## ④ Even & Odd DT signals:

$x[n]$ is even if $x[-n] = x[n]$

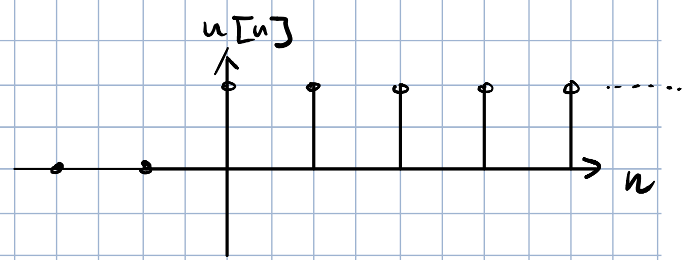
$x[n]$ is odd if $x[-n] = -x[n]$

$$x[n] = x_e[n] + x_o[n] \quad \begin{cases} \text{Even part: } \dfrac{x[n]+x[-n]}{2} = x_e[n] \\ \text{odd part: } \dfrac{x[n]-x[-n]}{2} = x_o[n] \end{cases}$$

## ⑤ Basic DT signals:

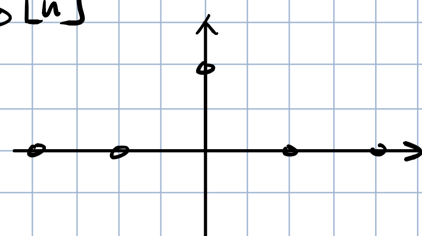
### 1. Unit Step function: $u[n]$

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

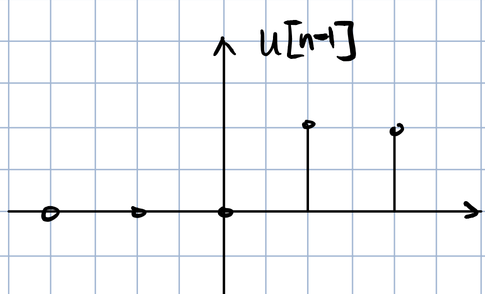



### 2. Unit Impulse Function: $\delta[n]$

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Relation btw $u[n]$ & $\delta[n]$:




$$u[n] - u[n-1] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} = \delta[n]$$

Recall: for CT: $\delta(t) = \dfrac{du(t)}{dt}$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$$u[n] = \sum_{m=-\infty} \delta[m] \quad , \quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

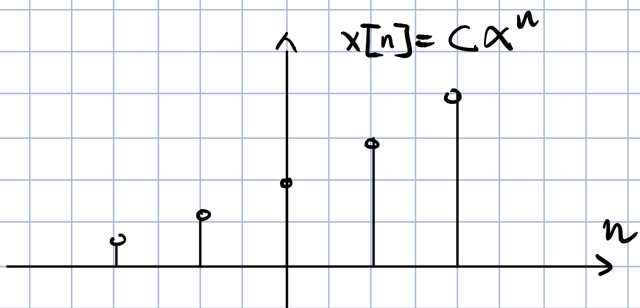
## 3. Complex Exponential Signal:

$$x[n] = C\alpha^n \qquad \alpha = e^{\beta} \qquad x[n] = Ce^{\beta n}$$

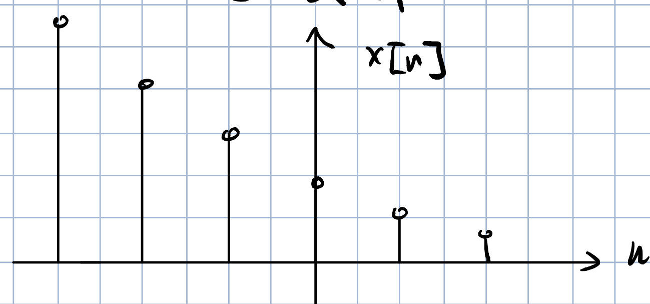
$C$ and $\alpha$ are, in general complex #s

a.) Real exponential signal: $C$ and $\alpha$ are real.

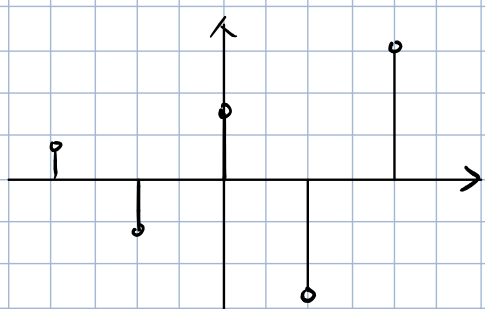
$\alpha > 1$




$0 < \alpha < 1$




$\alpha < -1$



$-1 < \alpha < 0$

same as left

but decaying.

b.) sinusoidal signals

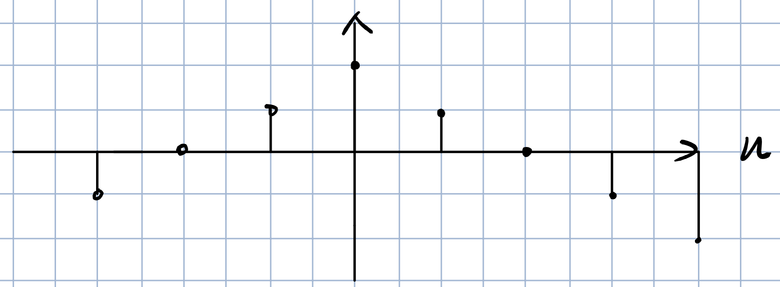
$$x[n] = C\alpha^n \xrightarrow{\alpha = e^{\beta}} x[n] = Ce^{\beta n}$$

If $\beta$ is purely imaginary: $\beta = j\omega_0$

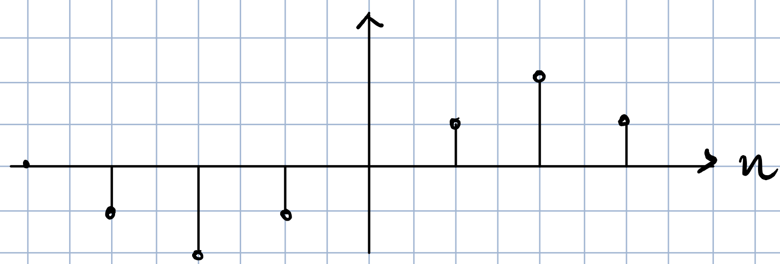
$$x[n] = e^{j\omega_0 n} \quad \text{(Assuming } C=1)$$

$$x[n] = \cos(\omega_0 n) + j\sin(\omega_0 n)$$

$Re\{x[n]\} = \cos(\omega_0 n)$




$Im\{x[n]\} = \sin(\omega_0 n)$




c) General Complex Exponential signal.

$$C = |C| e^{j\theta} \quad , \quad \alpha = |\alpha| e^{j\omega_0}$$

$$x[n] = C\alpha^n = |C| |\alpha|^n e^{j(\omega_0 n + \theta)}$$

$$x[n] = \underbrace{|C| |\alpha|^n \cos(\omega_0 n + \theta)}_{Re\{x[n]\}} + j \underbrace{|C| |\alpha|^n \sin(\omega_0 n + \theta)}_{Im\{x[n]\}}$$

Periodicity of Complex Exponential signals:

$x[n] = Ce^{\beta n}$, If $Re\{\beta\} \neq 0$, $x[n]$ is aperiodic.

1. Periodicity in the time domain.

$x[n] = e^{j\omega_0 n}$ is periodic if $\frac{2\pi}{\omega_0}$ is a rational #.

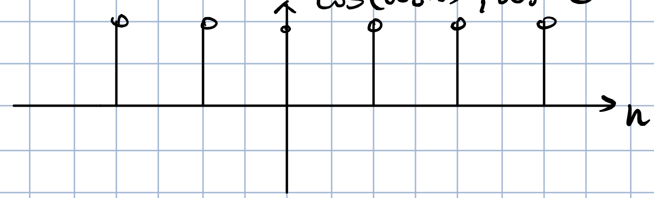
2. Periodicity in the frequency domain.

$X[n]$ is periodic in the freq. domain, and the period is $2\pi$.

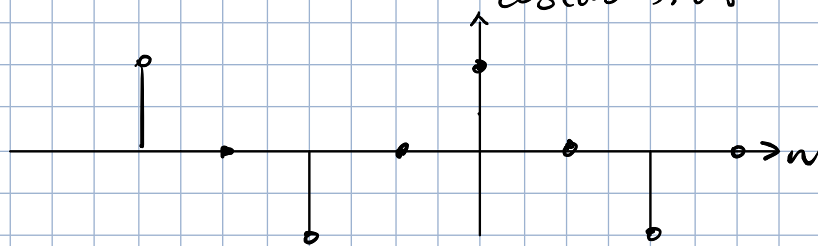
$$x[n] = e^{j\omega_0 n} = \cos(\omega_0 n) + j\sin(\omega_0 n)$$

$$Re\{x[n]\} = \cos(\omega_0 n)$$

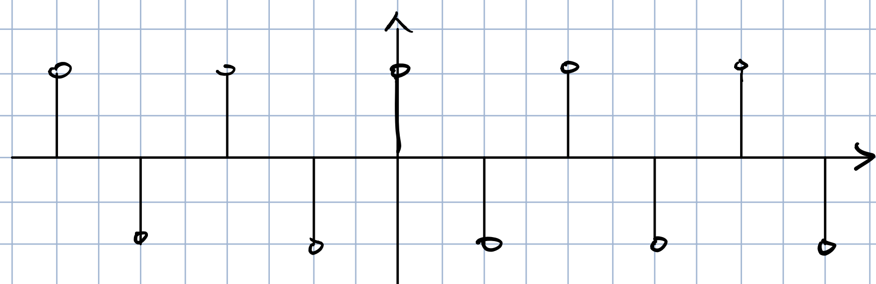
$\cos(\omega_0 n)$, $\omega_0 = 0$



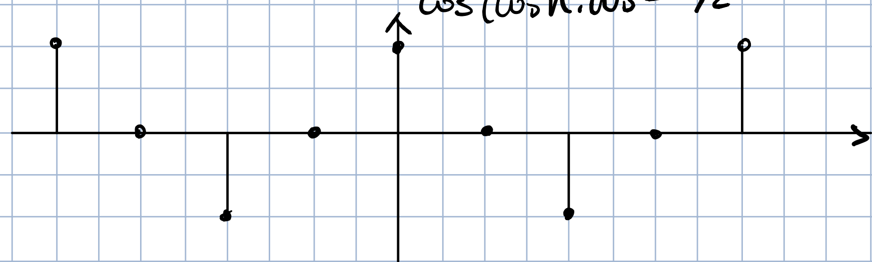
$\cos(\omega_0 n)$, $\omega_0 = \pi/2$



$\cos(\omega_0 n)$, $\omega_0 = \pi$



$\cos(\omega_0 n, \omega_0 = 3\pi/2$



$\cos(\omega_0 n, \omega_0 = 2\pi)$