Lecture 10: Materials and Currents

ECE221: Electric and Magnetic Fields

Prof. Sean V. Hum

Winter 2019

Outline

1. Properties of Materials
2. Currents and Current Densities
3. Behaviour of Conductors in Electric Fields

Properties of Materials

siemens.

ε constitutive parameter.

μ

σ

We can broadly characterize materials using their *conductivity* σ [S/m or ℧/m or $\Omega^{-1}\mathrm{m}^{-1}$].

- Materials with *high conductivity* $(\sigma \gg 1)$ are *conductors*.
- Materials with *low conductivity* $(\sigma \ll 1)$ are *dielectrics*. (a.k.a insulators)
- Materials between these extremes are *semiconductors*.

We will be mainly concerned with conductors and dielectrics in this course.

A perfect electrical conductor (PEC) has $\sigma \to \infty$.
A perfect dielectric has $\sigma = 0$.

Current I and Current Density $\boldsymbol{J}$

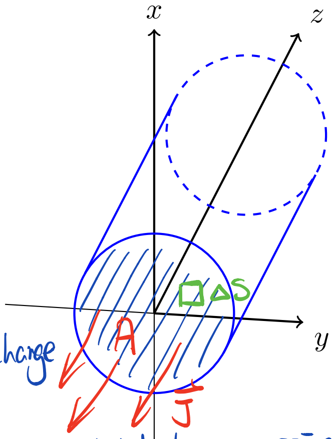
- **Voltage** (or potential difference) and **current** are fundamental quantities in EE.
- Consider the current flowing in a wire as an example.

current = rate of movement of charge across a reference plane.

$I = [C/s]$, $\underbrace{[C/s]}_{A = \text{Amperes.}}$

$$\boxed{I = \frac{dQ}{dt}}$$

Q = total charge passing through ref. plane.



$\vec{J}$ = current density $[A/m^2]$

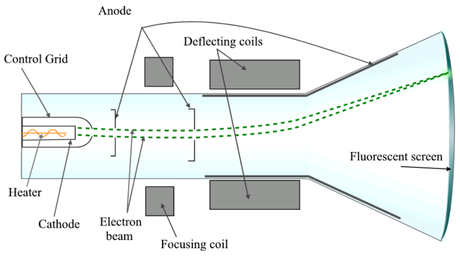
$\Delta I = J_N \Delta S$ ← normal component

$= \vec{J} \cdot \Delta \vec{S}$

$$I = \iint_S \vec{J} \cdot d\vec{s}$$

Convection Current

- Convection current does **not** involve conductors.
- It occurs when charges flow through an insulator (e.g. vacuum), such as an electron beam in a cathode ray tube, electron microscope, linear accelerator, etc.

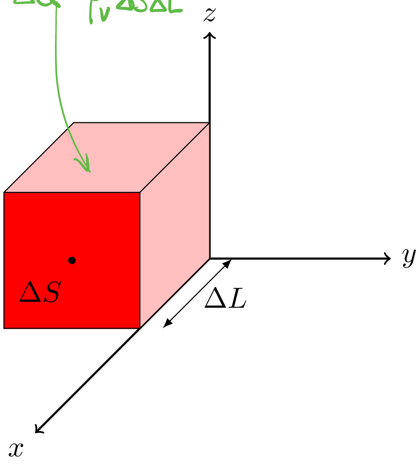


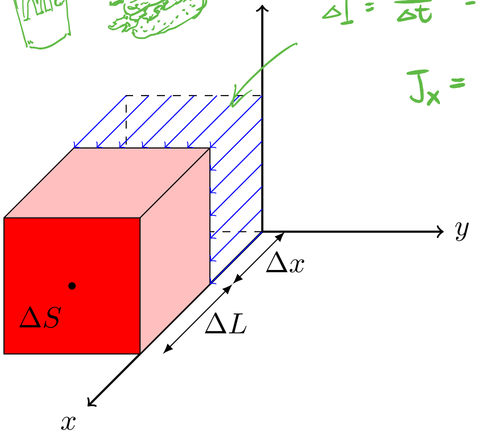
Source: Wikipedia

Convection Current

$\Delta V = \Delta S \Delta L$

$\Delta Q = \rho_v \Delta S \Delta L$





$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \Delta S \boxed{\frac{\Delta x}{\Delta t}}_{u_x}$$

$$J_x = \lim_{\Delta S} \frac{\Delta I}{\Delta S} = \rho_v u_x$$

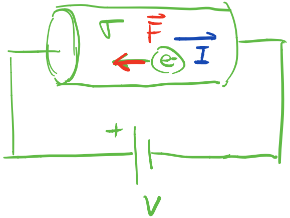
$\downarrow$ generalize.

$$\boxed{\vec{J} = \rho_v \vec{u}}$$

convection current density.

Conduction Current and Ohm's Law in Point Form

In conductors, free (valence) electrons move under the influence of an applied $\vec{E}$ field.



$$\vec{F} = q\vec{E} = -e\vec{E}$$

e = electron charge = 1.602×10^{-19} C

electrons does not accelerate in the conductor, it constantly collides w/ the lattice structure of the conductor, slowing it down, to a constant average velocity called drift velocity

Properties of Materials

Newton's Law says avg. change in momentum of the free e^- must be equal to the applied force on e^-

momentum = mass × velocity.

Table 4-1: Conductivity of some common materials at 20°C.

Material	Conductivity, σ (S/m)
Conductors	
Silver	6.2×10^7
Copper	5.8×10^7
Gold	4.1×10^7
Aluminum	3.5×10^7
Iron	10^7
Mercury	10^6
Carbon	3×10^4
Semiconductors	
Pure germanium	2.2
Pure silicon	4.4×10^{-4}
Insulators	
Glass	10^{-12}
Paraffin	10^{-15}
Mica	10^{-15}
Fused quartz	10^{-17}

mass of e^-

$$\frac{m_e \vec{u}}{\tau} = -e\vec{E} \rightarrow \vec{u} = \text{drift velocity} = \boxed{\frac{-e\tau}{m_e}\vec{E}}$$

τ = average time btw collisions.

μ_e = electron mobility in the conductor

If there're n electrons per unit volume

$$\rho_v = -ne$$

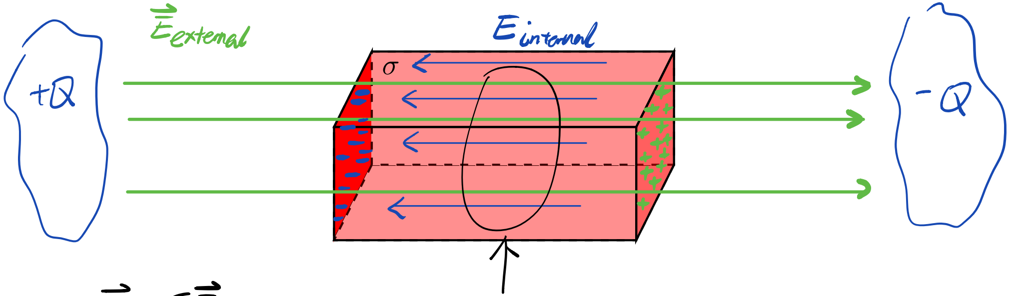
$$\vec{J} = \rho_v \vec{u} = \frac{ne^2\tau}{m_e}\vec{E}$$

σ = conductivity.

$$\boxed{\vec{J} = \sigma \vec{E}}$$ ohm's law in point form.

like I, like G $\left(\frac{1}{R}\right)$, like V

Behaviour of Conductors Under an Applied Electric Field



$\vec{J} = \sigma \vec{E}$

$\vec{E} = \vec{J}/\sigma$

PEC: $\lim_{\sigma \to \infty} \vec{J}/\sigma = 0$

$\vec{E}_T = \vec{E}_{external} + \vec{E}_{internal} = 0$

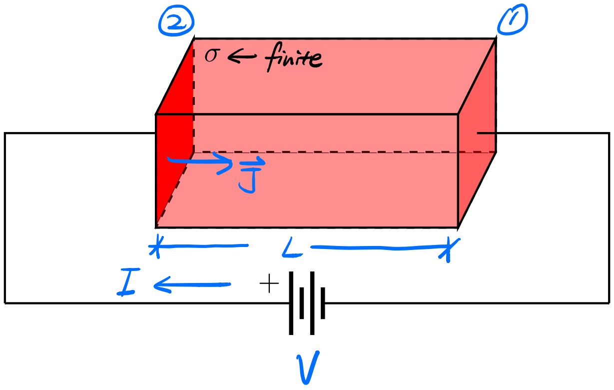
In <u>perfect electric conductors</u>, the electrostatic field $\vec{E} = 0$

PEC

$\Delta V = 0$

(PEC's are equipotential surfaces)

Behaviour of Conductors Under an Applied Electric Field



$$I = \iint_S \vec{J}\cdot d\vec{S}$$

if conductivity is uniform in this block

$$I = JA$$

$$J = I/A = \sigma E = \sigma \frac{V}{L}$$

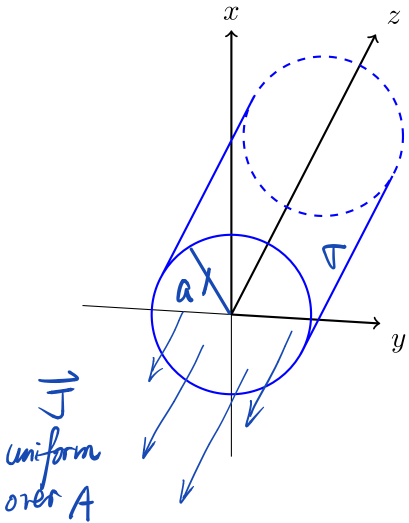
$$V = \frac{L}{\sigma A} I$$

$$\frac{V}{I} = \boxed{\frac{L}{\sigma A} = R}$$

$$V_{21} = -\int_1^2 \vec{E}\cdot d\vec{l} = EL$$

In general $R = \frac{V}{I} = \frac{-\int_1^2 \vec{E}\cdot d\vec{l}'}{\iint \sigma \vec{E}\cdot d\vec{S}'}$

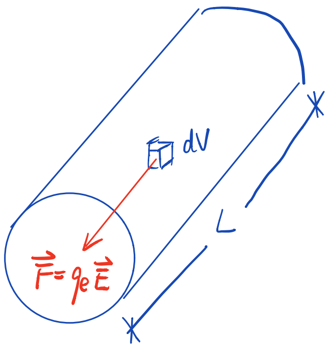
Example: Cylindrical Resistor



$A = \pi a^2$

$R = \dfrac{L}{\sigma \pi a^2}$

Joule's Law



electron charge contained in ΔV

$$q_e = \rho_v \Delta V$$

$$F = \vec{E} = q_e \vec{E} = \rho_v \Delta V \vec{E}$$

Total work expended by $\vec{E}$ in moving Δq a distance $\Delta \vec{L}$

$$\Delta W = \vec{F} \cdot \Delta \vec{L}$$

$$\text{power} = \text{work per unit time} = \frac{\Delta W}{\Delta t}$$

$$= \vec{F} \cdot \frac{\Delta \vec{L}}{\Delta t}$$

$$= \rho_v \Delta V \vec{E} \cdot \vec{u}$$

$$\Delta P = \vec{E} \cdot \vec{J} \Delta V$$

Integrate over volume of conductor.

$$P = \iiint_V \vec{E} \cdot \vec{J} \, dv \quad [W] \quad = \iiint_V \sigma |\vec{E}|^2 \, dv$$