① Transformations of the independent variable.

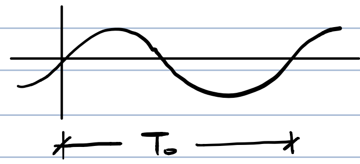
Time shifting. Time Scaling, Time reversal.

② Periodicity : Signal $x(t)$ is periodic if for every t, there is a T that satisfied. : $x(t) = x(t+T)$ and T is called the period.

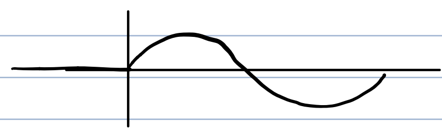
(The smallest positive T is referred to as the fudamental period T_0.)

Aperiodic → not periodic.

periodic. $x(t) = A\sin(\omega t + \alpha)$



Aperiodic. $x(t) = \begin{cases} A\sin(\omega t + \alpha) & t>0 \\ 0 & t<0 \end{cases}$



"because we cannot find a "T" for all values of t.

Example: Find the period of $x(t) = A\sin(\omega t + \alpha)$

$\because$ $x(t) = x(t+T) \longrightarrow A\sin(\omega t+\alpha) = A\sin(\omega(t+T)+\alpha)$

$\because$ $\sin(\beta) = \sin(\beta + 2\pi k)$, $\omega t + \alpha = \beta$ $\quad \omega T = 2\pi k$

$$\Rightarrow T = \frac{2\pi k}{\omega}$$

$$T_0 = \frac{2\pi}{\omega}$$ Fundamental period.

Proposition: If $x(t)$ and $y(t)$ are periodic signals w/ periods T_x and T_y,

then $z(t) = x(t) + y(t)$ is periodic if T_x / T_y is a rational number.

The period of $z(t)$, T_z, is often given by $T_z = LCM(T_x, T_y)$ (LCM: least common multiplier.)

The above proposition is valid also for $z(t) = x(t)y(t)$.

Example. Find the Fundamental period of $x(t) = \cos(t) + \sin(\pi t)$

period of $\cos(t)$: 2π

period of $\sin(\pi t)$: 2

$$\frac{T_{x_1}}{T_{y_1}} = \frac{2\pi}{2} = \pi$$ ← irrational number.

$\Rightarrow x(t)$ is a aperiodic

$$y(t) = \cos\left(\frac{3\pi}{4}t + \frac{\pi}{6}\right) + \sin\left(\frac{2\pi}{3}t - \frac{\pi}{4}\right)$$

$$\frac{2\pi}{\omega} = \frac{2\pi}{\frac{3\pi}{4}} = 2\pi \times \frac{4}{3\pi} = \frac{8}{3} \qquad \frac{2\pi}{\omega} = 2\pi \times \frac{3}{2\pi} = 3$$

$$T_y = \mathrm{LCM}\left(\frac{8}{3}, 3\right) = 24$$

Instead of the time it takes for a signal to be repeated. we are often interested in the # of times that a signal is repeated during a given period. of time.

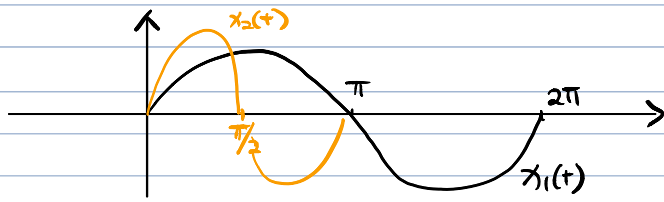
Fundamental frequency : $f_0 = \dfrac{1}{\text{fundamental period. } (T_0)}$ $[1/\text{sec}] = [\text{Hz}]$

$f = \dfrac{2\pi}{T_0}$ Rad/sec. (if want freq. in rad/sec.)

Harmonic. : $x_1(t) = A_1 \sin(\omega_1 t + \alpha_1)$, $x_m(t) = A_m \sin(m\omega_1 t + \alpha_m)$

$m \in \mathbb{N}$ $x_m(t)$ is the m^{th} harmonic of $x(t)$.

$x_1(t) = \sin(t)$, $x_2(t) = \sin(2t)$



③ Energy and power of signals.:

Energy of $x(t)$ between. t_1 and t_2 :

$$E \triangleq \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

Power of $x(t)$ between t_1 & t_2

$$P \triangleq \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt.$$

for total. energy and power., t_1 & t_2 approach $-\infty$ & $+\infty$

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt \qquad P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt.$$

Energy signal: A finite non-zero total energy. The total power of an energy signal is zero.

Power signal: The total power of a power signal is finite and non-zero.
The total energy of a power. signal. is not finite.

$\boxed{\text{Remark:}}$ periodic signals are normally power signals. The total power of a periodic signal is equal to the power of the signal over one period.