Lecture 29: Self Inductance and Mutual Inductance

ECE221: Electric and Magnetic Fields

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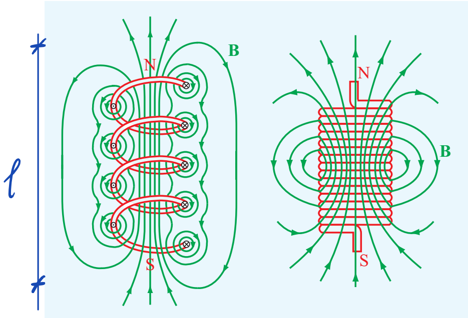
Outline

Example: Inductance of a Solenoid



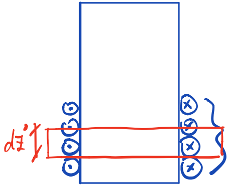
(a) Loosely wound solenoid

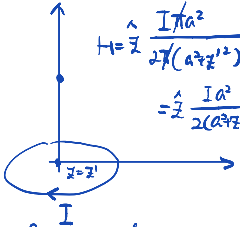
(b) Tightly wound solenoid

Source: Ulaby and Ravaoili, *Fundamentals of Electromagnetics*

Note:

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x/a^2}{\sqrt{x^2+a^2}} + C$$





$$H = \hat{z}\,\frac{I\pi a^2}{2\pi(a^2+z'^2)^{3/2}} = \hat{z}\,\frac{Ia^2}{2(a^2+z'^2)^{3/2}}$$

Let's treat an incremental length of solenoid: contains $n\,dz'$ turns if $n = \#$ of turns per unit length. dz' carries a current $I n\,dz'$

$$d\vec{B} = \mu\, d\vec{H} = \hat{z}\,\frac{I n\,dz'\,a^2}{2(a^2+z'^2)^{3/2}}\,\mu \longrightarrow \vec{B} = \int_{-\ell/2}^{\ell/2} d\vec{B} = \hat{z}\,\frac{\mu I n}{2}\,\frac{\ell}{\sqrt{a^2+(\ell/2)^2}}$$

Assume that solenoid is long

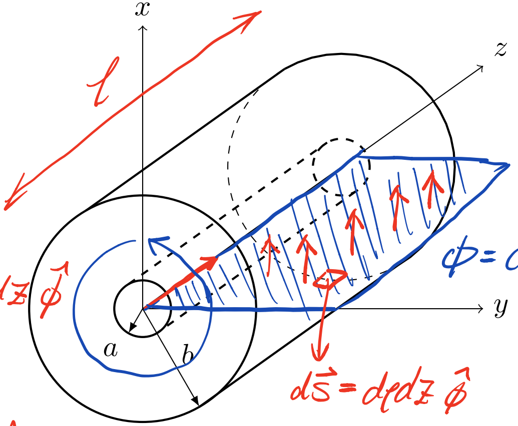
Example: Inductance of an Infinitely Long Coaxial Cable

$H_\phi = \frac{I}{2\pi\rho} \quad a<\rho<b$

$\vec{B} = \mu \frac{I}{2\pi\rho}\hat{\phi}$

$L = \frac{N\psi}{I}$

$\psi = \int_{z=0}^{\ell}\int_{\rho=a}^{b} \frac{\mu I}{2\pi\rho}\hat{\phi}\cdot d\rho\, dz\, \hat{\phi}$



$d\vec{S} = d\rho\, dz\, \hat{\phi}$

$\phi = \text{const.} \ (0^\circ)$

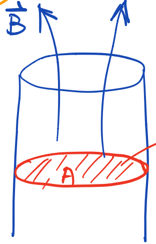
$\psi = \frac{\mu I \ell}{2\pi}\ln(b/a) = \Lambda$

$L = \frac{\Lambda}{I} = \frac{\mu\ell}{2\pi}\ln(b/a)$

inductance per unit length: $\boxed{L' = L/\ell = \frac{\mu}{2\pi}\ln(b/a)}$

$\ell >> a$ (radius)

$\vec{B} \approx \frac{\mu I N}{\ell} = \frac{\mu I N_{total}}{\ell}$



flux passing through?

$\psi = \int \vec{B}\cdot d\vec{S} \approx BA$

$\psi \approx \frac{\mu I N_{total}}{\ell} A$

$\Lambda = N_{total}\,\psi = \frac{\mu I (N_{total})^2 A}{\ell}$

$\boxed{L = \frac{\Lambda}{I} = \frac{\mu (N_{total})^2 A}{\ell}}$

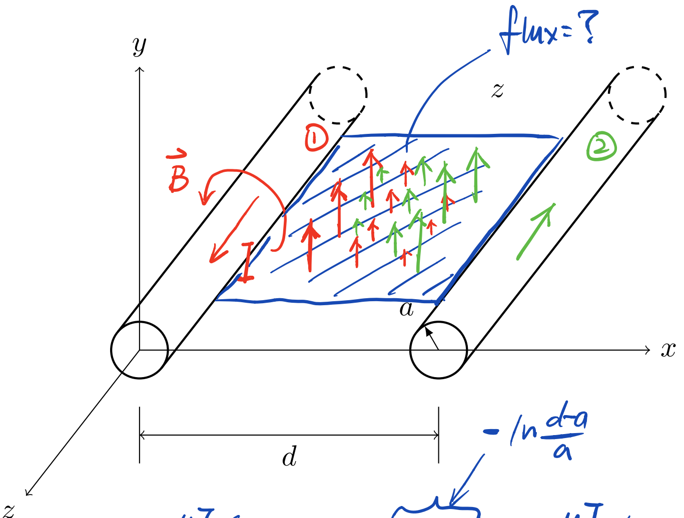
Example: Inductance Two Parallel Wires

For wire ①:

$$\vec{B} = \frac{\mu I}{2\pi\rho}\hat{\phi}$$

$$B_{y1}(z=0) = \frac{\mu I}{2\pi x}$$

$$B_{y2}(z=0) = \frac{\mu I}{2\pi(d-x)}$$



Flux linkage per unit length.

$$\Lambda' = \int_a^{d-a} (B_{y1} - B_{y2})\,dx$$

$$= \frac{\mu I}{2\pi}\int_a^{d-a} \frac{1}{x} + \frac{1}{d-x}\,dx = \frac{\mu I}{2\pi}\left(\ln\frac{d-a}{a} - \overbrace{\ln\frac{a}{d-a}}^{-\ln\frac{d-a}{a}}\right) = \frac{\mu I}{\cancel{2}\pi}\cancel{2}\ln\frac{d-a}{a} = \frac{\mu I}{\pi}\ln\frac{d-a}{a}$$

if $d >> a$

$\mu I \ln d$

Mutual Inductance

$$\boxed{L' = \frac{\Lambda'}{I} = \frac{\mu}{\pi}\ln(d/a)}$$

Mutual flux linkage

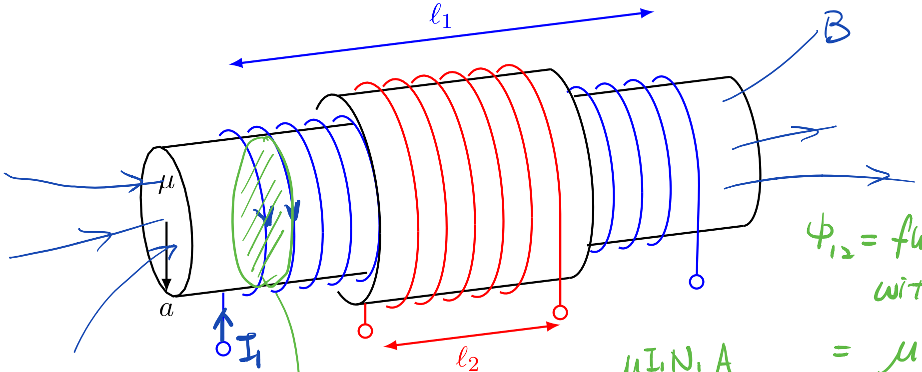
$$\Lambda_{12} = N_2 \Psi_{12}$$

Mutal inductance

$$L_{12} = \frac{\Lambda_{12}}{I_1}$$

Example: Solenoid with Two Windings

Determine the mutual inductances between two coaxial solenoids, with turns N_1 and N_2, wound on the same magnetic core.



ψ_{12} = flux linking inner coil ① with outer coil ②

$= \mu \left(\frac{N_1}{\ell_1}\right)(\pi a^2) I_1$

$\psi = \text{flux} = \int_S \vec{B} \cdot d\vec{s} \approx BA = \frac{\mu I_1 N_1 A}{\ell_1}$

since outer coil only

has N_2 turns.

Example: Mutual Inductance Between Wire and Loop

$$\lambda_{12} = N_2 \psi_{12}$$

Determine the mutual inductance between an infinitely long wire along the z-axis and the triangular loop as shown.

$$= \frac{\mu}{l_1} N_1 N_2 (\pi a^2) I_1$$

$$\boxed{L_{12} = \text{mutal inductance} = \frac{\mu}{l_1} N_1 N_2 (\pi a^2)}$$