat{z}}{(x^2+y^2+(z-h)^2)^{3/2}}$$

@ equipotential plane only $\hat{z}$ direction

$$Q_{induced} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{-Qh}{2\pi(\cdot)^{3/2}}\,dxdy$$

$$= \frac{-Qh}{2\pi}\int^{2\pi}\int^{\infty}\frac{\rho d\rho d\phi}{}$$

Electrostatics Recap

Attribute	Electrostatics
Fields	**E** [V/m]
Flux densities	**D** [C/m^2]
Sources	Stationary charges or current densities $\rho_{v\|s\|l}$
Constitutive parameter(s)	ϵ [F/m] and σ [S/m]
Constitutive relations	$\boldsymbol{D} = \epsilon \boldsymbol{E}$, $\boldsymbol{J} = \sigma \boldsymbol{E}$
Divergence relation	$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_v$
Curl relation	$\boldsymbol{\nabla} \times \boldsymbol{E} = 0$
Surface integral relation	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q$
Contour integral relation	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = 0$
Circuit components	C and R
Force on a charge	$\boldsymbol{F} = q\boldsymbol{E}$

$$(\rho^2+h^2)^{3/2}$$

$$= \frac{-Qh}{2\pi} \cdot 2\pi \int_0^\infty [\rho^2+h^2]^{3/2} \tfrac{1}{2} d(\rho^2)$$

$$= \frac{Qh}{(\rho^2+h^2)^{1/2}}\Big|_0^\infty = -Q$$

Preview of Magnetostatics

Attribute	Electrostatics	Magnetostatics
Fields	**E** [V/m]	**H** [A/m]
Flux densities	**D** [C/m^2]	**B** Wb/m^2
Sources	Stationary charges or charge densities $\rho_{v\|s\|l}$	Steady (DC) currents $\boldsymbol{J}$
Constitutive parameter(s)	ϵ [F/m] and σ [S/m]	μ [H/m]
Constitutive relations	$\boldsymbol{D} = \epsilon \boldsymbol{E}$, $\boldsymbol{J} = \sigma \boldsymbol{E}$	$\boldsymbol{B} = \mu \boldsymbol{H}$
Divergence relation	$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_v$	$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$
Curl relation	$\boldsymbol{\nabla} \times \boldsymbol{E} = 0$	$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}$
Surface integral relation	$\oint_S \boldsymbol{D} \cdot d\boldsymbol{s} = Q$	$\oint_S \boldsymbol{B} \cdot d\boldsymbol{s} = 0$
Contour integral relation	$\oint_C \boldsymbol{E} \cdot d\boldsymbol{l} = 0$	$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I$
Circuit components	C and R	L (inductance)
Force on a charge	$\boldsymbol{F} = q\boldsymbol{E}$	Next!

Force Relations

Electric force

$$\boldsymbol{F}_e = q\boldsymbol{E}$$

Magnetic force

$$\boldsymbol{F}_m = q\boldsymbol{u} \times \boldsymbol{B}$$