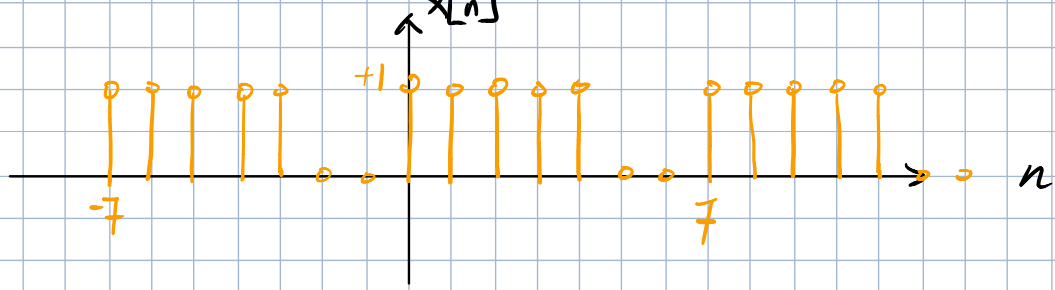
Example

Find the fourier Series Coefficients of



$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\underbrace{(2\pi/N)}_{\omega_0} n} \quad , N=7$$

$$= \frac{1}{7}\sum_{0}^{4} \underbrace{(+1)\, e^{-jk\omega_0 n}}_{\alpha} \rightarrow h$$

$$\sum_{h=0}^{M} \alpha^h = \frac{1-\alpha^{M+1}}{1-\alpha} \quad \leftarrow \text{Geometric Progression.}$$

$$a_k = \frac{1}{7}\,\frac{1-\overbrace{e^{-jk\omega_0 5}}^{\alpha,\ M+1}}{1-\underbrace{e^{-jk\omega_0}}_{\alpha}}$$

$$a_k = \frac{1}{7}\,\frac{e^{-jk\omega_0 5/2}}{e^{-jk\omega_0 1/2}} \cdot \frac{e^{+jk\omega_0 5/2} - e^{-jk\omega_0 5/2}}{e^{jk\omega_0 1/2} - e^{-jk\omega_0 1/2}}$$

$$\sin(\beta) = \frac{1}{2j}\left(e^{j\beta} - e^{-j\beta}\right)$$

$$a_k = \frac{1}{7}\, e^{-jk\omega_0 2} \cdot \frac{2j\sin(k\omega_0 5/2)}{2j\sin(k\omega_0 1/2)}$$

$$a_k = 1/7\, e^{-jk(4\pi/7)} \cdot \frac{\sin(k5\pi/7)}{\sin(k\pi/7)}$$

$$a_0 = 1/7\,(1+1+1+1+1+0+0) = 5/7$$

Properties of DT FS:

1) linearity : The period of $x[n]$ and $y[n]$ is N

$$x[n] \xleftrightarrow{fs} a_k$$

$$\Rightarrow \quad A x[n] + B y[n] \xleftrightarrow{fs} A a_k + B b_k$$

$$y[n] \xleftrightarrow{fs} b_k$$

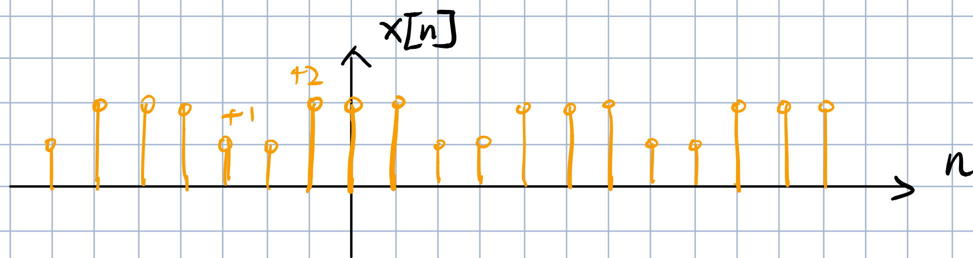
A & B are constant complex #s.

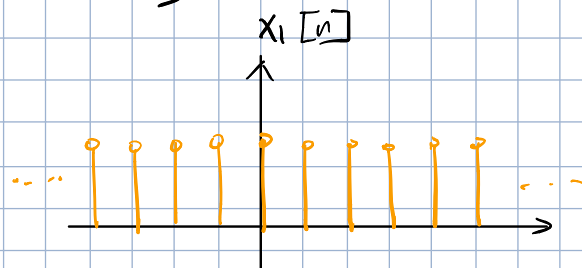
2) Time Shift : period N

$$x[n] \xleftrightarrow{fs} a_k$$

$$x[n-n_0] \xleftrightarrow{fs} e^{-jk(2\pi/N)n_0} a_k$$

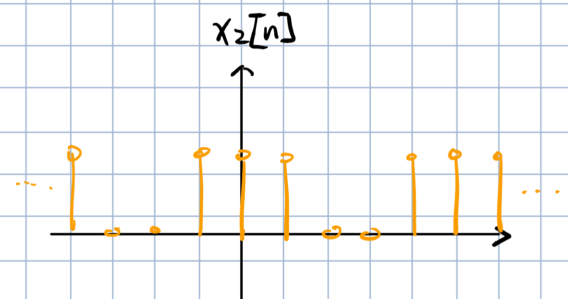
⊛ Find the FS coefficients of :



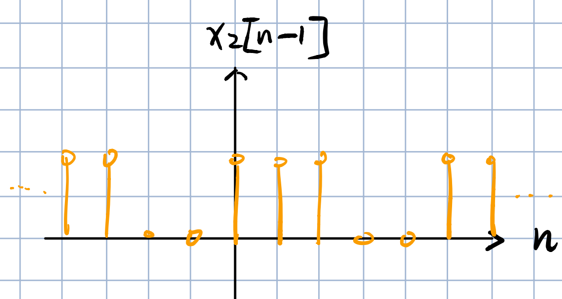


$$x[n] = x_1[n] + x_2[n]$$

for $x_1[n] \Rightarrow \quad a_0 = 1$



for $x_2[n]$:



$$x_2[n] \xleftrightarrow{fs} b_k = \begin{cases} \frac{1}{5} \dfrac{\sin(3\pi k/5)}{\sin(\pi k/5)} & k \neq 0, \pm 5h, \pm 10h \\ 3/5 & k = 5h,\ h \in \mathbb{Z} \end{cases}$$

$$x[n] = \begin{cases} \frac{1}{5} \dfrac{\sin(3\pi k/5)}{\sin(\pi k/5)} & k \neq 5h \quad h \in \mathbb{Z} \end{cases}$$

$8/5 \qquad k=5h,\ h\in\mathbb{Z}$

3) Frequency Shifting.

$$x[n] \xleftrightarrow{fs} a_k$$

$$e^{jn(2\pi/N)M}x[n] \xleftrightarrow{fs} a_{k-M}$$