(*) Find the Fourier Series coefficients of

a) $x(t) = \cos(2t + \frac{\pi}{4})$

$$x(t) = \underbrace{\frac{1}{2} e^{-j\frac{\pi}{4}}}_{a_{-1}} e^{\overset{jk\omega_0 t}{\cancel{-j2t}}} + \underbrace{\frac{1}{2} e^{j\frac{\pi}{4}}}_{a_1} e^{\overset{jk\omega_0 t}{\cancel{j2t}}}$$

$k = -1$ $\quad$ $k = 1$

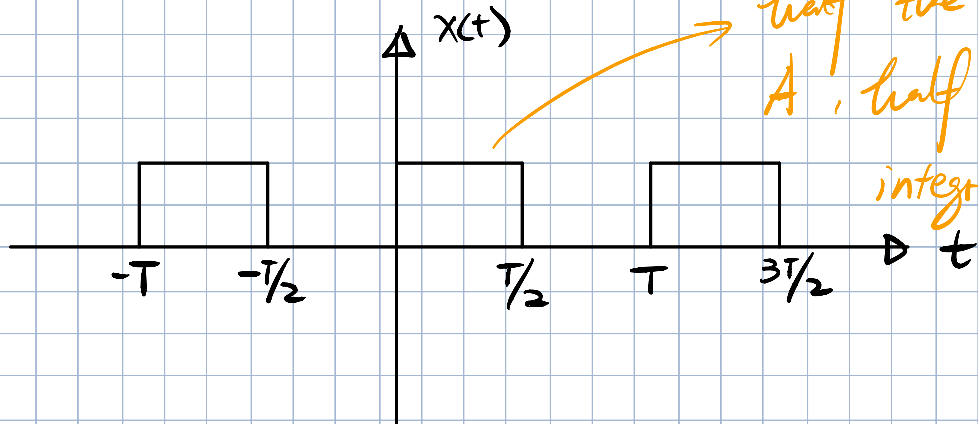
$$a_{-1} = \frac{1}{2} e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$

$$a_1 = \frac{1}{2} e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k : \begin{cases} a_{-1} = \frac{1}{2} e^{-j\frac{\pi}{4}} \\ a_1 = \frac{1}{2} e^{j\frac{\pi}{4}} \\ a_k = 0 \ , \text{else.} \end{cases}$$

b) $x(t) = \begin{cases} A & 0 < t < \frac{T}{2} \\ 0 & \frac{T}{2} < t < T \end{cases}$ $\quad$ periodic w/ period T



half the period is A, half is "0"

integrating 0 is 0

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} \qquad a_k = \frac{1}{T} \int x(t) e^{-jk(2\pi/T)t}$$

$$a_k = \frac{1}{T} \int^{T/2} A e^{-jk(2\pi/T)t} dt$$

$$a_k = \frac{1}{T}\int_0 A e \quad dt$$

$$= \frac{A}{T}\frac{1}{-jk2\pi/T} e^{-jk(2\pi/T)t}\Big|_0^{T/2}$$

$$= \frac{A}{T}\frac{1}{-jk2\pi/T}\left(e^{-jk\pi} - 1\right)$$

$$= \frac{Aj}{2\pi k}\left(e^{-jk\pi} - 1\right)$$

integer π

$\sin = 0$

$$= \frac{Aj}{2\pi k}\left(\underbrace{\cos(-k\pi) + j\sin(-k\pi)}_{= \cos(k\pi)} - 1\right) \qquad \cos(-k\pi) \to \cos(k\pi), \quad j\sin(-k\pi) \to 0$$

$$= \cos(k\pi) = \begin{cases} 1 & k \text{ is even} \\ -1 & k \text{ is odd} \end{cases}$$

k is an integer

$$= \begin{cases} 0 & k \text{ is even.} \\ \frac{-jA}{\pi k} & k \text{ is odd.} \end{cases}$$

for $k = 2m+1 \quad k \in \mathbb{Z}, m = \mathbb{Z}$

$$a_{2m+1} = \frac{-jA}{\pi(2m+1)}, \quad a_{2m} = 0, \quad m \neq 0$$

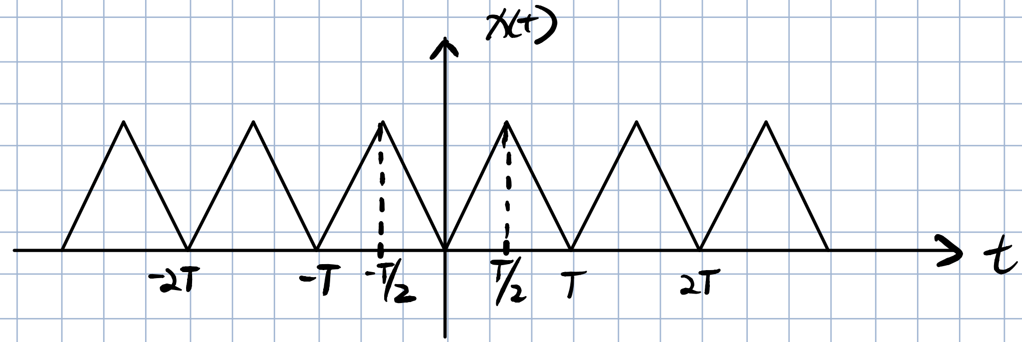
$$a_0 = \frac{1}{T}\int_T x(t)dt = \frac{1}{T}\int_0^{T/2} A\, dt = \frac{A}{T}(T/2 - 0) = \frac{A}{2}$$

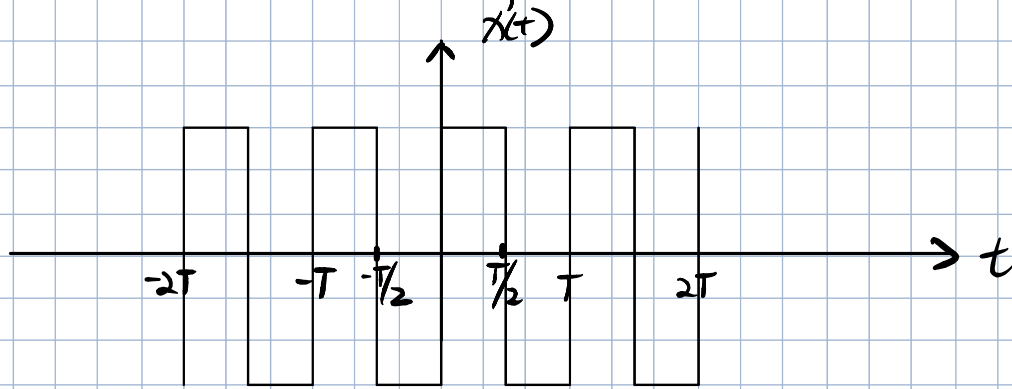
↑ 单独算.

$$a_0 = \frac{A}{2}.$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} = \left(\sum_{m=-\infty}^{\infty} \frac{-jA}{\pi(2m+1)} e^{j(2m+1)(2\pi/T)t}\right) + \frac{A}{2}$$

c)





call x(t) of the last problem y(t)

$$x'(t) = \left(y(t) \Big|_{A=\frac{4A}{T}} \right) - \frac{2A}{T}$$

only replacing A w/ $\frac{4A}{T}$

$$x'(t) = \cancel{\frac{4A/T}{2}} + \sum_{m=-\infty}^{\infty} \frac{-j(4A/T)}{\pi(2m+1)} e^{j(2m+1)(2\pi/T)t} \cancel{- \frac{2A}{T}}$$

$$x'(t) = \sum_{m=-\infty}^{\infty} \frac{-j(4A/T)}{\pi(2m+1)} e^{j(2m+1)(2\pi/T)t}$$

$$x(t) = \int x'(t)\,dt + C$$

$$= \sum_{m=-\infty}^{\infty} \frac{-j(4A/T)}{\pi(2m+1)} \cdot \frac{1}{j(2m+1)(2\pi/T)} e^{j(2m+1)(2\pi/T)t} + \underset{\substack{\uparrow \\ a_0}}{C}$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{AT}{2} = \frac{A}{2}$$

$$x(t) = \sum \frac{-2A}{\pi^2(2m+1)^2} e^{j(2m+1)(2\pi/T)t} + \frac{A}{2}$$

$$a_{2m} = 0 \quad m \neq 0 \qquad a_0 = \frac{A}{2}$$

$$a_{2m+1} = \frac{-2A}{\pi^2(2m+1)^2}$$