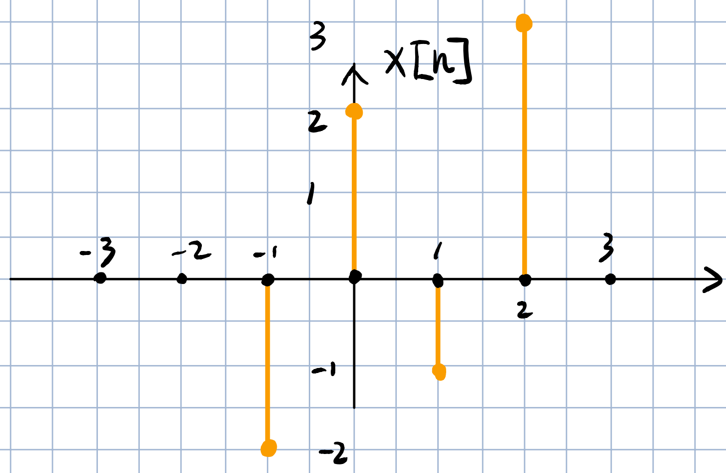
Discrete Time Signals. (DT signals)

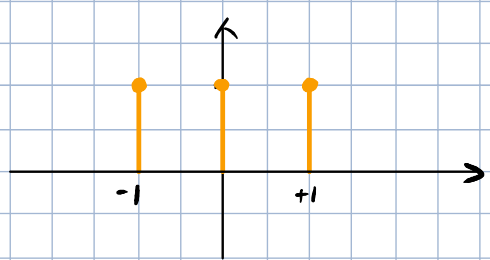
$x[n]$, $n \in \mathbb{Z}$



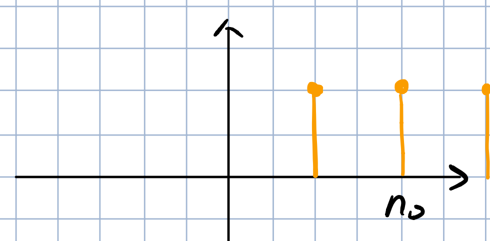
$$x[n] = \begin{cases} 2 & n = 0 \\ -1 & n = 1 \\ 3 & n = 2 \\ -2 & n = -1 \\ 0 & \text{else.} \end{cases}$$

① Transformations of the independent variable:

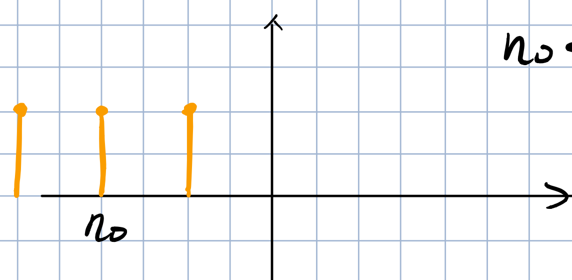
a) Time-shift $x[n - n_o]$



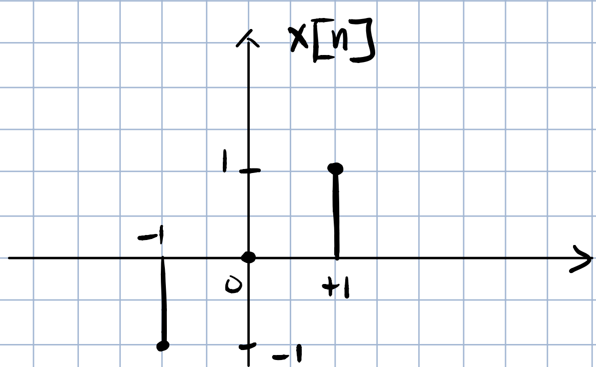
$n_o > 0$

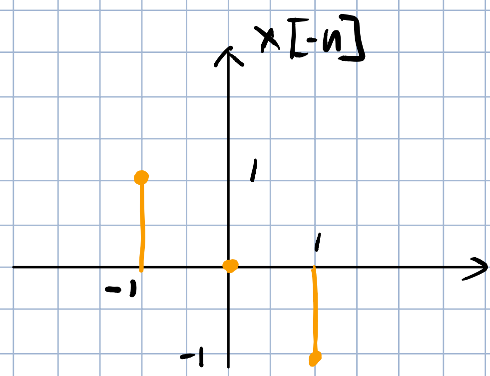


$n_o < 0$

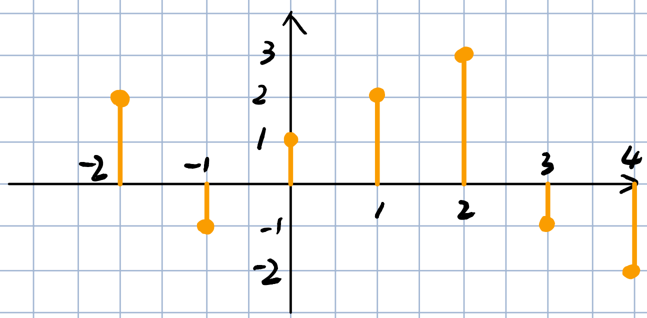


b) Time-reversal.

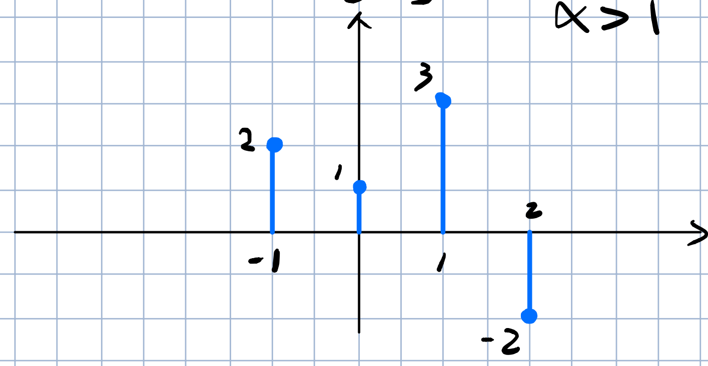




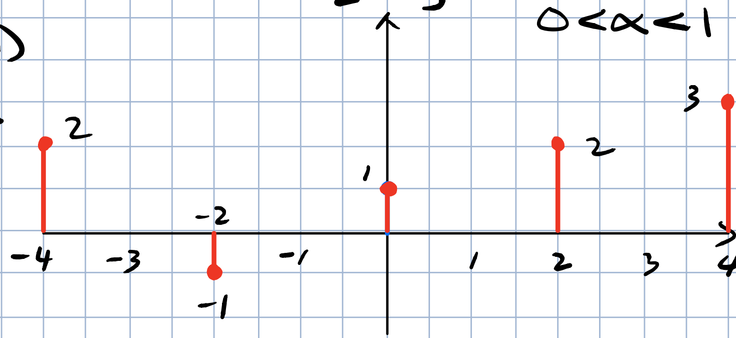
c) Time-Scaling $x[\alpha n]$



$x[2n]$ $\alpha > 1$



$x[0.5n]$ $0 < \alpha < 1$



- You lose some signal (information) when you compress the signal (i.e. $\alpha > 1$)
- Some signal will be undefined when you stretch the signal (i.e. $0 < \alpha < 1$) b/c discrete time only allow integer time.

② Periodicity: $x[n]$ is a periodic DT signal if for every n, there's is an N that satisfies.

$$x[n] = x[n+N] \quad n, N \in \mathbb{Z}$$

N is called the period of $x[n]$. The smallest positive N is called the fudamental period, N_0

✳ Check $y[n] = \sin(10n + \frac{\pi}{3})$ for periodicity. If it's periodic, find the period.

$$x[n] = x[n+N]$$

$\sin(10n + \frac{\pi}{3}) = \sin(10(n+N) + \frac{\pi}{3})$

$\sin(\beta) = \sin(\beta + 2\pi k)$

$$\cancel{10n} + \cancel{\frac{\pi}{3}} + 2\pi k = \cancel{10n} + 10N + \cancel{\frac{\pi}{3}}$$

$$2\pi k = 10N \Rightarrow N = \frac{2\pi k}{10} = \frac{\pi k}{5}$$ Not a integer.

$\therefore \Rightarrow$ signal is not periodic.

Remark:

$\Rightarrow$ $x[n] = \sin(\omega_0 n + \alpha)$ is periodic if $\frac{\omega_0}{2\pi} = \frac{k}{N}$, $k \& N \in \mathbb{Z}$

($\frac{\omega_0}{2\pi}$ is a rational #)

(*) Repeat the last example for:

$$x[n] = \underbrace{2\cos(\frac{\pi}{4}n + 2)}_{x_1[n]} + \underbrace{\sin(\frac{\pi}{8}n)}_{x_2[n]} - \underbrace{2\cos(\frac{\pi}{2}n)}_{x_3[n]}$$

$x_m[n] = \sin(\omega_{0m} n + \alpha_m)$ is periodic if $\frac{2\pi}{\omega_{0m}} = \frac{N}{k}$.

$x_1[n]$: $N_1 = \frac{2\pi k}{\omega_{01}} = \frac{2\pi k}{\pi/4} = 8k \Rightarrow 8$

$x_2[n]$: $N_2 = \frac{2\pi k}{\omega_{02}} = \frac{2\pi k}{\pi/8} = 16k \Rightarrow 16$

$x_3[n]$: $N_3 = \frac{2\pi k}{\omega_{03}} = \frac{2\pi k}{\pi/2} = 4k \Rightarrow 4$

$$N = LCM[N_1, N_2, N_3] = 16 \;//$$

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(3) Power and Energy of DT signals.

The energ. and power @ $x[n]$ bet n_1 and n_2 is:

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2 \quad , \quad P = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

Total Energy and power. of $x[n]$:

$$E_{Tot} = \lim_{N \to \infty} \sum_{n=-N}^{+N} |x[n]|^2 \quad P_{Tot} = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$