Continuous-Time Fourier Transform.

$$A_1 e^{j\omega_1 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = A_1 e^{j\omega_1 t} * h(t) = \int_{-\infty}^{\infty} A_1 e^{j\omega_1 \tau} h(t-\tau)\, d\tau$$

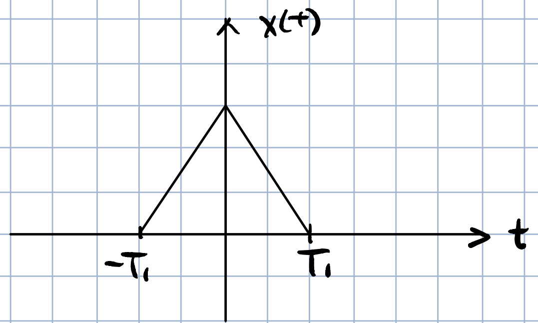
commutative prop.

$$= \int_{-\infty}^{+\infty} A_1 e^{j\omega_1 (t-\tau)} h(\tau)\, d\tau$$

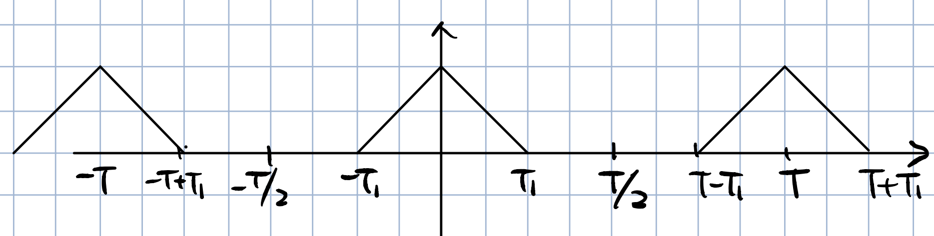
$$= A_1 e^{j\omega_1 t} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-j\omega_1 \tau}\, d\tau}_{\text{eg. } A_1' e^{j\phi'}} = A_1 A_1' e^{j(\omega_1 t + \phi')}$$

$$x(t) = \sum_{k=-\infty}^{+\infty} A_k e^{jk\omega t} \longrightarrow \boxed{\begin{matrix} h(t) \\ \text{LTI} \end{matrix}} \longrightarrow y(t) = \sum_{k=-\infty}^{+\infty} B_k e^{jk\omega t}$$

Consider arbitrary $x(t)$ that is zero for $t > T_1$



$x(t)$ can be used to develop a periodic signal., $\tilde{x}(t)$ with period T, where $T/2 > T_1$



$$\tilde{x}(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T} \qquad (1)$$

synthesis fourier Series.

$$a_k = \frac{1}{T}\int_T \hat{x}(t)e^{-jk\omega_0 t}dt \quad ②$$ Analysis fourier series.

replace $\hat{x}(t)$ with $x(t)$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-\infty}^{+\infty} x(t)e^{-jk\omega_0 t}dt \quad ③$$

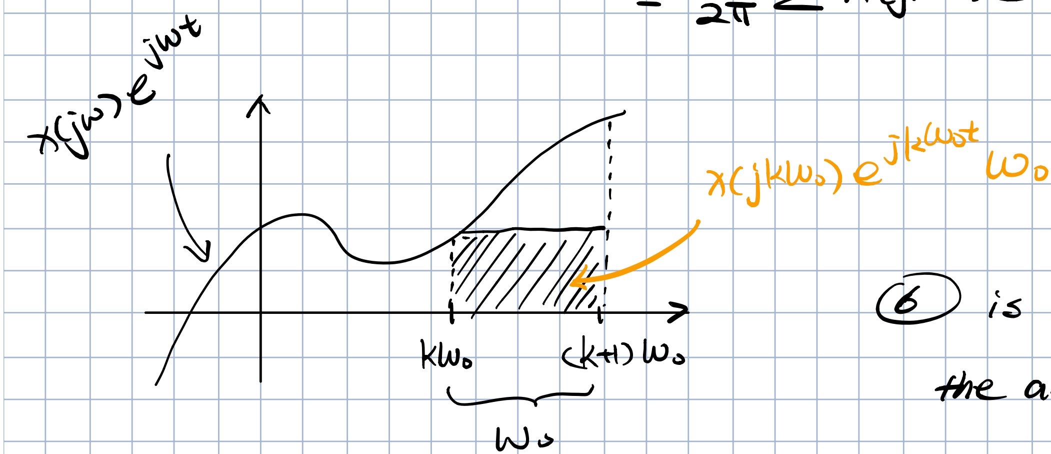
If we define $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$ ④

→ Then $a_k = \frac{1}{T}X(jk\omega_0)$ ⑤

By plugging ⑤ into ①: $\hat{x}(t) = \sum \frac{1}{T}X(jk\omega_0)e^{jk\omega_0 t}$

$$= \frac{1}{2\pi}\sum X(jk\omega_0)e^{jk\omega_0 t}\omega_0 \quad ⑥$$

Since $T = \frac{2\pi}{\omega_0}$, $\frac{1}{T} = \frac{\omega_0}{2\pi}$



⑥ is an approximation of the area under $X(j\omega)e^{j\omega t}$

If $\omega_0 \to 0$ then Σ in ⑥ turns into $\int$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega} \quad ⑦$$

$\omega_0 \to 0$

$\frac{2\pi}{\omega_0} = T \to \infty$

$\hat{x}(t)$ turns to $x(t)$

Synthesis relation.
(Inverse Fourier transform relation)

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt} \quad ⑧ \quad (\text{see } ④)$$

Analysis Relation.
(Fourier transform relation)

→ $X(j\omega)$ is called the fourier transform of $x(t)$, and uniquely describes $x(t)$ in the frequency domain.

→ $X(j\omega)$ is also called the frequency spectrum of $x(t)$

→ If $x(t)$ conforms to Dirichlet Conditions, the integral in ⑧ converges to a finite number.

⊛ Find the Fourier transform of:

a) $x(t) = e^{-at}u(t)$, $a > 0$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} e^{-at} u(t) e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{+\infty} u(t)\, e^{(-a-j\omega)t}\, dt$$

$$= \int_{0}^{\infty} e^{(-a-j\omega)t}\, dt = \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= -\frac{1}{a+j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty}$$

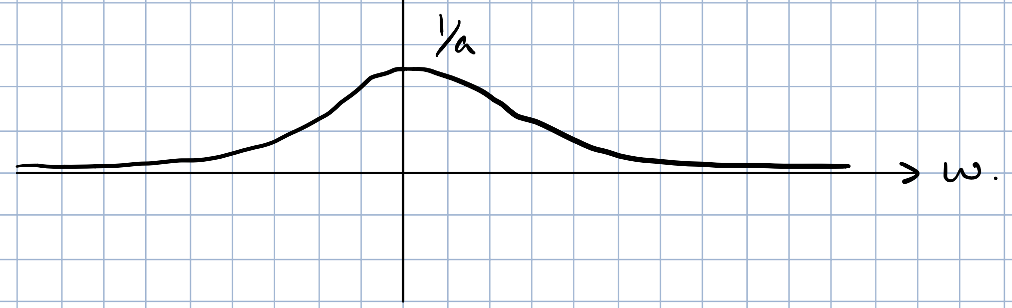
$$e^{-a}\, \underbrace{e^{-j\omega t}}_{\cos(\omega t) - j\sin(\omega t)}$$

must be number (finite)

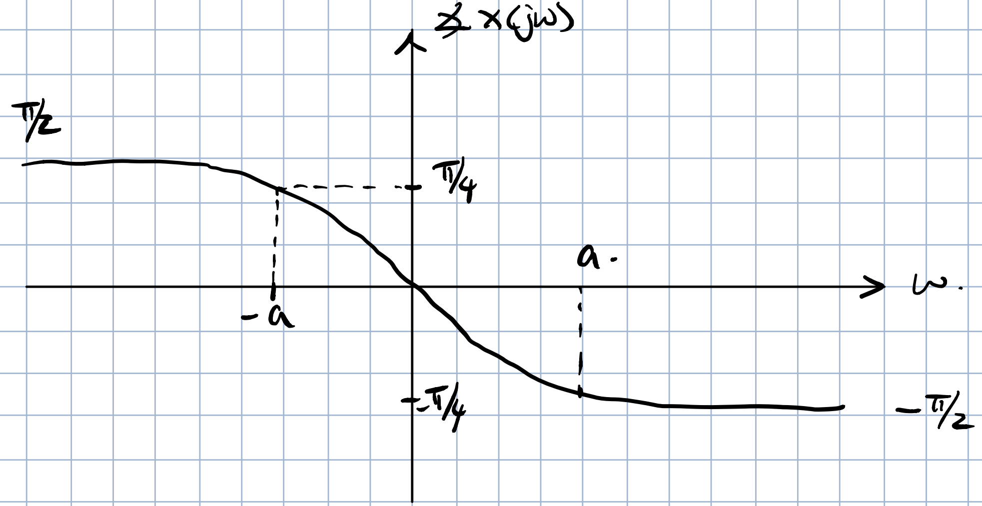
$$= -\frac{1}{a+j\omega}(0-1) = \frac{1}{a+j\omega} \,//$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \measuredangle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

↑ $|X(j\omega)|$



1/a
ω.



∡ X(jω)
π/2
π/4
a.
ω.
-a
-π/4
-π/2