Little Tricky Logic:
Misconceptions in the Understanding of LTL

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‹Programming› 2023

LTL = **L**inear **T**emporal **L**ogic

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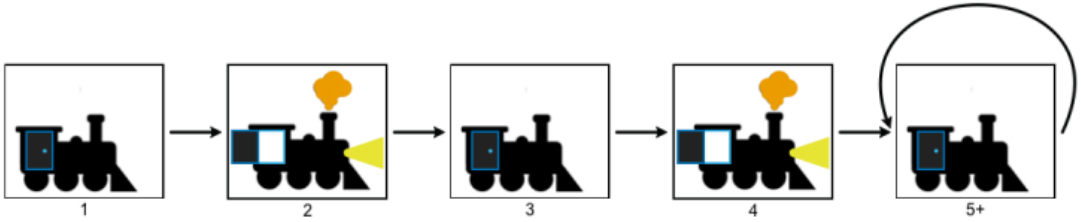
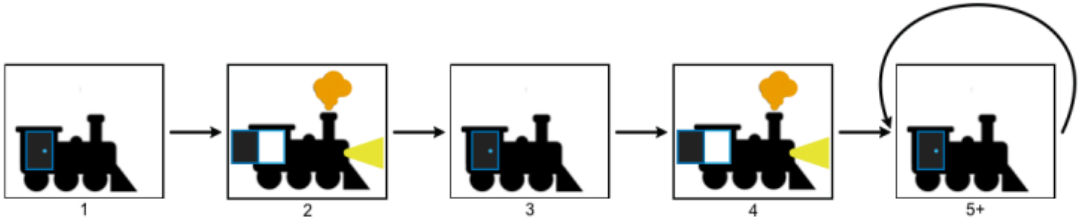


For systems that change over time

LTL = **L**inear **T**emporal **L**ogic



For systems that change over time

- ✓ Expressive
- ✓ Supports good decision procedures
- ✓ Small

.... and easy to learn?

RQ. In **what ways** is LTL tricky, and **what can we do** about it?

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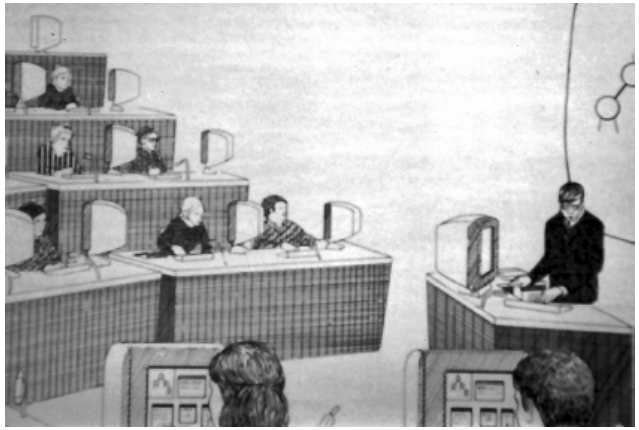
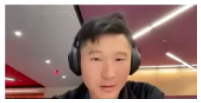
2 years of studies with researchers and students
4 survey rounds





qualtrics

Quiz Time!

Quiz Format:

one question,
possible answers,
you decide yes/no

✓ ✗

Question

✓ Possible Answer 1

✗ Possible Answer 2

✓ Possible Answer 3

LTL Operators:

`always`	`(G)`
`eventually`	`(F)`
`after`	`(X)`
`until`	`(U)`

Part 1:

Formulas vs. Traces

Q. Do the traces below satisfy this formula?

`{eventually Red} and {eventually Green}`

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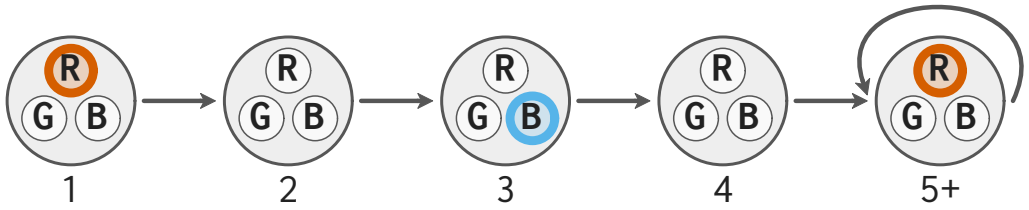

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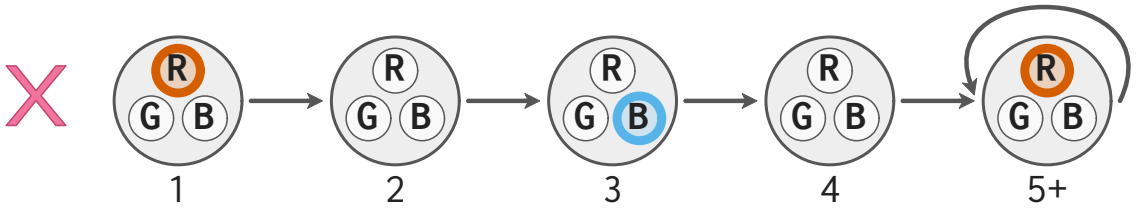

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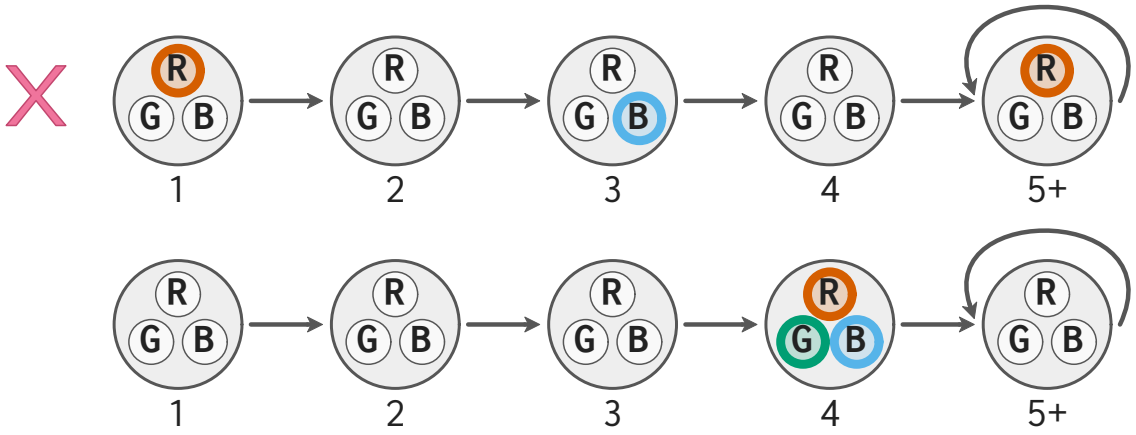

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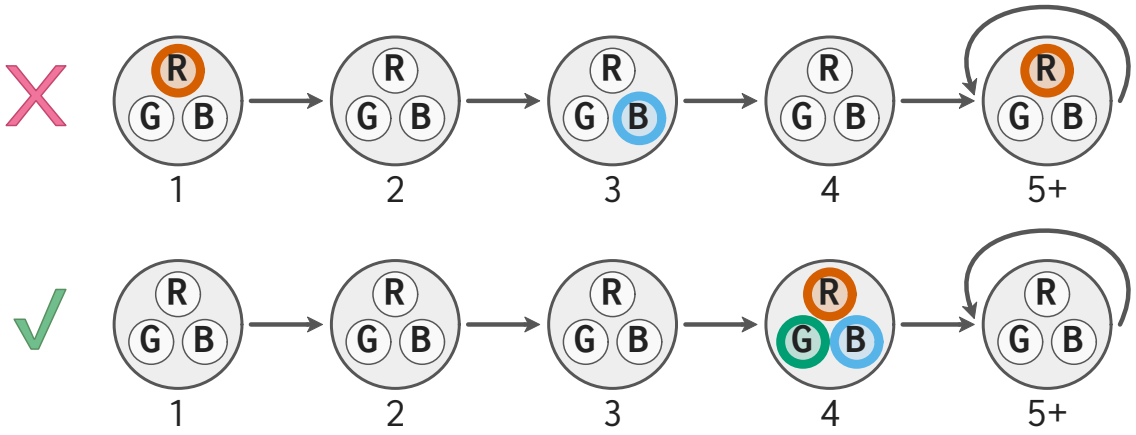

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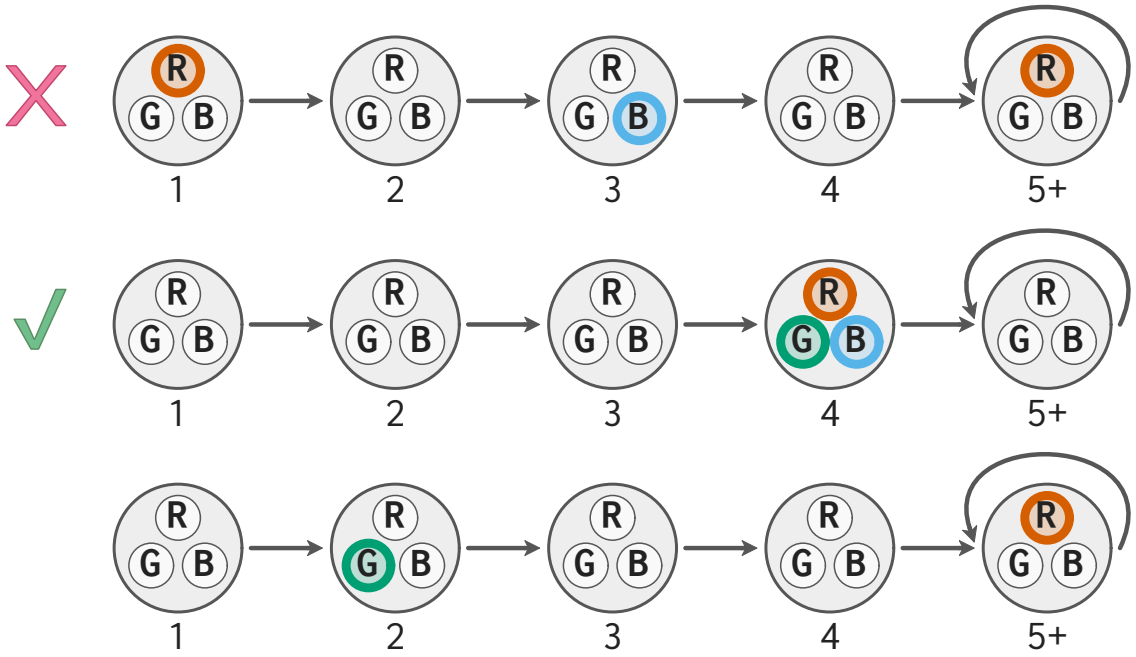

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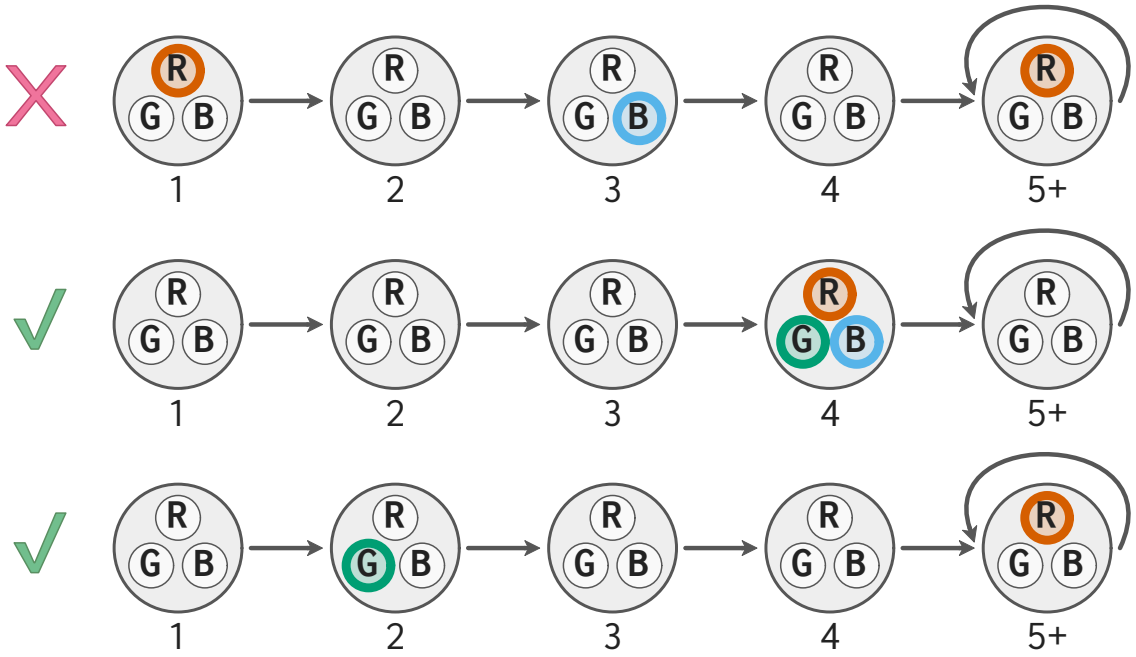

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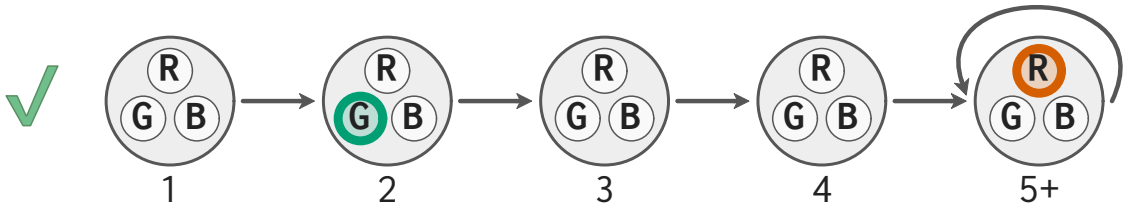
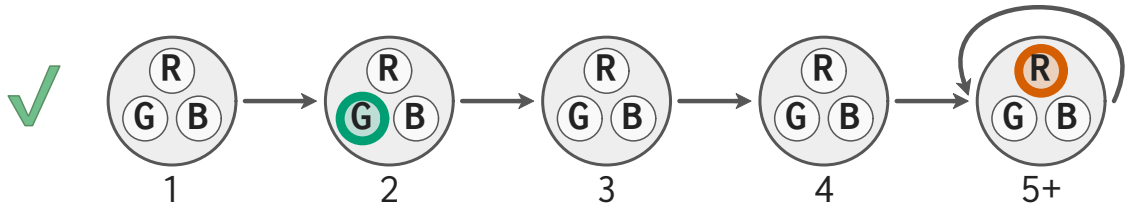


Q. Do the traces below satisfy this formula?

`{eventually Red} and {eventually Green}`

Not satisfied, because Green comes before Red

`Bad Prop` misconception



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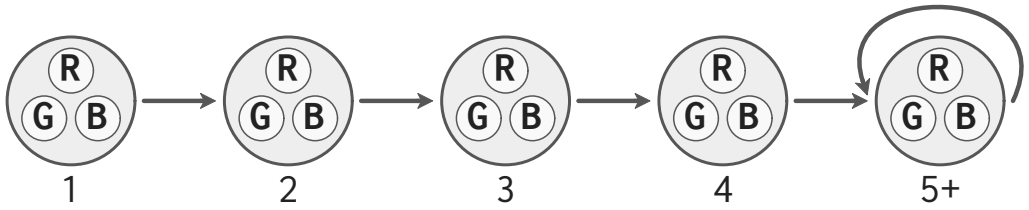

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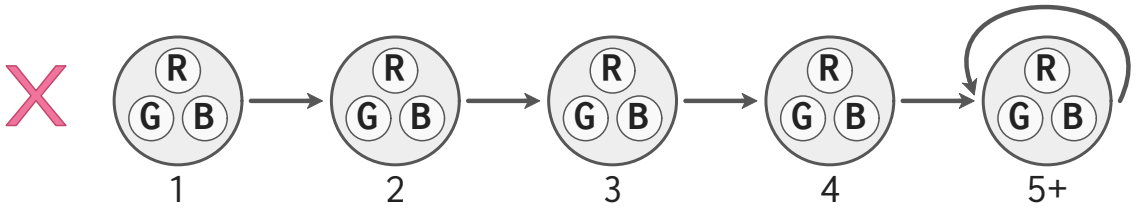

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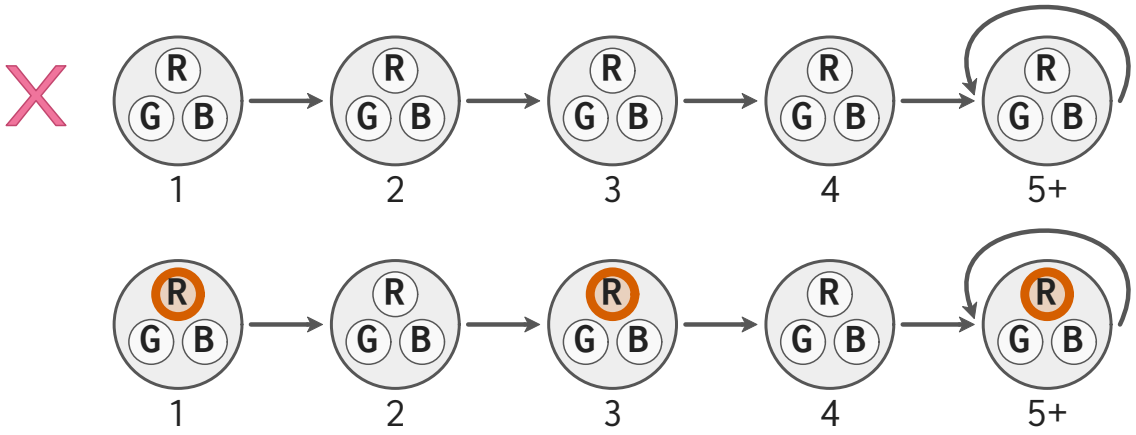

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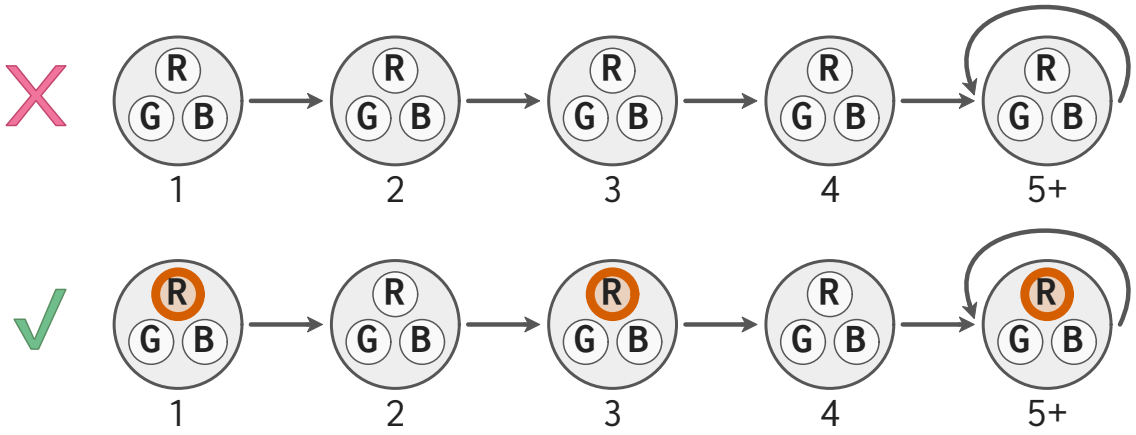

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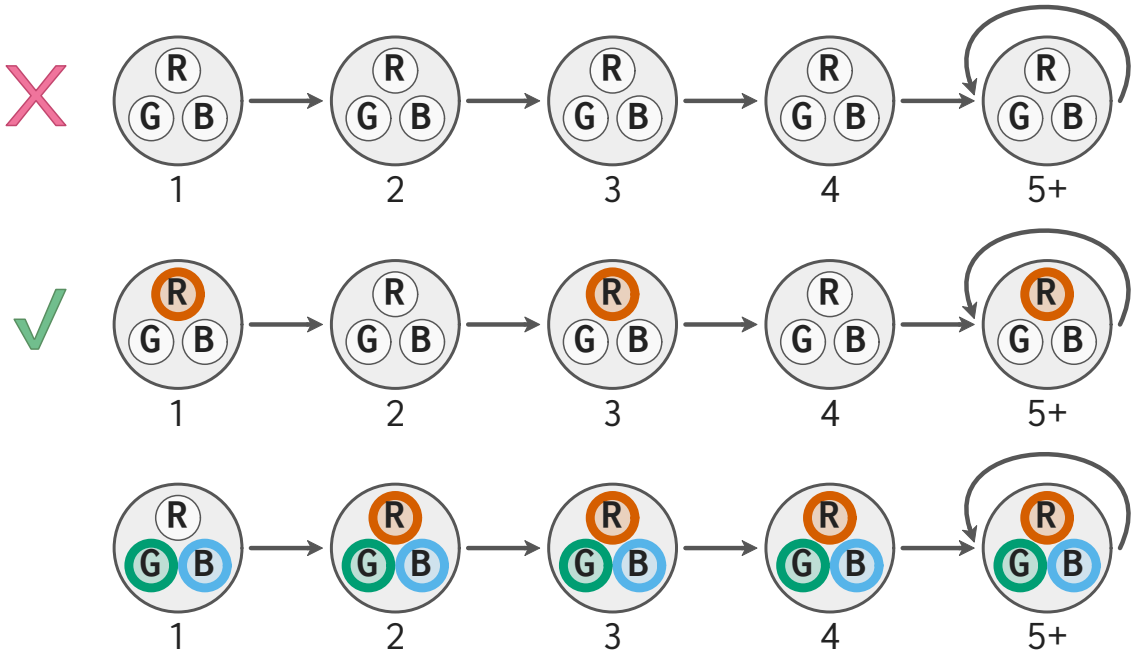

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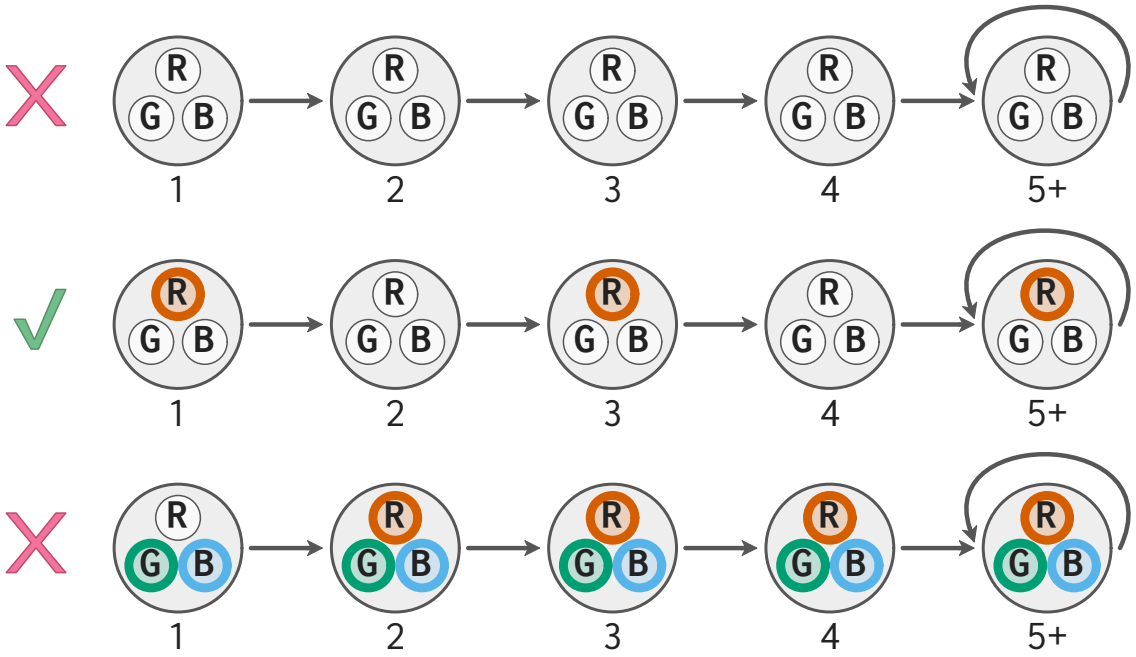

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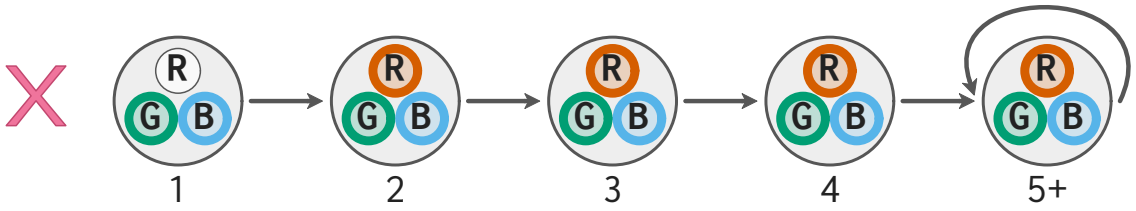
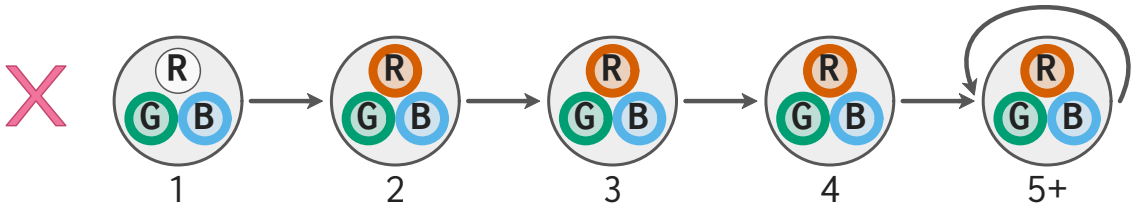


Q. Do the traces below satisfy this formula?

Red

Satisfied because Red is on at some point

`Implicit F` misconception



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`Red until Blue` [strong until]

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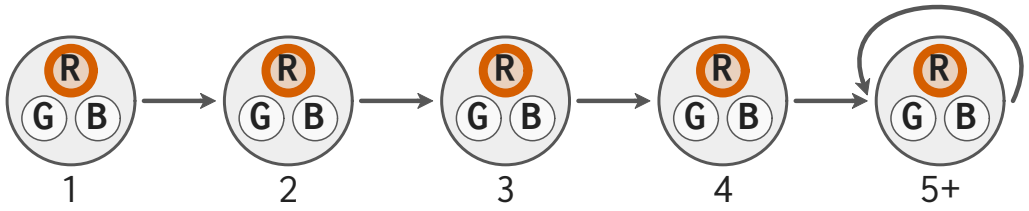

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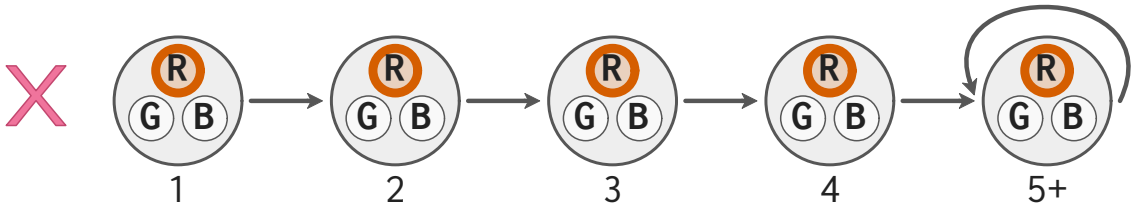

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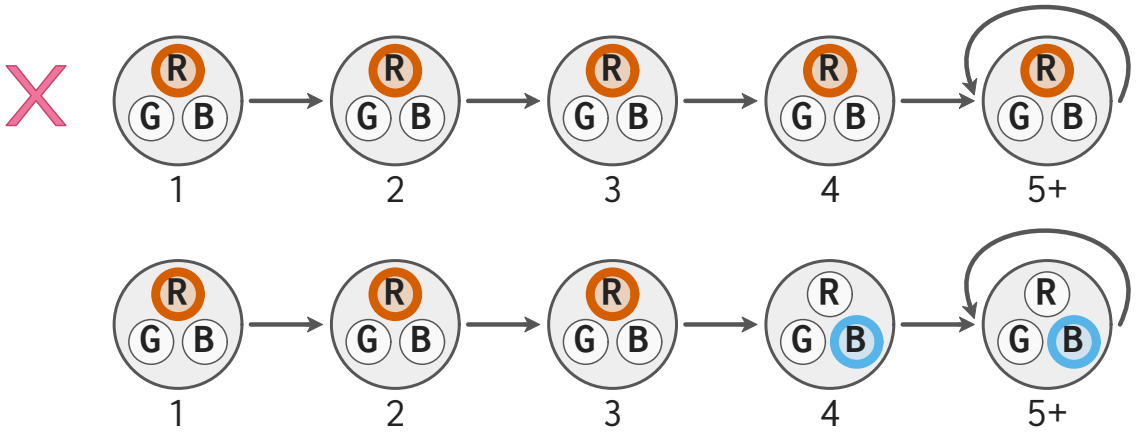

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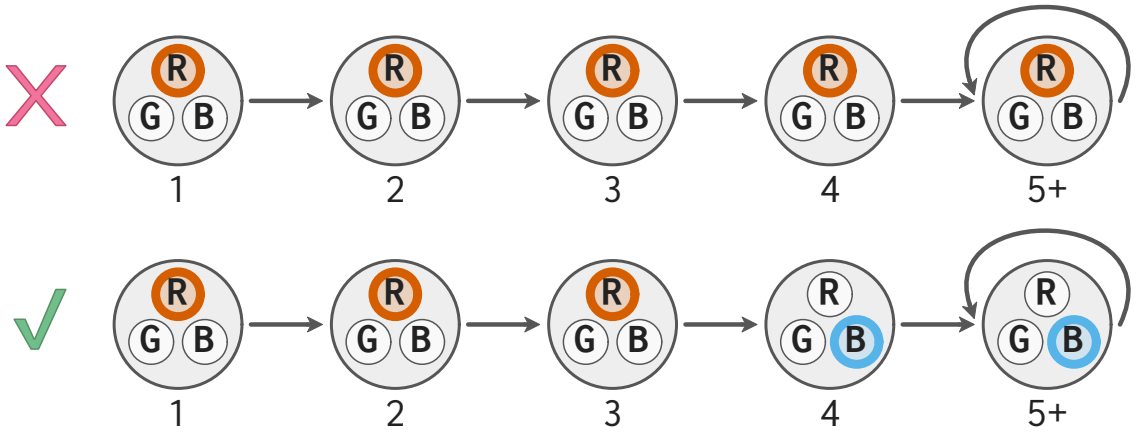

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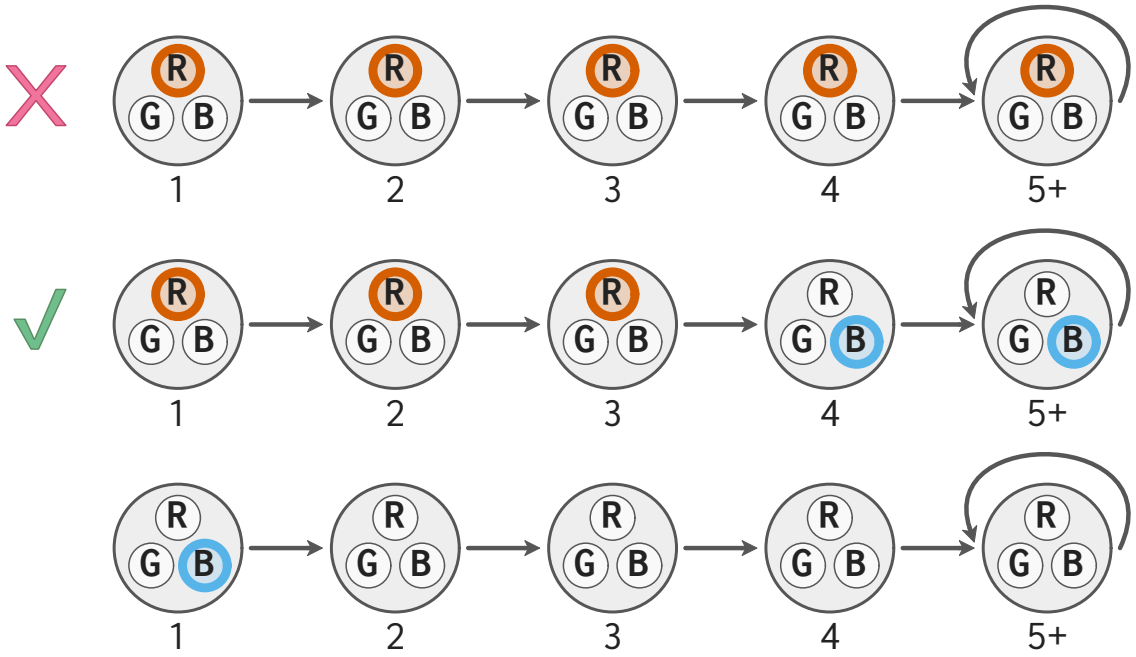

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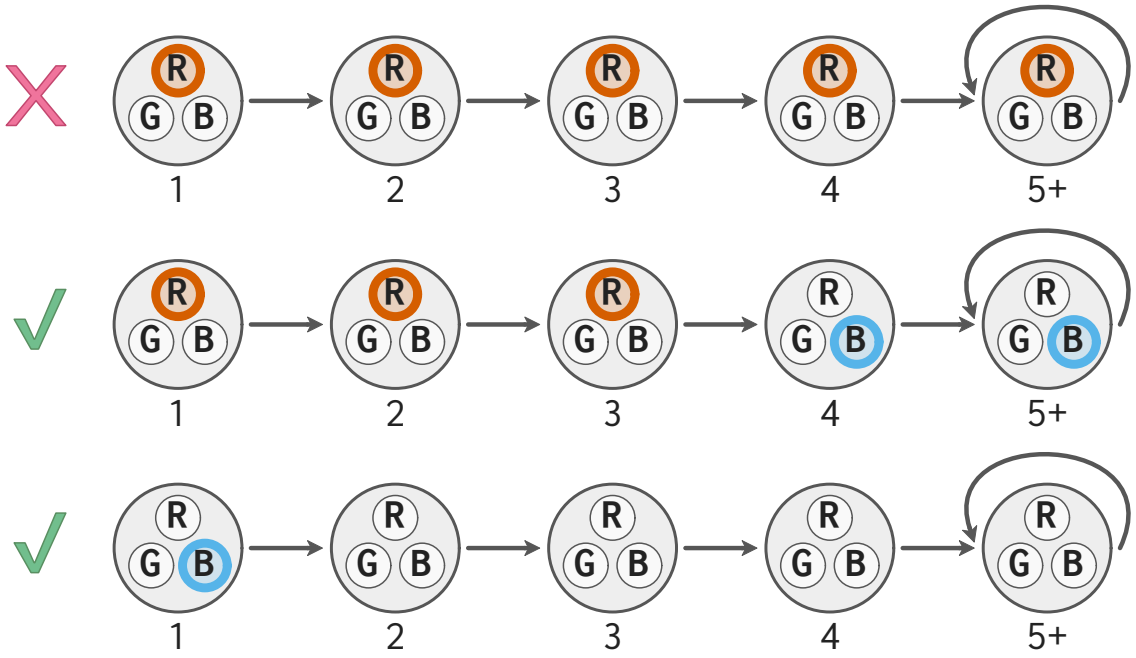

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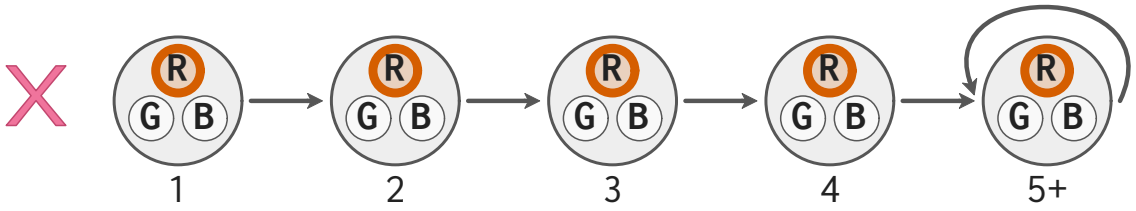

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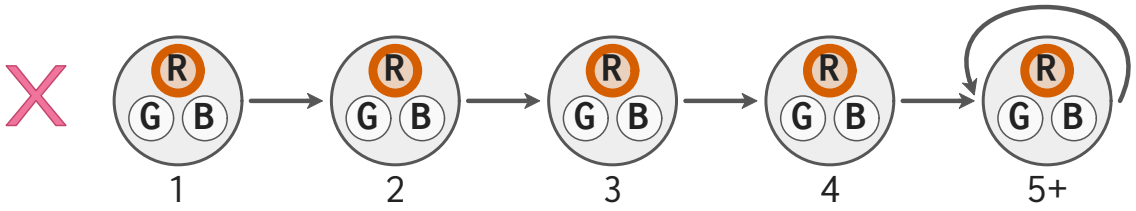

Satisfied because Blue may stay off

Even among researchers!

`Weak U` misconception

Part 2:

LTL to English

Q. Translate to English

{Red until Blue} and {always Red}

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{Red until Blue} and {always Red}

"Red is always on"

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{Red until Blue} and {always Red}

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Q. Translate to English

{Red until Blue} and {always Red}

"Red is always on"

"Red is always on and Blue is eventually on"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

"This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

✗ "Red is always on"

✓ "Red is always on and Blue is eventually on"

✗ "This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

"This statement can never be satisfied"

Q. Translate to English

{Red until Blue} and {always Red}

When Blue turns on, Red **must** be off

`Exclusive U` misconception

✗ "This statement can never be satisfied"

Q. Translate to English

{eventually Red} implies {always Blue}

Q. Translate to English

`{eventually Red} implies {always Blue}`

"if Red is ever on, then Blue is always on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

"Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✓ "if Red is ever on, then Blue is always on"

✗ "Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

✗ "Red is on at some point, after which Blue is on"

Q. Translate to English

`{eventually Red} implies {always Blue}`

Red **will** turn on

`Bad Prop` misconception

"Red is on at some point, after which Blue is on"

Part 3:

English to LTL

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

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```
Impossible!
```

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`Impossible!`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

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Impossible!
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```
{eventually Red} and {always {Red => always !Red}}
```

Q. Translate to LTL

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Impossible!
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Impossible!
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```
{eventually Red} and {always {Red => always !Red}}
```

```
{eventually Red} and {always {Red => after {always !Red}}}
```

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

✗ `Impossible!`

✗ `{eventually Red} and {always {Red => always !Red}}`

✓ `{eventually Red} and {always {Red => after {always !Red}}}`

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

```
{eventually Red} and {always {Red => always !Red}}
```

Q. Translate to LTL

The Red light is on in exactly one state, but not necessarily the first state

An implication constrains the **next state**

`Bad State Index` misconception

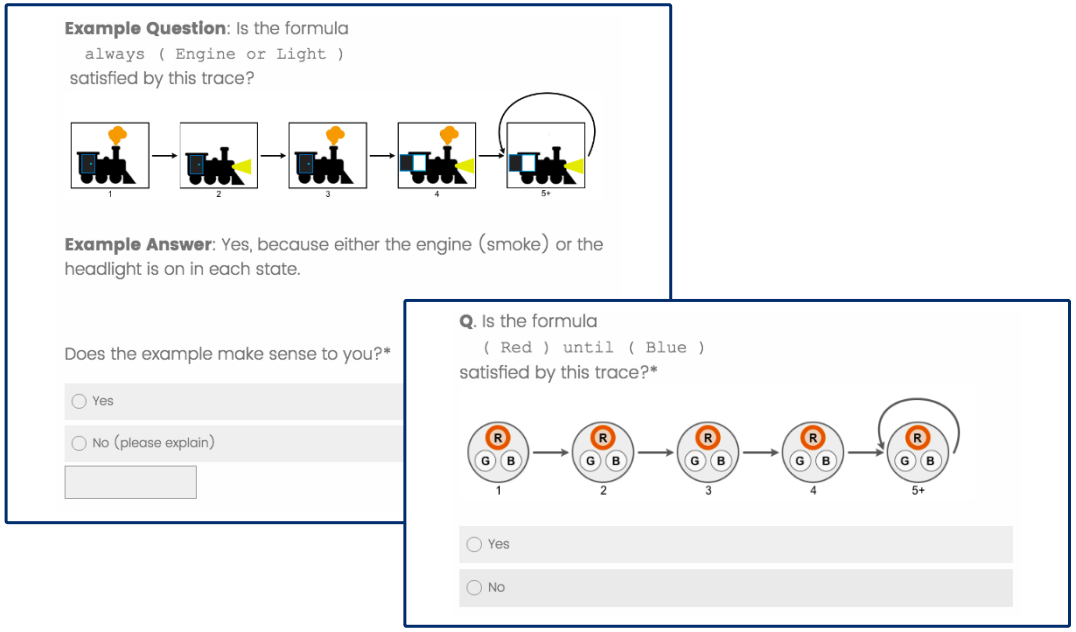
```
{eventually Red} and {always {Red => always !Red}}
```

All Done!

Simple formulas, yet
subtle issues and **expert blind spots**

Quiz Q's Based on 3 Instruments

- Trace Satisfaction
- LTL to English
- English to LTL



cs.brown.edu/~bgreenma/ltl-instruments.pdf

Code Book for Analysis

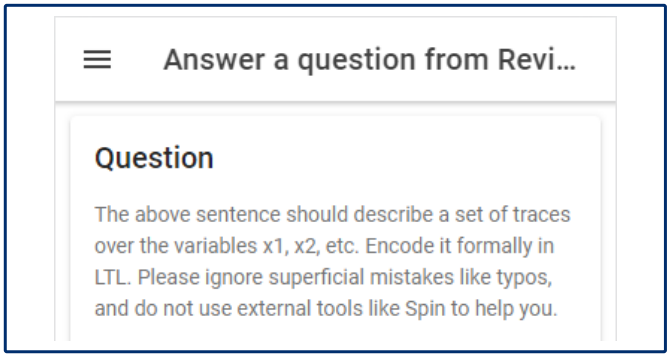
Bad Prop	Implicit F
Bad State Index	Implicit G
Bad State Quantification	Other Implicit
Exclusive U	Weak U

Coding Rubric in paper, past versions in artifact

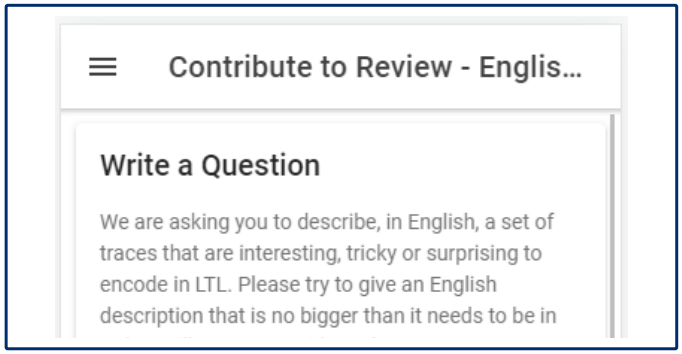
Software: Quizius

Class-sourcing to discover misconceptions

1. Answer Top Q's



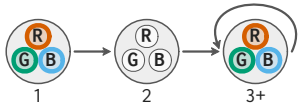
2. Submit New Q's



What Next?

What Next?

1. Teach Better



our instruments can help!

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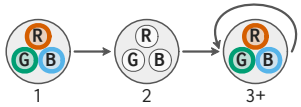

our instruments can help!

... but learners are everywhere
not just in classrooms

amazon Meta

NETFLIX

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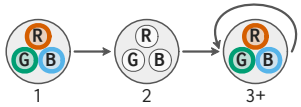

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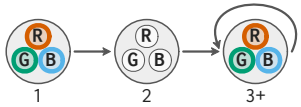

our instruments can help!

2. Build Tools

guard against misconceptions

What Next?

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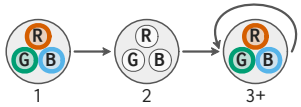

our instruments can help!

2. Build Tools

guard against misconceptions

3. Design Logics

Alloy 6 **Electrum**

our findings have inspired changes

Thank You!

benjamin.l.greenman@gmail.com