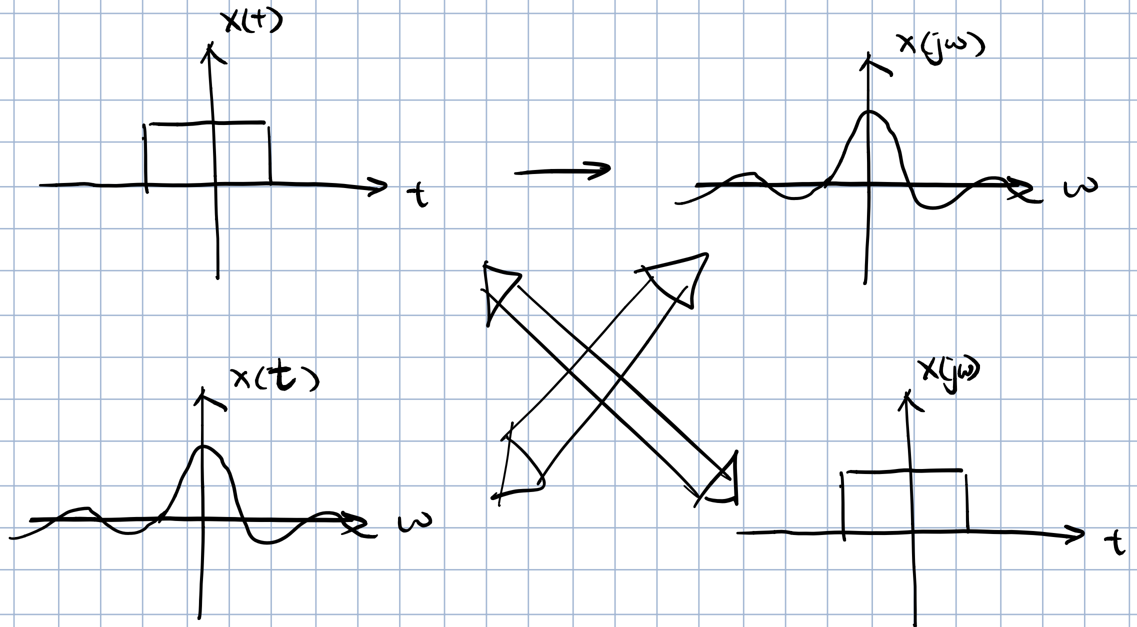
Properties of Fourier Transform:

⑤ Duality:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x(j\omega)e^{j\omega t}\,d\omega \qquad x(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$



(*) Given that $\mathcal{F}\{e^{-|t|}\} = \dfrac{2}{1+\omega^2}$

Find $\mathcal{F}\left\{\dfrac{2}{1+t^2}\right\}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} x(j\omega)\, e^{j\omega t}\, d\omega$$

$\times 2\pi$

$\rightarrow \quad e^{-|t|} = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{+\infty}\left(\frac{2}{1+\omega^2}\right)e^{j\omega t}\,d\omega$

$t \to -t$

$$2\pi e^{-|t|} = \int_{-\infty}^{+\infty}\frac{2}{1+\omega^2}e^{-j\omega t}\,d\omega$$

$t \to \omega$

$\omega \to t$

$$2\pi e^{-|\omega|} = \int_{-\infty}^{+\infty}\frac{2}{1+t^2}e^{-j\omega t}\,dt$$

$$x(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\boxed{X(j\omega) = 2\pi e^{-|\omega|}}$$

⑥ Frequency Differentiation.

Time Differentiation.

$$\left(\begin{array}{l} x(t) \overset{FT}{\longleftrightarrow} X(j\omega) \\ \dfrac{dx(t)}{dt} \overset{FT}{\longleftrightarrow} j\omega X(j\omega) \end{array} \right)$$

$$x(t) \overset{FT}{\longleftrightarrow} X(j\omega)$$

$$-jt\,x(t) \overset{ft}{\longleftrightarrow} \frac{dX(j\omega)}{d\omega}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$\frac{dX(j\omega)}{d\omega} = \int_{-\infty}^{+\infty} (-jt\,x(t)) e^{-j\omega t} dt$$

⊛ Find the FT of $x(t) = te^{-at}u(t) \quad a>0$

$$e^{-at}u(t) \overset{ft}{\longleftrightarrow} \frac{1}{a+j\omega}$$

$$\rightarrow -jte^{-at}u(t) \overset{ft}{\longleftrightarrow} \frac{d}{d\omega}\left(\frac{1}{a+j\omega}\right)$$

$$\rightarrow -jte^{-at}u(t) \overset{ft}{\longleftrightarrow} \frac{-j}{(a+j\omega)^2}$$

$$\rightarrow \boxed{te^{-at}u(t) \overset{ft}{\longleftrightarrow} \frac{1}{\ldots}}$$

$(a+j\omega)^2$

$$-\frac{1}{jt}x(t) + \pi x(0)\delta(t) \overset{ft}{\longleftrightarrow} \int_{-\infty}^{\omega} x(\tau)\,d\tau$$

⑦ Parseval's Theorem.

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega.$$

⑧ Convolution.

$$x(t) \overset{ft}{\longleftrightarrow} X(j\omega), \quad h(t) \overset{ft}{\longleftrightarrow} H(j\omega)$$

$$y(t) = x(t) * h(t) \overset{ft}{\longleftrightarrow} Y(j\omega) = X(j\omega)H(j\omega)$$

$$\begin{array}{ccccc} & & \text{LTI} & & \\ x(t) & \longrightarrow & \boxed{h(t)} & \longrightarrow & y(t) = x(t) * h(t) \\ \updownarrow ft & & \downarrow ft & & \\ X(j\omega) & & H(j\omega) & \longrightarrow & Y(j\omega) = X(j\omega)H(j\omega) \end{array}$$

$h(t)$: Impulse response. $H(j\omega)$: Frequency Response

↑ ft of the Impulse response

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

$$\parallel$$

$$x(t) \; — \boxed{h_1(t) * h_2(t)} — \; y(t)$$

$$X(j\omega) \; — \boxed{H_1(j\omega)H_2(j\omega)} — \; Y(j\omega) \qquad y(t) = \mathcal{F}^{-1}\{Y(j\omega)\}$$

⊛ Find the frequency response of a CT differentiator.

$$y(t) = \frac{dx(t)}{dt} \longrightarrow Y(j\omega) = j\omega \, X(j\omega)$$

$$Y = H \quad X$$

$$H(j\omega) = j\omega$$

⊛ The Impulse response of an LTI system is $h(t) = e^{-at}u(t)$, $a>0$, What is the output for $x(t) = e^{-bt}u(t)$, $b>0$

$$\mathcal{F}\{h(t)\} = \mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega}$$

$$\mathcal{F}\{x(t)\} = \frac{1}{b+j\omega}$$

$$Y(j\omega) = X(j\omega)H(j\omega) \rightarrow Y(j\omega) = \frac{1}{(a+j\omega)(b+j\omega)}$$

If $a \neq b$

$$\frac{A}{a+j\omega} + \frac{B}{b+j\omega} = \frac{1}{(a+j\omega)(b+j\omega)}$$

$$\rightarrow A(b+j\omega) + B(a+j\omega) = 1$$

$$\text{if } j\omega = -a \Rightarrow A(b-a) = 1$$

$$A = \frac{1}{b-a}$$

$$\text{if } j\omega = -b, \rightarrow B(a-b) = 1$$

$$B = \frac{1}{a-b}$$

$$y(j\omega) = \frac{1/(b-a)}{a+j\omega} + \frac{1/(a-b)}{b+j\omega}$$

$$y(t) = \frac{1}{b-a} e^{-at} u(t) + \frac{1}{a-b} e^{-bt} u(t)$$

$$= \frac{u(t)}{a-b}\left(e^{-bt} - e^{-at}\right)$$

<u>If $a=b$</u>

$$y(j\omega) = \frac{1}{(a+j\omega)^2}$$

$$y(t) = te^{-at}u(t) \; //$$

we found this @ the beginning of the lecture.