Lecture 27: Magnetic Materials and Inductance

ECE221: Electric and Magnetic Fields

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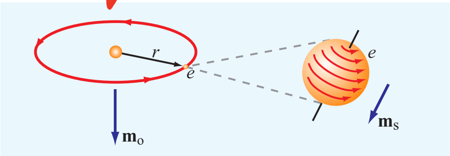
Outline

Nature of Magnetic Materials

- Just as we did for dielectrics, we now consider the magnetic properties of "magnetic" materials
- We gain invoke a simple atomic model where there is a positively charged nucleus around which there are electronics in various orbits

bound current



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Magnetization

- From a macroscopic perspective we now differentiate between free currents and bound currents
- A bound current I_b circulates around a path enclosing a differential area $d\boldsymbol{s}$

$$\boldsymbol{m} = I_b d\boldsymbol{s}$$

- If there are N magnetic dipoles per unit volume and we consider a volume $\Delta\nu$, then the total magentic dipole moment is

$$\boldsymbol{m}_{total} = \sum_{i=1}^{N\Delta\nu} \boldsymbol{m}_i$$

- Define **magnetization** as magnetic dipole moment per unit volume, just as we did for **polarization**

$$\boldsymbol{M} = \lim_{\Delta\nu \to 0} \frac{1}{\Delta\nu} \sum_{i=1}^{N\Delta\nu} \boldsymbol{m}_i$$

Generalized Amperes Law

$$\oint \vec{B}/\mu_0 \cdot d\vec{l} = I_{Total} = I_b + I_f$$

$$= \oint_c \vec{M} \cdot d\vec{l} + \oint_c \vec{H} \cdot d\vec{l}$$

Rearrange: $\boxed{\vec{B} = \mu_0 (\vec{M} + \vec{H})}$

$I_b = \oint_c \vec{M} \cdot d\vec{l}$

free current

$I_f = \oint_c \vec{H} \cdot d\vec{l}$

Differential Analysis

For a differential volume dv', the magnetic moment is $d\boldsymbol{m} = \boldsymbol{M}dv'$. What is the vector magnetic potential produced by the volume?

Just like

$\vec{D} = \varepsilon_0(\vec{E} + \vec{P})$

Let $\vec{M} = \chi_m \vec{H}$

magnetic susceptibility

$$\boxed{\vec{B} = \mu_0(\vec{H} + \chi_m \vec{H}) = \mu_0 \underbrace{(\chi_m + 1)}_{\mu_r}\vec{H}}$$

μ_r - relative permeability

Point form:

$$\nabla \times \frac{B}{\mu_0} = \vec{J_b} + \vec{J_f}$$

$\vec{J_b}$: $\nabla \times \vec{M}$; $\vec{J_f}$: $\nabla \times \vec{H}$

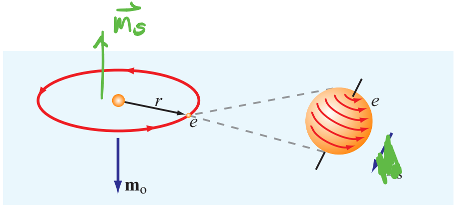
$$\boxed{J_b = \nabla \times \vec{M}}$$

Almost all dielectrics have $\mu_r = 1 \Rightarrow \mu = \mu_0$

Types of Magnetic Materials: Diamagnetic Materials

Diamagnetic materials:

- Have no permanent magnetic moments
- Electron orbit moment and spin moment nearly cancel
- An applied $\boldsymbol{B}$ field causes a very slight reduction in $\boldsymbol{m}_o$
- In most diamagnetic materials, $\chi_m \approx -10^{-5}$ and $\mu_r \approx 1$
- In very special materials (superconductors), the internal magnetic field can completely cancel the external one, producing $\mu_r = -1$



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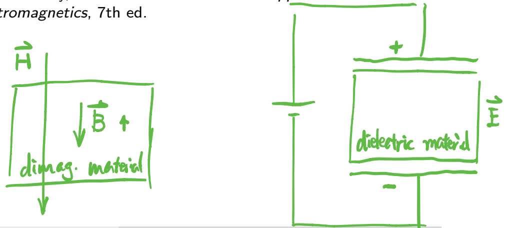
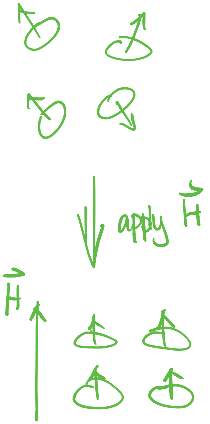
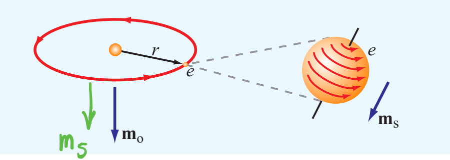


Types of Magnetic Materials: Paramagnetic Materials



Paramagnetic materials:

- Have no permanent magnetic moments
- Electron spin moment very slightly larger than electron orbit moment
- An applied $\boldsymbol{B}$ field torques the moments, causing them to align with the applied field → boost $\vec{B}$-field inside material.
- In most paramagnetic materials, $\chi_m \approx 10^{-5}$ and $\mu_r \approx 1$



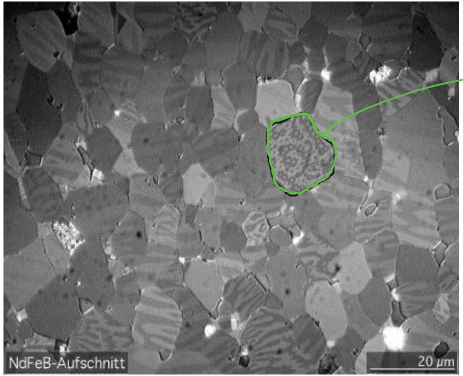
Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

$\mu_r = 1 + \chi_m \approx 1$

Types of Magnetic Materials: Ferromagnetic Materials

Ferromagnetic materials:

- Have permanent magnetic moments resulting from electron spin moments
- Interatomic forces cause these moments to line up in parallel over regions containing large numbers of atoms called **domains**
- An applied B field **magnetized** the material to cause regular and semi-permanent alignment of the dipoles

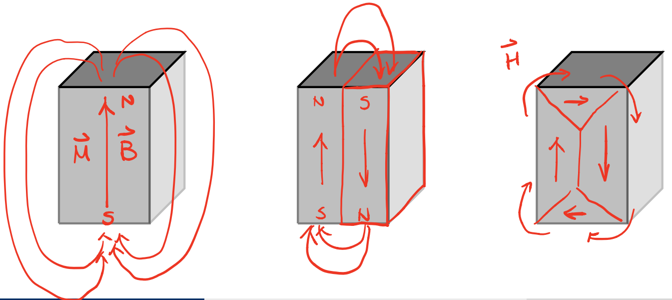


Source: Wikipedia

magnetic "domain"

Types of Magnetic Materials: Ferromagnetic Materials

- A magnetic materials spontaneously divides into domains to minimize the *magnetostatic energy* stored in the internal field.
- Domains can be reoriented by an external magnetic field
- Domains will remain aligned when external field is removed, since domain walls become pinned to defects in the crystal structure
- The material can be *demagnetized* by applying another field, or heating the material passed its *Curie temperature*



Summary of Magnetic Materials

	Diamagnetism	Paramagnetism	Ferromagnetism
Permanent magnetic dipole moment	No	Yes, but weak	Yes, and strong
Primary magnetization mechanism	Electron orbital magnetic moment	Electron spin magnetic moment	Magnetized domains
Direction of induced magnetic field (relative to external field)	Opposite	Same	Hysteresis*
Common substances	Bismuth, copper, diamond, gold, lead, mercury, silber, silicon	Aluminmum, chromium, magnesium, niobium, platinum, tungsten	Iron, nickel, cobalt
Typical value of χ_m	$\approx -10^{-5}$	$\approx 10^{-5}$	$\lvert\chi_m\rvert \gg 1$ and hysteretic
Typical value of μ_r	≈ 1	≈ 1	$\lvert\mu_r\rvert \gg 1$ and hysteretic

10000 or more

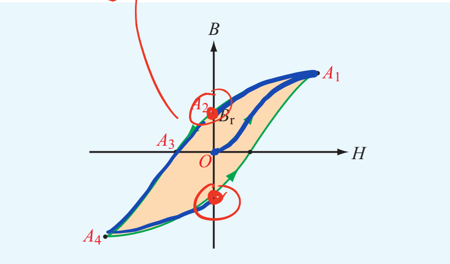
Hysteresis of Magnetic Materials

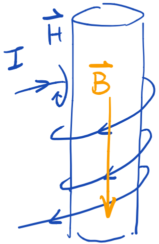
- A **magnetization curve** describes the relationship between B and H in a material
- You might think it is a line because $B = \mu H$, but this is not true in ferromagnetic materials where the relationship is **nonlinear**

$$B = \mu(H)H$$

- Ferromagnetic materials have magnetic *hysteresis* (*to lag behind*)

hysteresis loop.





1. Apply $\vec{H}$ field by ramping up I in the loop.

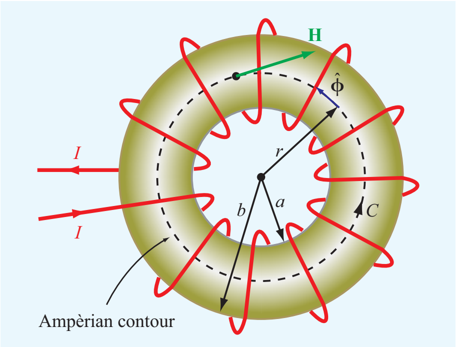
2. Increase $\vec{H}$ → to pt. A_1

3. Reduce $\vec{H}$ → to O

Magnetic Flux

$$\Psi = \iint_S \boldsymbol{D} \cdot d\boldsymbol{s} = \oint_C \boldsymbol{A} \cdot d\ell$$

Recall: toroid example



Source: Ulaby and Ravaoili, *Fundamentals of Electromagnetics*

What flux **links** each turn?

$$\boldsymbol{H} = \hat{\phi}\frac{NI}{2\pi r}$$

$$\boldsymbol{B} = \hat{\phi}\frac{\mu NI}{2\pi r}$$

(ramp down current to 0) → A_2

4. Further reduce $\vec{H}$ (reverse I) → A_3 (demagnetized)

5. A_4 (saturation)

6. Reverse I

Inductance

Flux linkage Λ is the product of the number of turns N and the flux Ψ linkng each of them.

$$\Lambda = N\Psi$$

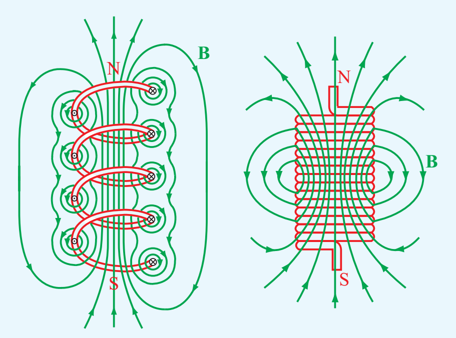
Example: for a single-turn toroid, the flux linkage is equal to the total flux.

Self inductance

$$L = \frac{N\Psi}{I} = \frac{\Lambda}{I}$$

Units of inductance are **Henrys**.

Example: Inductance of a Solenoid



(a) Loosely wound solenoid

(b) Tightly wound solenoid

Source: Ulaby and Ravaoili, *Fundamentals of Electromagnetics*