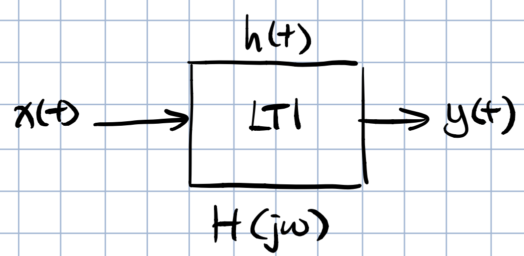
Sampling



$$y(t) = x(t) * h(t)$$

$$Y(j\omega) = X(j\omega)H(j\omega)$$

i.e.

$$x(t) = e^{j30t}$$

$$y(t) = H(j30)e^{j30t}$$

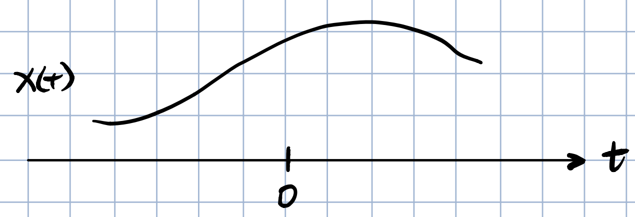
$$x(t) = e^{j30t} + e^{j60t}$$

$$y(t) = H(j30)e^{j30t} + H(j60)e^{j60t}.$$

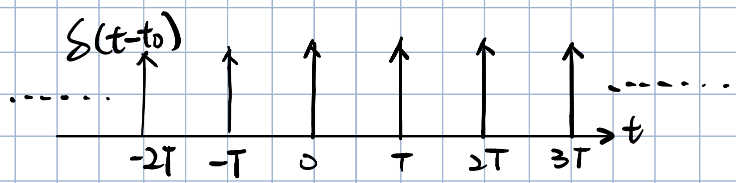
Sampling:

Can we sample a CT signal such that no information is lost?

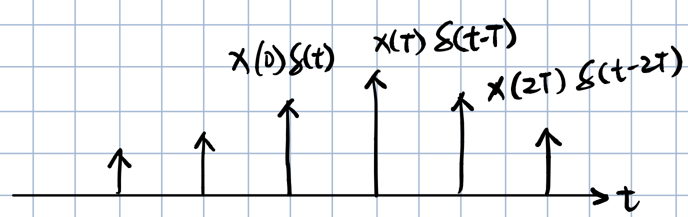
Can we sample a CT signal such that the CT signal can be reconstructed using the sampled values?



$x(t)$



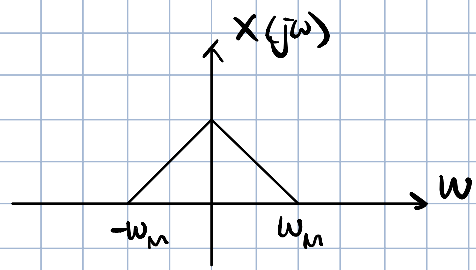
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT)$$



$$x_p(t) = x(t)p(t)$$

Objective: Finding $X_p(j\omega)$

Assume $x(t)$ is a band-limited signal.



$X(j\omega)=0$ for $|\omega|>W_M$

Reminder :

FS coefficients of $p(t)$: $a_k=\frac{1}{T}$ for every k

Another reminder: $y(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jw_0kt}$, $W_0=\frac{2\pi}{T}$, T is the period of $y(t)$

$$Y(j\omega)=\sum_{k=-\infty}^{+\infty} 2\pi a_k\,\delta(\omega-kW_0)$$

$$P(j\omega)=\sum_{k=-\infty}^{+\infty} 2\pi\left(\frac{1}{T}\right)\delta(\omega-kW_s)$$

$$=\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\omega-kW_s)$$

Convolution property: $x(t)*h(t)\xleftrightarrow{f}X(j\omega)H(j\omega)$

multiplication property: $x(t)\cdot h(t)\xleftrightarrow{f}\frac{1}{2\pi}(X(j\omega)*H(j\omega))$

$$X_p(j\omega)=\frac{1}{2\pi}\left(X(j\omega)*\left(\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\omega-kW_s)\right)\right)$$

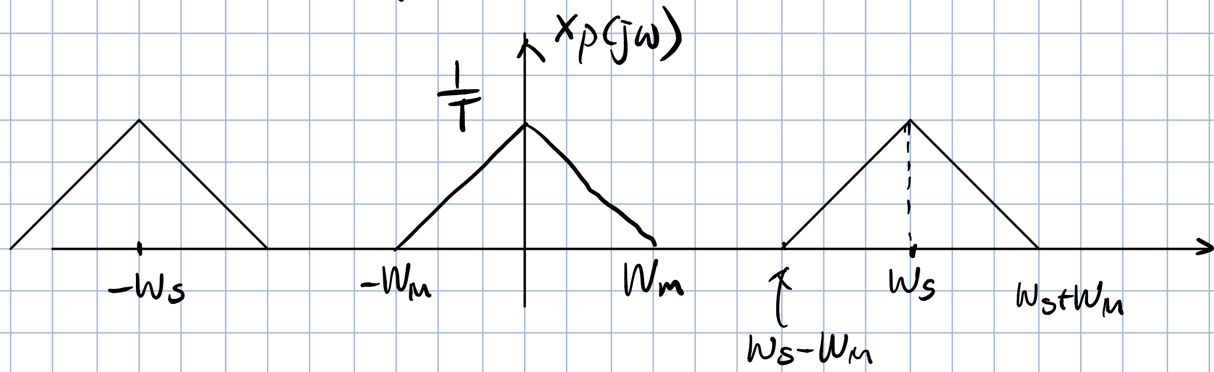
$$=\frac{1}{T}\left(X(j\omega)*\sum_{k=-\infty}^{+\infty}\delta(\omega-kW_s)\right)$$

Reminder: $x(t)=x(t)*\delta(t)=\int_{k=-\infty}^{+\infty}x(\tau)\,\delta(t-\tau)\,d\tau$

$t-t_0$ $\quad$ $t-t_0$

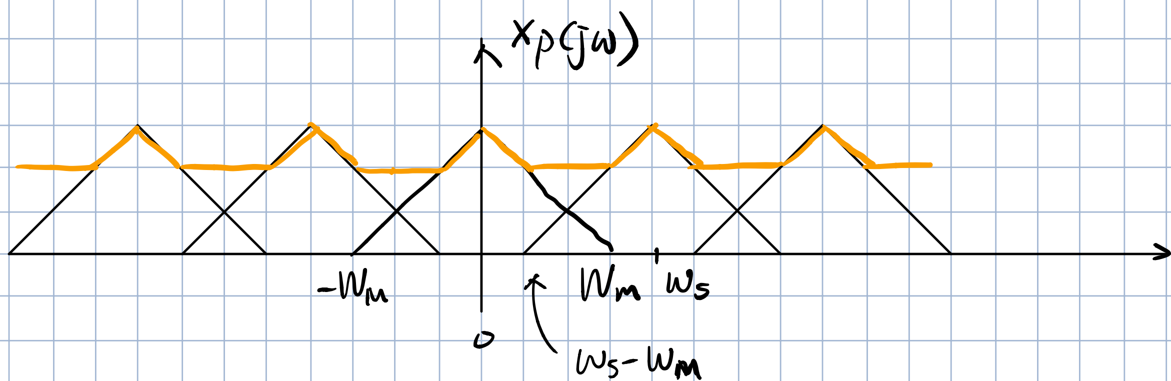
$$\therefore\quad X_p(j\omega)=\frac{1}{T}\sum_{k=-\infty}^{+\infty}X(j(\omega-kW_s))$$

① $W_s > 2W_M$



if we want to recover $x(t)$ from $x_p(t)$. (or equivalently, $X(jw)$ from $X_p(jw)$), we have to apply an ideal low pass filter with $W_M < W_c < W_s - W_M$

② $W_s < 2W_M$



$x(t)$ can not be recovered from $X(jw)$ because $\delta(w-kW_s)$ for $k \in \mathbb{Z}$ are going to have overlaps in $X_p(jw)$

$T_s = \frac{2\pi}{W_s}$ ← sampling freq.

↑ sampling period.