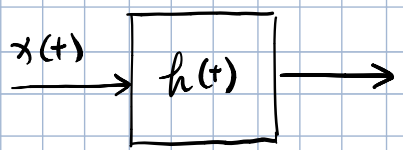
Properties of LTI systems :

$x[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

$x(t)$



$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$

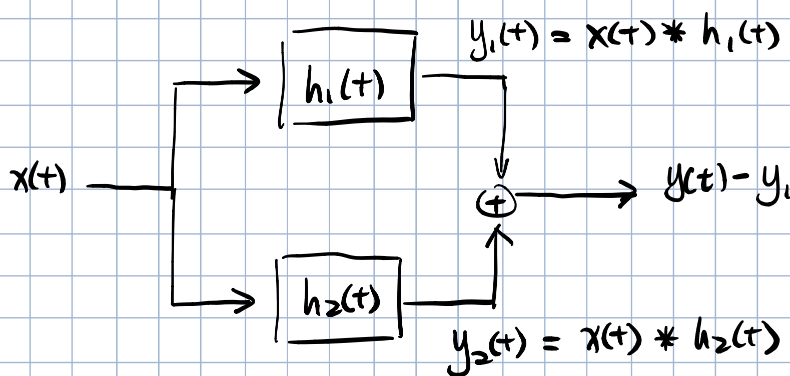
① Commutative property:

$$x(t) * h(t) = h(t) * x(t)$$

② Distributive property:

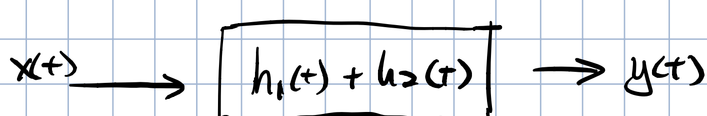
$$x(t) * (h_1(t) + h_2(t)) = x(t) * h_1(t) + x(t) * h_2(t)$$

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$



$$y(t) = y_1(t) + y_2(t) = x(t) * h_1(t) + x(t) * h_2(t)$$

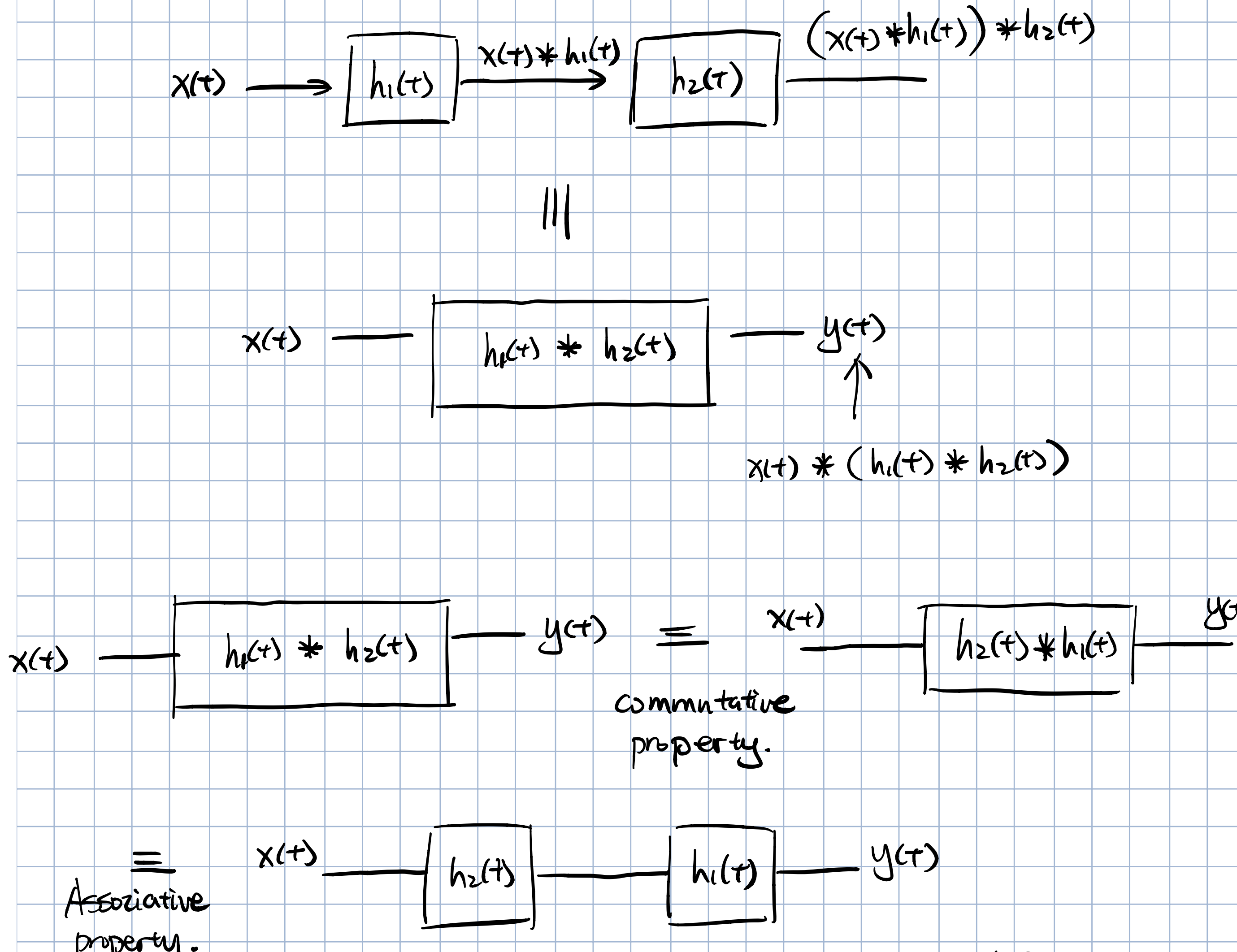
Distributive property: $y(t) = x(t) * (h_1(t) + h_2(t))$



③ Associative property:

$$(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t))$$

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$



(this means you can change the order)

④ LTI systems with and without memory.

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

If the system is memory-less, $y[n]$ depends only on $x[n]$.

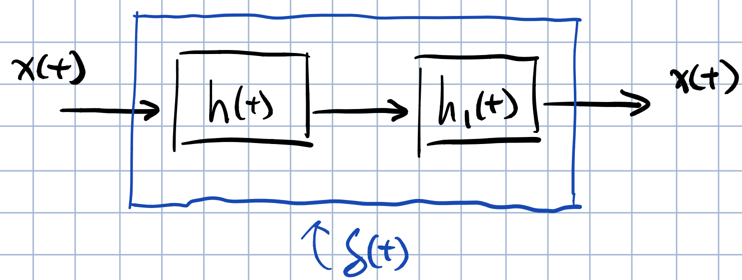
For any k that is not equal to n, $h[n-k] = 0$.

$\Rightarrow h[n]=0$ for $n\neq 0$ $\Rightarrow h[n] = m\,\delta[n]$

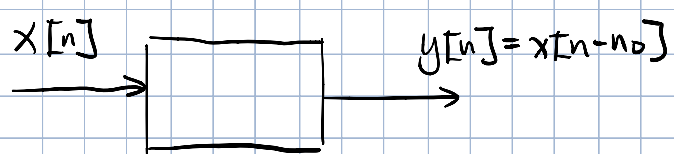
⑤ Invertibility: If an LTI system w/ impulse response $h(t)$ is invertible then there is an $h_1(t)$ that satisfies.

$$h(t) * h_1(t) = \delta(t)$$

DT version: $h[n] * h_1[n] = \delta[n]$



e.g.



$h[n] = \delta[n-n_0]$ invertible.

$h_1[n] = \delta[n+n_0]$

$\delta[n-n_0] * \delta[n+n_0] = \delta[n]$

⑥ Causality of LTI systems:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$

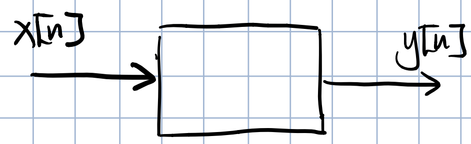
For a causal system, $y[n]$ depends of $x[k]$, $k \le n$ (current and past input)

$\longrightarrow$ $x[k]$ for $k>n$ should be multiplied by zero in the above sum.

↳ $h[n-k]=0$ if $k>n$

↳ $h[n]=0$, for $n<0$

7) Stability of LTI systems:



bounded input gives bounded output.

$$|x[n]| \le B \longrightarrow |y[n]| \le C$$

For a LTI system, $$|y[n]| = \Bigg|\underbrace{\sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]}_{x[n]*h[n] \,=\, h[n]*x[n]}\Bigg| = \left|\sum_{k=-\infty}^{+\infty} h[k]\,x[n-k]\right|$$

$$\le \sum_{k=-\infty}^{+\infty} |h[k]|\,|x[n-k]|$$

$$\le B\underbrace{\sum_{k=-\infty}^{+\infty} |h[k]|}_{D} \le C$$

For a stable system,

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$