Properties of DT FT:

⑤ Differencing and accumulation:

$$x[n] \xleftrightarrow{f} X(e^{j\omega})$$

CT: differentiation $\frac{dx(t)}{dt}$

DT: differencing $x[n]-x[n-1]$

$$x[n]-x[n-1] \xleftrightarrow{f} (1-e^{-j\omega})X(e^{j\omega})$$

$$x[n-1] \xleftrightarrow{f} e^{-j\omega}X(e^{j\omega})$$

$$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{ft} \frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta[\omega-2\pi k]$$

$$\delta[n] \longleftrightarrow 1$$

$$\Rightarrow u[n] \xleftrightarrow{ft} \frac{1}{1-e^{-j\omega}}\cdot 1 + \pi\cdot 1\sum_{k=-\infty}^{+\infty}\delta[\omega-2\pi k]$$

⑥ Time Reversal:

$$x[n] \xleftrightarrow{f} X(e^{j\omega}) \Rightarrow x[-n] \longleftrightarrow X(e^{-j\omega})$$

(Remainder: conjugation property
$x^*[n] \longleftrightarrow X^*(e^{-j\omega})$)

If $x[n]$ is a real signal $\rightarrow x^*[n]=x[n]$

$$\rightarrow X(e^{j\omega}) = X^*(e^{-j\omega}) \Rightarrow X(e^{-j\omega}) = X^*(e^{j\omega})$$

<u>If $x[n]$ is real:</u>

$$\mathcal{E}v\{x[n]\} = \frac{x[n]+x[-n]}{2} \xleftrightarrow{ft} \frac{X(e^{j\omega})+X(e^{-j\omega})}{2}$$

$\rightarrow Ev\{x[n]\} \xleftrightarrow{ft} \dfrac{X(e^{j\omega})+X^*(e^{j\omega})}{2} = Re\{X(e^{j\omega})\}$

If $x[n]$ is a real even signal, then $X(e^{j\omega})$ is a real signal.

If $x[n]$ is real:

$$odd\{x[n]\} = \frac{x[n]-x[-n]}{2} \xleftrightarrow{ft} \frac{X(e^{j\omega})-X(e^{-j\omega})}{2}$$

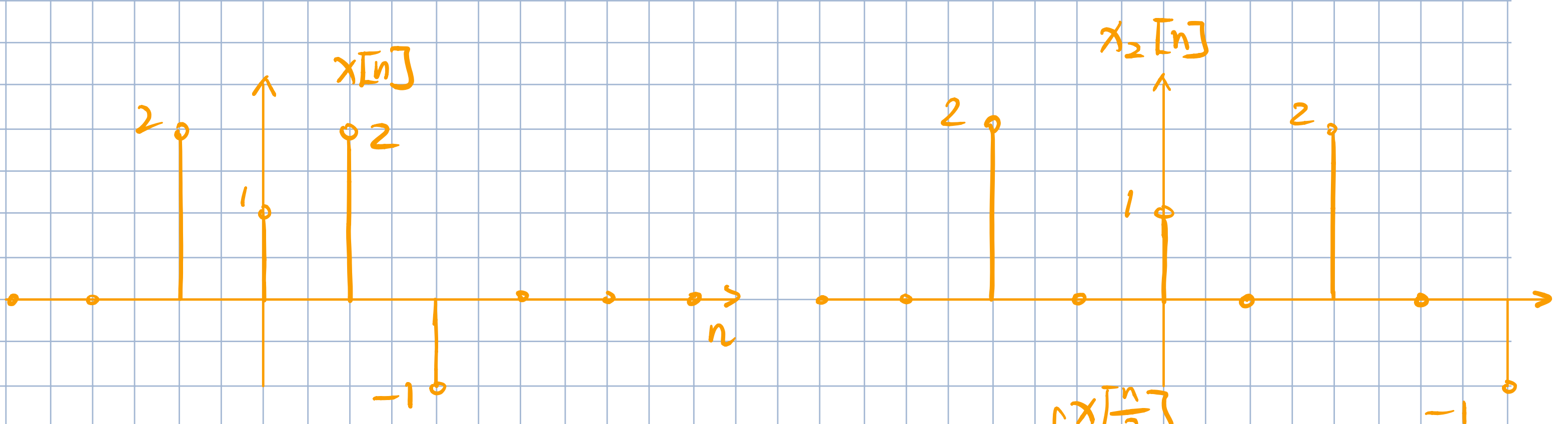
$$odd\{x[n]\} = \frac{X(e^{j\omega})-X^*(e^{j\omega})}{2} = \frac{j2\,Im\{X(e^{j\omega})\}}{2}$$

$$odd\{x[n]\} \xleftrightarrow{ft} j\,Im\{X(e^{j\omega})\}$$

$\rightarrow$ If $x[n]$ is a real odd signal, the $X(e^{j\omega})$ is a purely imaginary signal.

⑦ Time expansion :

$$x_{(k)}[n] \triangleq \begin{cases} x[\frac{n}{k}] & n \text{ is an integer multiple of } k \\ 0 & \text{otherwise.} \end{cases}$$

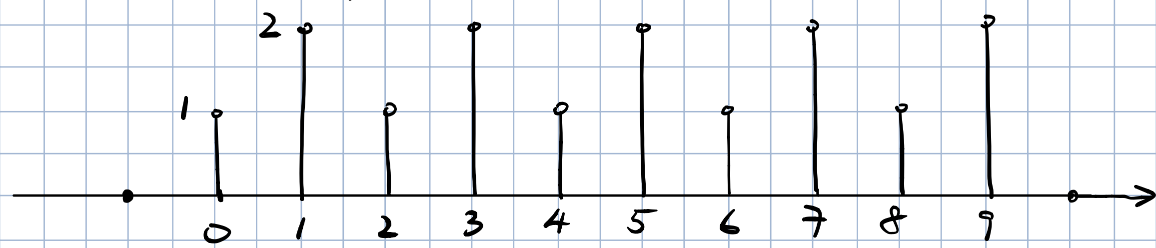


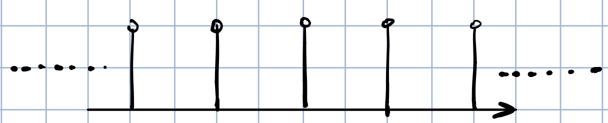
$$x_2[n] = \begin{cases} x[\frac{n}{2}] \\ 0 \end{cases}$$

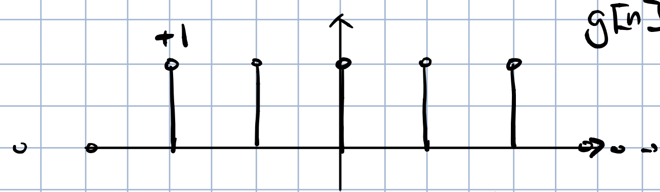
$$x[n] \xleftrightarrow{ft} X(e^{j\omega}) \Rightarrow x_k[n] \xleftrightarrow{ft} X(e^{jk\omega})$$

Since $k \in \mathbb{Z} \to X(e^{jk\omega})$ is a compressed version of $X(e^{j\omega})$ (Remainder of the scaling transformation)

* Find the DT FT of:



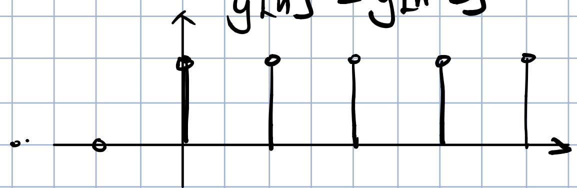




$$g[n] = \frac{\sin\left(\frac{2N+1}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$$

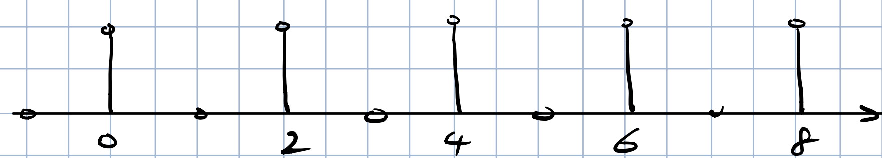
$$G(e^{j\omega}) = \frac{\sin(2.5\omega)}{\sin(0.5\omega)}$$

$y[n] = g[n-2]$



$$y[n] \Rightarrow Y(e^{j\omega}) = e^{-j2\omega}\frac{\sin(2.5\omega)}{\sin(0.5\omega)}$$

$y_2[n]$



$$x[n] = y_2[n] + 2y_2[n-1]$$

$$y_2[n] \longleftrightarrow Y(e^{j2\omega}) = e^{-j4\omega}\frac{\sin(5\omega)}{\sin(\omega)}$$

$$X(e^{j\omega}) = e^{-j4\omega}\frac{\sin(5\omega)}{\sin(\omega)} + 2e^{-j\omega}e^{-j4\omega}\frac{\sin(5\omega)}{\sin(\omega)} \quad //$$