Periodic of $e^{j\omega_0 n}$:

1) Periodicity in the time domain:

$e^{j\omega_0 n}$ is periodic if $2\pi/\omega_0$ is a rational number.

2. Periodicity in the frequency domain:

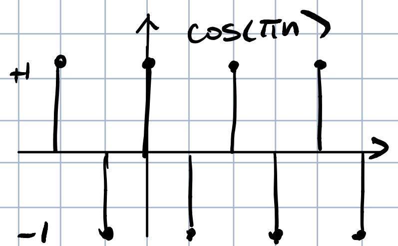
i.e. $\cos(\omega_0 n)$ $\omega_0 = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

一样.

continuous time: $x(t) = e^{j\omega_0 t}$ $\omega_0 \uparrow$ $T \downarrow$, the higher is the freq of oscillation.

discrete time: $x[n] = e^{j\omega_0 n}$ ← the highest rate of oscillation is obtained when $\omega_0 = (2k+1)\pi$, $k \in \mathbb{Z}$

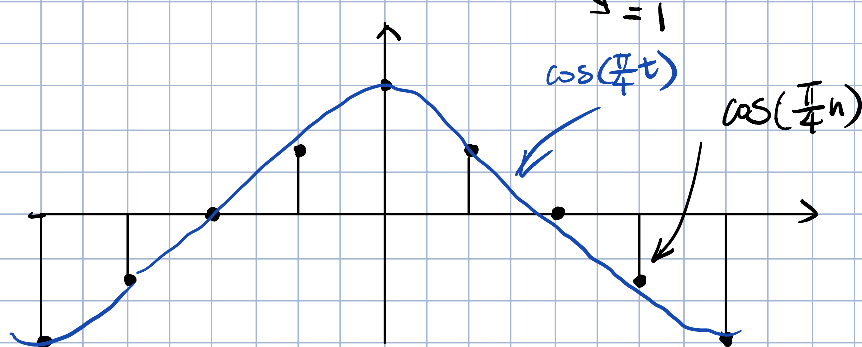
the lowest rate of oscillations for the signal is obtained for $\omega_0 = (2k)\pi$, $k \in \mathbb{Z}$



$x[n] = e^{j\frac{\pi}{4}n}$, $\omega_0 = \frac{\pi}{4}$

$$y[n] = e^{j(\frac{\pi}{4}+2\pi)n} = e^{j\frac{\pi}{4}n} \cdot \underbrace{e^{j2\pi n}}_{=1} = e^{j\frac{\pi}{4}}$$

$$z[n] = e^{j(\frac{\pi}{4}+10\pi)n} = e^{j\frac{\pi}{4}n} \cdot \underbrace{e^{j10\pi n}}_{=1} = e^{j\frac{\pi}{4}n}$$



Fourier Series Representations of DT periodic signals.

If $x[n]$ is periodic w/ period N,

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$$ ⇐ "Synthesis Relation"

a_k is called Fourier Series coefficient of $x[n]$ is given by:

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk(2\pi/N)n}$$

$$a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n] e^{-jk\omega_0 n}$$

$$x[n] = a_0 e^{j0\omega_0 n} + a_1 e^{j1\omega_0 n} + a_2 e^{j2\omega_0 n} + \cdots + a_{N-1} e^{j(N-1)\omega_0 n}$$

$$x[n] = a_1 e^{j1\omega_0 n} + a_2 e^{j2\omega_0 n} + \cdots + a_N e^{jN\omega_0 n}$$ 相等

$$\Rightarrow a_0 = a_N$$

→ So unlike CT signal, the Fourier coefficient of DT signal a_k, is periodic for a periodic $x[n]$

→ Unlike CT FS synthesis relation, adding a finite # of properly weighted exponentials is enough for synthesizing $x[n]$

Example: Find the DT FS coefficients of $x[n] = \sin(\omega_0 n)$, where $2\pi/\omega_0$

is a real number.

→ First Assume $2\pi/\omega_0 = N$ is an integer.

$$\sin(\omega_0 n) = \frac{1}{2j} e^{j\omega_0 n} - \frac{1}{2j} e^{-j\omega_0 n}$$

$$= \frac{1}{2j} e^{j(2\pi/N)n} - \frac{1}{2j} e^{-j(2\pi/N)n}$$

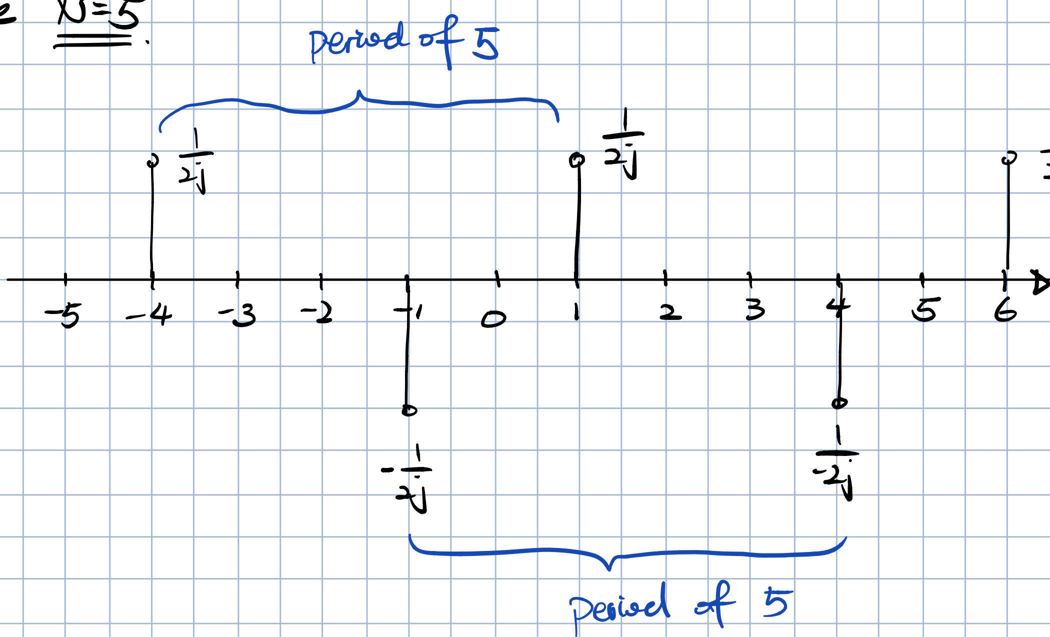
Compare with:

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n} ,$$

$$e^{j(2\pi/N)n}, \quad k=1, \Rightarrow \frac{1}{2j} = a_1$$

$$e^{-j(2\pi/N)n}, \quad k=-1, \Rightarrow \frac{-1}{2j} = a_{-1}$$

Assume $\underline{\underline{N=5}}$.

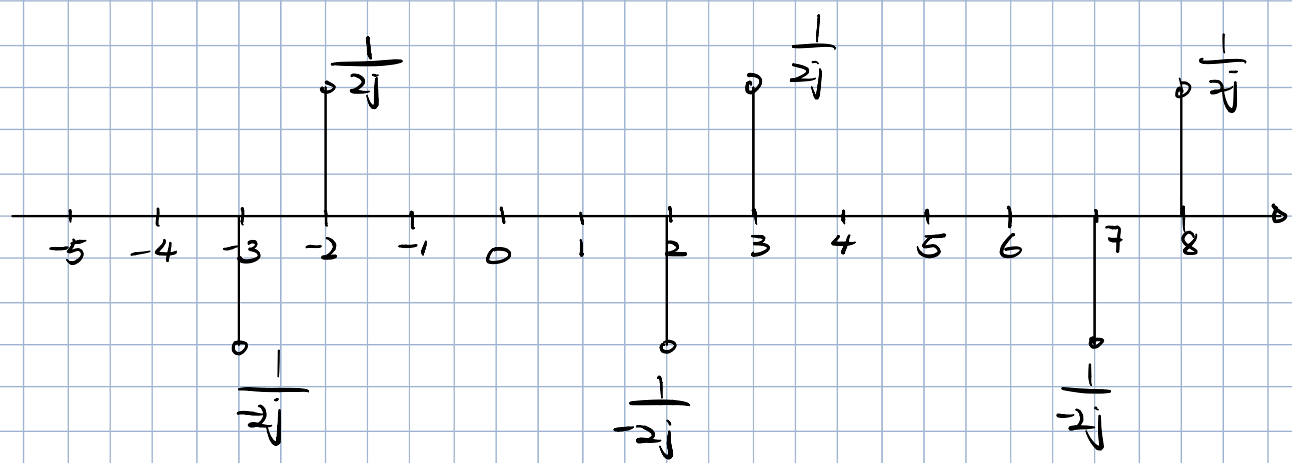


Now assume $2\pi/\omega_0 = \frac{N}{M}$, $N \,\&\, M \in \mathbb{Z}$

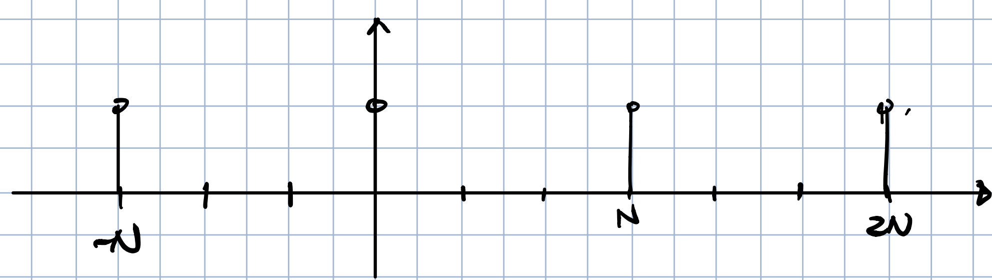
$$\sin(\omega_0 n) = \frac{1}{2j} e^{j\omega_0 n} - \frac{1}{2j} e^{-j\omega_0 n} = \left(\frac{1}{2j}\right) e^{j(2\pi/N)Mn} \left(-\frac{1}{2j}\right) e^{-j(2\pi/N)Mn}$$

a_{+M} a_{-M}

Assume $N=5$ $M=3$



example: Find the FS coefficients of.



$$a_k = \frac{1}{N}\left[x[n]e^{-jk(2\pi/N)n}\right] = \frac{1}{N}$$