Lecture 19: Introduction to Magnetostatics

ECE221: Electric and Magnetic Fields

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Outline

Force Relations

Electric force

$$\boldsymbol{F}_e = \cancel{q}\,Q\boldsymbol{E} \qquad \vec{F_e} \parallel \vec{E}$$

Magnetic force

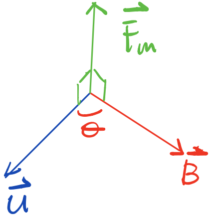
u= velocity of charge.

$$\boldsymbol{F}_m = Q\boldsymbol{u} \times \boldsymbol{B} \qquad \vec{F_m} \perp \vec{u} \perp \vec{B}$$

cross product.

magnitute.

$$|\vec{F_m}| = QuB\sin\theta$$

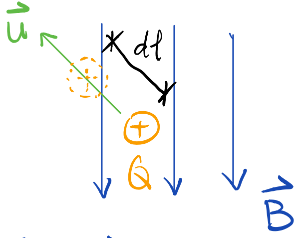


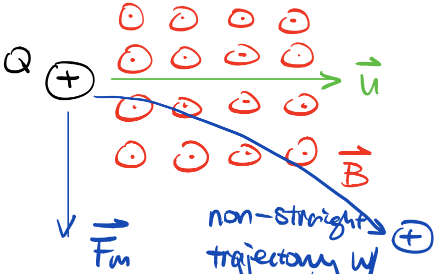
Lorentz force equation

$$\boldsymbol{F} = \boldsymbol{F}_e + \boldsymbol{F}_m = \cancel{q}\,Q(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$$

Work Done by Fields

$$\text{Work} = \text{force} \times \text{distance}$$





non-straight trajectory w/ same initial velocity.

$$dw = \vec{F}_m \cdot d\vec{l}$$

$$d\vec{l} = \vec{u}\,dt$$

距离 = 速度 × 时间

$$dw = (\vec{F}_m \cdot \vec{u}\,dt) = 0\,!$$

$\therefore$ $\vec{B}$ does not do any work on Q at all.

But $\vec{B}$ can change the direction of motion of a charged particle. (But not velocity)

Biot-Savart's Law

Biot-Savart's Law is the magnetic equivalent (*dual*) of Coulomb's Law for electric fields.

Elementary Source of Electric Field $\boldsymbol{E}$	Elementary Source of Magnetic Field $\boldsymbol{H}$
A point charge Q	An infinitesimal current element $Id\boldsymbol{\ell}$

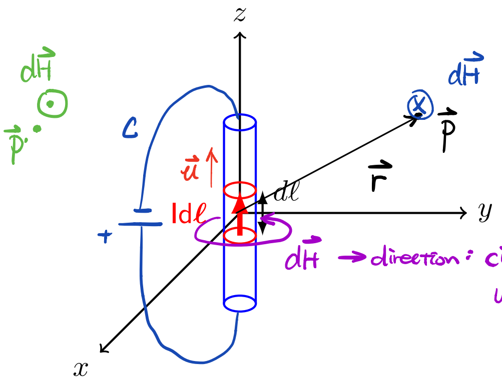
$I\, d\vec{\ell}$

scalar vector

Magnetic Field Produced by an Infinitesimal Current Element

$$d\boldsymbol{H} = \frac{Id\boldsymbol{l} \times \hat{\boldsymbol{R}}}{4\pi R^2} = \frac{Id\boldsymbol{l} \times \boldsymbol{R}}{4\pi R^3}$$

Magnetic Field from a Current Element $I d\vec{\ell}$



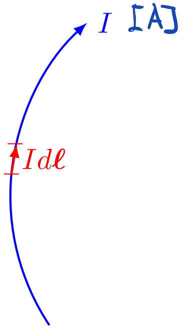
$d\vec{H}$ → direction: circulating around the wire.

$$\vec{H} = \oint_c \frac{I d\vec{\ell} \times \vec{R}}{4\pi R^3}$$

$$d\boldsymbol{H} = \frac{I d\boldsymbol{l} \times \boldsymbol{R}}{4\pi R^3}$$

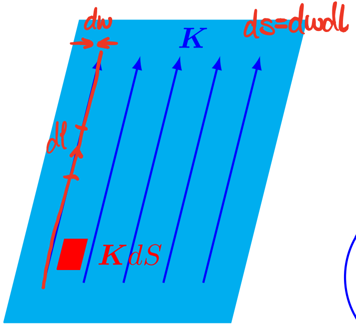
Types of Current Distributions

Line Current



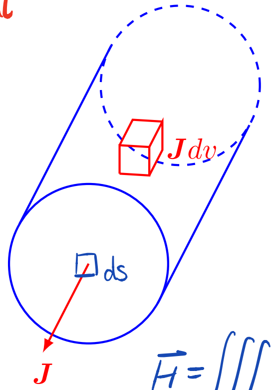
$$\vec{H} = \int_c \frac{I\, d\vec{\ell} \times \vec{R}}{4\pi R^3}$$

Surface Current [A/m]



$$\vec{H} = \iint_S \frac{\vec{K} \times \vec{R}\, ds}{4\pi R^3}$$

Volume Current [A/m²]



$$\vec{H} = \iiint_V \frac{\vec{J} \times \vec{R}\, dv}{4\pi R^3}$$

Wire Along z-axis

$$\tan\alpha = \rho / z'$$

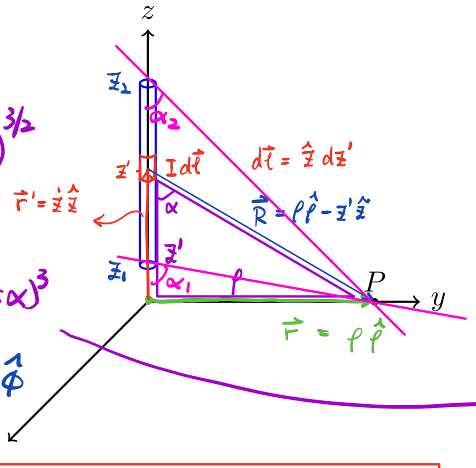
$$z' = \rho / \tan\alpha = \rho\cot\alpha$$

$$dz' = -\rho\csc^2\alpha\, d\alpha$$

$$(\rho^2 + z'^2)^{3/2} = (\rho^2 + \rho^2\cot^2\alpha)^{3/2}$$

$$= (\rho^2(1+\cot^2\alpha))^{3/2}$$

$$= (\rho^2\csc^2\alpha)^{3/2} = (\rho\csc\alpha)^3$$



$$d\vec{H} = \frac{I\, d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$d\vec{l} \times \vec{R} = \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ 0 & 0 & dz' \\ \rho & 0 & -z' \end{vmatrix} = \rho\, dz'\, \hat{\phi}$$

$$\boxed{\vec{H} = \int_{z_1}^{z_2} \frac{I\rho\, dz'\, \hat{\phi}}{4\pi(\rho^2 + z'^2)^{3/2}}}$$

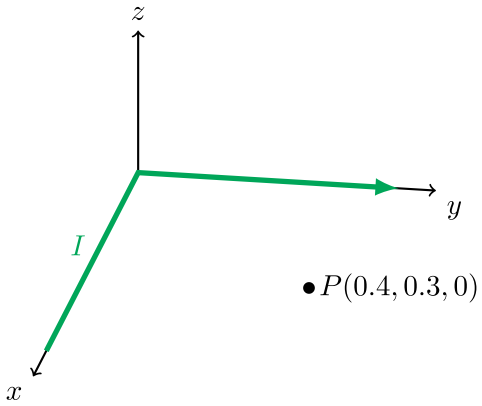
$$H = \frac{I}{4\pi}\int \frac{\rho(-\rho\csc^2\alpha)\, d\alpha}{\rho^3\csc^3\alpha}\hat{\phi}$$

$$H = \frac{-I\hat{\phi}}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha = \boxed{\frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\,\hat{\phi} = H}$$

$H \propto$ wire length $\propto (\cos\alpha_2 - \cos\alpha_1)$

$H \propto I$ $H \propto 1/\rho$

Example: Two Semi-Infinite Current Segments



Example: Circular Loop