Lecture 31: Faraday's Law of Induction

ECE221: Electric and Magnetic Fields

Prof. Sean V. Hum

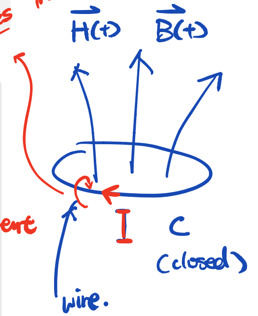
Winter 2019

Outline

1. Faraday's Law
2. Transformers and Transformer EMF
3. Motional EMF
4. Total EMF

Faraday's Law

correct direction for
$\vec{B}$ produced by I
opposes increasing in
$\vec{B}$



Faraday's Law
A time-varying magnetic field produces an *electromotive force (emf)* that may establish a current in a suitable closed circuit.

I

motivates current to flow.

$$\text{emf} = \boxed{-}\frac{d\Psi}{dt} \ [\text{Wb/s} = \text{V}]$$

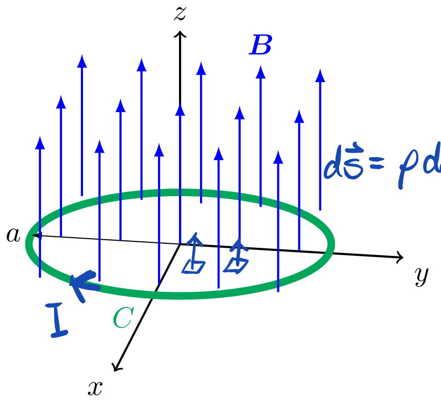
If the closed path has N turns of a filamentary conductor, then

$$\text{emf} = -\frac{d\Lambda}{dt} = -N\frac{d\Psi}{dt}$$

!! Lenz's law: The emf has to be induced in such a direction as to produce a current

Example

whose magnetic flux,
if added to the original
flux, would reduce the
magnetude of the emf.



$d\vec{S} = \rho\, d\phi\, d\rho\, \hat{z}$

Determine the EMF induced into a closed circuit C in the $z = 0$ plane as shown, across which $\boldsymbol{B}$ is uniform and descibed by

$$\boldsymbol{B} = B_0 e^{kt} \hat{\boldsymbol{z}}$$

$$emf = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$$

$$= -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$$

$$emf = -\int_0^{2\pi}\int_0^{a} kB_0 e^{kt} \rho\, d\rho\, d\phi = -2\pi k B_0 e^{kt}\,\frac{a^2}{2}$$

$$= -\pi k B_0 e^{kt}(a^2)$$

$$\frac{\partial \vec{B}}{\partial t} = kB_0 e^{kt}\,\hat{z}$$

$$\oint_C \vec{E}\cdot d\vec{l} = \underbrace{-\pi k B_0 e^{kt}(a^2)}_{\text{not zero.}}$$

Transformer EMF

$V_{emf}^{tr} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

loop area does not change with time.

Electrostatics: $\oint_C \vec{E}_{static} \cdot d\vec{l} = 0$

Generalize: $\boxed{emf = \oint_C \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}}$

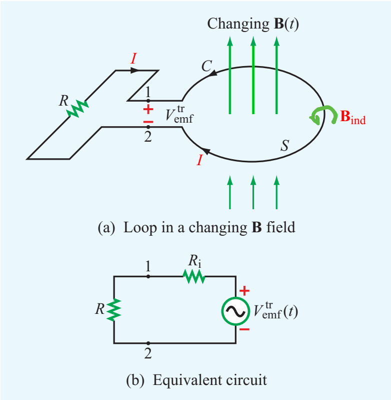
faraday's law in integral form.

EMF produced by a stationary closed path and a time-varying B-field is called **transformer emf** V_{emf}^{tr}.

$-\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = \oint \vec{E} \cdot d\vec{l} = \int_S (\nabla \times \vec{E}) \cdot d\vec{s}$

$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$

point / differential form.

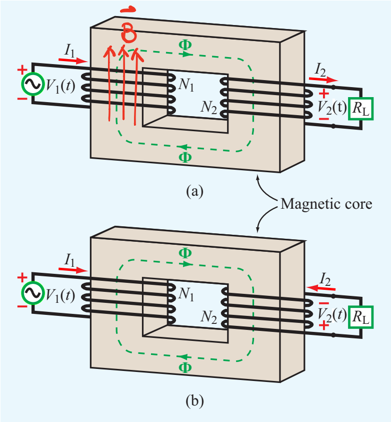


(a) Loop in a changing **B** field

(b) Equivalent circuit

Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Transformers



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

- An ideal transformer has:
  1. A core with very large (ideally infinite) permeability
  2. Magnetic flux completely confined within the core
- The winding on the left side is called the **primary winding** of the transformer. The right is the **secondary winding**.
- $V_1(t)$ is an alternating current (AC) source.

same.

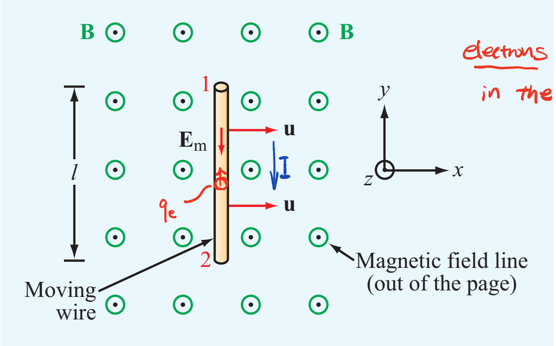
$$V_1 = -N_1 \frac{d\psi}{dt} \qquad V_2 = -N_2 \frac{d\psi}{dt}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Motional EMF

$\vec{B}$ 是 constant, bar move w.r.t t

- We now consider a second case: relative motion between a closed path and a steady field
- Example: sliding conductive bar moving with a constant velocity moving through a steady (DC) magnetic field $\boldsymbol{B} = \hat{\boldsymbol{z}} B_0$



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

electrons in the wire move in the direction of $-\vec{E}_m$

$F_m = q_e(\vec{u}\times\vec{B})$
motion

$E_m = \vec{u}\times\vec{B}$
motion

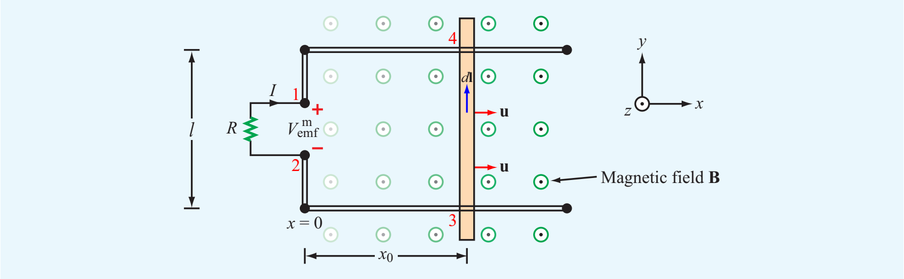
$$V_{21} = \int_2^1 \vec{E}_m\cdot d\vec{l} = \int_2^1 (\vec{u}\times\vec{B})\cdot d\vec{l} = \oint (\vec{u}\times\vec{B})\cdot d\vec{l} \equiv V_{emf}^{m}$$

motion.

$$= \oint \vec{E}\cdot d\vec{l}$$

Vc

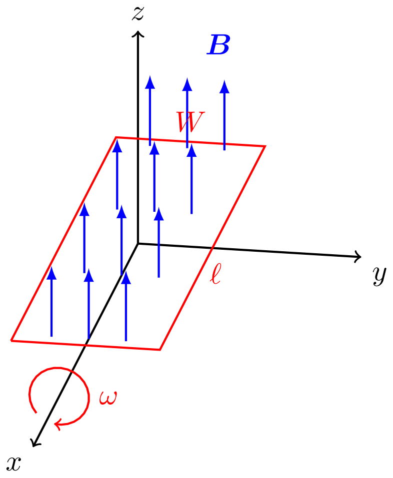
Example: Sliding Bar



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

A rectangular loop has a constant width l but its length x_0 increases with time as the conducting bar slides with a uniform velocity $\boldsymbol{u}$ in a static magenetic field $\boldsymbol{B} = \hat{\boldsymbol{z}} B_0 x$. The bar starts from $x = 0$ at $t = 0$. Find the motional emf between terminals 1 and 2.

Electromagnetic Generator

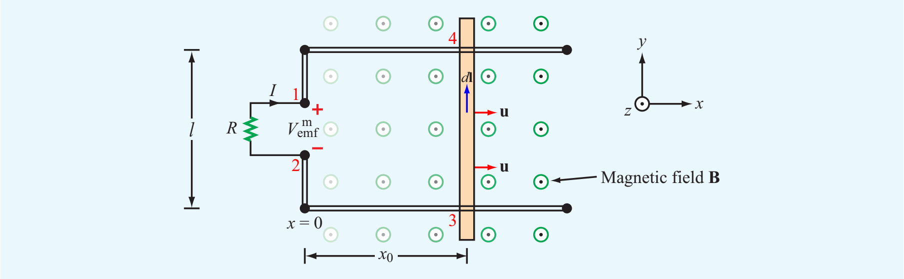


Find the induced EMF in the loop if it rotates at an angular velocity ω within a constant magnetic field $\boldsymbol{B}$.

Total EMF

$$\text{Total EMF} = V_{\text{emf}}^{tr} + V_{\text{emf}}^{m}$$

Example: Sliding Bar Revisited



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

A rectangular loop has a constant width l but its length x_0 increases with time as the conducting bar slides with a uniform velocity $\boldsymbol{u}$ in a **time-varying** magnetic field $\boldsymbol{B} = \hat{\boldsymbol{z}} B_0 \sin(\omega t)$. The bar starts from $x = 0$ at $t = 0$. Find the total emf between terminals 1 and 2.