Lecture 26: Magnetic Dipole and Magnetic Materials

ECE221: Electric and Magnetic Fields

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Outline

The Magnetic Dipole

Recall the circular loop we analyzed before. If the loop is **small** such that $r \gg a$, then

$$\boxed{\boldsymbol{A} \approx \frac{\mu_0 I \underbrace{\pi a^2}_{A_{loop}} \sin\theta\,\hat{\boldsymbol{\phi}}}{4\pi r^2}}$$

define $\vec{m}$ = magnetic moment

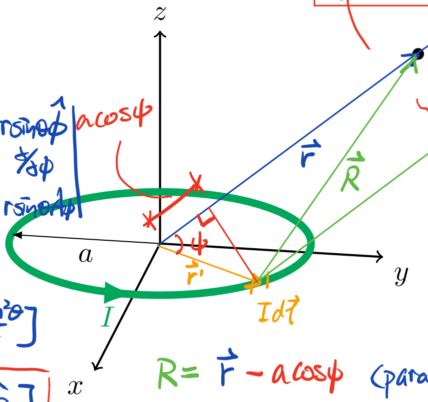
$= I \cdot \pi a^2 \cdot \hat{z}$

$$\boxed{A = \mu_0 \frac{\vec{m} \times \vec{r}}{4\pi r^2}}$$

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{r\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left[\hat{r}\frac{d}{d\theta}\left(\frac{\mu_0 m \sin^2\theta}{4\pi r}\right) - r\hat{\theta}\frac{d}{dr}\left[\frac{\mu_0 m \sin^2\theta}{4\pi r}\right]\right]$$

$$= \boxed{\frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]}$$



parallel if blue line is far far away.

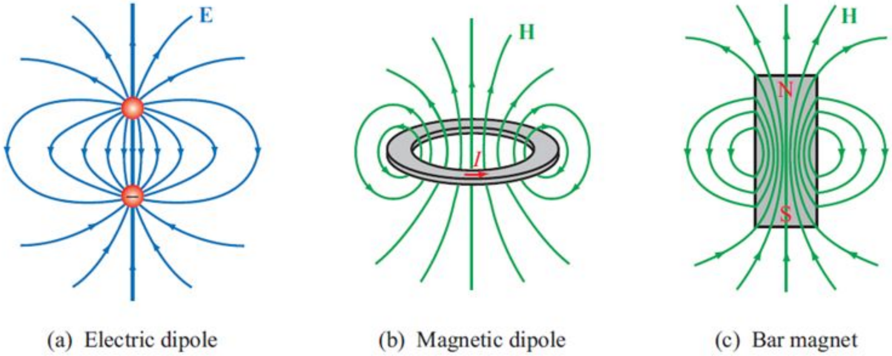
$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}}{R}$$

$R = \vec{r} - a\cos\phi$ (parallel ray approximation), w/ this approx we get:

$= \vec{B}$ for $r \gg a$

Dipoles

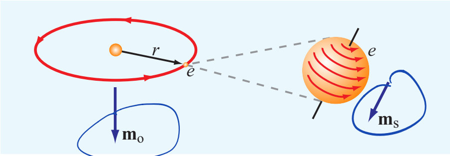
Recall electrostatic dipole. $\vec{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Nature of Magnetic Materials

- Just as we did for dielectrics, we now consider the magnetic properties of "magnetic" materials
- We gain invoke a simple atomic model where there is a positively charged nucleus around which there are electronics in various orbits



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

If orbital radius is r and electron e^- completes one orbit in time T

electron current $= -\frac{q_e}{T} = \frac{q_e u}{2\pi r}$

$T = \frac{2\pi r}{u}$ $\quad u = e^-$ velocity.

$m_o = I A_{loop} = \frac{-q_e u}{2\pi r} \pi r^2 = \frac{-q_e u r}{2} \equiv \frac{q_e}{2 m_e} L_e$

where $L_e = m_e U r = \frac{nh}{2\pi}$ where h is plank's constant

Magnetization

$$\boxed{m_o = \frac{n q_e h}{2m_e \cdot 2\pi}}$$

$n = 0, 1, 2, 3, \ldots$

e.g. $m_o = -9 \times 10^{-24} A \cdot m^2$

- From a macroscopic perspective we now differentiate between free currents and bound currents
- A bound current I_b circulates around a path enclosing a differential area $d\boldsymbol{s}$

$$\boldsymbol{m} = I_b d\boldsymbol{s}$$

$$m_S = \pm \frac{n q_e h}{2m_e \cdot 2\pi}$$

$$= \pm 9 \times 10^{-24} A \cdot m^2$$

- If there are N magnetic dipoles per unit volume and we consider a volume $\Delta\nu$, then the total magentic dipole moment is

$$\boldsymbol{m}_{total} = \sum_{i=1}^{N\Delta\nu} \boldsymbol{m}_i$$

- Define **magnetization** as magnetic dipole moment per unit volume, just as we did for **polarization**

$$\boldsymbol{M} \lim_{\Delta\nu \to 0} \frac{1}{\Delta\nu} \sum_{i=1}^{N\Delta\nu} \boldsymbol{m}_i$$

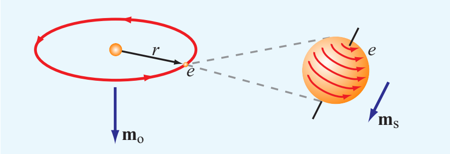
Differential Analysis

For a differential volume dv', the magnetic moment is $d\boldsymbol{m} = \boldsymbol{M}\,dv'$. What is the vector magnetic potential produced by the volume?

Types of Magnetic Materials: Diamagnetic Materials

Diamagnetic materials:

- Have no permanent magnetic moments
- Electron orbit moment and spin moment nearly cancel
- An applied $\boldsymbol{B}$ field causes a very slight reduction in $\boldsymbol{m}_o$
- In most diamagnetic materials, $\chi_m \approx -10^{-5}$ and $\mu_r \approx 1$
- In very special materials (superconductors), the internal magnetic field can completely cancel the external one, producing $\mu_r = -1$

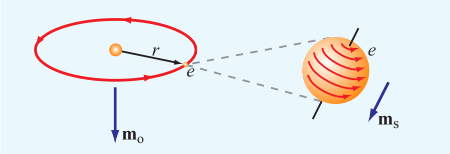


Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Types of Magnetic Materials: Paramagnetic Materials

Paramagnetic materials:

- Have no permanent magnetic moments
- Electron spin moment very slightly larger than electron orbit moment
- An applied $\boldsymbol{B}$ field torques the moments, causing them to align with the applied field
- In most paramagnetic materials, $\chi_m \approx 10^{-5}$ and $\mu_r \approx 1$

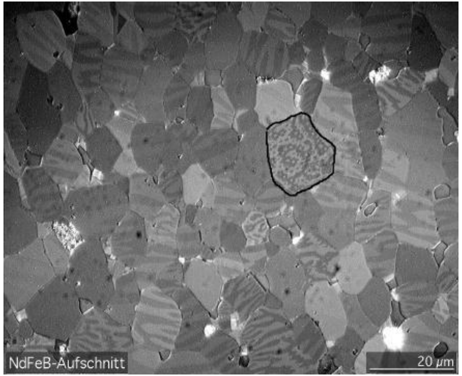


Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Types of Magnetic Materials: Ferromagnetic Materials

Ferromagnetic materials:

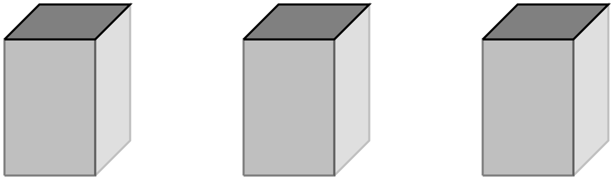
- Have permanent magnetic moments resulting from electron spin moments
- Interatomic forces cause these moments to line up in parallel over regions containing large numbers of atoms called **domains**
- An applied B field **magnetized** the material to cause regular and semi-permanent alignment of the dipoles



Source: Wikipedia

Types of Magnetic Materials: Ferromagnetic Materials

- A magnetic materials spontaneously divides into domains to minimize the *magnetostatic energy* stored in the internal field.
- Domains can be reoriented by an external magnetic field
- Domains will remain aligned when external field is removed, since domain walls become pinned to defects in the crystal structure
- The material can be *demagnetized* by applying another field, or heating the material passed its *Curie temperature*



Summary of Magnetic Materials

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis*
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silber, silicon	Aluminmum, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m **Typical value of μ_r**	$\approx -10^{-5}$ ≈ 1	$\approx 10^{-5}$ ≈ 1	$\lvert\chi_m\rvert \gg 1$ and hysteretic $\lvert\mu_r\rvert \gg 1$ and hysteretic

Hysteresis of Magnetic Materials

- A **magnetization curve** describes the relationship between B and H in a material
- You might think it is a line because $B = \mu H$, but this is not true in ferromagnetic materials where the relationship is **nonlinear**

$$B = \mu(H)H$$

- Ferromagnetic materials have magnetic *hysteresis* (*to lag behind*)