Lecture 24: Magnetic Potentials, Magnetic Forces

ECE221: Electric and Magnetic Fields

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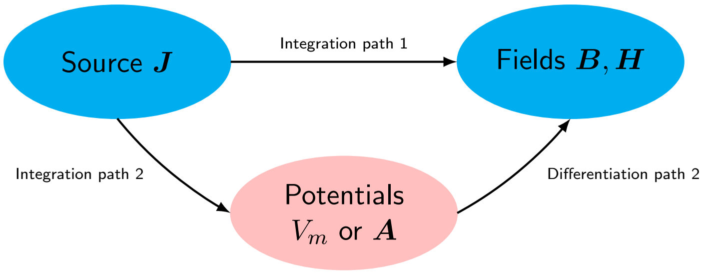
Winter 2019

Outline

Poisson's Equation

Magnetic Circuits	Ampère's Law
$\boldsymbol{\nabla}^2 \boldsymbol{A} = -\mu_0 \boldsymbol{J}$	$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$
Easy to solve for $\boldsymbol{A}$ Then find $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$	Hard to solve for $\boldsymbol{B}$ from point form



Vector Potential and Magnetic Flux

$$\Psi = \iint_S \boldsymbol{B} \cdot d\boldsymbol{S}$$

Apply Stokes theorem.

Example: Infinitely Long Coaxial Cable

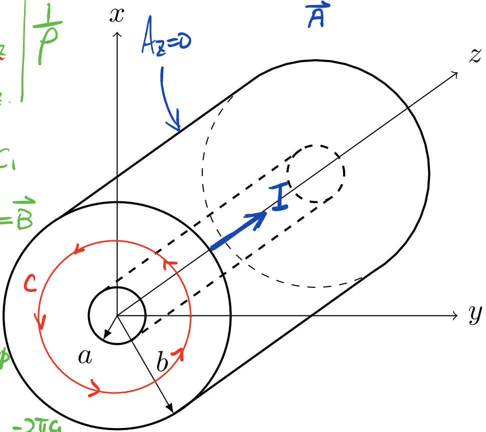
field sources.

Need to relate C_1 to sources in the problem.

Find the vector magnetic potential in the region $a < \rho < b$.

$\nabla^2 \vec{A} = -\mu_0 \vec{J} \Rightarrow \begin{cases} \nabla^2 A_x = -\mu_0 J_x \\ \nabla^2 A_y = -\mu_0 J_y \\ \nabla^2 A_z = -\mu_0 J_z \end{cases}$

only z component, $\cancel{A_x}$ $\cancel{A_y}$



$\nabla \times \vec{A} = \vec{B} = \begin{vmatrix} \hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \partial/\partial\rho & \cancel{\partial/\partial\phi} & \cancel{\partial/\partial z} \\ \cancel{A_\rho} & \cancel{\rho A_\phi} & A_z \end{vmatrix} \frac{1}{\rho}$

$= -\frac{1}{\rho}\rho\frac{\partial A_z}{\partial \rho}\hat{\phi} = -C_1$

$= -\frac{1}{\rho}C_1\frac{b}{\rho}\cdot\frac{1}{b}\hat{\phi}\cdot\rho = \vec{B}$

$\vec{H} = \frac{\vec{B}}{\mu_0} = -\frac{C_1}{\rho\mu_0}\hat{\phi}$

we know $= \oint_C H \cdot d\vec{l} = I = 2\pi\rho H_\phi$

$= \oint_C \frac{-C_1}{\mu_0\rho}\hat{\phi}\cdot\rho d\phi\,\hat{\phi} = \frac{-2\pi C_1}{\mu_0} = I$

$\boxed{A_z = \frac{-\mu_0 I}{2\pi}\ln\left(\frac{\rho}{b}\right)}$

$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_z}{\partial\rho}\right) + \cancel{\frac{1}{\rho^2}\frac{\partial^2 A_z}{\partial\phi^2}} + \cancel{\frac{\partial^2 A_z}{\partial z^2}}$

In source free region

$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_z}{\partial\rho}\right) = 0$

$\rho\frac{dA_z}{d\rho} = C_1 \rightarrow A_z = C_1\ln\rho + C_2$

Let $A_z(\rho=b)=0$. $\rightarrow C_2 = -C_1\ln b$

$A_z = C_1\ln\rho - C_1\ln b = C_1\ln(\rho/b)$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

左边继续

Example: Two Parallel Wires

let $A_z = 0$ @ a distance 4cm away from each conductor.

Consider two parallel wires of 1 cm radius, each carrying 12 A. Find $\boldsymbol{A}$ at a) $(0, 0, z)$; b) $(0, 8\text{ cm}, z)$; c) $(4\text{ cm}, 4\text{ cm}, z)$; d) $(2\text{ cm}, 4\text{ cm}, z)$.

a) $A_z = 0$

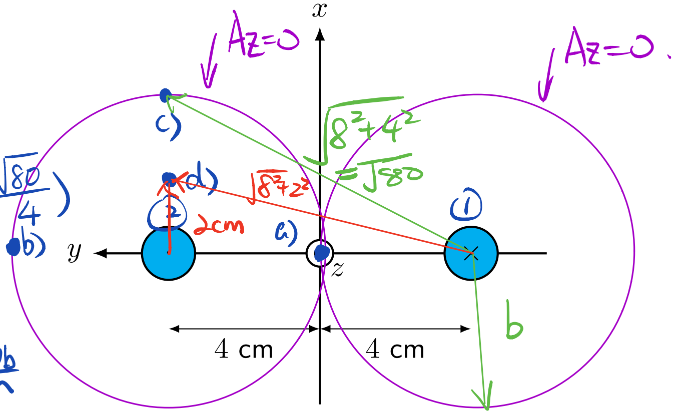
b) $A_{z_2} = 0 \quad A_{z_1} = \frac{-\mu_0 I}{2\pi} \ln\left(\frac{12}{4}\right)$

c) $A_z^{total} = A_{z_1} + \cancel{A_{z_2}}$

$$A_{z_1} = -\frac{\mu_0 I}{2\pi} \ln(\rho/b) = \frac{-\mu_0 \times 12}{2\pi} \ln\left(\frac{\sqrt{80}}{4}\right)$$

$$= 1.93\ \frac{\mu\text{Wb}}{\text{m}}$$

d) $A_z^{total} = A_{z_1} + A_{z_2} = 3.40\ \frac{\mu\text{Wb}}{\text{m}}$

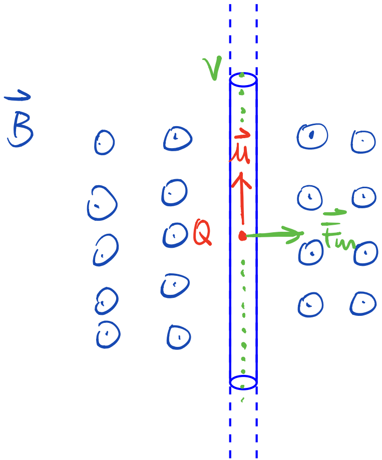


Magnetic Force on a Current-Carrying Conductor

Recall magnetic force on a moving charge is:

$$\boldsymbol{F}_m = Q\boldsymbol{u} \times \boldsymbol{B}$$

$\rho_v \, dV$



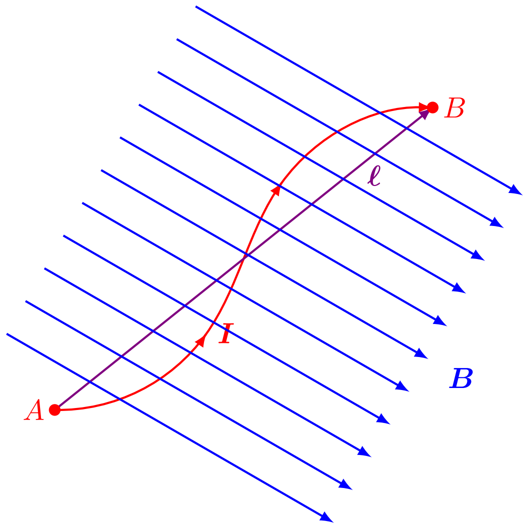
$[\frac{C}{m^3}]$ $[\frac{m}{s}]$

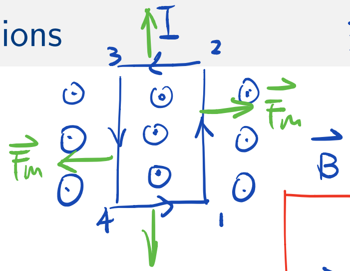
$$\vec{F}_m = \iiint_V \rho_v \vec{u} \times \vec{B} \, dv = \iiint_V \vec{J} \times \vec{B} \, dv$$

$$\vec{F}_m = \iint_S \vec{K} \times \vec{B} \, dS$$

$$\vec{F}_m = \int_C I d\vec{l} \times \vec{B}$$

Contour Integrals for Force Calculations





$$\vec{F}_m = \int_1^2 + \int_2^3 + \int_3^4 + \int_4^1$$

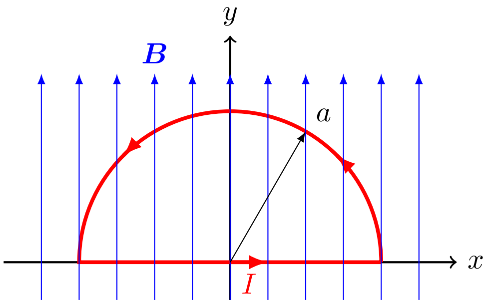
$$= 0 \quad \text{(all cancels out)}$$

if $\vec{B}$ is uniform.

$$\vec{F}_m = I B \oint d\vec{l} \rightarrow 0$$

Example: Force on a Semicircular Conductor

A semicircular loop lies in a uniform field $\boldsymbol{B} = \hat{\boldsymbol{y}} B_0$. Calculate the force on the loop if it carries a current I.



Example: Force on a Loop

Calculate the force on the loop if it carries a current as shown.