(*) Find the CT FT of: $x(t) = e^{-a|t|}$, $a>0$

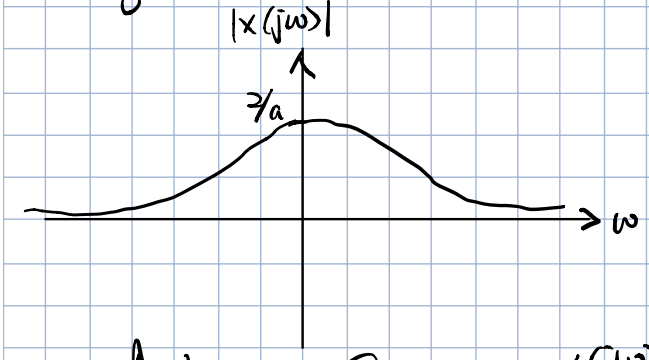
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\, dt \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t}\, d\omega$$

$$e^{-a|t|} = \begin{cases} e^{-at} & t>0 \\ e^{at} & t<0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{0} e^{+at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt.$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt.$$

magnitude.



$$= \frac{1}{a-j\omega} e^{(a-j\omega)t}\Big|_{-\infty}^{0} + \frac{-1}{a+j\omega} e^{-(a+j\omega)t}\Big|_{0}^{\infty}$$

$$= \frac{1}{a-j\omega}(1-0) + \frac{-1}{a+j\omega}(0-1)$$

Angle is 0 //

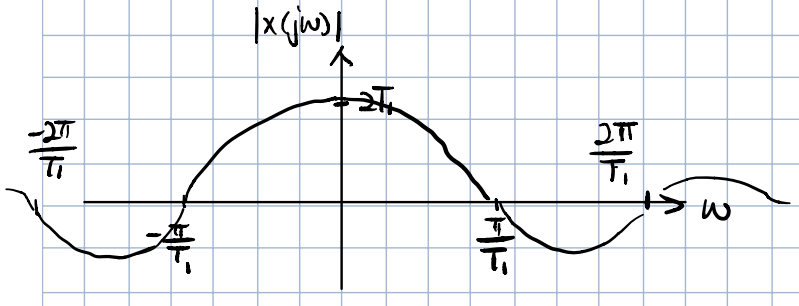
$$X(j\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$ //

(*) Find the CT FT of: $x(t) = \begin{cases} 1 & |t|<T_1 \\ 0 & |t|>T_1 \end{cases}$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\, dt$$

$$= \int_{-T_1}^{T_1} e^{-j\omega t}\, dt = \frac{1}{-j\omega} e^{-j\omega t}\Big|_{-T_1}^{T_1}$$

$$= \frac{-1}{j\omega}\left(e^{-j\omega T_1} - e^{j\omega T_1}\right)$$



$$= \frac{1}{j\omega} 2j\sin(j\omega T_1)$$

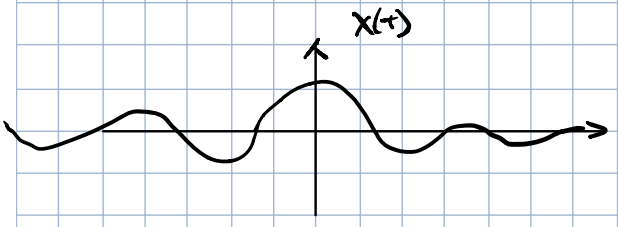
$$X(j\omega) = \frac{2\sin(\omega T_1)}{\omega}$$

(*) Find the inverse FT of:

$$X(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases} \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$$

$$x(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t} d\omega = \frac{1}{2\pi}\,\frac{1}{jt} e^{j\omega t}\Big|_{-W}^{W}$$

$$= \frac{1}{2\pi}\left[\frac{1}{jt}\left(e^{jWt} - e^{-jWt}\right)\right]$$



$$= \frac{1}{2\pi}\left[\frac{1}{jt}\, 2j \sin(Wt)\right]$$

$$= \frac{1}{2\pi}\,\frac{2}{jt}\sin(Wt)$$

$$= \frac{\sin(Wt)}{\pi t} \quad //$$

FT of periodic signals.

$$X(j\omega) = 2\pi\,\delta(\omega - \omega_0)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 2\pi\,\delta(\omega-\omega_0)\, e^{j\omega t} d\omega = e^{j\omega_0 t}\int_{-\infty}^{+\infty}\delta(\omega-\omega_0)\, d\omega$$

$$\rightarrow x(t) = e^{j\omega_0 t}$$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k\, \delta(\omega - k\omega_0)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$$

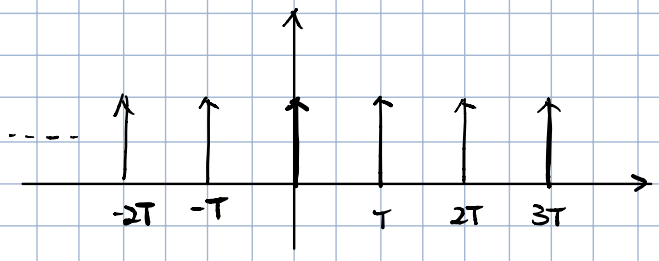
Synthesis Relation for FS analysis.

$$x(t) = \sin(t) \qquad a_1 = \frac{1}{2j} \qquad a_{-1} = \frac{-1}{2j}$$

$$X(j\omega) = \frac{\pi}{j}\,\delta(\omega-\omega_0) - \frac{\pi}{j}\,\delta(\omega+\omega_0)$$

(k) Find the FT of:

$$x(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)$$



$$a_k = \frac{1}{T}\int_T x(t) e^{-j\omega t}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t) e^{-j\omega t}\, dt = 1/T$$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{2\pi}{T}\, \delta(\omega - k\omega_0)$$

$\omega_0 = \frac{2\pi}{T}$