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PHYSICA A

Crossover in the Cont–Bouchaud percolation model for market fluctuations

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Abstract

Monte Carlo simulations of the Cont–Bouchaud herding model for stock market traders show power-law distributions for short times and exponential truncation for longer time intervals, if they are made at the percolation threshold in two to seven dimensions.

The fluctuations of the stock market – the price changes per unit time – are believed [1] to follow a Gaussian distribution for long time intervals but to deviate from it for short time steps. Power laws, exponentials, and multifractal descriptions have been offered to explain this short-time behavior [1].

Microscopic models dealing with the decisions of single traders on the market have tried to reproduce this behavior [2]. Possibly the simplest of these models is the herding approach of Cont and Bouchaud [3]. Here traders cluster together randomly as in percolation theory on a random graph, with infinitely long interactions instead of the usual nearest-neighbor percolation on lattices [4]. (Ref. [3] has therefore the critical exponents of the Flory–Stockmayer percolation theory [5].) Each cluster trades with probability a (called activity); if it trades, it gives with equal probability a buying or selling demand proportional to the cluster size. The total demand is then the difference between the sum of all buying orders and the sum of all selling orders, received during the time step Δt. The price P changes from one time step to the next by an amount proportional to this demand (which can be positive or negative). Thus with n_s clusters containing s traders each, and with cluster i described by $\Phi_i = +1$ if it wants to buy, by $\Phi_i = -1$ if it wants to sell, and by $\Phi_i = 0$ if it does not trade, the price

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change $\Delta P = P(t+\Delta t) - P(t)$ is

$$\Delta P \propto \sum_i s_i \Phi_i \,, \tag{1}$$

where s_i is the number of traders in cluster i. Since $n_s \propto s^{-5/2} \exp(-\text{const.} \cdot s)$ in this Flory–Stockmayer limit, Cont and Bouchaud could solve their model analytically without Monte Carlo simulation.

Instead, we consider this model on the nearest-neighbor lattice in d dimensions, i.e. on the square lattice ($d=2$), simple cubic lattice ($d=3$), or hypercubic lattice ($d=4$–7). This requires computer simulations with random numbers ("Monte Carlo") and a precise definition of the updating rule connected with the time step Δt.

For both the original model [3] and our modification, the main simplification is to ignore the history of the price changes as well as the limitations in the disposable capital of each investor. No fundamental economic information on the traded stocks or currencies enters into the model. Instead, the traders cluster together randomly by sharing their random opinions.

In our simulation, each site of the d-dimensional lattice is randomly occupied with probability p and empty with probability $1-p$; clusters are groups of occupied nearest neighbors. For p below some threshold p_c, all clusters are finite; for $p > p_c$ also one spanning cluster exists which connects one end of the sample to the other. Thus, depending on the parameter p most of the traders are isolated (small p), most of them form one huge group (large p), or clusters of all sizes occur (p near percolation threshold).

For large but finite sizes s the cluster numbers behave as

$$n_s(p<p_c) \propto s^{-\theta} \exp(-\text{const.} \cdot s) \,, \tag{2a}$$

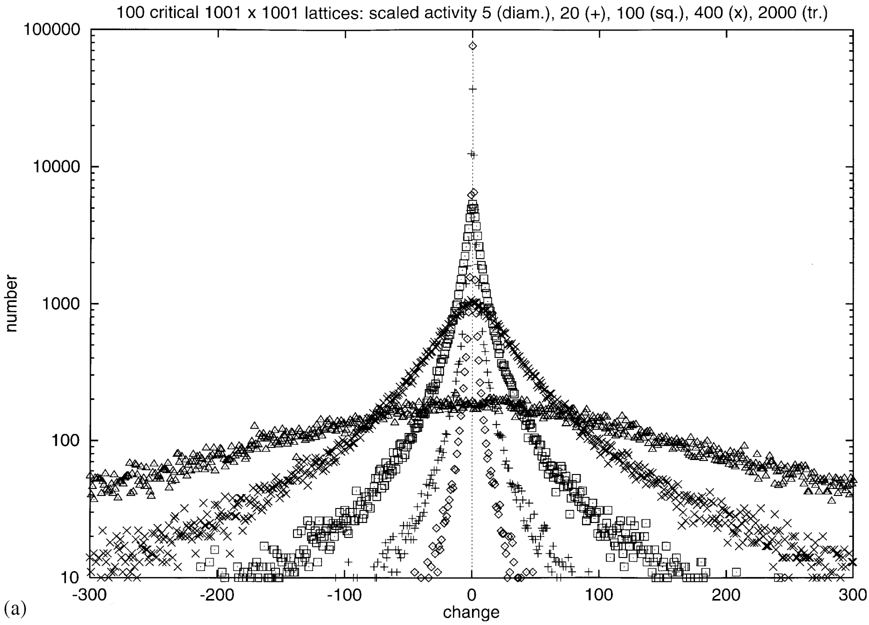
$$n_s(p=p_c) \propto s^{-\tau} \,, \tag{2b}$$

$$n_s(p>p_c) \propto s^{-\theta'} \exp(-\text{const.} \cdot s^{1-1/d}) \tag{2c}$$

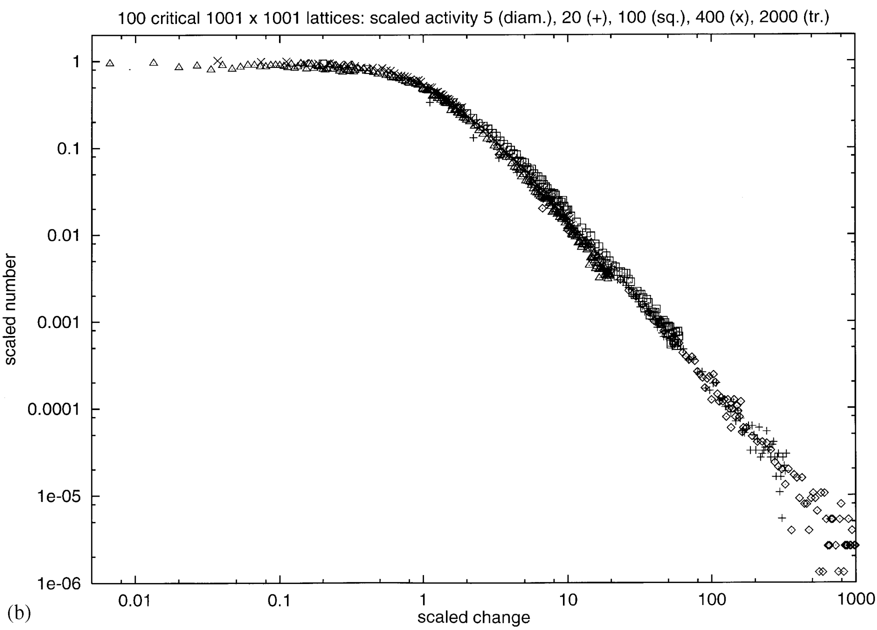
according to standard percolation theory [4]; only for $d>6$ we have $\theta=\theta'=\tau=5/2$ as used in Ref. [3].

If we take our time step Δt so small that exactly one cluster of traders issues a demand during this time interval, then the probability distribution function ("histogram") for the price changes ΔP is proportional to the cluster numbers n_s and nothing new remains to be simulated. If Δt is large so that all clusters are dealt with once in each time interval, then numerous clusters issue their random buy or sell orders and the histogram for the price changes becomes Gaussian. Novel non-Gaussian histograms thus can be expected if the time step is so small that the number of clusters trading within it is of the order of unity, on average. Since we take Δt as our time unit, we achieve this limit by taking the activity a as very small, proportional to $1/L^d$, the reciprocal number of sites in our lattice of linear extent L.

Our computer simulation first distributes sites randomly on the lattice, then determines the resulting clusters. Now for each time step we let each of these clusters determine randomly whether it is active or does not trade. The trading clusters choose

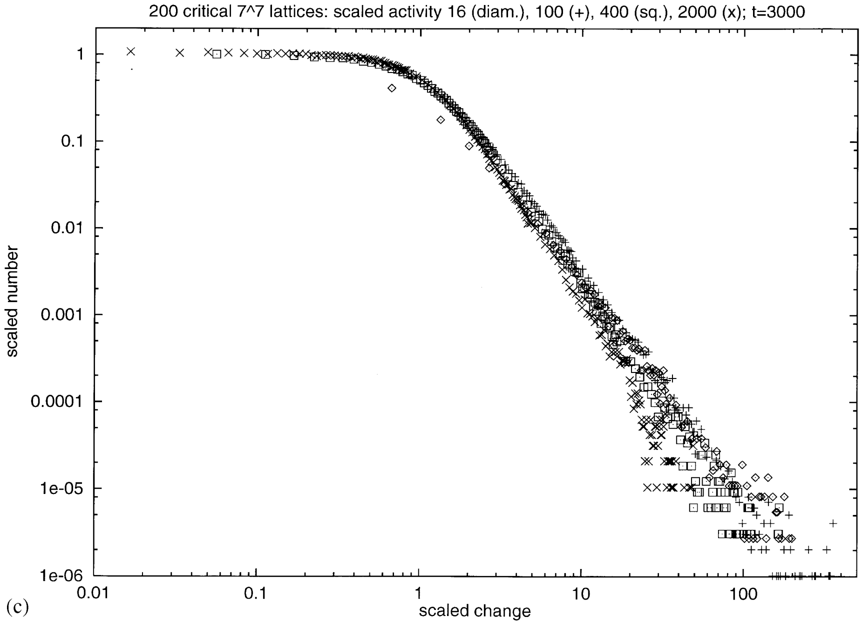


(a)



(b)

Fig. 1. Histogram of price changes obtained from simulations at the percolation threshold. (a) $N = 1001^2$ traders on 100 square lattices, at scaled activity $A = aN = 5$–2000 as given on top of the figure, from 1000 time intervals. (b) Same data in log-log scaling plot. (c) Double-logarithmic scaling plot in seven dimensions; $N = 7^7$, 200 lattices, 3000 time intervals.



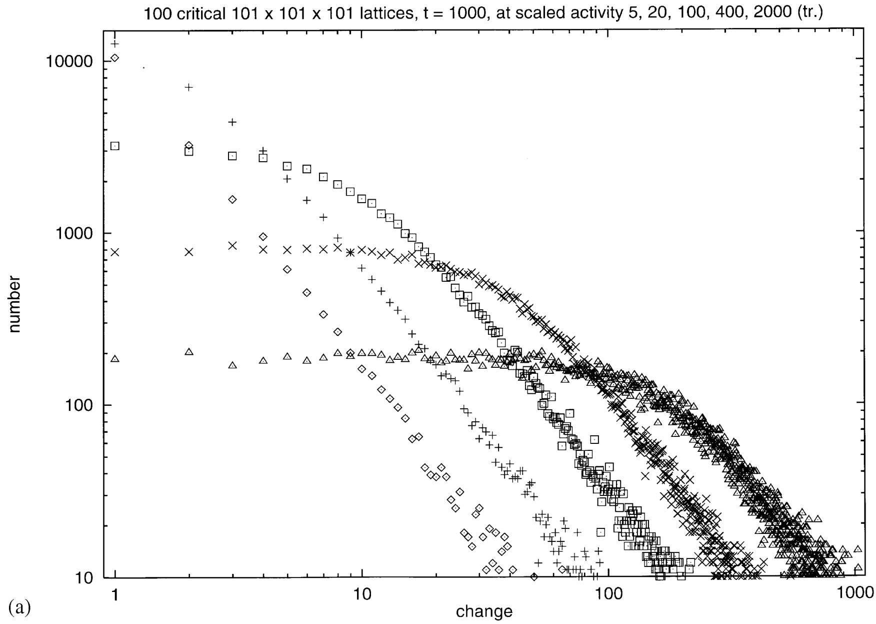
(c)

Fig. 1. Continued.

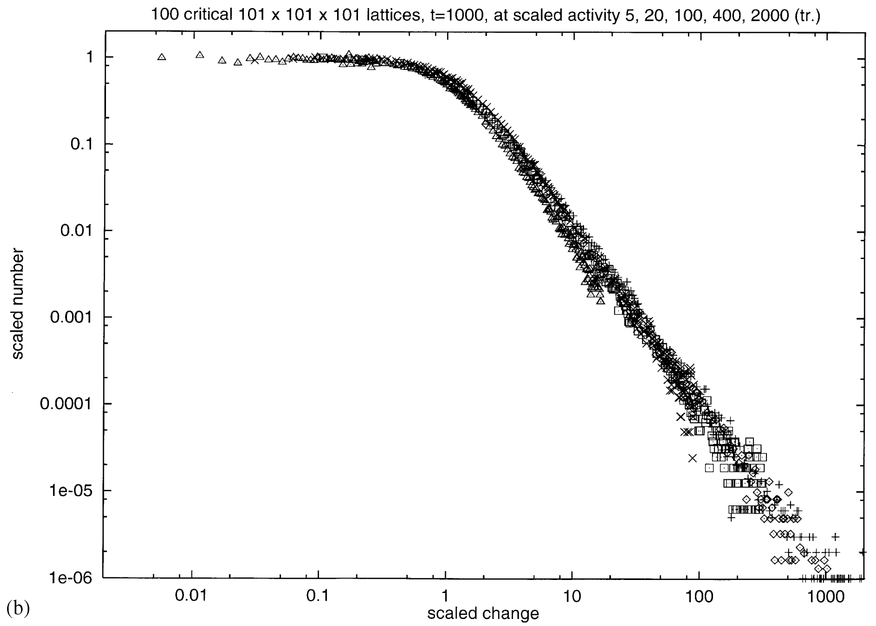
randomly to buy or to sell, and then the sum (1) determines the price change. We average over many different lattices to find the histogram. A 215-line Fortran program, based on the Hoshen–Kopelman algorithm [4] in variable dimension d, is available from the authors. (For computational efficiency we used site instead of bond percolation and a mixture of free and helical boundary conditions.)

Fig. 1 shows the histogram for $d=2$ and 7, and Fig. 2 for $d=3$, with p_c = 0.592746, 0.0888, and 0.3116, respectively. The critical behavior in four to six dimensions was similar and is not shown. (Finite size effects in a comparison of $20^3, 50^3, 101^3$, 150^3 and 201^3 sites were seen only at p_c for small changes.) We see for increasing activity a, which can be identified with increasing time steps Δt at fixed activity, a crossover from a power law to a bell-shaped behavior, within the accuracy of these simulations. Thus the Cont–Bouchaud model seems to be realistic in this aspect on finite lattices.

If we normalize for $p=p_c$ the observed price change by the half-width of the change distribution and normalize its maximum (for zero change) to unity, the histograms for different activities roughly collapse to a single curve, as seen in Figs. 1b, 1c and 2b. For large enough changes always a power law decay with the cluster decay exponent τ seems to be obeyed, but the larger the activity a is, the larger is the minimal price change for which this power law is seen. For $p<p_c$ we could not see such simple scaling, as shown in Fig. 2c. Also the crossover to Gaussians, Fig. 3, seen when the fraction of active clusters no longer is very small, is no longer described by this simple scaling law.

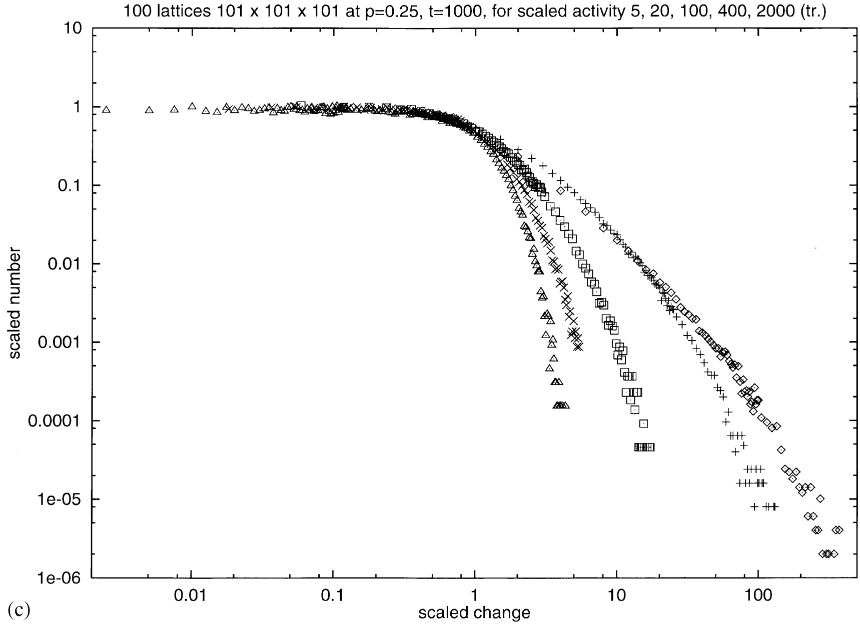


(a)



(b)

Fig. 2. Log–log plots for $d=3$; same symbols as in two dimensions for the same scaled activities 5–2000. (a) $N=101^3$ traders in 100 simple cubic lattices. (b) Same data in scaling plot. (c) Analogous data for $p=0.25<p_c$; they do not scale in the way of part b.



(c)

Fig. 2. Continued.

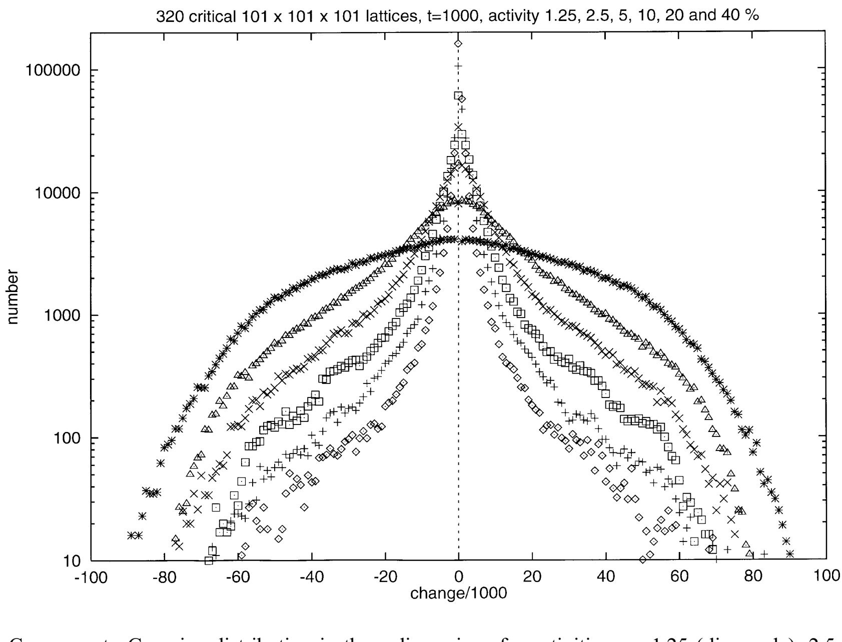


Fig. 3. Crossover to Gaussian distribution in three dimensions for activities $a = 1.25$ (diamonds), 2.5 (+), 5 (squares), 10 ($\times$), 20 (triangles) and 40 (stars) percent. Similar crossover was seen in seven dimensions.

In summary, when we increase the number of active clusters from about one to larger values, we switch from a simple power law distribution of price changes (governed by the percolation exponent τ) to a smooth peak with this power law restricted to the tails. This behavior seems analogous to Lévy flights where the single step (the demand from one cluster) follows a power-law distribution. A simple scaling behavior is seen in Figs. 1b, 1c and 2b. For even larger activities, when an appreciable fraction of all clusters participates, a crossover to a Gaussian is seen in Fig. 3. This increase in the activity parameter can also be interpreted as an increase in the time unit, since a is the fraction of traders which are active per unit time.

Of course, the model could be made more realistic by including history effects and a finite capital for each investor, as well as learning by increasing successful clusters at the expense of less lucky ones. Work along these lines is in progress.

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