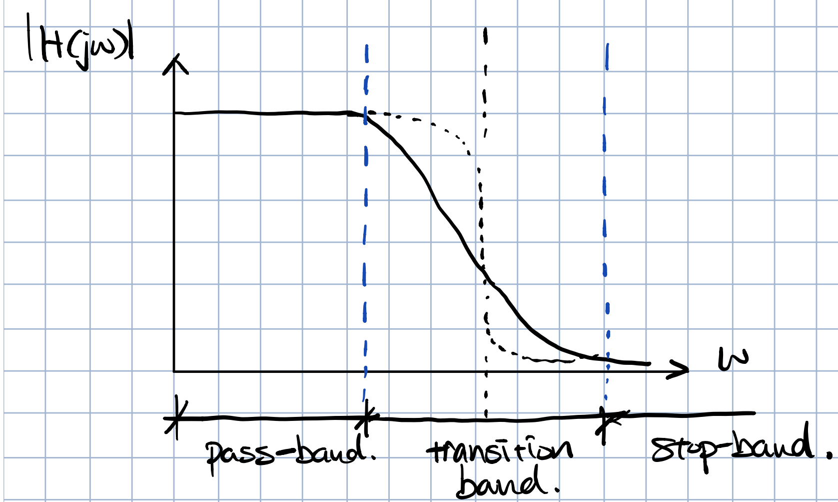
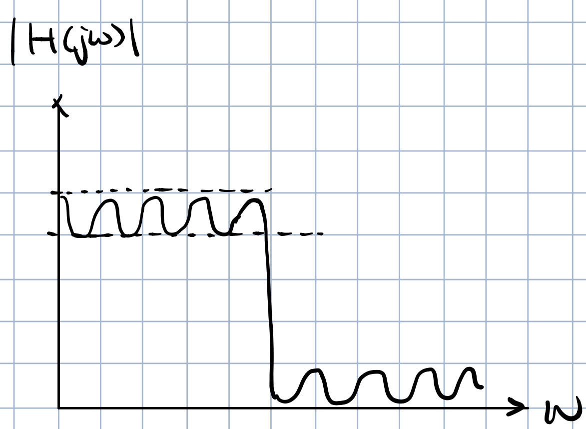
CT Filters

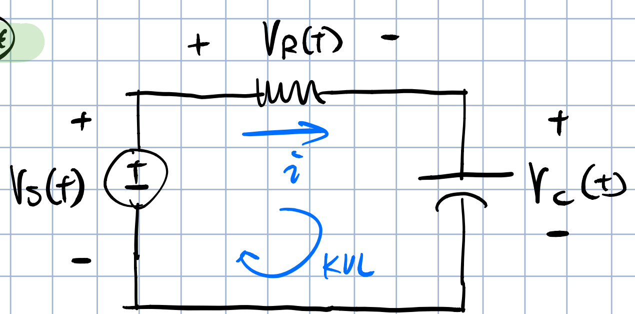
① Butterworth Filter:



② Elliptic Filter:



⊛



Assume $V_s(t)$ and $V_c(t)$ are the input and output.

* Find the frequency response.
* Find the step response.

Determine what type of filter this system is.

KVL: $-V_s(t) + V_R(t) + V_c(t) = 0$

$V_R = Ri$, $i = C\dfrac{dV_c(t)}{dt}$

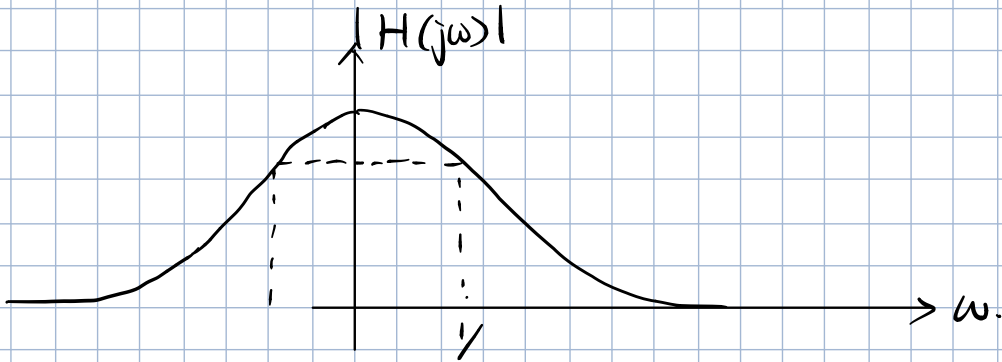
$$RC\frac{dV_c(t)}{dt} + V_c(t) = V_s(t)$$

$$\xrightarrow{FT} jRC\omega\, V_c(j\omega) + V_c(j\omega) = V_s(j\omega)$$

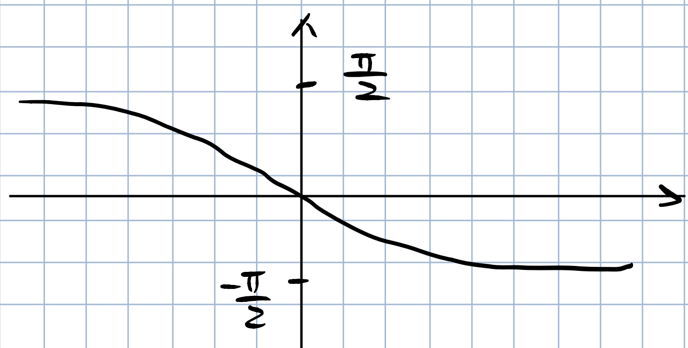
$$V_c(j\omega)\,(1 + j\omega RC) = V_s(j\omega)$$

$$H(j\omega) = \frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1}{1+j\omega RC}$$

$$\begin{cases} \omega = 0 \Rightarrow H(j\omega) = 1 \\ \omega = \infty \Rightarrow H(j\omega) = 0 \end{cases}$$

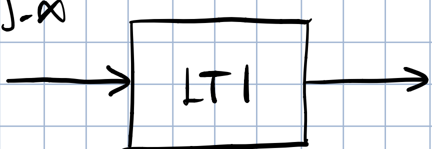


$$\measuredangle H(j\omega) = -\tan^{-1}(\omega RC)$$



$$\boxed{H(j\omega) = \frac{1}{1+j\omega RC}}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$



$$s(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$\delta(t)$

$h(t)$

$H(j\omega)$

since $H(j\omega) = \frac{1}{1+j\omega RC}$

and $e^{-at}u(t) \xleftrightarrow{f} \frac{1}{a+j\omega}$

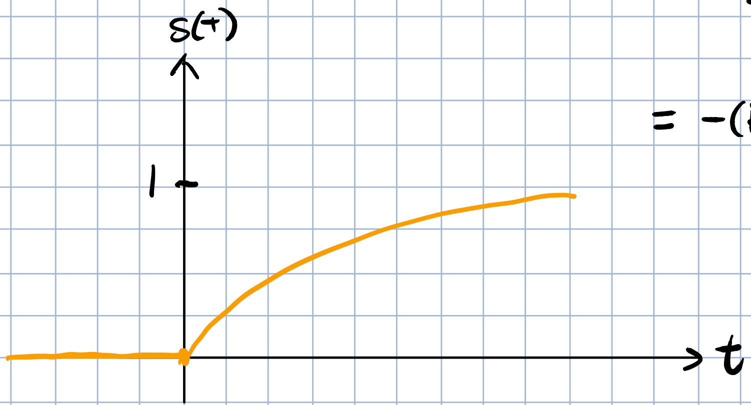
$$H(j\omega) = \frac{1/RC}{\ldots} = \left(\frac{1}{RC}\right) * \frac{1}{\ldots}$$

$$\frac{1}{RC} + j\omega \qquad \frac{1}{RC} \qquad \frac{1}{RC} + j\omega$$

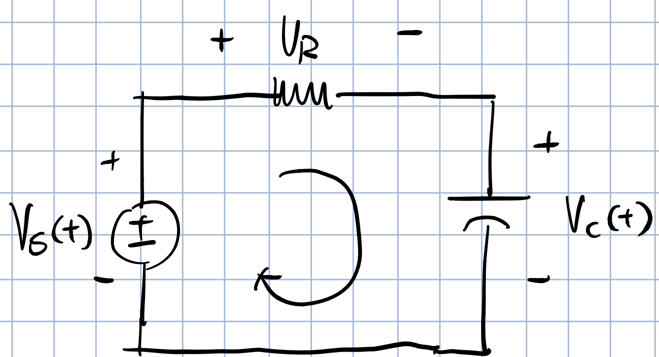
$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

$$S(t) = \int_{-\infty}^{t} h(\tau)d\tau = \frac{1}{RC}\int_{0}^{t} e^{-\tau/RC} d\tau$$

$$= -(RC)\frac{1}{RC} e^{\frac{-\tau}{RC}}\Big|_0^t = -e^{\frac{-t}{RC}} + 1 \;//$$



$$s(t) = 1 - e^{-t/RC} u(t)$$



$$x(t) = V_s(t)$$

$$y(t) = V_R(t)$$

$$-V_s(t) + V_R(t) + V_c(t) = 0$$

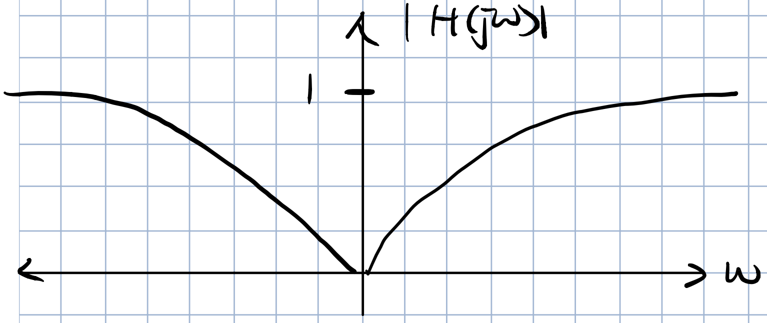
$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau + V_c(-\infty)$$

$$V_R(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau + V_c(-\infty) = V_s(t)$$

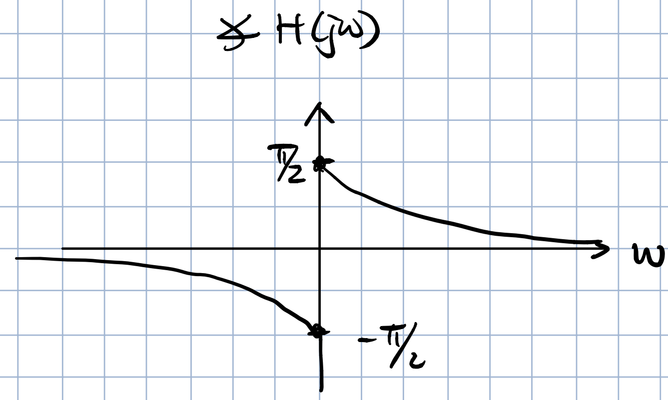
$$\xrightarrow{i = \frac{V_R}{R}} \quad \frac{dV_R(t)}{dt} + \frac{1}{RC} V_R(t) = \frac{dV_s(t)}{dt}$$

$$\xrightarrow{FT} \quad j\omega V_R(j\omega) + \frac{1}{RC} V_R(j\omega) = j\omega V_s(j\omega)$$

$$H(j\omega) = \frac{j\omega}{\frac{1}{RC} + j\omega} \qquad \begin{cases} H(j0) = 0 \\ H(j\infty) = 1 \end{cases}$$



|H(jω)|
1
ω



∡ H(jω)
π/2
−π/2
ω