DT FT

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw})e^{jwn}\,dw$$

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-jwn}$$

Find the DT FT of:

$$x[n] = a^{|n|} \qquad |a|<1$$

$$|n| = \begin{cases} +n & n>0 \\ -n & n<0 \end{cases}$$

$$X(e^{jw}) = \sum_{n=-\infty}^{-1} a^{-n}e^{-jwn} + \sum_{n=0}^{+\infty} a^{n}e^{-jwn}$$

$m=-n$

$$= \sum_{m=+1}^{+\infty} a^{+m}e^{jwm} + \sum_{n=0}^{+\infty} a^{n}e^{-jwn}$$

$$= \frac{1-\cancel{(ae^{jw})^{+\infty}}}{1-ae^{jw}} - 1 + \frac{1-\cancel{(ae^{-jw})^{+\infty}}}{1-ae^{-jw}}$$

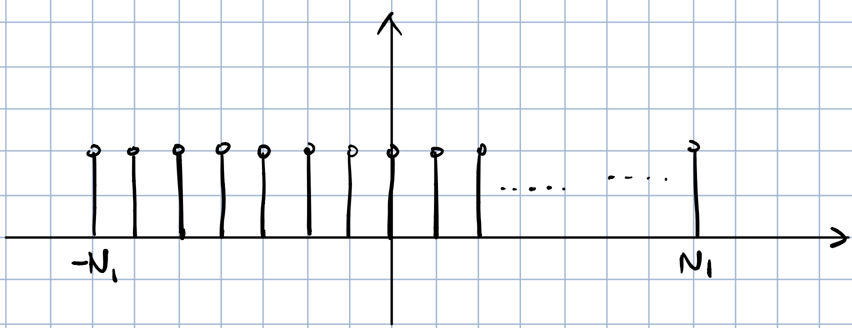
since $|a|<1$

$$= \frac{1}{1-ae^{jw}} - 1 + \frac{1}{1-ae^{-jw}}$$

$$= \frac{ae^{jw}-a^2+1-ae^{jw}}{1-ae^{-jw}-ae^{jw}+a^2}$$

$$X(e^{jw}) = \frac{1-a^2}{1-2a\cos(w)+a^2}$$

Find the DT FT of $x[n] = \begin{cases} 1 & |n| \le N_1 \\ 0 & |n| > N_1 \end{cases}$



$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} = \sum_{n=-N_1}^{+N_1} 1 e^{-j\omega n}$$

$m = n + N_1$

$n = m - N_1$

$$\longrightarrow X(e^{j\omega}) = \sum_{m=0}^{2N_1} e^{-j\omega(m-N_1)}$$

$$= e^{j\omega N_1} \sum_{m=0}^{2N_1} e^{-j\omega m}$$

$$X(e^{j\omega}) = e^{j\omega N_1} \frac{1 - e^{-j\omega(2N_1+1)}}{1 - e^{-j\omega}}$$

$$= e^{j\omega N_1} \frac{e^{-j\omega\left(\frac{2N_1+1}{2}\right)} \cdot \left(e^{+j\omega\left(\frac{2N_1+1}{2}\right)} - e^{-j\omega\left(\frac{2N_1+1}{2}\right)}\right)}{e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}$$

$$= \cancel{e^{j\omega N_1}} \frac{\cancel{e^{-j\omega N_1}} \cancel{e^{-j\frac{\omega}{2}}}}{\cancel{e^{-j\omega/2}}} \cdot \frac{\cancel{j2} \sin\left(\frac{2N+1}{2}\omega\right)}{\cancel{j2} \sin\left(\frac{\omega}{2}\right)}$$

$$= \frac{\sin\left(\frac{2N+1}{2}\omega\right)}{\sin\left(\frac{\omega}{2}\right)}$$

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Properties of DT FT

1. Periodicity: $X(e^{j\omega}) = X(e^{j(\omega+2\pi)})$

2. Linearity:

$$x_1[n] \xleftrightarrow{FT} X_1(e^{j\omega})$$

$$x_2[n] \xleftrightarrow{ft} X_2(e^{j\omega})$$

$a_1 \,\&\, a_2 \in C$

$$a_1 x_1[n] + a_2 x_2[n] \xleftrightarrow{ft} a_1 X_1(e^{j\omega}) + a_2 X_2(e^{j\omega})$$

3. Time shifting and frequency shifting:

$$x[n] \xleftrightarrow{ft} X(e^{j\omega})$$

$\rightarrow$ $x[n-n_0] \xleftrightarrow{ft} e^{-j\omega n_0} X(e^{j\omega})$ time shift.

$\rightarrow$ $e^{j\omega_0 n} x[n] \xleftrightarrow{ft} X(e^{j(\omega-\omega_0)})$ freq. shift.

4. Conjugation.

$$x[n] \xleftrightarrow{ft} X(e^{j\omega})$$

$\rightarrow$ $x^*[n] \xleftrightarrow{ft} X^*(e^{-j\omega})$

{If $x[n]$ is real,} then $X(e^{-j\omega}) = X^*(e^{j\omega})$

$\rightarrow$ $Re\{X(e^{j\omega})\}$ and $|X(e^{j\omega})|$ are even signals.

$Im\{X(e^{j\omega})\}$ and $\measuredangle X(e^{j\omega})$ are odd signals.