

Principles of Superposition

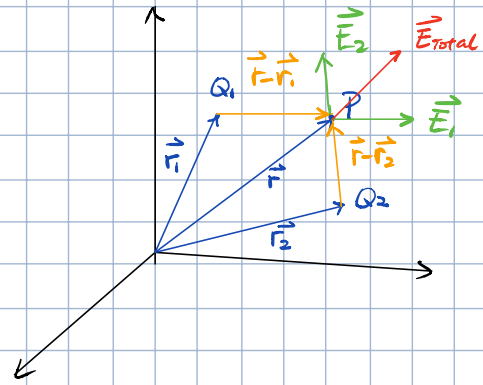
- Consider two charges placed in space.
- What's the E-field produced at P by Q_1 & Q_2 ?

eg. $Q_1, Q_2 > 0$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{|\vec{r} - \vec{r}_1|^3} (\vec{r} - \vec{r}_1)$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{|\vec{r} - \vec{r}_2|^3} (\vec{r} - \vec{r}_2)$$

By Principle of Superposition: $\vec{E}_{\text{Total}} = \vec{E}_1 + \vec{E}_2$

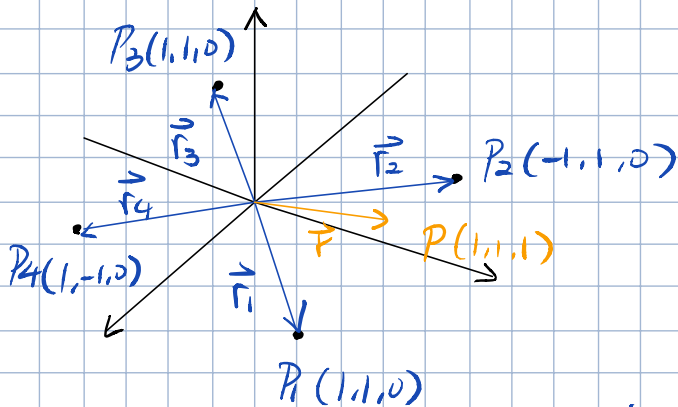


Extension to N charges

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{Q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

Example.

Find \vec{E} at $P(1,1,1)$ caused by four identical $3nC$ charges $P_1 - P_4$



$$\vec{r} = \hat{x} + \hat{y} + \hat{z}$$

$$\vec{r}_1 = \hat{x} + \hat{y}$$

$$\vec{r}_1 = \hat{z}$$

$$\vec{r}_2 = -\hat{x} + \hat{y}$$

$$\vec{r}_2 = 2\hat{x} + \hat{z}$$

$$R_2 = \sqrt{5}$$

$$\vec{r}_3 = -\hat{x} - \hat{y}$$

$$\vec{r}_3 = 2\hat{x} + 2\hat{y} + \hat{z}$$

$$R_3 = \sqrt{9}$$

$$\vec{r}_4 = \hat{x} - \hat{y}$$

$$\vec{r}_4 = 2\hat{y} + \hat{z}$$

$$R_4 = \sqrt{5}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^4 \frac{Q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

$$= \frac{3nC}{4\pi\epsilon_0} \sum_{i=1}^4 \frac{\vec{r}_i}{R_i^3}$$

$$= 29.96 \cdot \left[\frac{\hat{z}}{1} + \frac{2\hat{x} + \hat{z}}{(\sqrt{5})^3} + \frac{2\hat{x} + 2\hat{y} + \hat{z}}{(\sqrt{9})^3} + \frac{2\hat{y} + \hat{z}}{(\sqrt{5})^3} \right]$$

$$= 6.82 \hat{x} + 6.82 \hat{y} + 32.8 \hat{z} \text{ [V/m]}$$

