Lecture 20: Biot-Savart's Law, Ampère's Law

ECE221: Electric and Magnetic Fields

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Outline

1. Examples of Applying Biot-Savart's Law

2. Ampère's Law

Example: Two Semi-Infinite Current Segments

contribution to $\vec{H}$ from wire wire ①

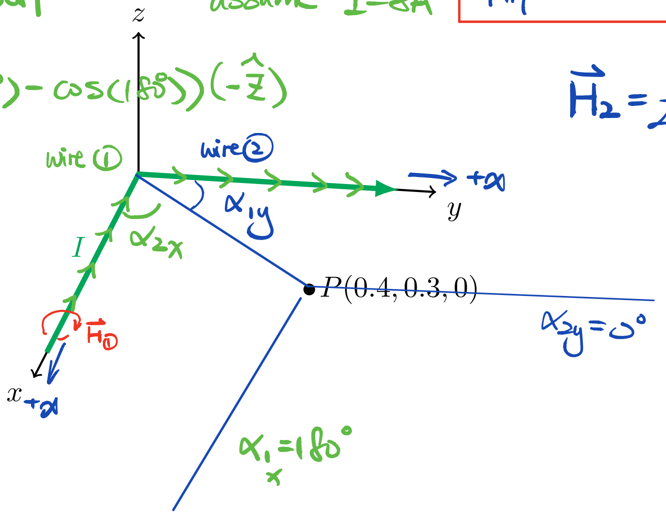
$\alpha_{2x} = \tan^{-1}\left(\frac{0.3}{0.4}\right) = 36.9^\circ$

assume $I = 8A$

$$\boxed{\frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{\phi} = H}$$

$$\vec{H} = \frac{8}{4\pi(0.3)}(\cos(36.9^\circ) - \cos(180^\circ))(-\hat{z})$$

$$\vec{H}_1 = \frac{-12}{\pi}\hat{z} \left[\frac{A}{m}\right]$$



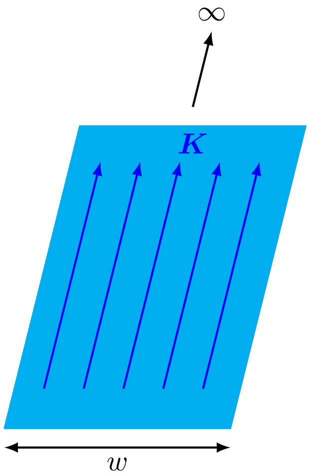
$$\vec{H}_2 = \frac{8}{4\pi(0.4)}\left(\cos(0^\circ) - \cos\left(\tan^{-1}\left(\frac{0.4}{0.3}\right)\right)\right)(-\hat{z})$$

$$= -\frac{2}{\pi}\hat{z} \left[\frac{A}{m}\right]$$

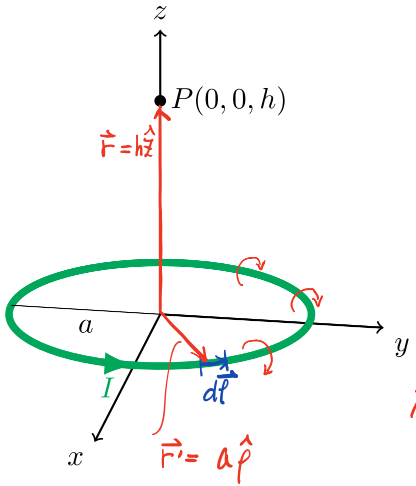
Apply superposition:

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{-14}{\pi}\left[\frac{A}{m}\right]$$

Example: Infinitely Long Current Strip or Sheet



Example: Circular Loop



$$d\vec{H} = \frac{I}{4\pi R^3}\, d\vec{l} \times \vec{R}$$

$$d\vec{l} = \rho\, d\phi\, \hat{\phi} = a\, d\phi\, \hat{\phi}$$

$$\vec{R} = \vec{r} - \vec{r}' = h\hat{z} - a\hat{\rho}$$

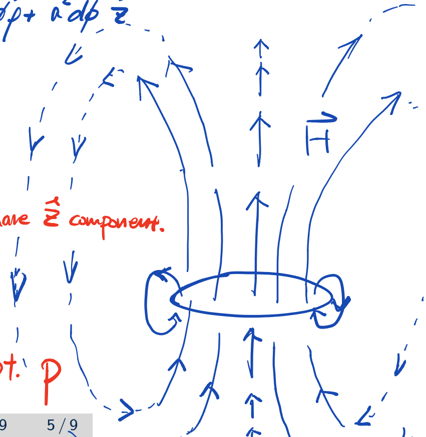
$$d\vec{l} \times \vec{R} = \begin{vmatrix} \hat{\rho} & \hat{\phi} & \hat{z} \\ 0 & a\,d\phi & 0 \\ -a & 0 & h \end{vmatrix} = ah\, d\phi\, \hat{\rho} + a^2 d\phi\, \hat{z}$$

$$d\vec{H} = \frac{I(ah\, d\phi\, \hat{\rho} + a^2 d\phi\, \hat{z})}{4\pi (a^2+h^2)^{3/2}}$$

→ @ point P, $\vec{H} = \oint_c d\vec{H}$ will only have $\hat{z}$ component.

$$\vec{H} = \oint_c d\vec{H} = \hat{z}\, \frac{I a^2}{4\pi (a^2+h^2)^{3/2}} \int_0^{2\pi} d\phi$$

$$\boxed{\vec{H} = \hat{z}\, \frac{I a^2}{2(a^2+h^2)^{3/2}} \left[\frac{A}{m}\right]}$$ at pt. P



Ampère's Law

$\nabla \cdot \vec{H} = 0$

$\vec{H}$ is non-divergent

$\vec{H}$ is circulating field

aka solenoidal field

- We saw in electrostatics that applying Coulomb's law to problems was very tedious and was simplified by the use of Gauss' Law.
- Is there the same kind of thing for magnetic fields?
- Yes $\rightarrow$ Ampère's Law, which can be derived from Biot-Savart Law (advanced topic involving *magnetic potential* [later])

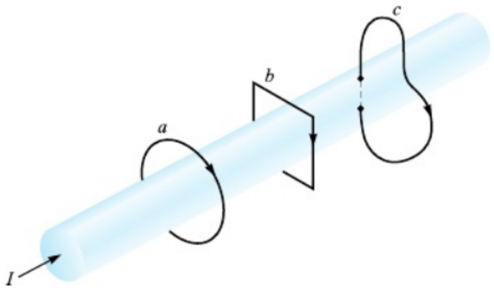
Ampère's Law

The line integral of $\boldsymbol{H}$ about any closed path is equal to the current enclosed by that path,

$$\oint_C \boldsymbol{H} \cdot dl = I$$

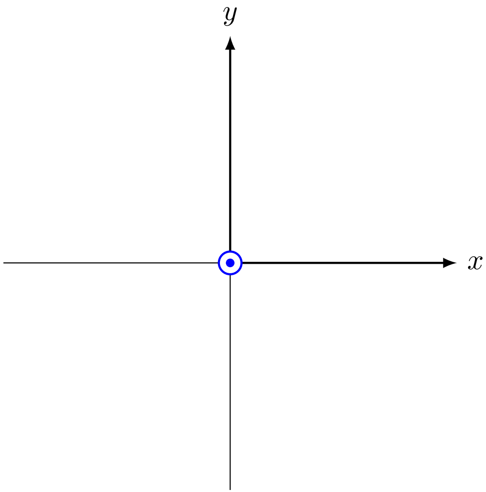
Ampèrian Contours

Examples of Ampèrian Contours



Source: Hayt and Buck, *Engineering Electromagnetics*, 8/e

Example: Filamentary Wire Along z-axis



$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I$$

Example: Thick Wire

$$\oint_C \boldsymbol{H} \cdot d\boldsymbol{l} = I$$