DT FT of $\cos(\omega_0 n)$ and $\sin(\omega_0 n)$

$y[n]=1$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} y[n]e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} e^{-j\omega n}$$

$$y[n] = 1 = u[n] + u[-n] - \delta[n]$$

$$F\{\delta[n]\} = \sum_{k=-\infty}^{+\infty} \delta[n]e^{-j\omega n} = 1$$

Accumulation property:

$$\sum_{k=-\infty}^{+\infty} x[k] \xleftrightarrow{f} \frac{1}{1-e^{-j\omega}} X(e^{j\omega}) + \pi X(e^{j\omega}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$$

$$f\{u[n]\} = \frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$$

Time reversal property: $x[-n] \xleftrightarrow{f} X(e^{-j\omega})$

$$f\{u[-n]\} = \frac{1}{1-e^{+j\omega}} + \pi \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$$

$$f\{y[n]\} = \frac{1}{1-e^{-j\omega}} + \pi \sum \delta(\omega - 2\pi k) + \frac{1}{1-e^{+j\omega}} + \pi \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k) - 1$$

$$= 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$$

$$1 \xleftrightarrow{f} 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$$

Frequency shifting: $e^{j\omega_0 n} x[n] \xleftrightarrow{f} X(e^{j(\omega-\omega_0)})$

$$e^{j\omega_0 n} \quad 1 \quad \xleftrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi k)$$

$$\cos(\omega_0 n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}$$

$$\mathcal{F}\{\cos(\omega_0 n)\} = \frac{1}{2}\left[2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi k) + 2\pi \sum_{k=-\infty}^{+\infty} \delta(\omega + \omega_0 - 2\pi k)\right]$$

$$= \pi \sum_{k=-\infty}^{+\infty} \left[\delta(\omega - \omega_0 - 2\pi k) + \delta(\omega + \omega_0 - 2\pi k)\right]$$

$$\mathcal{F}\{\sin(\omega_0 n)\} = \frac{\pi}{j} \sum_{k=-\infty}^{+\infty} \left(\delta(\omega - \omega_0 - 2\pi k) - \delta(\omega + \omega_0 - 2\pi k)\right)$$

DT FT of periodic signals.:

Consider $x[n]$ to be periodic w/ period N

FS Synthesis & Analysis relations.

Synthesis: $x[n] = \sum_{k=\langle N \rangle} a_k e^{j(2\pi/N)kn} = \sum_{k=\langle N \rangle} a_k e^{j\omega_0 kn}$

Analysis: $a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-j\omega_0 kn}$

$$\rightarrow \mathcal{F}\{x[n]\} = \mathcal{F}\left\{\sum_{k=\langle N \rangle} a_k e^{j\omega_0 kn}\right\}$$

$$\rightarrow X(e^{j\omega}) = \sum_{k=\langle N \rangle} a_k \mathcal{F}\{e^{j\omega_0 kn}\} \quad \text{(Linearity)}$$

$$\rightarrow X(e^{j\omega}) = \sum_{k=\langle N \rangle} a_k 2\pi \sum_{m=-\infty}^{+\infty} \delta(\omega - k\omega_0 - 2\pi m)$$

$$X(e^{j\omega}) = \sum_{m=-\infty}^{+\infty} \sum_{k=\langle N\rangle} 2\pi a_k \delta(\omega - k\omega_0 - 2\pi m)$$

For $0 \leq \omega < 2\pi \rightarrow m = 0$

$$X(e^{j\omega}) = \sum_{k=\langle N\rangle} a_k \delta(\omega - k\frac{2\pi}{N})$$

a_k (FS coefficients are periodic w/ N)

$$\boxed{X(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - \frac{2\pi k}{N})}$$ for the entire ω axis.