Properties of CT Fourier Transform:

Analysis: $$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\, dt$$

synthesis: $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t}\, d\omega$$

$$x(t) \xleftrightarrow{ft} X(j\omega)$$

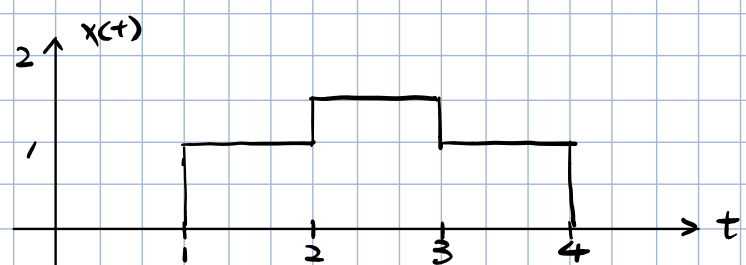
① linearity.

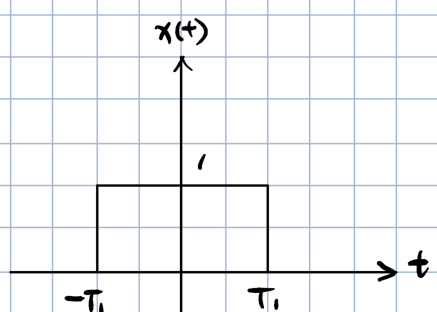
$$\left.\begin{array}{l} x_1(t) \xleftrightarrow{ft} X_1(j\omega) \\ x_2(t) \xleftrightarrow{ft} X_2(j\omega) \\ \text{and} \quad a_1 \,\&\, a_2 \in \mathbb{C} \end{array}\right\} \rightarrow a_1 x_1(t) + a_2 x_2(t) \xleftrightarrow{ft} a_1 X_1(j\omega) + a_2 X_2(j\omega)$$

② Time-shift:

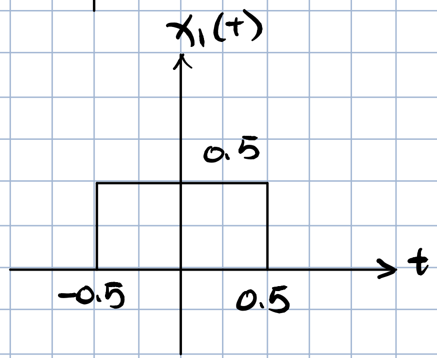
$$x(t) \xleftrightarrow{ft} X(j\omega) \longrightarrow x(t-t_0) \xleftrightarrow{ft} e^{-j\omega t_0} X(j\omega)$$

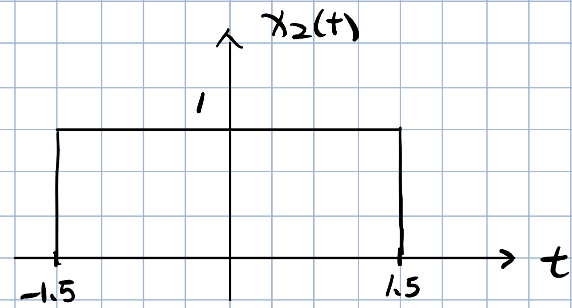
⊛ Find the FT of:

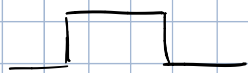




$$X_0(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$





$x(t) =$  $= x_1(t-2.5) + x_2(t-2.5)$

$$x_1(t) \xleftrightarrow{ft} \frac{1}{2}\frac{2\sin(0.5\omega)}{\omega} = X_1(j\omega)$$

$$x_2(t) \xleftrightarrow{ft} \frac{2\sin(1.5\omega)}{\omega} = X_2(j\omega)$$

$$x_1(t-2.5) \xleftrightarrow{ft} \frac{1}{2}e^{-j2.5\omega}\frac{2\sin(0.5\omega)}{\omega}$$

$$x_2(t-2.5) \xleftrightarrow{ft} e^{-j2.5\omega}\frac{2\sin(1.5\omega)}{\omega}$$

$$\therefore \boxed{X(j\omega) = \frac{e^{-j2.5\omega}}{\omega}\left(\sin(0.5\omega) + 2\sin(1.5\omega)\right)}$$

③ Conjugation

$x(t) \xleftrightarrow{ft} X(j\omega)$ $\qquad$ $x^*(t) \xleftrightarrow{ft} X^*(-j\omega)$

If $x(t)$ is real $(x(t) = x^*(t)) \rightarrow X(j\omega) = X^*(-j\omega)$

$$\Rightarrow X(-j\omega) = X^*(j\omega)$$

$$\rightarrow \mathrm{Re}\{X(-j\omega)\} + j\,\mathrm{Im}\{X(-j\omega)\} = \mathrm{Re}\{X(j\omega)\} - j\,\mathrm{Im}\{X(j\omega)\}$$

$$\mathrm{Re}\{X(-j\omega)\} = \mathrm{Re}\{X(j\omega)\}$$

$$\mathrm{Im}\{X(-j\omega)\} = -\mathrm{Im}\{X(j\omega)\}$$

$\Rightarrow$ $\mathrm{Re}\{X(j\omega)\}$ is an even signal

$\mathrm{Im}\{X(j\omega)\}$ is an odd signal.

$$\Rightarrow |X(-j\omega)| e^{j\measuredangle X(-j\omega)} = |X(j\omega)| e^{-j\measuredangle X(j\omega)}$$

$$\rightarrow |X(-j\omega)| = |X(j\omega)| \quad \text{(Even signal)}$$

$$\measuredangle X(-j\omega) = -\measuredangle X(j\omega) \quad \text{(odd signal)}$$

④ Time differentiation and integration.

$$x(t) \xleftrightarrow{ft} X(j\omega) \Rightarrow \frac{dx(t)}{dt} \xleftrightarrow{ft} j\omega X(j\omega)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega \Rightarrow \frac{dx}{dt} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} j\omega X(j\omega) e^{j\omega t} d\omega$$

$$\int_{-\infty}^{+\infty} x(\tau) d\tau \xleftrightarrow{ft} \frac{X(j\omega)}{j\omega} + \pi \underbrace{X(0)}_{X(j\omega)\text{所带 }0} \delta(\omega)$$

(*) Find the FT of $u(t)$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \longrightarrow U(j\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t}\, dt$$

$$FT\{\delta(t)\} = ? \qquad \delta(t) = \frac{du(t)}{dt}$$

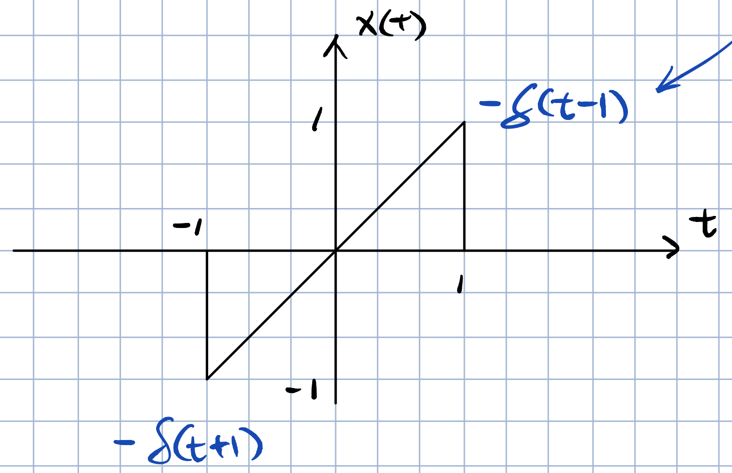
$$FT\{\delta(t)\} = \int_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = 1$$

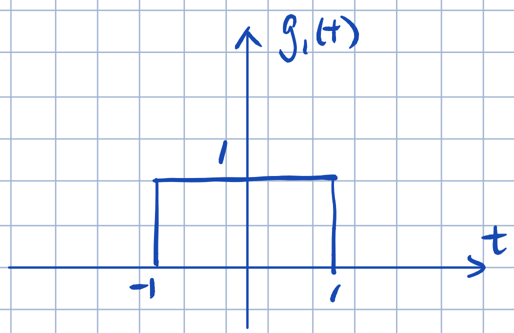
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau \xleftrightarrow{ft} \frac{1}{j\omega}(1) + \pi \overbrace{1}^{X(0)} \delta(\omega)$$

$$u(t) \xleftrightarrow{ft} \frac{1}{j\omega} + \pi\delta(\omega)$$

(*) Find the FT of

dropping from 1 to 0, so "-ve"





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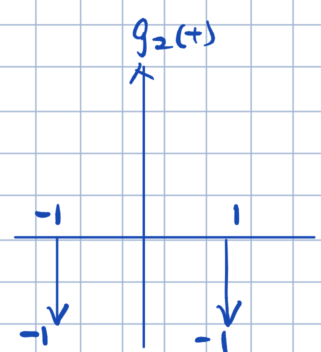

$$= g(t) = \frac{d\,x(t)}{dt}$$

$$FT\{g_1(t)\} = \frac{2\sin w}{w} \qquad FT\{\delta(t)\} = 1 \rightarrow FT\{g_2(t)\}$$

$$= -e^{j\omega}\cdot 1 - e^{+j\omega}\cdot 1$$

$$\Rightarrow FT\{g(t)\} = \frac{2\sin(\omega)}{\omega} - e^{-j\omega} - e^{+j\omega} = G(j\omega)$$

$$\lim_{\omega \to 0} G(j\omega) = 0 \longrightarrow FT\{x(t)\} = \frac{2\sin(\omega)}{j\omega(\omega)} - \frac{e^{-j\omega}}{j\omega} - \frac{e^{+j\omega}}{j\omega}$$

$$= \frac{2\sin(\omega)}{j\omega^2} - \frac{1}{j\omega} 2\cos(\omega)$$