Matrix Representation of DT FS coefficients:

Analysis eqn. : $$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n}$$

$$a_0 = \frac{1}{N}\Big[x[0]e^{-j0\omega_0 0} + x[1]e^{-j0\omega_0 1} + x[2]e^{-j0\omega_0 2} + \cdots\cdots + x[N-1]e^{-j0\omega_0 (N-1)} \Big]$$

$$a_1 = \frac{1}{N}\Big[x[0]e^{-j1\omega_0 0} + x[1]e^{-j1\omega_0 1} + x[2]e^{-j1\omega_0 2} + \cdots\cdots + x[N-1]e^{-j1\omega_0 (N-1)} \Big]$$

$$a_{N-1} = \frac{1}{N}\Big[x[0]e^{-j(N-1)\omega_0 0} + x[1]e^{-j(N-1)\omega_0 1} + x[2]e^{-j(N-1)\omega_0 2} + \cdots\cdots + x[N-1]e^{-j(N-1)\omega_0 (N-1)} \Big]$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{N-1} \end{bmatrix} = \frac{1}{N} \underbrace{\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{-j\omega_0} & e^{-j\omega_0 2} & \cdots & e^{-j\omega_0 (N-1)} \\ 1 & e^{-j\omega_0 2} & e^{-j\omega_0 4} & \cdots & e^{-j\omega_0 2(N-1)} \\ \vdots & \vdots & \vdots & & \\ 1 & e^{-j\omega_0 (N-1)} & e^{-j\omega_0 2(N-1)} & \cdots & e^{-j\omega_0 (N-1)(N-1)} \end{bmatrix}}_{G \quad e^{-jk\omega_0 n}} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix}$$

Synthesis Relation: $$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\omega_0 n}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ \vdots \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & e^{j\omega_0} & e^{j\omega_0 2} & \cdots & e^{j\omega_0 (N-1)} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} a_0 \\ a_1 \\ \vdots \end{bmatrix}$$

$$
\begin{bmatrix} x[2] \\ \vdots \\ x[N-1] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & e^{j\omega_0 2} & e^{j\omega_0 4} & \cdots & e^{j\omega_0 2(N-1)} \\ \vdots & \vdots & \vdots & & \\ 1 & e^{j\omega_0 (N-1)} & e^{j\omega_0 2(N-1)} & \cdots & e^{j\omega_0 (N-1)(N-1)} \end{bmatrix}}_{H \qquad e^{jk\omega_0 n}} \begin{bmatrix} a_2 \\ \vdots \\ a_{N-1} \end{bmatrix}
$$

$$G = H^*$$

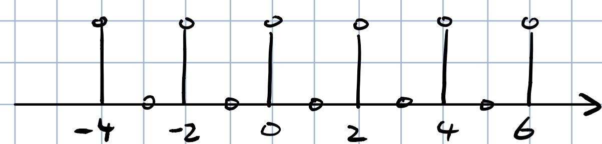
$$GH = NI \rightarrow \frac{1}{N} G = H^{-1}$$

I ← identity matrix = $\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & \cdots & \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & & \end{bmatrix}$

$N \times N$

Example:

$$x[n] = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$$



→ Find the DT FS coefficients of $x[n]$ using its projection onto the basis signals. :

$$y_0[n] = e^{-j\frac{2\pi}{2} 0 n} \quad \forall n$$

$$y_1[n] = e^{-j\frac{2\pi}{2} 1 n} \quad \forall n$$

$N=2$

$$a_0 = \frac{\langle x[n], y_0[n] \rangle}{\langle y_0[n], y_0[n] \rangle} = \frac{\sum_{n=0}^{1} x[n] y_0^*[n]}{\sum_{n=0}^{1} y_0[n] y_0^*[n]}$$

$$y_0[n] = 1 \qquad = \frac{1\cdot 1 + 0\cdot 1}{1\cdot 1 + 1\cdot 1} = \frac{1}{2}$$

$$y_1[n] = e^{-j\pi n} = \begin{cases} 1 & \text{even } n \\ -1 & \text{odd } n \end{cases}$$

$$a_1 = \frac{\langle x[n],\ y_1[n] \rangle}{\langle y_1[n],\ y_1[n] \rangle} = \frac{1\cdot 1 + 0\cdot(-1)}{\underbrace{1\cdot 1}_{n=0} + \underbrace{(-1)\cdot(-1)}_{n=1}} = \frac{1}{2}$$

Systems.

Def: A process to transform signals.

$$x[n] \longrightarrow \boxed{\begin{matrix}\text{system.}\\ S\end{matrix}} \xrightarrow{\ y[n]\ }$$

DT system

$$y[n] = T\{x[n]\}$$

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{System}\\ S\end{matrix}} \xrightarrow{\ y(t)\ }$$

CT system.

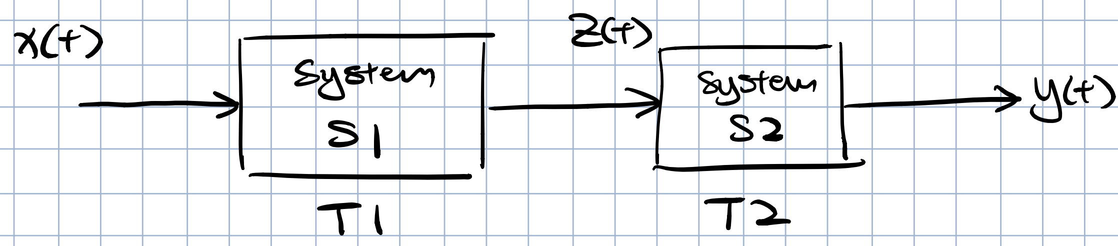
$$y(t) = T\{x(t)\}$$

eg. Differentiator.: $x(t) \longrightarrow \boxed{\text{Diff.}} \longrightarrow y(t) = \frac{dx}{dt}$

Averaging: $x[n] \longrightarrow \boxed{\begin{matrix}\text{Ave.}\\ N\end{matrix}} \longrightarrow y[n] = \frac{1}{N}\sum_{k=0}^{N-1} x[n-k]$

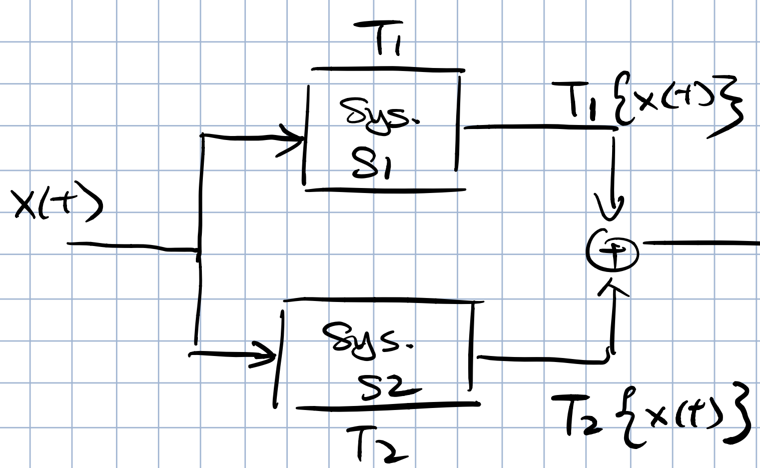
Connection of systems.

1. Series Connection:



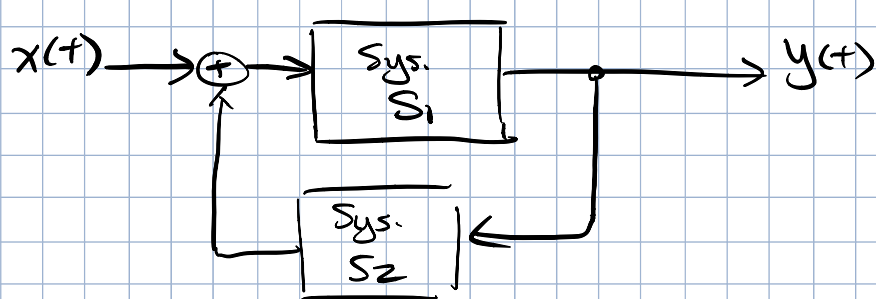
$$y(t) = T_2\{z(t)\} = T_2\{T_1\{x(t)\}\}$$

2. Parallel connection:



$$y(t) = T_1\{x(t)\} + T_2\{x(t)\}$$

3. Feedback Connection



Properties of Systems:

① Systems with and without memory:

For a memoryless system, the output at a given time depends on the input at the same time.

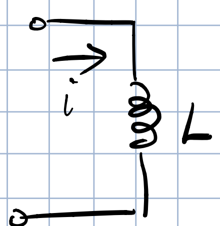
$$y[n] = 2x^2[n] - 3x[n] + 5$$ Memory less ✓

$$y(t) = \int_{\infty}^{t} x(\tau)\, d\tau$$ with memory.

$$y[n] = x[n+1]$$ with memory.

$$y(t) = x(2t)$$ with memory.

Physical systems with memory usually include energy storage elements.

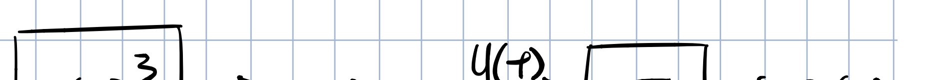


$$i(t) = \frac{1}{L} \int_{\infty}^{t} V_L(\tau)\, d\tau$$

② Invertibility:

A system is invertible if distinct inputs leads to distinct outputs If a system is invertible, then an inverse system exists.

$$y(t) = x^3(t)$$ Invertible. $$x(t) = \sqrt[3]{y(t)}$$



$x(t) \rightarrow \boxed{(\ )} \rightarrow y(t) \qquad \rightarrow \boxed{\sqrt[3]{(\ )}} \rightarrow x(t)$

$y(t) = x^2(t)$ **NOT** invertible.

$$x_1(t) = 2 \rightarrow y(t) = 4$$

$$x_2(t) = -2 \rightarrow y(t) = 4$$

For invertible systems, no horizon. line passes through the x-y graph more than once.