# DT Filters

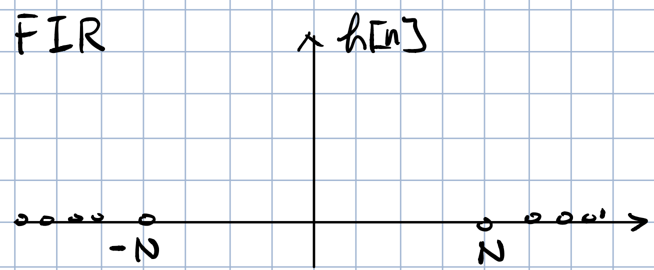
② Non-recursive filters: The output depends on only the input, not previous outputs

Causal: $y[n] = \sum_{k=0}^{N} b_k x[n-k]$ ← The only filter that can be used for real-time applications

Non-Causal: $y[n] = \sum_{-M}^{N} b_k x[n-k]$

Anti-causal: $y[n] = \sum_{k=-M}^{-1} b_k x[n-k]$

The impulse response of a non-recursive filter has a finite length: FIR




⊛ Example: Averaging system:

$$y[n] = \frac{1}{3}\left[x[n-1] + x[n] + x[n+1]\right]$$

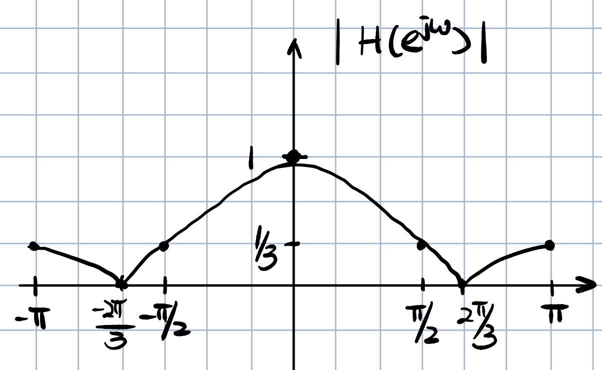
$$h[n] = \frac{1}{3}\left[\delta[n-1] + \delta[n] + \delta[n+1]\right]$$

$\delta[n] \xleftrightarrow{f} 1$,

$x[n-n_0] \xleftrightarrow{f} e^{-j\omega n_0} X(e^{j\omega})$

$$\Rightarrow H(e^{j\omega}) = \frac{1}{3}\left(e^{-j\omega} + 1 + e^{j\omega}\right)$$

$$= \frac{1}{3}\left(1 + 2\cos(\omega)\right)$$



For better freq. response, increase the length of the averaging window.

$$y[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} x[n-k]$$

$$h[n] = \frac{1}{2N+1} \sum_{k=-N}^{+N} \delta[n-k] \longrightarrow H(e^{j\omega}) = \frac{1}{2N+1} \sum_{k=-N}^{+N} e^{-j\omega k}$$

$$m = k+N \qquad k = m-N$$

$$H(e^{j\omega}) = \frac{1}{2N+1} \sum_{m=0}^{2N} e^{-j\omega(m-N)}$$

$$= \frac{e^{+j\omega N}}{2N+1} \sum_{m=0}^{2N} e^{-j\omega m}$$

$$= \frac{e^{j\omega N}}{2N+1} \frac{1-e^{-j\omega(2N+1)}}{1-e^{-j\omega}}$$

$$= \frac{\cancel{e^{j\omega N}}}{2N+1} \cdot \frac{e^{-j\omega \cancel{\frac{2N+1}{2}}}}{\cancel{e^{-j\omega/2}}} \cdot \frac{e^{j\omega\frac{2N+1}{2}} - e^{-j\omega\frac{2N+1}{2}}}{e^{j\omega/2} - e^{-j\omega/2}}$$

$$= \frac{1}{2N+1} \frac{\sin((2N+1)\omega/2)}{\sin(\omega/2)}$$

$\rightarrow$ $N=4$, $H_4(e^{j\omega}) = \dfrac{1}{9} \dfrac{\sin(9\omega/2)}{\sin(\omega/2)}$

$\rightarrow$ $N=8$, $H_8(e^{j\omega}) = \dfrac{1}{17} \dfrac{\sin(17\omega/2)}{\sin(\omega/2)}$

$\omega = \pi$ : $H_1(e^{j\pi}) = 1/3$

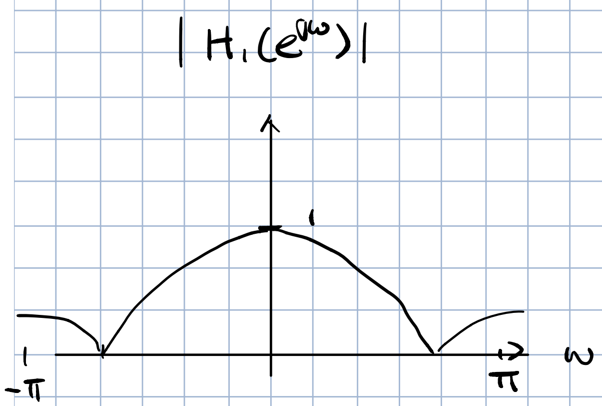
$$H_4(e^{j\pi}) = 1/9$$

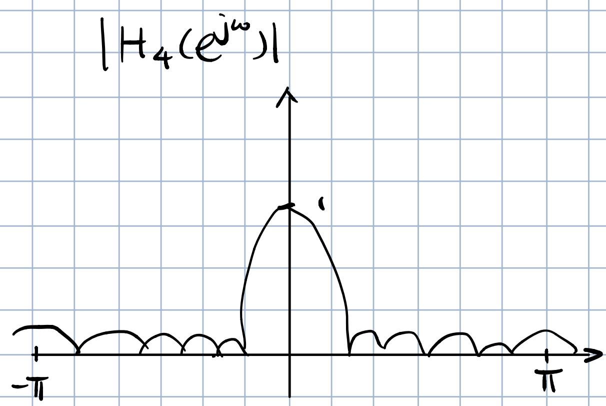
$$H_8(e^{j\pi}) = 1/17$$

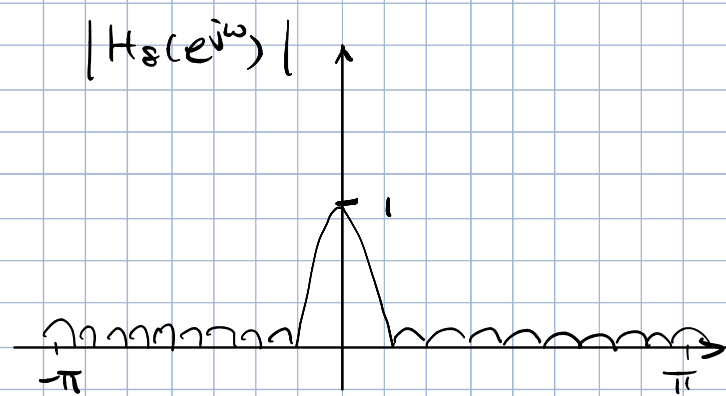
for $0<\omega<\pi$ $\quad H_1(e^{j\omega})=0$ for $\omega=\frac{2\pi}{3}$

for $0<\omega<\pi$ $\quad H_4(e^{j\omega})=0$ for $\omega=\frac{2\pi}{9},\frac{4\pi}{9},\frac{6\pi}{9},\frac{8\pi}{9}$

$H_8(e^{j\omega})=0$ for $\omega=\frac{2\pi}{17},\frac{4\pi}{17},\frac{6\pi}{17},\frac{8\pi}{17},\dots\frac{16\pi}{17}$




## ⊛ High-pass filter

$$y[n]=\tfrac{1}{2}x[n]-\tfrac{1}{2}x[n-1]$$

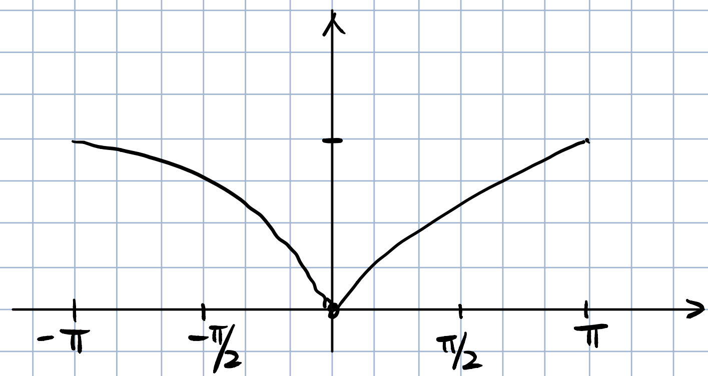
$$h[n]=\tfrac{1}{2}\delta[n]-\tfrac{1}{2}\delta[n-1]$$

$$H(e^{j\omega})=\tfrac{1}{2}-\tfrac{1}{2}e^{-j\omega}=\tfrac{1}{2}(1-e^{-j\omega})=\frac{e^{-j\frac{\omega}{2}}}{2}\left(e^{+j\omega/2}-e^{-j\frac{\omega}{2}}\right)$$

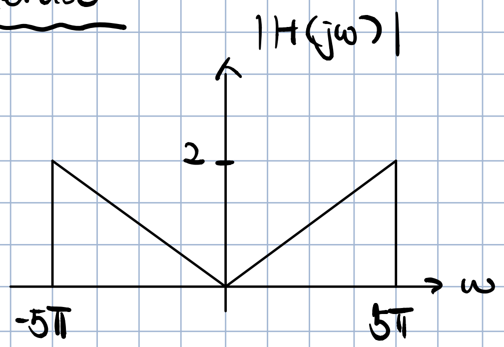
$$=\tfrac{1}{2}e^{-j\omega/2}\left(j2\sin(\omega/2)\right)$$

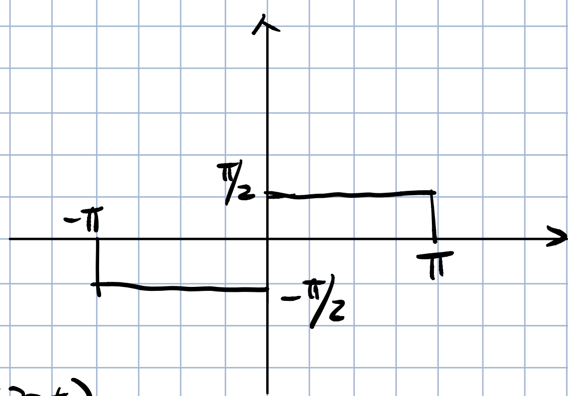
$$=je^{-j\omega/2}\sin(\omega/2)$$

$$|H(e^{j\omega})| = \left|\sin\left(\frac{\omega}{2}\right)\right|$$




Exercise:




$$x(t) = 30\cos(10t) + 10\sin(30t)$$

$$y(t) = \ ?$$