Fourier series Representation of CT Periodic signals. — CTFS:

Dirichlet conditions:

A periodic signal, $x(t)$ w/ period T, satisfies Dirichlet conditions:

1) $\int_T |x(t)|\,dt < \infty$ (Absolutely integrable)

2) $x(t)$ has a finite # of maxima & minima over every interval Δt

3) $x(t)$ has a finite # of discontinuities over every interval Δt, and each discontinuity is finite

If periodic $x(t)$ w/ period T satisfies Dirichlet conditions, it can be expressed as:

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t} \leftarrow$$ Synthesis Relation.

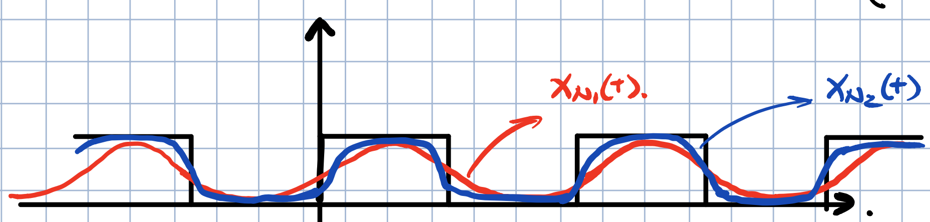
The set of coefficient $\{a_k\}$ is called Fourier Series coefficient of $x(t)$ and is obtained from:

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\,dt = \frac{1}{T}\int_T x(t) e^{-jk(2\pi/T)t}\,dt$$

Analysis Relation.

$$a_0 = \frac{1}{T}\int_T x(t)\,dt \text{ , for } k=0$$

(The Average (DC) value of $x(t)$)



$$x(t) = \sum^{+\infty} a_k e^{jk\omega_0 t}, \Rightarrow x_N(t) = \sum^{+N} a_k e^{jk\omega_0 t}$$

$$\begin{cases} x_{N_1}(t) & \text{lower order exponential} \\ \quad N_2 > N_1 & \\ x_{N_2}(t) & \text{higher order exponential} \end{cases}$$

(more close to the $x(t)$)

$$e^{jk\omega_0 t} = \cos(k\omega_0 t) + j\sin(k\omega_0 t) \quad \{\text{Euler's theorem}\}$$

Trignometric Fourier Series Representation of CT periodic signals:

The periodic signal $x(t)$ w/ period T that satisfies Dirichlet conditions can be expressed by:

$$x(t) = \sum_{k=1}^{+\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)\right) + a_0$$

$$= \sum_{k=1}^{+\infty} \left(a_k \cos(k(2\pi/T)t) + b_k \sin(k(2\pi/T)t)\right) + a_0$$

$\{a_k\}$ and $\{b_k\}$ are called trignometric Fourier series coefficients.

$$a_k = \frac{2}{T}\int_T x(t)\cos(k\omega_0 t)\,dt = \frac{2}{T}\int_T x(t)\cos(k(2\pi/T)t)\,dt$$

$$b_k = \frac{2}{T}\int_T x(t)\sin(k\omega_0 t)\,dt = \frac{2}{T}\int_T x(t)\sin(k(2\pi/T)t)\,dt$$

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

Example: Find the Fourier series coefficient of:

a) $x(t) = \cos(2t + \frac{\pi}{4})$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}, \qquad a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}$$

$$\sum_{k=-\infty} \qquad \int_T$$

$$a_k = \frac{1}{T}\int_T \cos\left(2t+\frac{\pi}{4}\right) e^{-jk2t}\,dt$$

$$e^{j\alpha} = \cos\alpha + j\sin(\alpha) \qquad e^{-j\alpha} = \cos(\alpha) - \sin(\alpha)$$

$$e^{j\alpha} + e^{-j\alpha} = 2\cos(\alpha) \Rightarrow \cos(\alpha) = \frac{e^{j\alpha}+e^{-j\alpha}}{2}$$

$$\cos\left(2t+\frac{\pi}{4}\right) = \frac{1}{2}\left(e^{j(2t+\frac{\pi}{4})} + e^{-j(2t+\frac{\pi}{4})}\right)$$

$$= \underbrace{\frac{1}{2}e^{-j\frac{\pi}{4}}}_{a_{-1}} e_{k=-1}^{\overset{\cancel{jk\omega_0 t}}{\cancel{-j2t}}} + \underbrace{\frac{1}{2}e^{j\frac{\pi}{4}}}_{a_1} e_{k=1}^{\overset{\cancel{jk\omega_0 t}}{\cancel{j2t}}}$$

$$a_{-1} = \frac{1}{2}e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$

$$a_1 = \frac{1}{2}e^{j\frac{\pi}{4}} = \frac{\sqrt{2}}{4} + j\frac{\sqrt{2}}{4}$$