Lecture 37: Review

ECE221: Electric and Magnetic Fields

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Outline

1. Electromagnetic Waves
2. Electric and Magnetic Dipoles
3. Midterms

Application 2: Electromagnetic Waves

Example: Determine $\boldsymbol{H}$ if

$$\boldsymbol{E} = \hat{\boldsymbol{x}} A \cos(\omega t - kz)$$

In the source-free region $(\vec{J}=0,\ \rho_v=0)$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = \hat{y} \frac{\partial E_x}{\partial z}$$

$$\frac{\partial E_x}{\partial z} = kA\sin(\omega t - kz) = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial t} = \frac{-kA}{\mu} \sin(\omega t - kz)$$

$$H_y = \frac{k}{\omega\mu} \underbrace{A\cos(\omega t - kz)}_{E_x}$$

Electric and Magnetic Dipoles

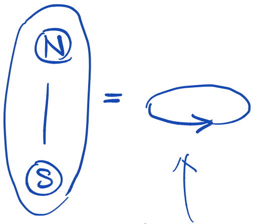
$\vec{p} \to \varepsilon_r$ $\vec{M} \to \mu_r$

⊕ fundamental $\vec{E}$ field source

Why are they important?

- The **electric** and **magnetic response** of a material can be described in the context of molecular dipole: dipoles form **elementary descriptions** of molecules.
- The **elementary magnetic unit** is a magnetic dipole: there are no magnetic monopoles.
- The electric dipole is the **simplest electromagnetic radiator** imaginable (ECE422).

We *usually* care about the fields far away from the dipole.



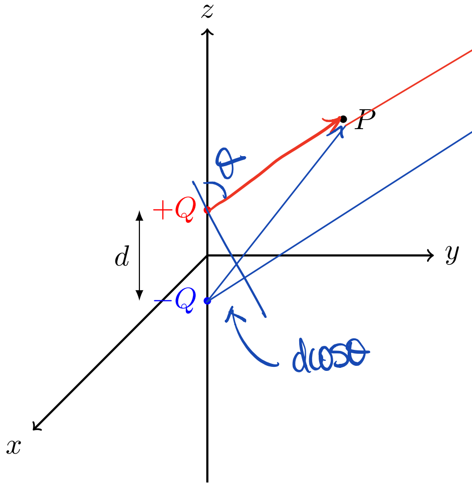
fundamental

B/H field source

$\cdot P$ (far)

Electric Dipole

What is $\boldsymbol{E}$ and V at P?



$$V_1 = \frac{Q}{4\pi\varepsilon_0 R_1}$$

$$V_2 = \frac{Q}{4\pi\varepsilon_0 R_2}$$

$$V = V_1 + V_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$R_2 - R_1 = d\cos\theta$$

$$V = \frac{\overbrace{Qd}^{p}\cos\theta}{4\pi\varepsilon_0 r^2}$$

p = electric dipole moment.

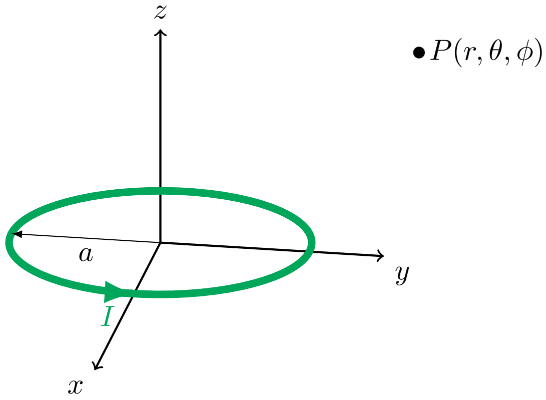
$$\vec{E} = -\nabla V = \frac{-Qd}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

(far from the dipole)

The Magnetic Dipole

Recall the circular loop we analyzed before. If the loop is **small** such that $r \gg a$, then

$$\boldsymbol{A} \approx \frac{\mu_0 I \pi a^2 \sin\theta \hat{\boldsymbol{\phi}}}{4\pi r^2}$$

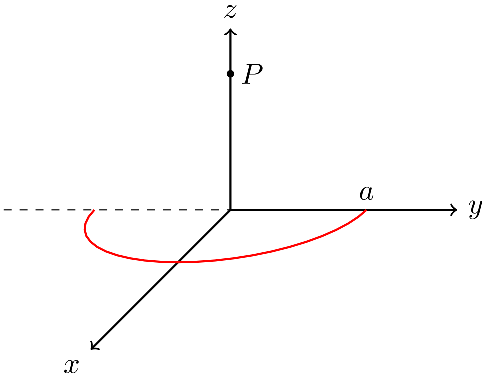


Example: Dipole Representation of a Charge Distribution

A semicircular line charge of radius a is situated in free space as shown. The line charge density is nonuniform, given by

$$\rho_l(\phi') = \rho_0 \sin\phi', \; -\pi/2 \leq \phi' \leq \pi/2.$$

Find the total charge of the semicircle and $\boldsymbol{E}$ along the z-axis.



Difficult Midterm Problems

1. Midterm 1 Problem 3
2. Midterm 2 Problem 3

Ask me anything!

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