System Properties.

① System w/ & w/o memory
② Invertibility
} last lec.

③ Causality : A system is causal if the output at any time depends on the input at the present & past times.

$$y[n] = \sum_{m=n-9}^{n} x[m]$$

Causual

$$y[n] = \sum_{m=n-4}^{n+4} x[m]$$

not causal

$$y(t) = x(-t)$$

not Causal
(if $t = -ve$. then we need future input)

$$y[n] = x[n-1]e^{j\omega_0(n+1)}$$

causal.

④ Stability : A system is stable if bounded inputs result in bounded outputs

$$y(t) = \sqrt{t}\, x(t) \Rightarrow x(t) = 1 \longrightarrow |y(t)| \text{ not bounded.}$$

$$t \to \infty \Rightarrow y(t) \to \infty$$

$$y(t) = e^{x(t)} \Rightarrow |x(t)| < B \Rightarrow -B < x(t) < +B$$

$$e^{-B} < e^{x(t)} < e^{+B} \longleftrightarrow |y(t)| < e^{B}$$

⑤ Time - Invarience : A system is time invarient if its characteristics (input - output relation) are fixed over time.

$y(t) = e^{x(t+1)}$

Time-Invarient

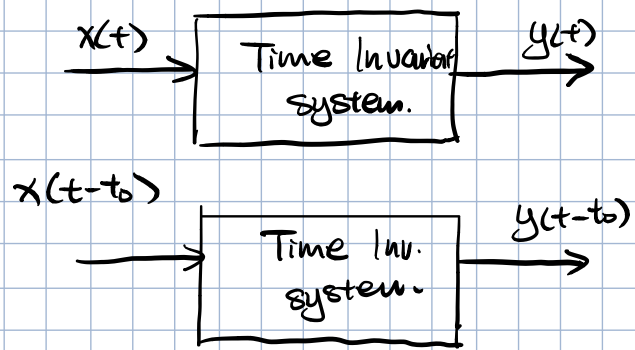
$y(t) = e^{t\,x(t+1)}$

$t=1$: $y(t) = e^{x(t+1)}$

$t=2$: $y(t) = e^{2x(t+1)}$

$t=t_0$: $y(t) = e^{t_0\,x(t+1)}$

Time-Varient.



⑥ Linearity :

If for a system S :

$$x_1(t) \xrightarrow{S} y_1(t) \quad \text{and} \quad x_2(t) \xrightarrow{S} y_2(t)$$

then S is lineary if these two cond. are met.

1) Additivity : $x_1(t) + x_2(t) \xrightarrow{S} y_1(t) + y_2(t)$

2) Scaling : $a x_1(t) \xrightarrow{S} a y_1(t)$

$$\longrightarrow \quad a_1 x_1(t) + a_2 x_2(t) \xrightarrow{S} a_1 y_1(t) + a_2 y_2(t)$$

$$a_1 \,\&\, a_2 \in C$$

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau \longrightarrow \int (a_1 x_1(t) + a_2 x_2(t))\,dt$$

$$= a_1 \int x_1(t)\,dt + a_2 \int x_2(t)\,dt$$

eg. 1. $y(t) = x^2(t)$

$$x_1(t) = 3 \xrightarrow{S} y_1(t) = 9.$$

$$x_2(t)=7 \xrightarrow{S} y_2(t)=49$$

$$x_1(t)+x_2(t)=10 \xrightarrow{S} (10^2)=100 \neq 9+49$$

↳ non-linear.

2. $y[n]=5x[n]+2$

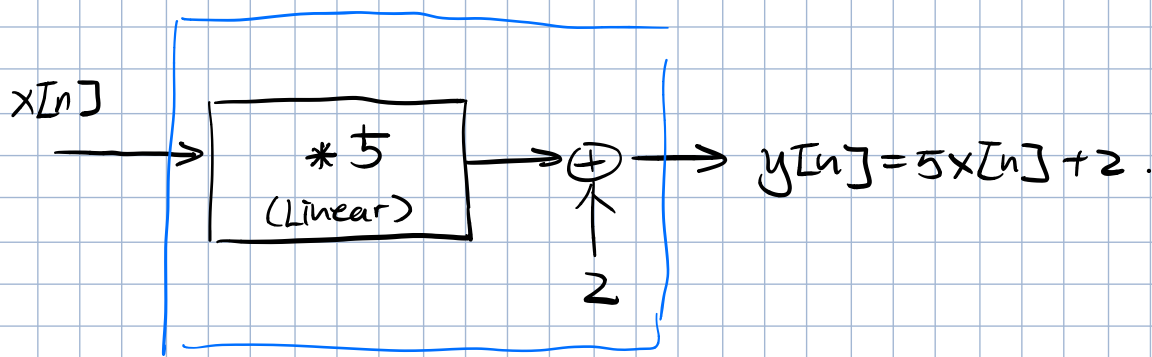
$$x_1[n]=4 \xrightarrow{S} y_1[n]=22$$

$$x_2[n]=6 \xrightarrow{S} y_2[n]=32$$

$$x_1[n]+x_2[n]=10 \rightarrow 52 \neq 22+32$$

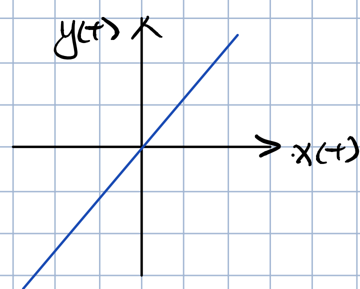
↳ non-linear.

Systems of the form $y[n]=a\,x[n]+b$ are called increamentally linear. : They can be realized as the series connection of a linear system and a constant.



increamentally linear system.

The input-output graph of a linear system is a straight line passes through the origin.



Another example...

$$y(t) = \sin(x(t))$$

$$x_1(t) = \frac{\pi}{2} \xrightarrow{S} y_1(t) = 1$$

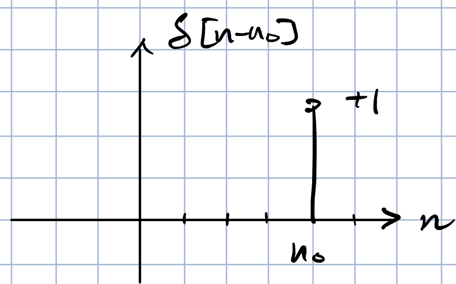
$$x_2(t) = \frac{\pi}{2} \xrightarrow{S} y_2(t) = 1$$

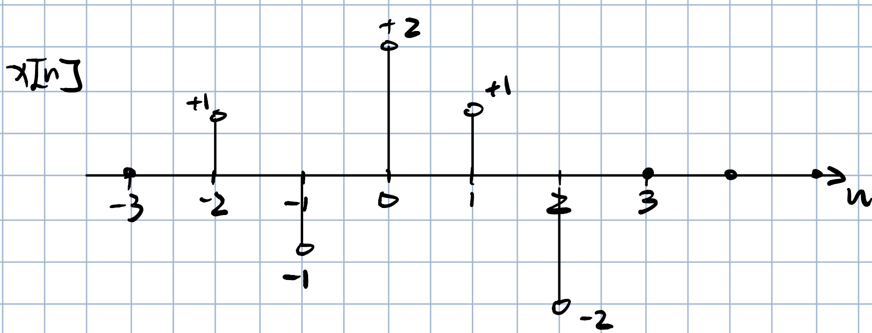
$$x_1(t) + x_2(t) = \pi \xrightarrow{S} 0 \neq 1+1$$

The main focus of this course is linear time-invarient system (LTI) systems.

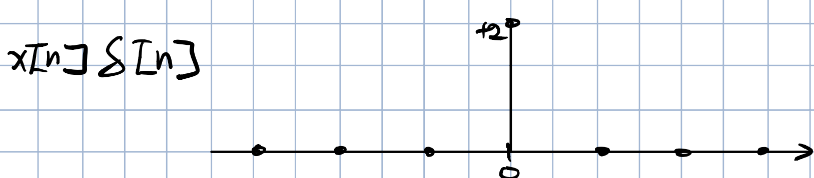
Objective: To find the output of an LTI system for an arbitrary input.

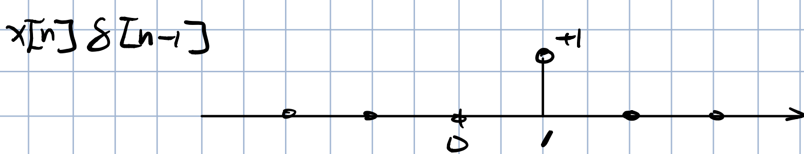
$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$$





$$x[n] = x[n]\,\delta[n-(-2)] + x[n]\,\delta[n-(-1)] + x[n]\,\delta[n] + x[n]\,\delta[n-1] + x[n]\,\delta[n-2]$$





$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k]$$

, An arbitrary DT signal can be written as the sum of some scaled - time - shifted versions of $\delta[n]$

这两替降.