③ Unit Impulse Function. : $\delta(t)$

It's a signal that satisfies :

1) $\delta(t) = 0$ for $t \neq 0$

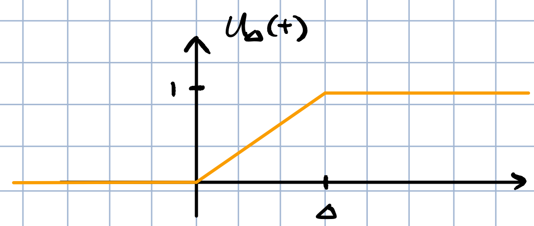
2) $\displaystyle\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$

Although $\delta(t)$ might define some mathematical principles

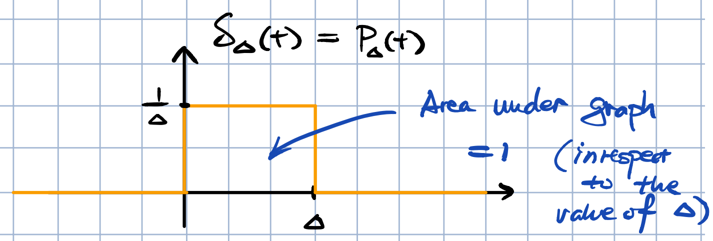
It's very helpful in analyzing signals & systems.

Also, some practical signals closely resemble $\delta(t)$;

$$U_\Delta(t) = \begin{cases} 0 & t<0 \\ \dfrac{t}{\Delta} & 0<t<\Delta \\ 1 & \Delta<t \end{cases}$$



$$\delta_\Delta(t) = \frac{d\,U_\Delta(t)}{dt} = \begin{cases} 0 & t<0 \\ \dfrac{1}{\Delta} & 0<t<\Delta \\ 0 & t>0 \end{cases}$$

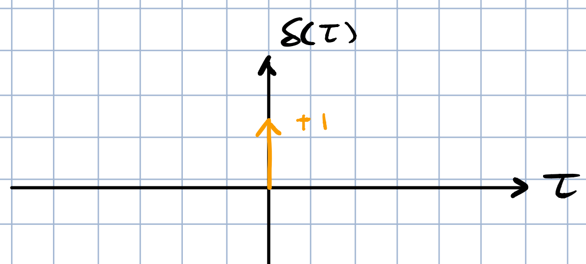


$\delta(t)$

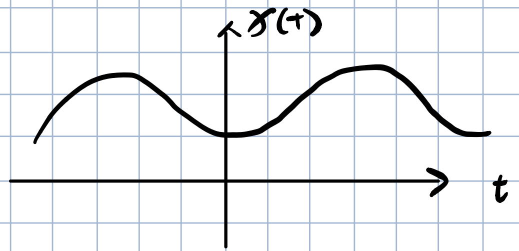
$$\lim_{\Delta \to 0} \delta_\Delta(t) = \lim_{\Delta \to 0} \frac{dU_\Delta(t)}{dt} \rightarrow \frac{du(t)}{dt}.$$

$$\delta(t) = \frac{du(t)}{dt}, \quad u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

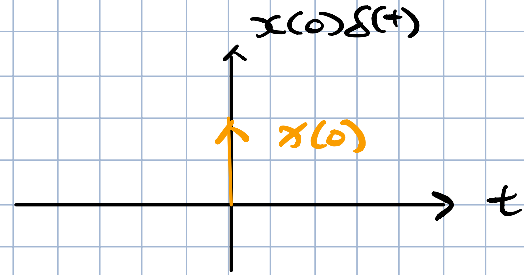
relationship btw delta function & unit step function



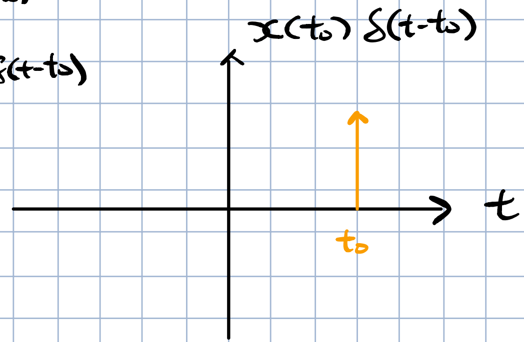
$$x(t)\,\delta(t) = x(0)\,\delta(t)$$



$\Longrightarrow$



$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$



$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt = x(0)\int_{-\infty}^{\infty}\delta(t)\,dt$$

$$= x(0)$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

Complex Number.

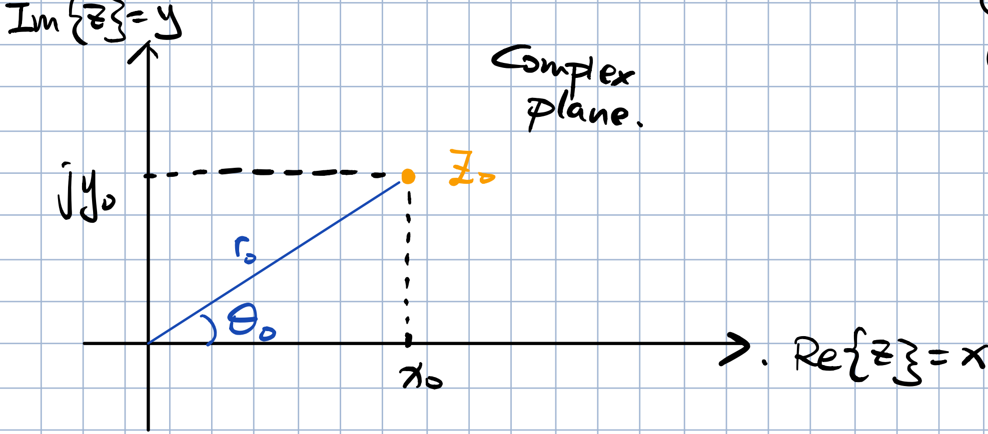
A complex number $z = x + jy$; x & $y \in \mathbb{R}$

j satisfies $j^2 = -1$, $\left.\begin{array}{l} x \to \text{real part} \\ y \to \text{imaginary part.} \end{array}\right\}$ of z.

$z = x + jy$ (Rectangular format / Cartesian)

$= re^{j\theta}$ (polar).

$z_0 = x_0 + jy_0$



$$= r_0 e^{j\theta_0}$$

$$x_0 = r_0 \cos(\theta_0)$$

$$y_0 = r_0 \sin(\theta_0)$$

$$r_0 = \sqrt{x_0^2 + y_0^2}$$

$$\theta_0 = \begin{cases} \tan^{-1}\left(\frac{y_0}{x_0}\right) & x_0 > 0 \\ \tan^{-1}\left(\frac{y_0}{x_0}\right) + \pi & x_0 < 0 \end{cases}$$

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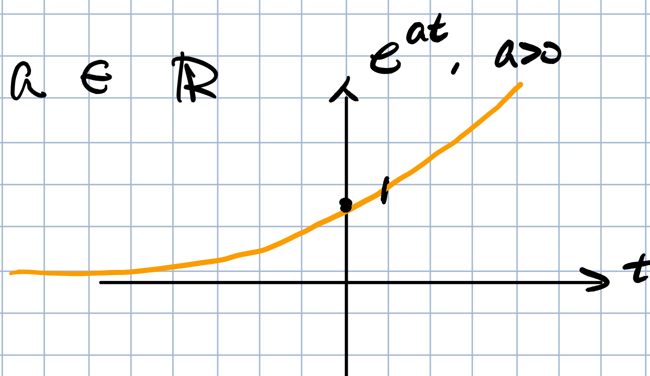
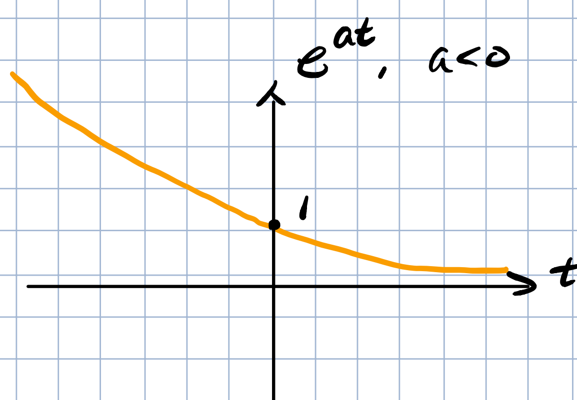
④ Complex Exponential.

$$x(t) = e^{at}$$

a) $a \in \mathbb{R}$



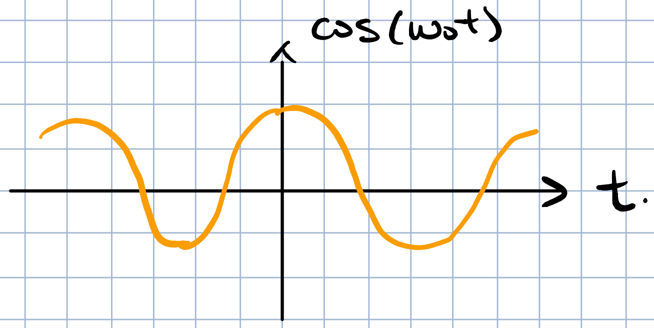
Growing Exp.

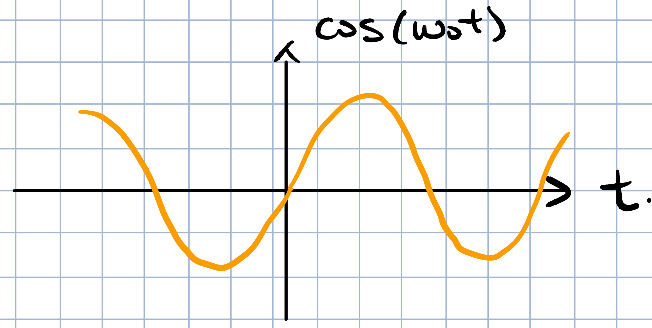


Decaying Exp.

b) $a = j\omega_0$, $x(t) = e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$

Real Imaginary.





$T_x = \frac{2\pi}{|\omega_0|}$ $T_y = \frac{2\pi}{|\omega_0|}$

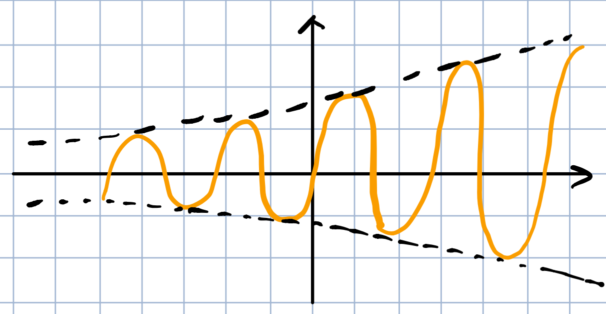
T for $(x(t) = e^{j\omega_0 t}) : \frac{2\pi}{|\omega_0|}$

c) $x(t) = e^{at}$, $a = r_0 + j\omega_0$

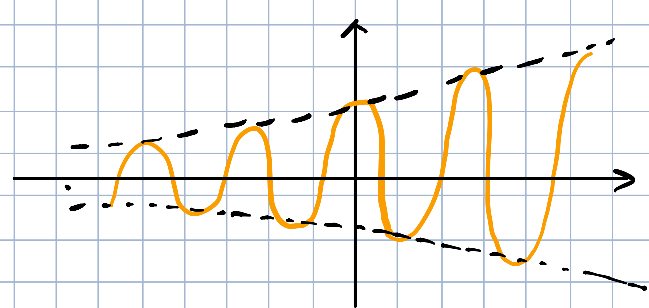
$$x(t) = e^{(r_0 + j\omega_0)t} = e^{r_0 t} e^{j\omega_0 t}$$

$r_0 > 0$

Growing Exp. Envelop.



sin



cos.