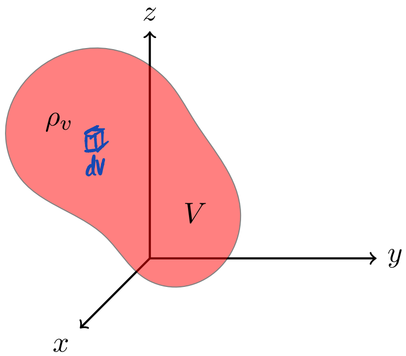
Divergence



$$\rho_v = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

$$Q = \iiint_V \rho_v(\vec{r})\, dV$$

Defn: Divergence is flux per unit volume over a small volume ΔV

$$\psi = \oint\!\!\oint_S \vec{D}(\vec{r})\; d\vec{s}\,'$$

$$\nabla \cdot \vec{D} = \lim_{\Delta V \to 0} \frac{\psi}{\Delta V} = \frac{\oint\!\!\oint \vec{D}\, d\vec{s}}{\Delta V}$$

$$= \frac{Q_{encl}}{\Delta V} = \rho_v(\vec{r}\,')\Delta V$$

Gauss' Law and the Divergence Theorem

- We can use the point form of Gauss' law back in the integral form:

$$Q = \iiint_V \rho_v(\boldsymbol{r}')dv' = \boxed{\iiint_V \boldsymbol{\nabla}\cdot\boldsymbol{D}(\boldsymbol{r}')dv' \equiv \oint_S \boldsymbol{D}(\boldsymbol{r}')\cdot d\boldsymbol{s}'}$$

- We have indirectly developed the *divergence theorem*, which actually holds for all vectors!

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field through the volume enclosed by the closed surface.

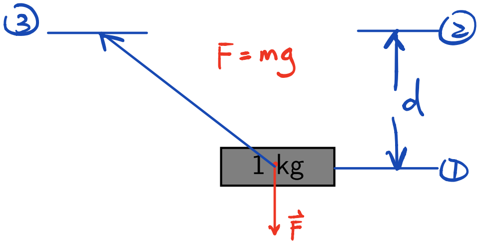
Δv

$= \rho_v(\vec{r})$

$$\boxed{\nabla\cdot\vec{D} = \rho_v}$$

Gauss's law in point (differential) form.

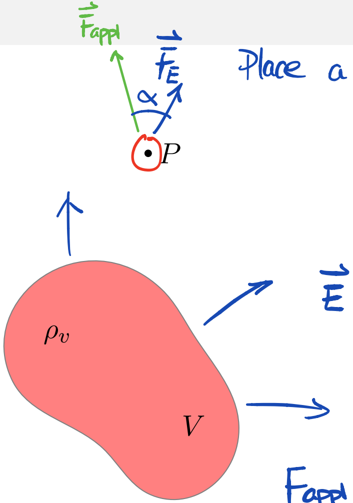
Energy and Work



Work done to move mass from

① to ② $W = Fd = mgd$

① to ③ is the same as ① to ②



Place a charge Q @ P

$$\vec{F}_E(\vec{r}) = Q\vec{E}(\vec{r})$$

Apply a force $\vec{F}_{apply}$ to move the charge.

$$F_{appl} = -Q\vec{E}(\vec{r}) \cdot \vec{a}_L$$

$$= -QE(\vec{r})\cos\alpha$$

Over a small path ...

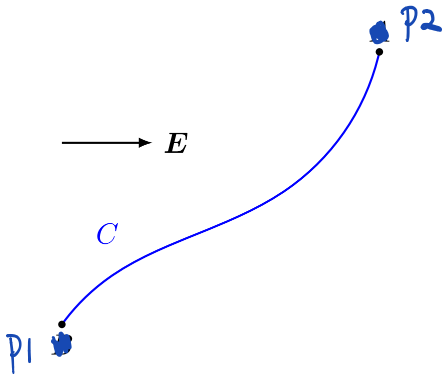
$dW = -Q\vec{E}(\vec{r})\cdot d\vec{l}$

Total Work Along a Contour

Total work done over a path C

$$W = -Q\int_{\text{initial position } P1}^{\text{initial position } P2} \vec{E}(\vec{r})\cdot d\vec{l}$$

What is the total work moving a charge along the contour C?

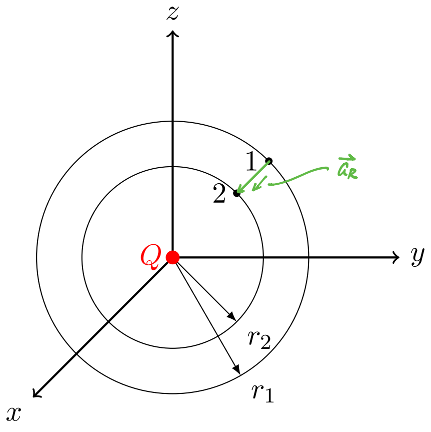


$$W = -Q\int_{P1}^{P2} \vec{E}(\vec{r})\cdot d\vec{l}$$

W does not depend on the path taken from P1 to P2, it only depends on P1 and P2 (path indep.)

Potential difference between P_1 and P_2 $\equiv V_{12} = \frac{W}{Q} = -\int_{P1}^{P2} \vec{E}(\vec{r})\cdot d\vec{l}$

Potential Between Two Points near a Point Charge



$R = r$

(source pt. is @ the origin)

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\vec{a}_R \qquad (\vec{a}_R \text{ is } \vec{a}_r \text{ in this case})$$

$$\vec{dl} = \vec{a}_r \text{ (radial direction)} \cdot dr$$

$$V_{12} = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0 \cancel{R^2}\, r^2}\vec{a}_r \cdot \vec{a}_r\, dr$$

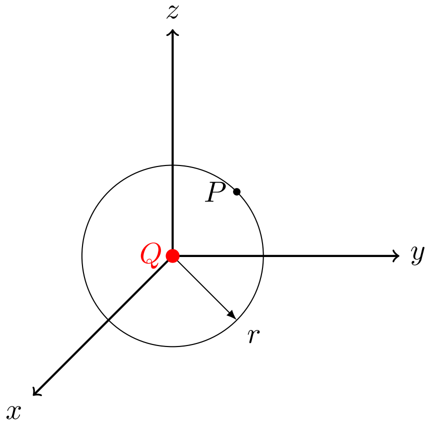
$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{1}{r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Potential From a Point Charge

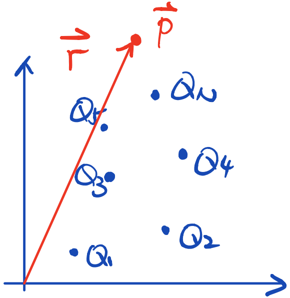
$= V(P_2) - V(P_1)$

Let $r_1 \to \infty$, $V_1 \equiv 0$, and $r_2 \equiv r$:

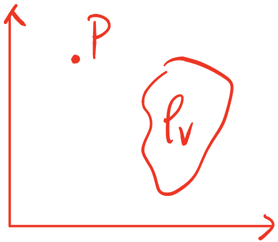


set $r_1 \to \infty$, $V_{21} \to V(\vec{r}) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r}\right)$

Generalization to Charge Distributions



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{Q_n}{R_n} \qquad R_n = |\vec{r} - \vec{r}_n{}'|$$



$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\rho_v(\vec{r}\,')\,dv'}{|\vec{r} - \vec{r}\,'|}$$

Conservative Property of the Electrostatic Field