Lecture 32: Motional EMF, Total EMF, and Displacement Current

ECE221: Electric and Magnetic Fields

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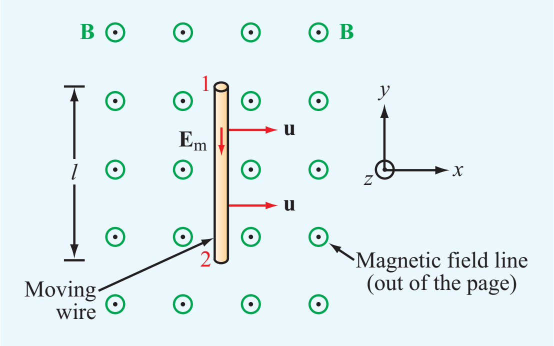
Outline

Motional EMF

- We now consider a second case: relative motion between a closed path and a steady field
- Example: sliding conductive bar moving with a constant velocity moving through a steady (DC) magnetic field $\boldsymbol{B} = \hat{\boldsymbol{z}} B_0$



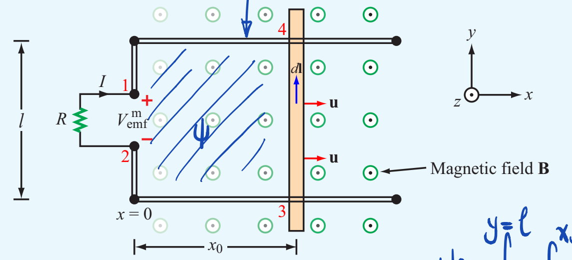
Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

Example: Sliding Bar

Even though $\vec{B}$ does not vary with time,

$\psi = \int_S \vec{B}\cdot d\vec{s}$ does $\psi(t)$

$\frac{\partial \psi}{\partial t} \neq 0$



$x_0 = ut$

$\psi = \int_S \vec{B}\cdot d\vec{s}$

Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

$$\psi = \int_{y=0}^{y=l}\int_{0}^{x_0} B_0 x\, \hat{z}\cdot\hat{z}\, dx\, dy = \frac{l B_0 x_0^2}{2} = \frac{l B_0 (ut)^2}{2} = \psi(t)$$

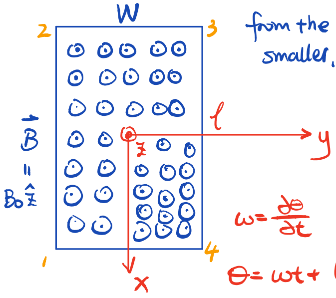
A rectangular loop has a constant width l but its length x_0 increases with time as the conducting bar slides with a uniform velocity $\boldsymbol{u}$ in a static magenetic field $\boldsymbol{B} = \hat{z}B_0 x$. The bar starts from $x = 0$ at $t = 0$. Find the motional emf between terminals 1 and 2.

nonuniform

$$\text{emf} = -\frac{\partial \psi}{\partial t} = -\frac{l B_0 2ut \cdot u}{2}$$

Electromagnetic Generator

top view



As it rotates, the width from the top view become smaller.

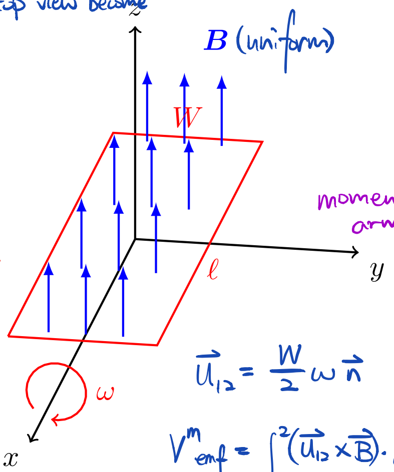
$\omega = \frac{d\theta}{dt}$

$\theta = \omega t + C$

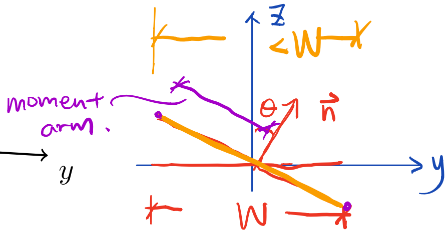
$\theta = 0$ @ $t=0$

$C = 0$

$\theta = \omega t$



Find the induced EMF in the loop if it rotates at an angular velocity ω within a constant magnetic field $\boldsymbol{B}$.



Only segments $\overline{12}$ and $\overline{34}$ cuts through $\vec{B}$, segment $\overline{12}$ moves w/ a linear velocity. = moment arm × angular velocity.

$$\vec{u}_{12} = \frac{W}{2}\omega\hat{n}$$

$$V^m_{emf} = \int_1^2 (\vec{u}_{12}\times\vec{B})\cdot d\vec{l}_{12} + \int_3^4 (\vec{u}_{34}\times\vec{B})\cdot d\vec{l}_{34} = \int_{pt.1}^{pt.2} \frac{W}{2}\omega\hat{n}\times(B_0\hat{z})\cdot(-\hat{x}\,dx)$$

$$= \ell\frac{W}{2}\omega B_0\sin\theta$$

Total EMF

$$\text{Total EMF} = V_{\text{emf}}^{tr} + V_{\text{emf}}^{m}$$

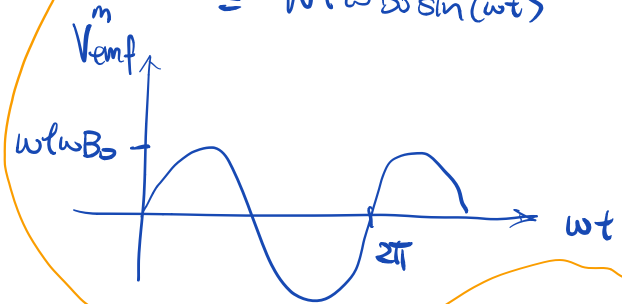
$$-\frac{\partial\Psi}{\partial t} = -\int_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S} + \oint (\vec{u}\times\vec{B})\cdot d\vec{l}$$

$$= \frac{wl}{2}\,\omega B_0 \sin(\omega t)$$

$+$

一样的

$$= wl\,\omega B_0 \sin(\omega t)$$



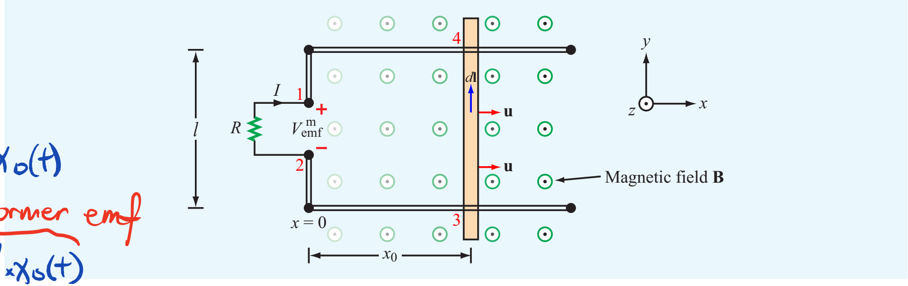
Example: Sliding Bar Revisited

$$\Phi(t) = \int_S \vec{B}\cdot d\vec{s}$$

$$= B\cdot A$$

$$= B_0 \sin(\omega t) \times l \times x_0(t)$$

$$-\frac{d\Phi}{dt} = -\Big[\overbrace{B_0\omega\cos(\omega t)\times l \times x_0(t)}^{\text{transformer emf}} + \underbrace{l\,\underbrace{\frac{dx_0}{dt}}_{u} B_0 \sin(\omega t)}_{\text{motion emf}}\Big]$$



Source: Ulaby, Ravaioli: *Fundamentals of Applied Electromagnetics*, 7th ed.

A rectangular loop has a constant width l but its length x_0 increases with time as the conducting bar slides with a uniform velocity $\boldsymbol{u}$ in a **time-varying** magnetic field $\boldsymbol{B} = \hat{\boldsymbol{z}} B_0 \sin(\omega t)$. The bar starts from $x = 0$ at $t = 0$. Find the total emf between terminals 1 and 2.

$\vec{B}$ is uniform in space but it varies with time.

Updating Maxwell's Equations

- We now revisit Ampère's law in the time-varying case.
- For statics, we know

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

wrong if there's time variation

- We know that the divergence of a curl must be zero (vector identity):

①

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{F}) = 0$$

- Let's take the divergence of Ampère's law.

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \boldsymbol{H} = \nabla \cdot \boldsymbol{J}. = -\frac{\partial \rho_v}{\partial t} \overset{?!!}{=} 0$$

- Recall the equation of continuity,

equation of continuity.

②

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\oint_S \vec{J} \cdot d\vec{s} = -\frac{\partial Q}{\partial t}$$

maxwell proposed (1865) to amend Ampere's law so that

$$\nabla \times \vec{H} = \vec{J} + \vec{G}$$

$$\nabla \cdot \vec{G} = \frac{\partial \rho_v}{\partial t}$$

Gauss's law says that $\nabla \cdot \vec{D} = \rho_v$ $\rightarrow$ $\frac{\partial}{\partial t}(\nabla \cdot \vec{D}) = \nabla \cdot \frac{\partial \vec{D}}{\partial t} = \nabla \cdot \vec{G}$

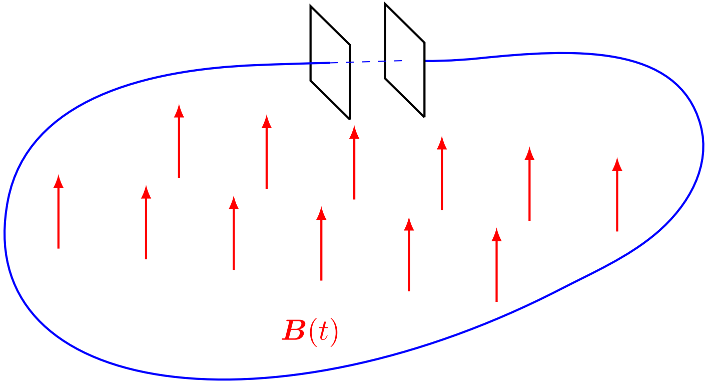
$$G = \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\nabla \times \vec{H} = \underbrace{\vec{J}}_{\text{conduction current density}} + \underbrace{\frac{\partial \vec{D}}{\partial t}}_{\text{displacement current density}}}$$

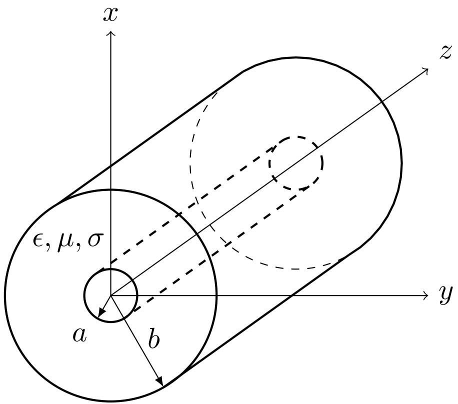
Ampere maxwell's law.

Displacement Current

Consider a closed circuit with a parallel-plate capacitor, in which the induced EMF is $V_0 \cos \omega t$.



Example: Coaxial Cable



Let the internal dimensions of a coaxial capacitor be $a = 1.2$ cm, $b = 4$ cm, and $l = 40$ cm. The material within the capacitor has the parameters $\epsilon = 10^{-11}$ F/m, $\mu = 10^{-5}$ H/m, and $\sigma = 10^{-5}$ S/m. The electric field intensity between the cylinders is

$$\boldsymbol{E} = \frac{10^6}{r} \cos 10^5 t \hat{\boldsymbol{\rho}}$$

Find (a) $\boldsymbol{J}$; (b) the total conduction current I through the capacitor; (c) the total displacement current through the capacitor I_d.