Distortion in Transmission Lines.

$$\gamma(\omega) = \alpha(\omega) + j\beta(\omega) = \sqrt{(R+j\omega L)(G+j\omega C)}$$

(Lossless) $(R=0,\ G=0)$

$$\alpha(\omega)=0 \qquad \beta(\omega) = \omega\sqrt{LC} \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

all freq. travel w/ the same velocity

(Lossy line)

$$\alpha(\omega) = \mathrm{Re}\left[\sqrt{(R+j\omega L)(G+j\omega C)}\right]$$

$$\beta(\omega) = \mathrm{Im}\left[\sqrt{(R+j\omega L)(G+j\omega C)}\right]$$

$$u(\omega) = \frac{\omega}{\beta} = \frac{\omega}{\mathrm{Im}\left[\sqrt{(R+j\omega L)(G+j\omega C)}\right]}$$

Distortionless Line.

Heaviside condition:

$$\frac{R}{L} = \frac{G}{C} \qquad G = \frac{RC}{L}$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{(R+j\omega L)\left(\frac{RC}{L}+j\omega C\right)} = \sqrt{\frac{R^2C}{L} + j2\omega RC - \omega^2 LC}$$

$$= \sqrt{\frac{C}{L}}(R+j\omega L)$$

$$\underbrace{\alpha = R\sqrt{\frac{C}{L}}}_{\text{freq. independent}} \qquad \underbrace{\beta = \omega\sqrt{LC}}_{\beta \text{ is linear function of freq.}}$$

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \text{const.}$$

More on Maxwell's Equation:

$$\boxed{\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t} = 0}$$ wave equation.