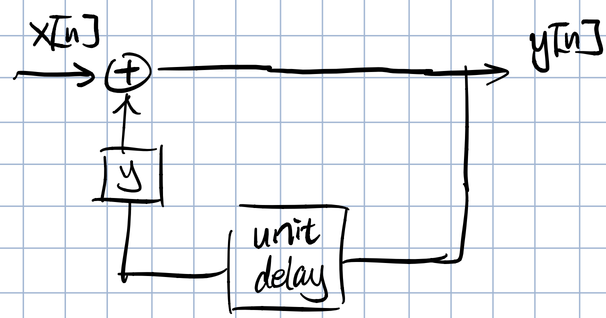
DT Filters.

⊛ Recursive Filters:

$$y(t) - a\frac{dy(t)}{dt} = x(t)$$

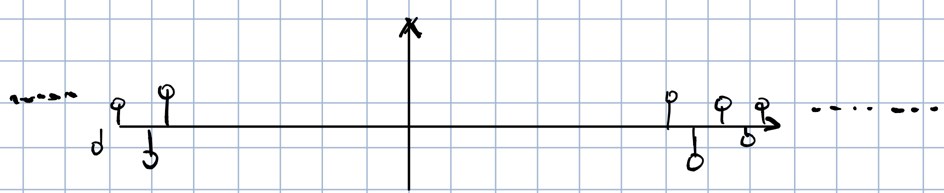
DT equivalent : $y[n] - ay[n-1] = x[n]$

Recursive filter : The output @ a given time depends on the input and the previous output.



The impulse response of recursive filters has infinite length

→ IIR (Infinite Impulse response)



$$y[n] - a\,y[n-1] = x[n]$$

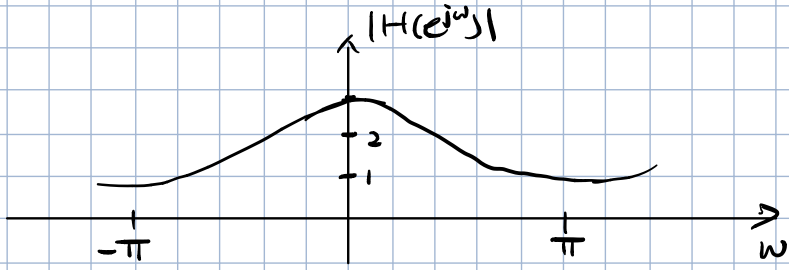
$$\rightarrow Y(e^{j\omega}) - ae^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$$

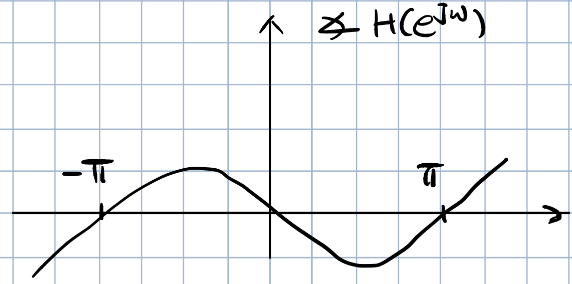
$$Y(e^{j\omega})(1 - ae^{-j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$$

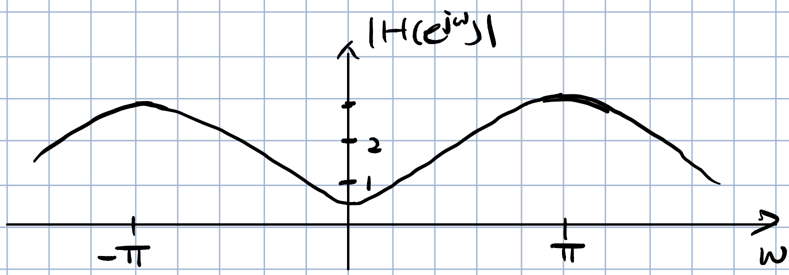
since $a^n u[n] \overset{f}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}} \rightarrow \underbrace{h[n] = a^n u[n]}_{\text{IIR}}$

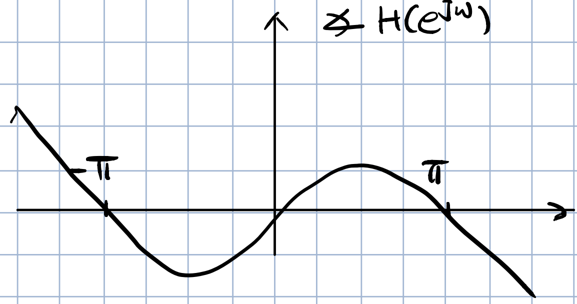
If $0 < a < 1$, $H(e^{j\omega})$ represents a low pass filter.





If $-1 < a < 0$ (High-pass)





For $-1 < a < 1$, the smaller is $|a|$, the broader is the passband.

$\rightarrow$ $y[n] - a\,y[n-1] = x[n]$, find the step response.

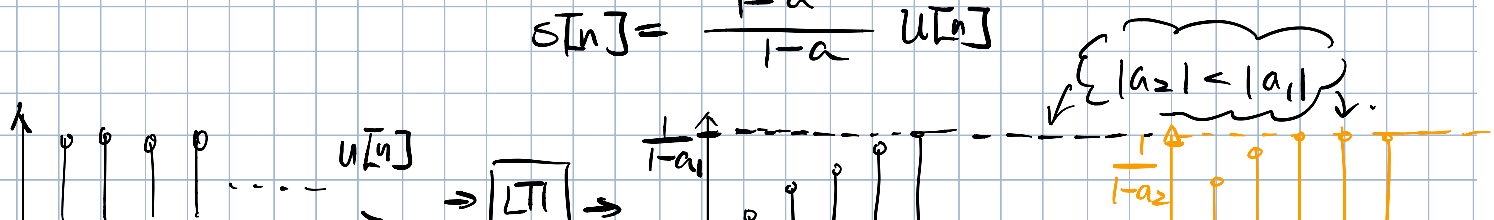
$$\delta[n] \longrightarrow \boxed{LTI} \longrightarrow h[n] = a^n u[n]$$

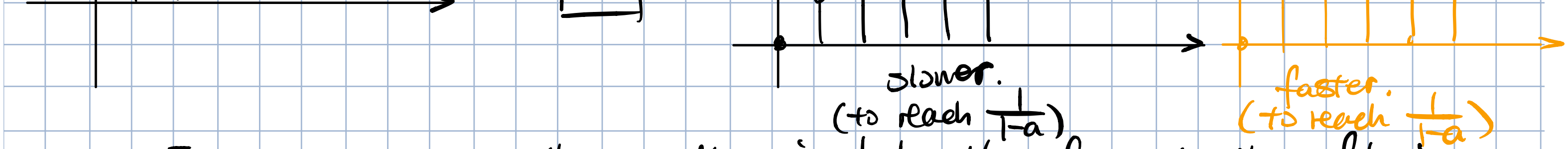
$$u[n] \longrightarrow \boxed{LTI} \longrightarrow s[n] \qquad u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

$$s[n] = \sum_{m=-\infty}^{n} h[m]$$

$$\rightarrow s[n] = \sum_{m=0}^{n} a^m = \frac{1-a^{n+1}}{1-a} \quad \text{for } n \geq 0$$

$$s[n] = \frac{1-a^{n+1}}{1-a}\,u[n]$$





For $-1 < a < 1$, the smaller is $|a|$, the faster is the filter's response in the domain.

For $|a| > 1$, this system is not stable

General format of a recursive DT filter:

$$y[n] = \sum_{k=1}^{M} a_k \, y[n-k] + \sum_{k=0}^{N} b_k x[n-k]$$

"Causal format"