Lecture 11: Dielectrics and the Equation of Continuity

ECE221: Electric and Magnetic Fields

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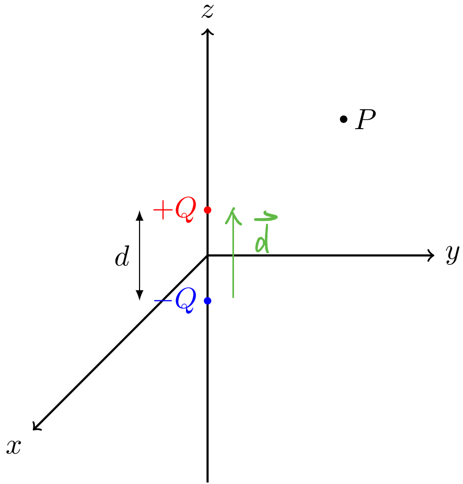
Outline

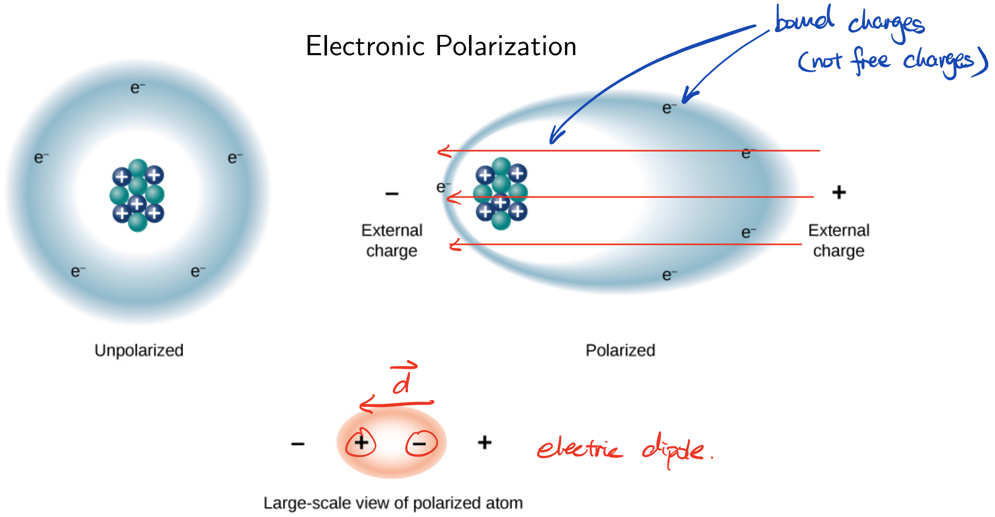
Review: Electric Dipole



$$\boldsymbol{E} = \frac{Qd}{4\pi\epsilon_0 R^3}(2\cos\theta\hat{\boldsymbol{R}} + \sin\theta\hat{\boldsymbol{\theta}})$$

$\vec{p}$ = dipole moment $= Q\vec{d}$

The Effect of an Applied Electric Field on Dielectrics



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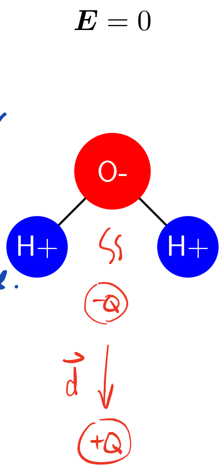
Effect of an Applied Electric Field on Polar Molecules

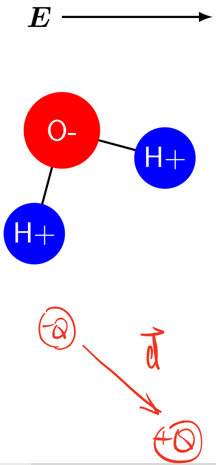
Say we have a mass of water. There are n dipoles per unit volume in the water

$$\vec{P}_{Total} = \sum_{i=1}^{n\Delta v} \vec{P}_i$$

dipole moment per unit volume.

$$\vec{P} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \sum_{i=1}^{n\Delta v} P_i \quad [C/m^2]$$





Polarization of a Dielectric

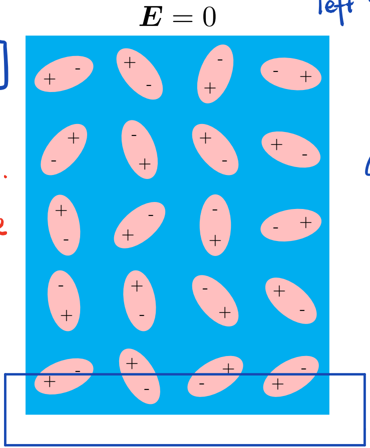
Dipole Moment per unit volume

$$\vec{P} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_{i=1}^{N\Delta V} \vec{P}_i \quad \left[\frac{C}{m^2}\right]$$

Generalized Gauss's law.
(accounts for both free charge & bound charges)

$$Q_T = Q_b + Q$$

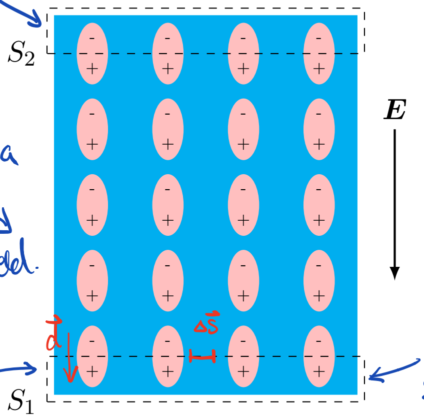
$$= \oint_S \varepsilon_0 \vec{E} \cdot d\vec{S} = Q_b + Q$$



apply a E-field.

+ charges has left S_2

polarized dielectric.



Gaussian Surface.

+ charges has crossed into S_1

$$\rightarrow \underset{\substack{\uparrow \\ \text{bound charge}}}{\Delta Q_b} = \underbrace{nQ\vec{d}}_{\vec{P}} \cdot \Delta\vec{s} \quad [C]$$

Dielectric Breakdown

$$Q = \oint_S \varepsilon_0 \cdot \vec{E} \cdot d\vec{s} - Q_b$$

$$= \oint_S \varepsilon_0 \vec{E} \cdot d\vec{s} + \oint_S \vec{P} \cdot d\vec{s}$$

$$= \oint_S \underbrace{(\varepsilon_0 E + \vec{P})}_{\vec{D}} \cdot d\vec{s}$$

only non-zero in dielectric

$$D = \varepsilon_0 \vec{E} + \vec{P}$$

Let $\vec{P} \equiv \chi_e \varepsilon_0 \vec{E}$

where χ_e = "electric susceptibility"

$$Q_b = -\oint_{S_2 \text{ or } S_1} \vec{P} \cdot d\vec{s}$$

charge in S_1 is increasing

looks kind of like

$$\boxed{\text{Gauss's law. } \quad Q = \oint_S \vec{D} \cdot d\vec{s}}$$

↑ free charge.

apply to free charge only.

The **dielectric strength** is the maximum electric field that a dielectric can withstand without electrical breakdown.

Table of Dielectrics

$\vec{D} = \underbrace{\varepsilon_0(1+\chi_e)}\vec{E}$

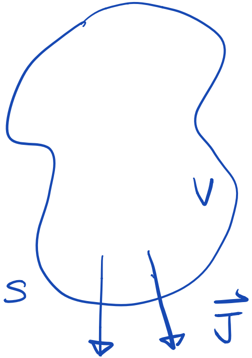
ε_r = relative permittivity of the dielectric.

$= \varepsilon \vec{E}$

$\varepsilon = \varepsilon_0 \varepsilon_r$

Material	Dielectric constant ϵ_r	Dielectric strength ($\times 10^6$ V/m)
Vacuum	1	∞
Dry air (1 atm)	1.00059	3.0
Teflon™	2.1	60 to 173
Paraffin	2.3	11
Silicon oil	2.5	10 to 15
Polystyrene	2.56	19.7
Nylon	3.4	14
Paper	3.7	16
Fused quartz	3.78	8
Glass	4 to 6	9.8 to 13.8
Concrete	4.5	–
Diamond	5.5	2,000
Mica	6.0	118
Water	80	–
Titanium dioxide	86 to 173	–
Strontium titanate	310	8
Barium titanate	1,200 to 10,000	–
Calcium copper titanate	$>$ 250,000	–

Conservation of Charge and the Equation of Continuity



Charges cannot be created or destroyed.

What is the circuit equivalent of this law?

Total current leaving closed surface S is

$$I = \oint_S \vec{J}\cdot d\vec{s} = \text{outward flow of positive charge.}$$

If the total charge in S is Q_i

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s} = \frac{-dQ_i}{dt}}$$

integral

Equation of continuity (of current)

Q_i is decreasing w time.

Div. theorem.

$$\iiint_V (\nabla\cdot\vec{J})\,dv = \frac{-d}{dt}\underbrace{\iiint_V \rho_v\,dv}_{Q_i}$$

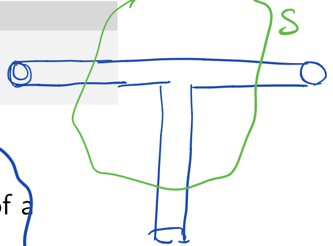
$$= -\iiint_V \frac{d\rho_v}{dt}\,dv$$

$$\Rightarrow \boxed{\nabla\cdot\vec{J} = -\frac{d\rho_v}{dt}}$$

point form.

form.

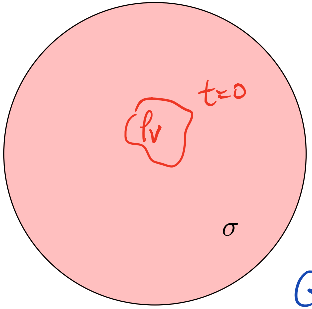
Relaxation Time

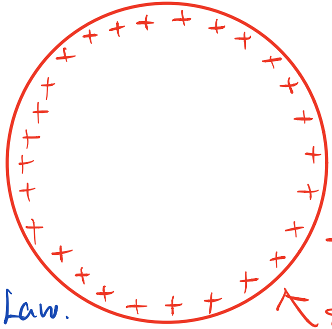


$\oint_S \vec{J}\cdot d\vec{s} = 0$

(KCL)

Consider a problem where charges are introduced into the interior of a conductor during the time $t < 0$. What happens for $t > 0$?





surface charge (density)

Gauss's Law.

$\nabla\cdot\vec{E} = \rho_v/\varepsilon_0$

$$\nabla\cdot\underbrace{\sigma\vec{E}}_{=\vec{J}} = \frac{\sigma\rho_v}{\varepsilon_0} = \frac{-d\rho_v}{dt}$$

Relaxation Time

$$\frac{\partial \rho_v}{dt} + \sigma \frac{\rho_v}{\varepsilon_0} = 0 \rightarrow \frac{\partial \rho_v}{\rho_v} = \frac{-\sigma}{\varepsilon_0} dt$$

Relaxation time is the time it takes a charge density placed in the interior of a material to drop to $e^{-1} = 36.8\%$ of its original value.

$$\ln(\rho_v) - \ln(\rho_{v_0}) = \frac{-\sigma}{\varepsilon_0} t$$

$$\rho_v = \rho_{v_0} e^{-t/T_r}$$

$$\boxed{T_r = \frac{\varepsilon}{\sigma}}$$ relaxation time.