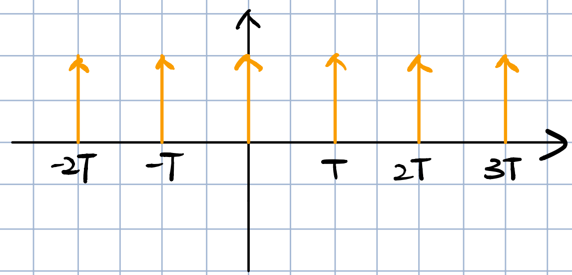
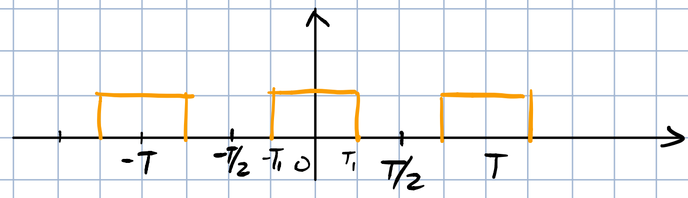
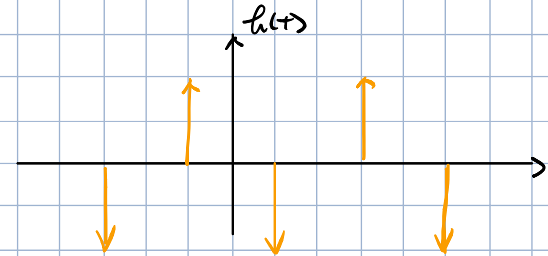
(*) Find the FS coefficients of $x(t)$ & $y(t)$







$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk(2\pi/T)t}$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2} \delta(t)\, e^{-jk(2\pi/T)t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\,dt = \frac{1}{T}$$

$$h(t) = x(t+T_1) - x(t-T_1)$$

$$fs\{h(t)\} = e^{+jk(2\pi/T)T_1}\frac{1}{T} - e^{-jk(2\pi/T)T_1}\frac{1}{T}$$

$$= \frac{1}{T}\left(e^{jk(2\pi/T)T_1} - e^{-jk(2\pi/T)T_1}\right)$$

$$= \frac{1}{T}\left(2j\sin(k(2\pi/T)T_1)\right)$$

$$m(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

$$\rightarrow \int_{-\infty}^{t} m(\tau)\,d\tau = \sum_{k=-\infty}^{+\infty}\int_{-\infty}^{t} a_k e^{jk(2\pi/T)\tau}\,d\tau$$

$$\int_{-\infty}^{t} m(\tau)\,d\tau = \sum_{k=-\infty}^{+\infty} \frac{a_k}{jk(2\pi/T)} e^{jk(2\pi/T)t}$$

$k \neq 0$

$$y(t) = \int_{-\infty}^{t} h(z)\,dz \longrightarrow fs\{y(t)\} = \frac{1}{jk(2\pi/T)}\frac{2j}{T}\sin(k(2\pi/T)T_1)$$

$$= \frac{1}{k\pi}\sin(k\underbrace{(2\pi/T)}_{w_0}T_1)$$

$$= \frac{\sin(kw_0T_1)}{k\pi} \qquad \boxed{k \neq 0}$$

for $k=0$, $a_0 = \frac{1}{\ }\int^{T/2}$ ...

$\int_{-T/2} \quad y(t) = \frac{1}{T} 2\pi = \frac{2\pi}{T}, \quad (k=0)$

(*) Exercise: Suppose we are given the following facts about a signal.

1 – $x(t)$ is a real signal.

2 – $x(t)$ is periodic w/ $T=4$ and FS coefficient a_k

3 – $a_k = 0$ for $|k| > 1$

4 – The signal with FS coefficients $b_k = e^{-j\pi k/2} a_{-k}$ is odd.

5 – $\frac{1}{4}\int_{4} |x(t)|^2 dt = \frac{1}{2}$.

Show that $x(t)$ can be determined w/ a $\underbrace{\text{sign factor}}$.

$x(t) = \pm$