Sampling theorem.

Let $x(t)$ be a band-limited signal (i.e. $X(j\omega)=0$ for $\omega > |W_M|$)
Then $x(t)$ can be uniquely determined by its samples
$x(nT)$, $n = 0, \pm 1, \pm 2, \cdots$ if $W_s > 2W_M$, where $W_s = \frac{2\pi}{T}$

$x(t)$ can be reconstucted by generating a impulse train w/ amplitudes equal to the sampled values. If a low-pass filter with cut-off freq btw W_M and W_s and gain T in the passband. is applied to this impulse train, then $X(t)$ is recovered.

$$p(t) = \sum_{k=-\infty}^{+\infty} \delta(t-nT)$$

The frequency $2W_M$ that must be exceeded by W_s (the sampling freq) is referred to as the Nyquiste rate.

Parsevals Theorem for CTFT:

$$\int_{-\infty}^{+\infty} |x(t)|^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

⊛ For $x(t)$ and $y(t)$:

$X(j\omega) = 0$ for $|\omega| > W_x$

$Y(j\omega) = 0$ for $|\omega| > W_y$

What's the Nyquist rate for :

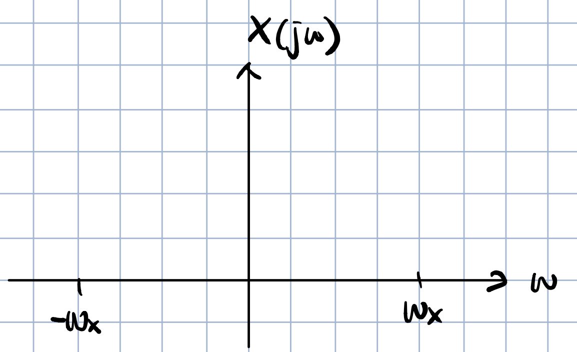
a) $x(t) + y(t)$

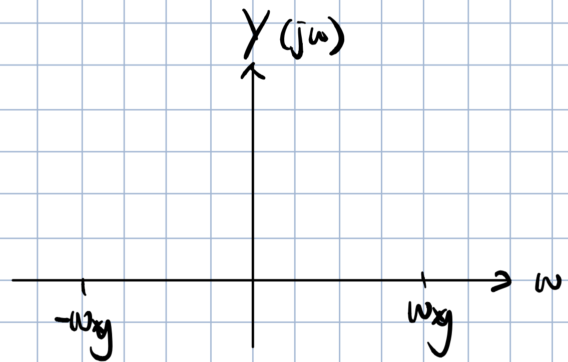
b) $x(t-1) + y(t+2)$

c) $x(t)\sin(w_0 t)$

d) $x(t)\,y(t)$

e) $x(t)\,u(t)$





a) $x(t) + y(t) \overset{f}{\longleftrightarrow} X(j\omega) + Y(j\omega)$

$X(j\omega) + Y(j\omega) = 0$ for $|w| > \max(W_x, W_y)$

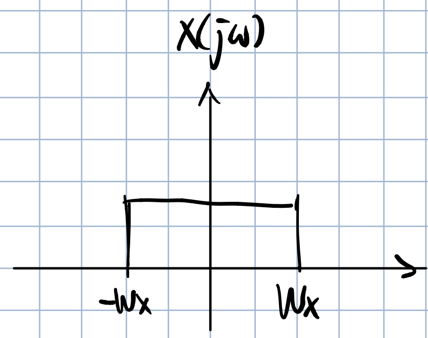
Nyquiste rate $= 2\max(W_x, W_y)$

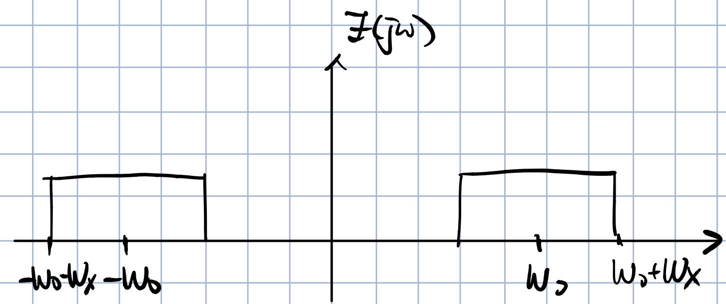
b) $x(t-1) + y(t+2) \overset{f}{\longleftrightarrow} \underbrace{e^{-j\omega} X(j\omega)}_{=0 \text{ if } |W| > W_x} + \underbrace{e^{j\omega 2} Y(j\omega)}_{=0 \text{ if } |w| > W_y}$

Nyquiste rate $= 2\max(W_x, W_y)$

c) $x(t)\sin(w_0 t) = \frac{1}{2j} x(t)\, e^{jW_0 t} - \frac{1}{2j} x(t)\, e^{-jW_0 t} = z(t)$

$f\{z(t)\} = \frac{1}{2j} X(j(w - w_0)) - \frac{1}{2j} X(j(w + w_0))$

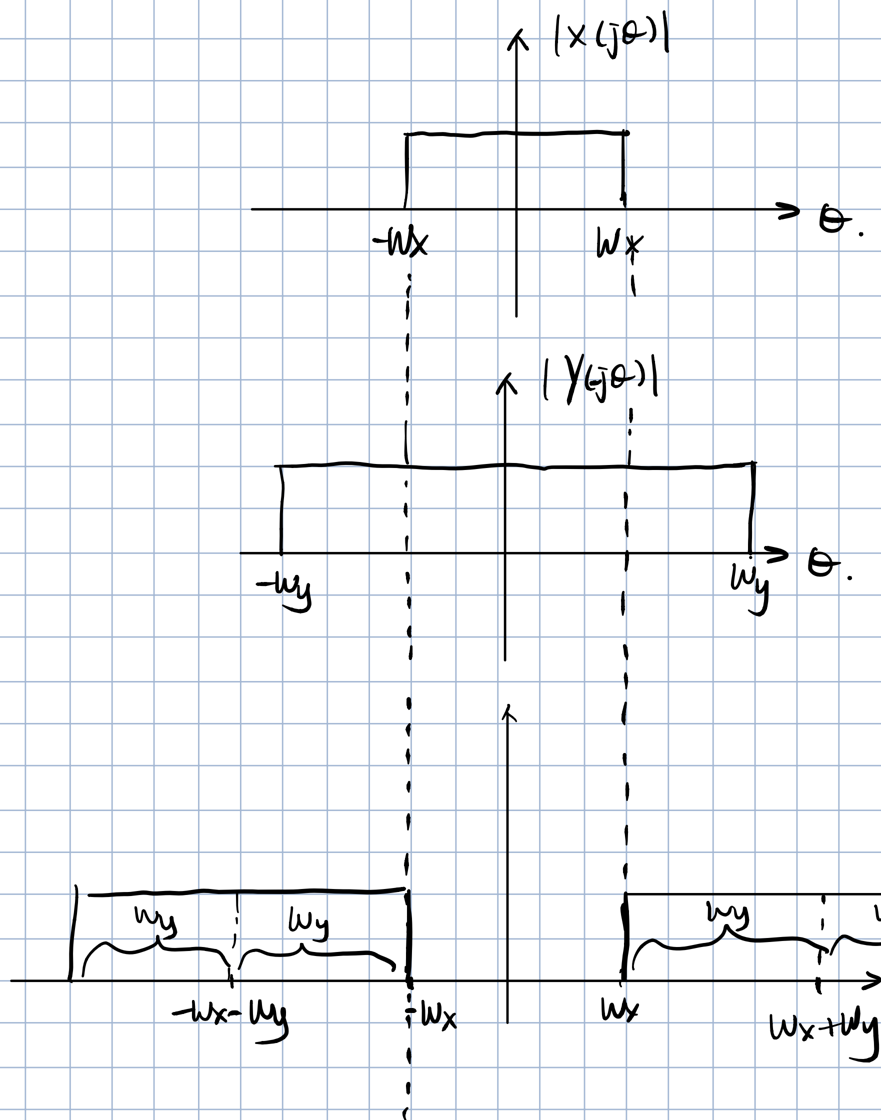




$Z(j\omega) = 0$ for $|\omega| > (\omega_0 + W_x)$

$\Rightarrow$ Nyquist rate = $2(\omega_0 + W_x)$

d) $x(t)\,y(t) = z(t)$ $\quad Z(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * Y(j\omega)\right)$



$$Z(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta)\, Y(j(\omega-\theta))\, d\theta$$

$\therefore$ $Z(j\omega) = 0$ for

$|\omega| > (W_x + W_y)$

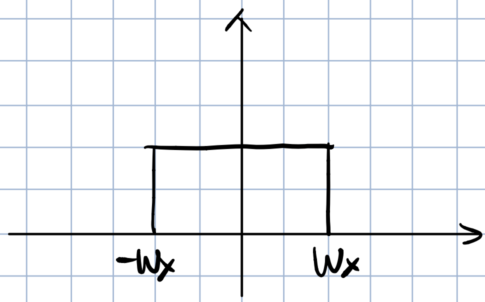
Nyquist rate =

$2(W_x + W_y)$

e)

$$\mathcal{F}\{u(t)\} = \frac{1}{j\omega} + \pi\delta(\omega)$$

$z(t) = x(t)\,u(t)$, $Z(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * U(j\omega)\right)$



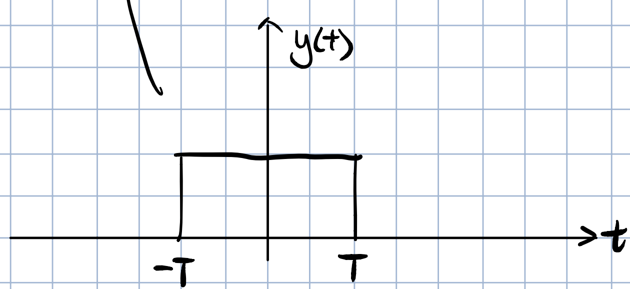
but u(t) is not band-limited,

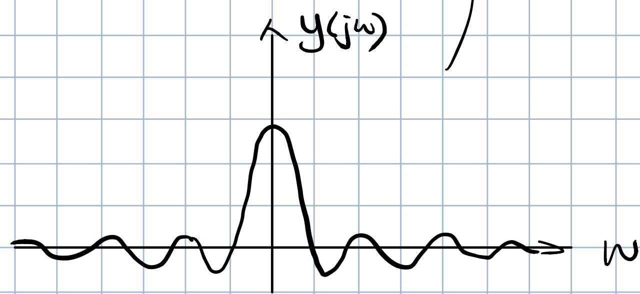
$\Rightarrow$ z(t) is not band-limited.

$\rightarrow$ No Nyquist rate.

$$y(t) = \frac{\sin(Wt)}{\pi t} \xleftrightarrow{f} Y(j\omega) = \begin{cases} 1 & |\omega| < W \\ 0 & |\omega| > W \end{cases}$$

$$y(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \xleftrightarrow{f} Y(j\omega) = \frac{2\sin(\omega T)}{\omega}$$





$$p(t) = \sum_{k=-\infty}^{+\infty} \delta(t - nT)$$

$$X(j\omega) = X_p(j\omega)\,H_{lp}(j\omega)$$

$$x(t) = x_p(t) * h_{lp}(t) \qquad h_{lp}(t) = \frac{2\sin(W_c t)}{\pi t}$$