For an arbitrary DT signal $x[n]$:

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

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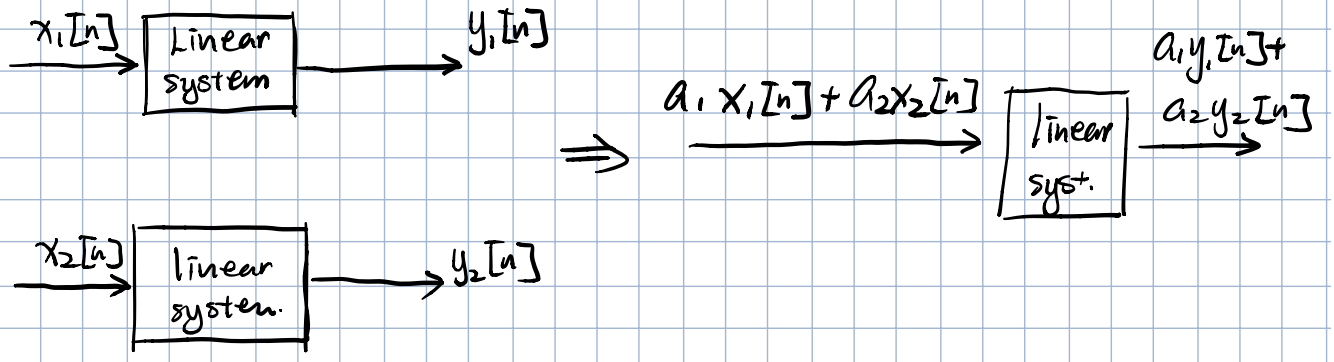
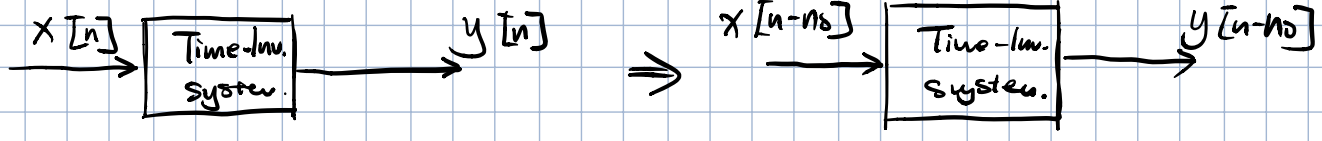
Linear System:




Time-Invariant System:




LTI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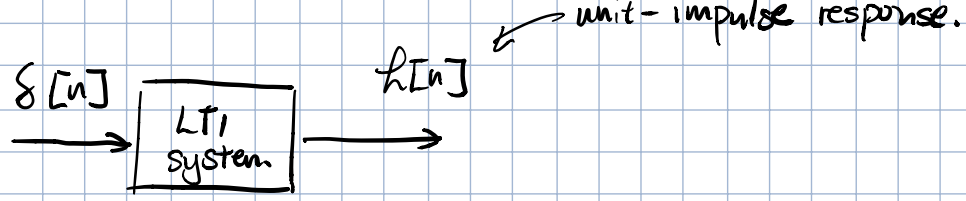
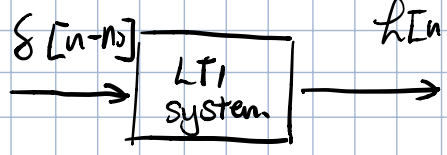
For an arbitrary LTI system:



$a_1\delta[n-n_1] + a_2\delta[n-n_2]$ → LTI system → $a_1 h[n-n_1] + a_2 h[n-n_2]$

$$+x[-2]\,\delta[n-(-2)] + x[-1]\,\delta[n-(-1)] + x[0]\,\delta[n-0] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2] + \cdots$$

→ LTI system. →

$$+x[-2]\,h[n-(-2)] + x[-1]\,h[n-(-1)] + x[0]\,h[n-0] + x[1]\,h[n-1] + x[2]\,h[n-2] + \cdots$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$

The characteristic of an LTI system are completely identified. (i.e. the output of a LTI system can be identified for any arbitrary input) if the impulse respond of that LTI system is known.

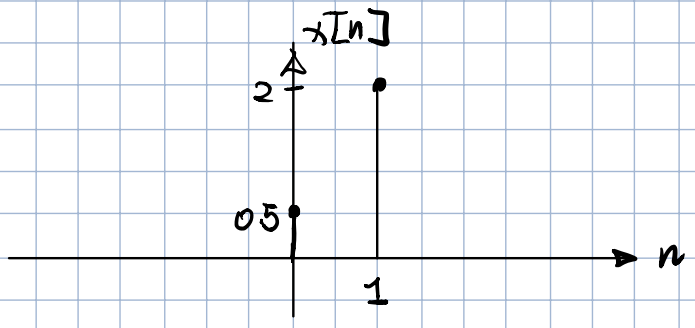
$\delta[n]$ → LTI → $h[n]$

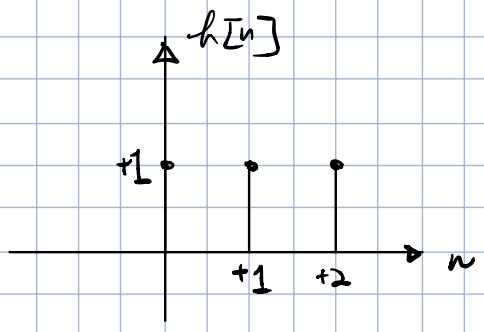
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$

$$= x[n] * h[n]$$

(convolution of $x[n]$ & $h[n]$
$x[n]$ convolved by $h[n]$)

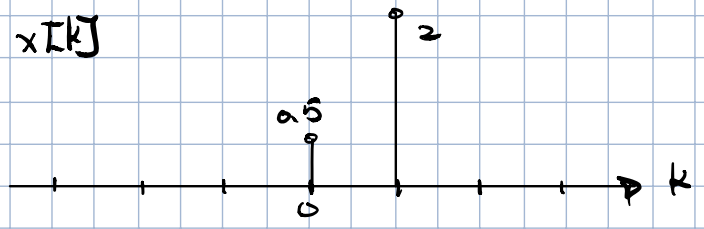
(*) The input $x[n]$ and impulse response $h[n]$ of an LTI system are given below. Find the output.

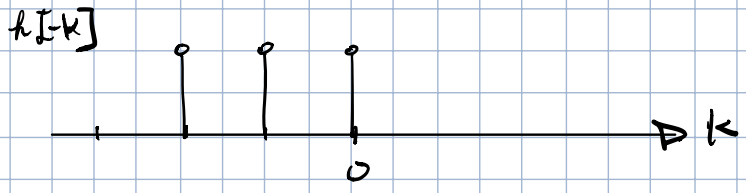


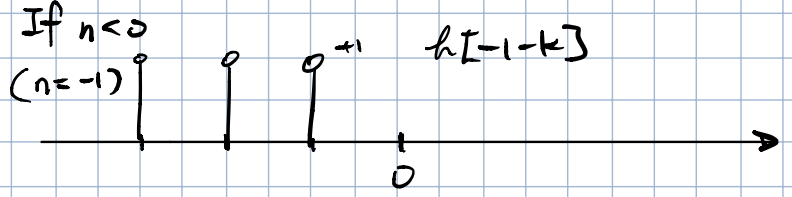

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$




If $n<0$ ($n=-1$)



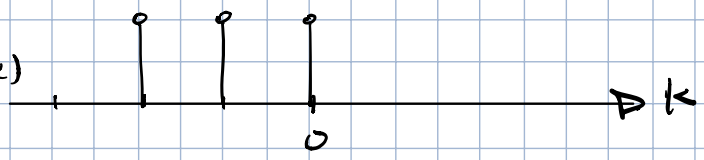

$\boxed{n<0 \Rightarrow y[n]=0}$

If $n=0$.

$h(n-k)$

$=$

$h(-k)$

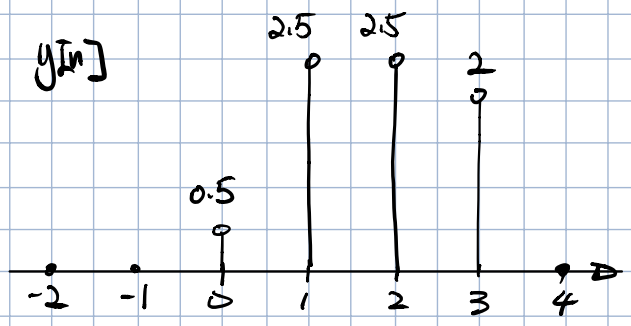



$y[0] = 0.5$

Similarly: $y[1] = 1.5$

$y[2] = 2.5$

$y[3] = 2$



y[4] = 0