2 The Vicsek model

How do flocks of birds or swarms of fishes work together to move so coherently in large groups? Do they have a leading fish or bird that directs the motion? Research indicates that this is not the case, instead collective motion is a spontaneous phenomenon that emerges from very local interactions of individuals with each other.

The first, and simplest, model of flocking in animal groups was proposed by Tamas Vicsek et al. (PRL 1995). Each bird is represented by a point particle that moves at constant velocity v_0 along its polar direction $\hat{n}$. At each discrete time step, each bird checks its neighbours within a radius R around its position, and then reorients itself along their mean direction. Since birds are imperfect, there will be noise added to that new direction. In a two-dimensional plane, the equations of motion for bird i at step $k+1$ are:

$$\vec{r}_i(k+1) = \vec{r}_i(k) + \hat{n}_i v_0 \tag{8}$$

$$\theta_i(k+1) = \text{angle}\left[\sum_{j=1}^{z_i} \hat{n}_j\right] + \eta_i \tag{9}$$

Here the polar direction is $\hat{n}_i = (\cos\theta_i, \sin\theta_i)$. The sum in the second equation is over all z_i neighbours within radius R, and we take the angle of the final vector. To this, we then add a normally distributed noise η_i, with mean 0 and variance σ, i.e. at each step, η_i is chosen from a $N(0,\sigma)$ normal distribution. Note that this is not a set of ODEs, but a model with a unit time step.

As a function of σ, the Vicsek model undergoes a phase transition from an aligned, moving flocking state at low σ to a randomly moving state at large values of noise σ. Simulate the Vicsek model for $R = 1$, $v_0 = 0.5$ and a number $N = 200$ individual birds for at least 1000 steps. At every step, compute the Vicsek order parameter

$$n = \frac{1}{N}\left|\sum_{i=1}^{N} \hat{n}_i\right|. \tag{10}$$

which is a measure of the alignment between birds and has values between 0 and 1. Detailed instructions; for :

1. [**core**] Create your birds with uniformly random positions and uniformly random initial orientations in a $L \times L$ container, with suggested $L = 15$, or in general roughly $L = \sqrt{N}$. You will crucially need periodic boundary conditions for the positions: When you birds cross the system boundary to the right, they should appear on the left again, going in the same direction. The same applies for birds that disappear at the top, they need to reappear at the bottom.

2. [**core**] For equation 9, you will need to determine the distances of *all* the other birds and then choose the ones closer than R. Again, you will need periodic boundary conditions here as birds will also interact with other birds "over the edge". Equation 9 is tricky to implement numerically! A clean way of doing so is to do the two-dimensional vector sum of the $\hat{n}_j$, normalize the result, and then take the angle of it (you won't need to divide by the number of neighbours z_i anymore, either).

3. [**core**] Run your system for a long time, at least $T = 5000$ steps, and compute and record the Vicsek order parameter at each step. Plot the Vicsek order parameter as a function of time. Also, make a couple of representative plots. The matplolib 'quiver' plot creates a field of arrows, and is well suited for this task.

4. [**core**] Make one or several movies of your flocking simulation at different σ.

5. Compute the $n(\sigma)$ phase transition plot: For each σ, run one simulation; your code so far should go into a function for this. Save the output n traces in a csv file for each run of your system. Finally, compute the average n as a function of σ. For this, remove the values of n for early times before the system reaches steady state from the average. Create a plot with appropriate error bars and labels.

6. Transform your flock of birds model into a model for a swarm of fish by designating a point in the middle of your system as an attractor: When your "fish" select their new direction, include a vector component toward that point. Depending on parameters, this will either give you a dense cloud of midges or a rotating swarm. Develop and compute an equivalent of the Vicsek order parameter for this system.

7. Add a bird of prey to your system, i.e. a particle that changes its orientation of motion towards the position of birds within a radius R_{pred}. Conversely, the other birds near enough the bird of prey will gain an extra term in equation 9 that turns them away from the position of the predator.