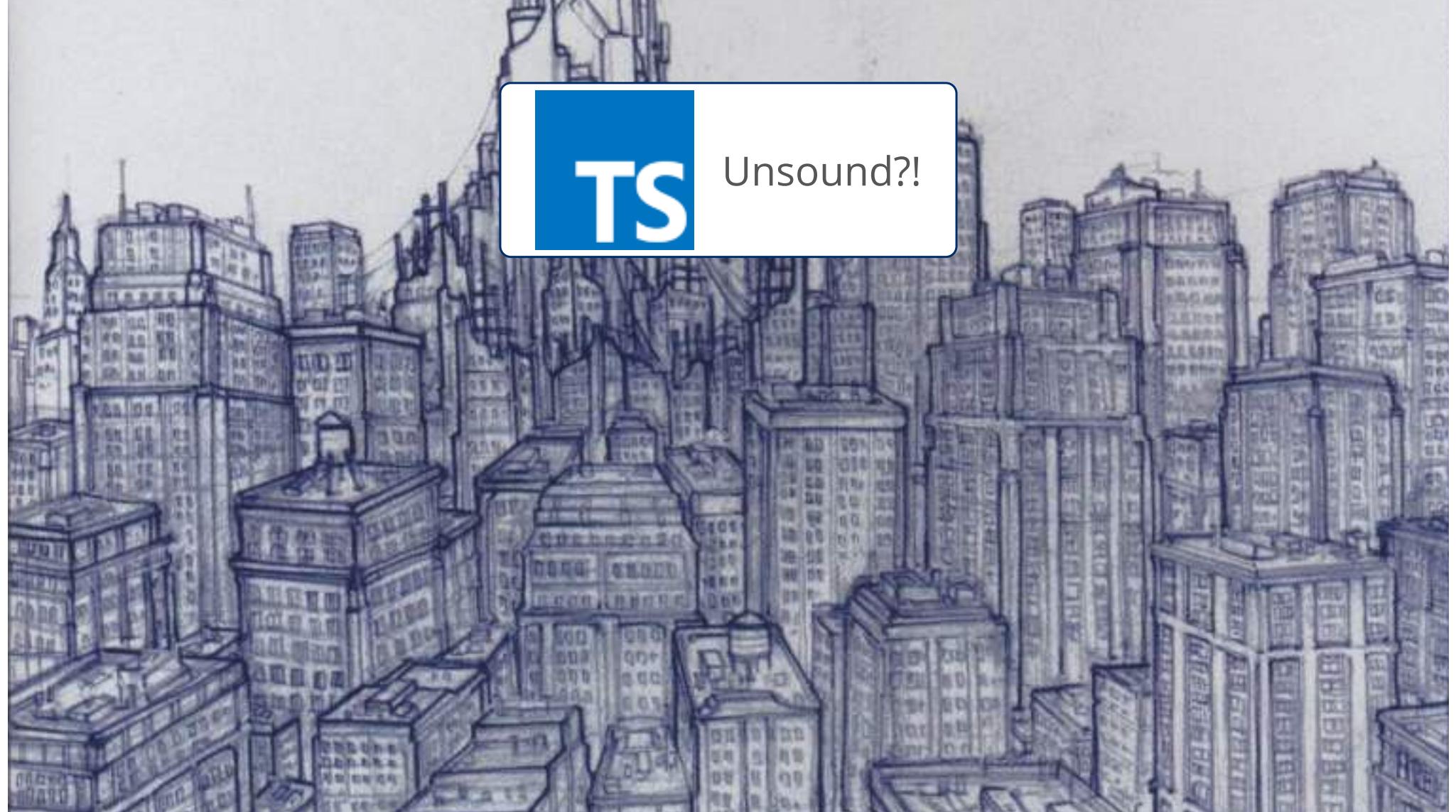




Type Narrowing the Hard Way

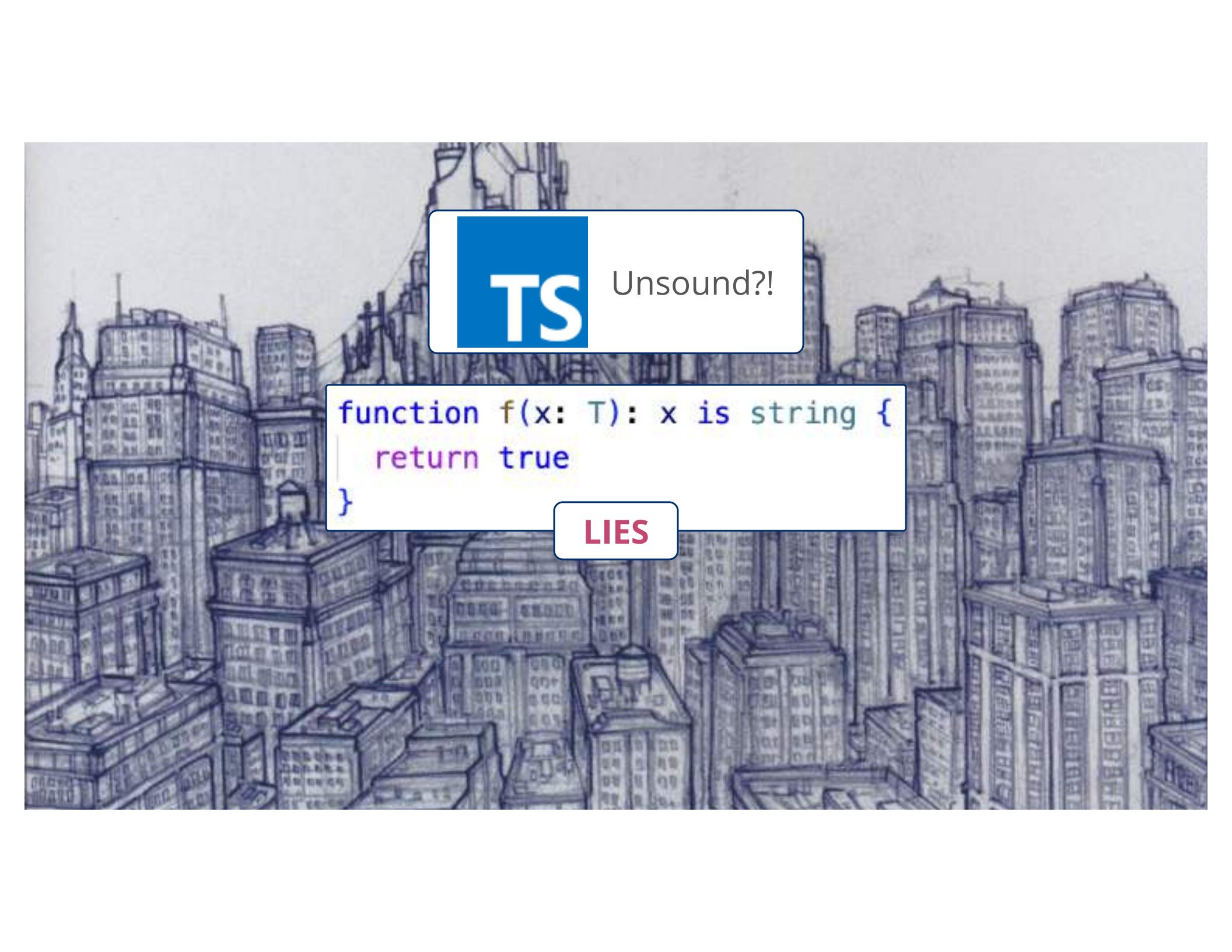
Ben Greenman
Hanwen Guo





TS

Unsound?!

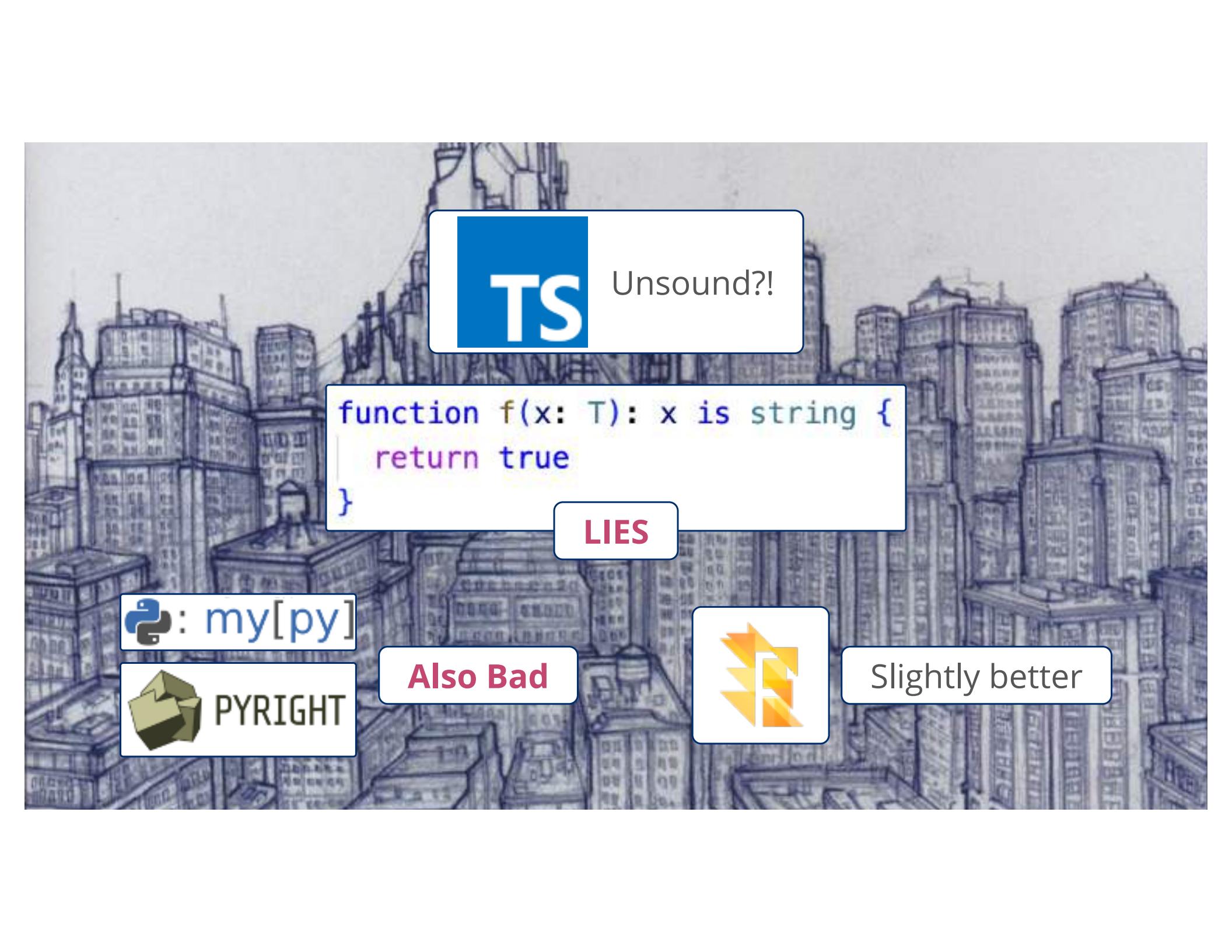


TS

Unsound?!

```
function f(x: T): x is string {  
    return true  
}
```

LIES



TS

Unsound?!

```
function f(x: T): x is string {  
    return true  
}
```

LIES



: my[py]



PYRIGHT

Also Bad



Slightly better

Type Narrowing

Refines types using tests

```
function add1(x: object): number {  
    if (typeof(x) === "number") {  
        return x + 1  
    } else {  
        return 0  
    }  
}
```



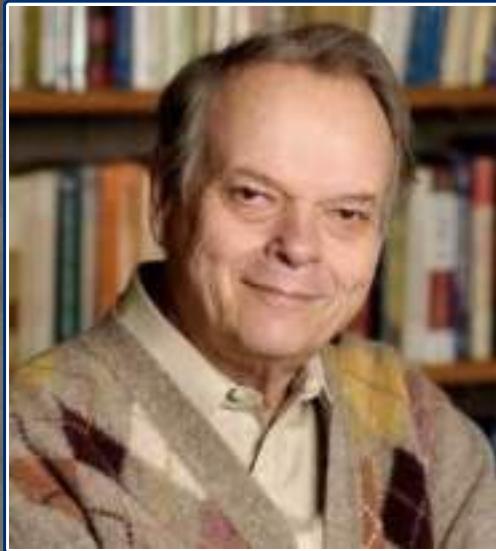
Fundamental to gradual typing

You don't have types for all the data in the world

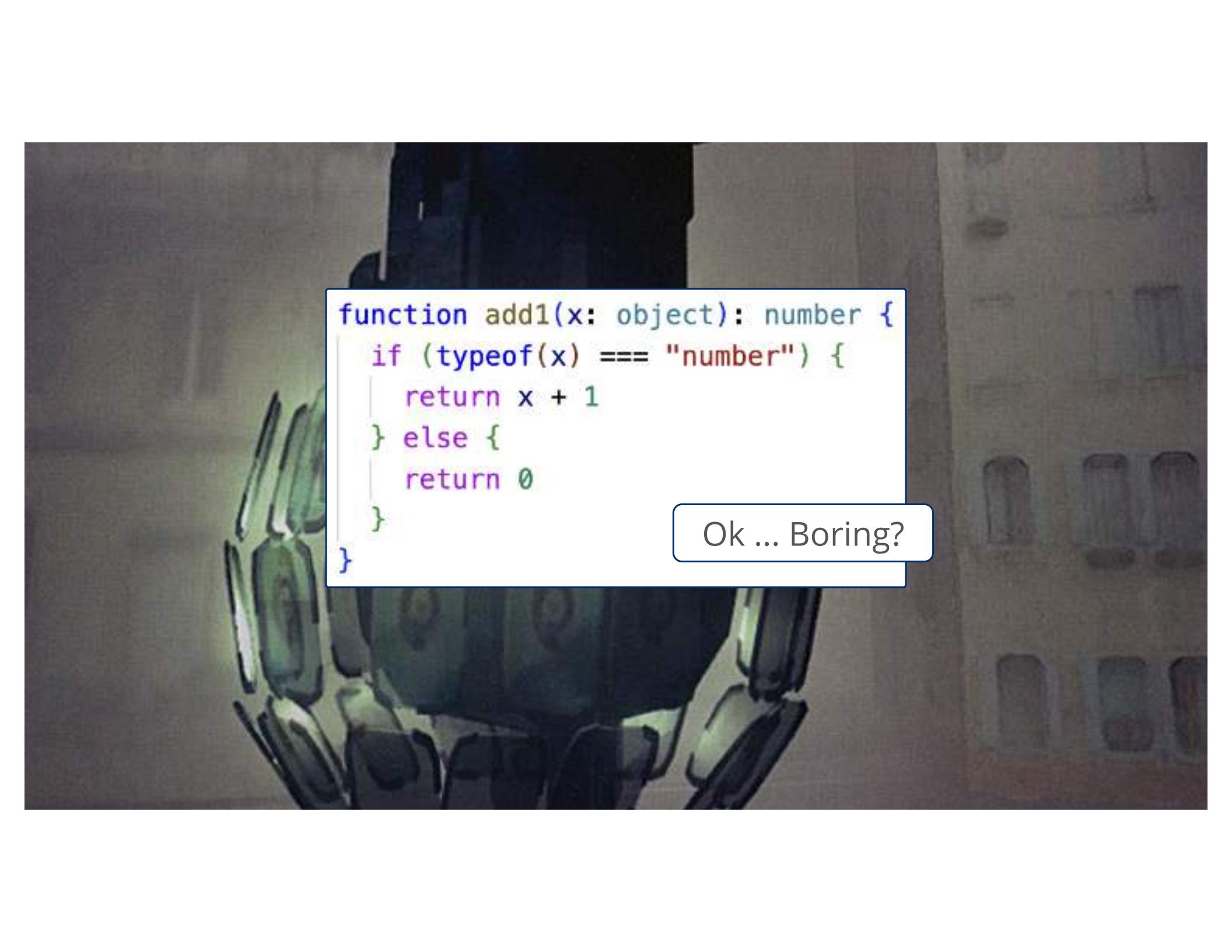
Ugly Data



TS



*"Some account should be taken
of the premises in
conditional expressions." (1968)*



```
function add1(x: object): number {  
    if (typeof(x) === "number") {  
        return x + 1  
    } else {  
        return 0  
    }  
}
```

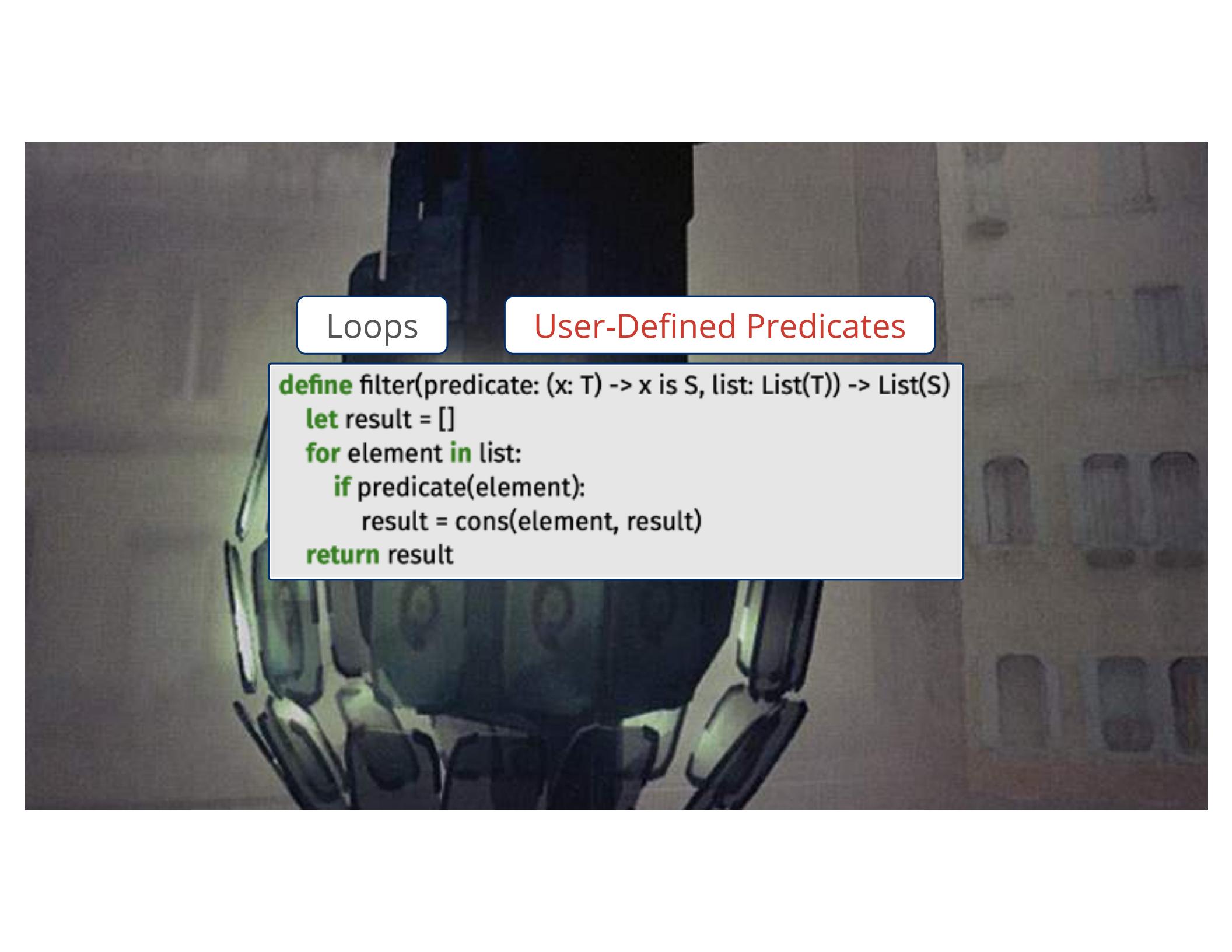
Ok ... Boring?

Indexing

Nested If

Aliasing

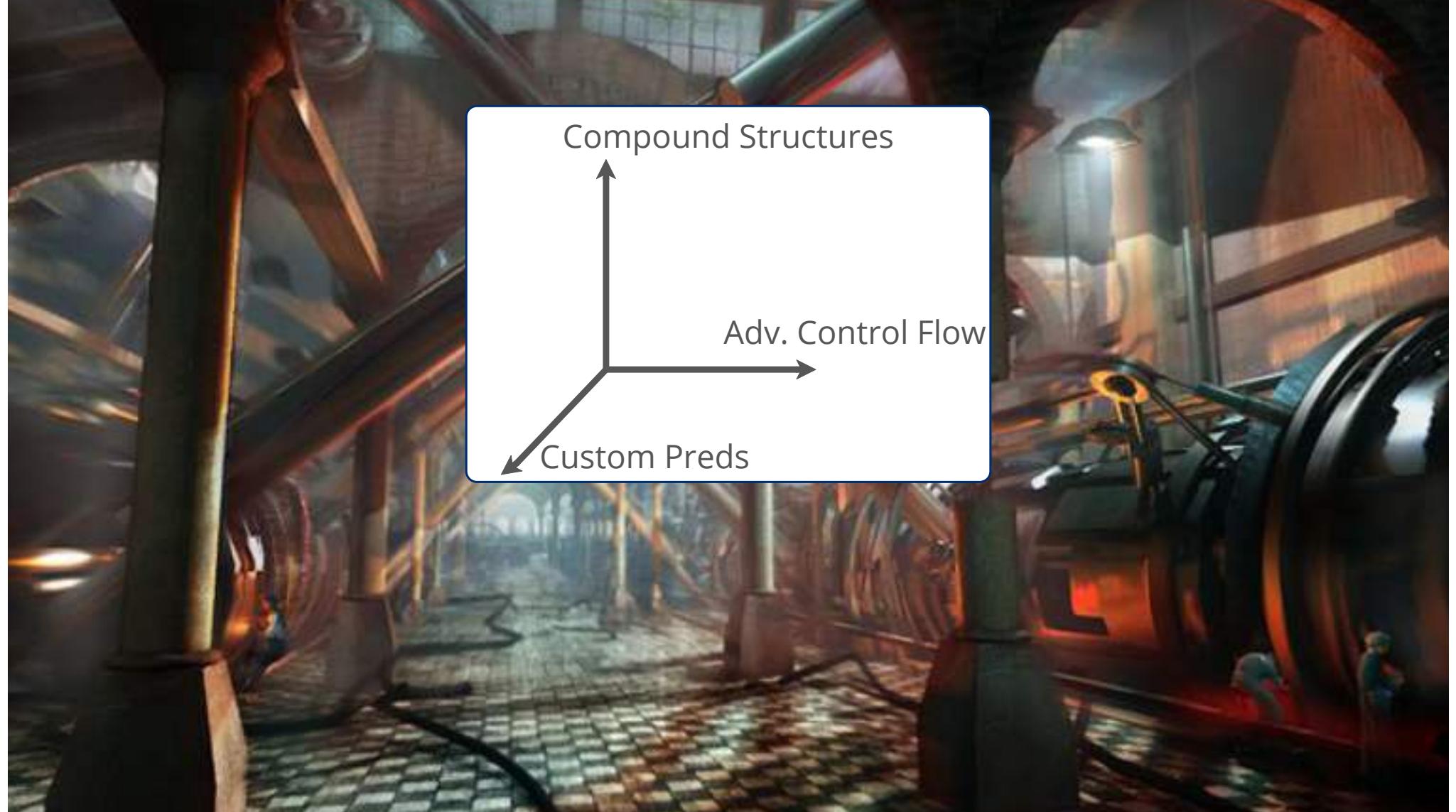
```
define avg_rainfall(weather_reports: List(JSON)) -> Number:  
  let total = 0, count = 0  
  for day in weather_reports:  
    if day is Object and has_field(day, "rainfall"):  
      let val = day["rainfall"]  
      if val is Number and 0 ≤ val ≤ 999:  
        total += day["rainfall"] // expected: no type error, right-hand expression is a number  
        count += 1  
  return (if count > 0: total / count else: 0)
```

A photograph of a person's hands resting on a laptop keyboard. The hands are positioned as if ready to type. The background is slightly blurred.

Loops

User-Defined Predicates

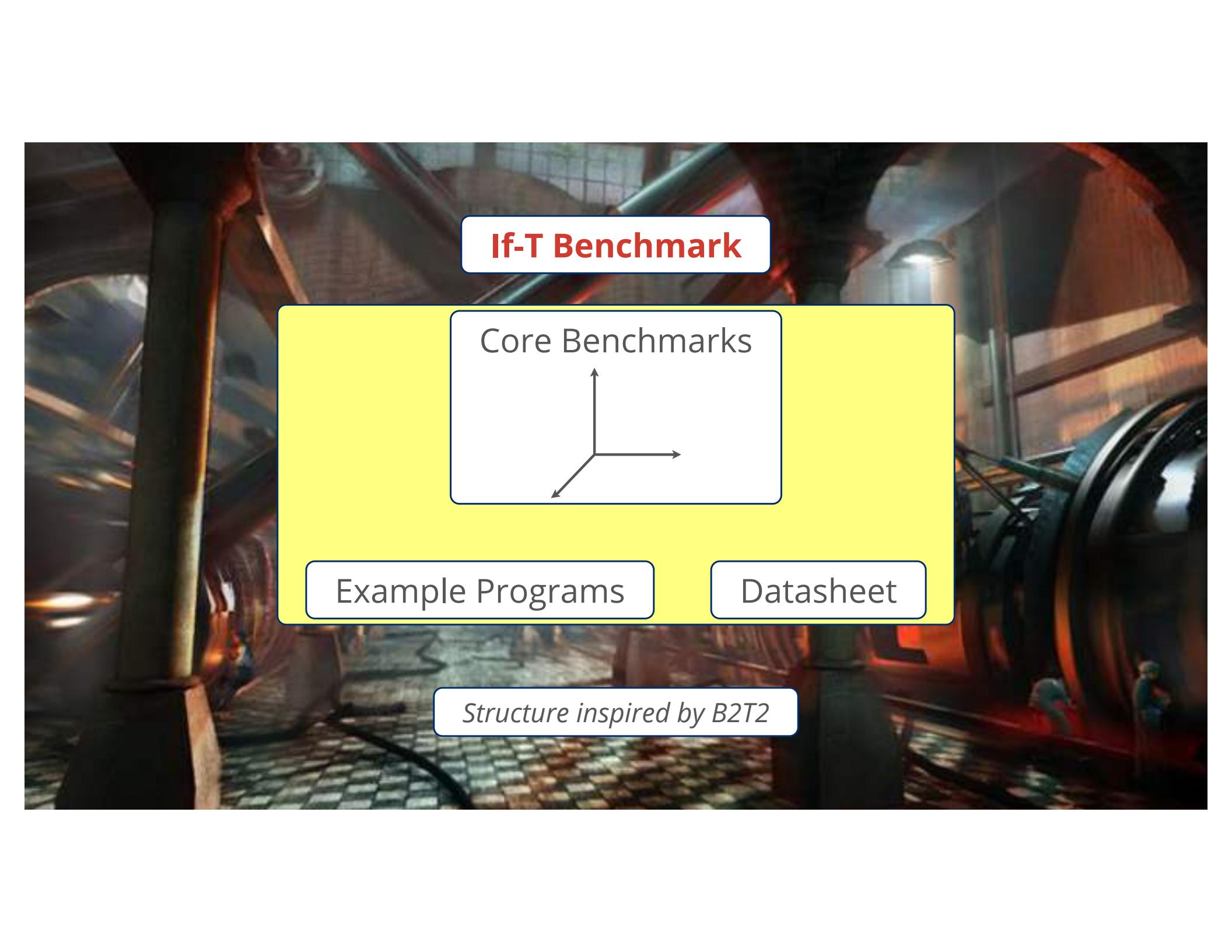
```
define filter(predicate: (x: T) -> x is S, list: List(T)) -> List(S)
let result = []
for element in list:
    if predicate(element):
        result = cons(element, result)
return result
```



Compound Structures

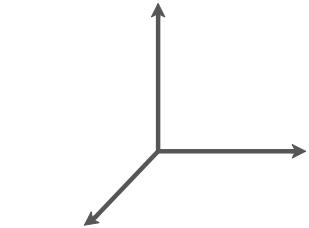
Adv. Control Flow

Custom Preds



If-T Benchmark

Core Benchmarks



Example Programs

Datasheet

Structure inspired by B2T2

Core

Basic

positive
negative
connectives
nesting_body

Compound Structures

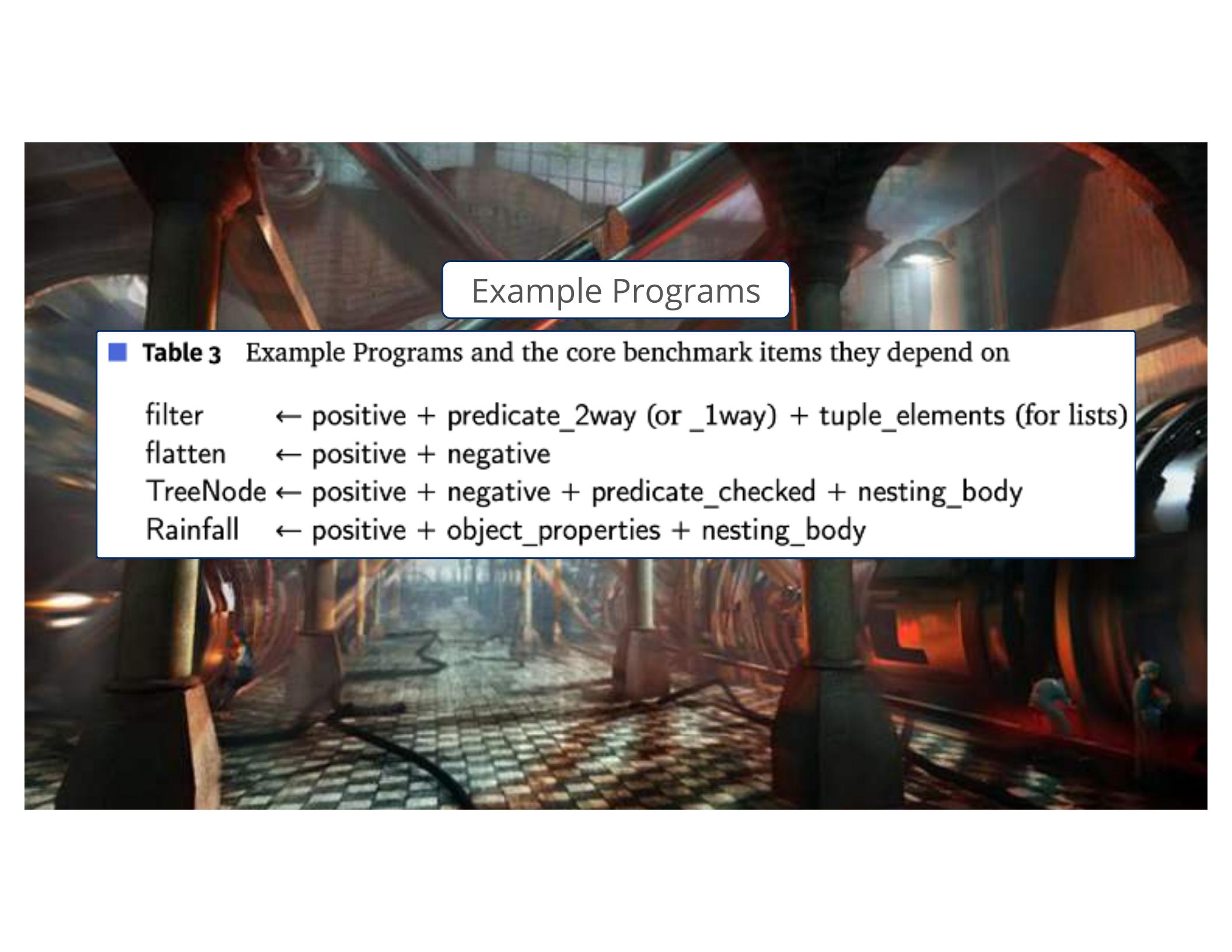
struct_fields
tuple_elements
tuple_length

Advanced Control Flow

alias
nesting_condition
merge_with_union

Custom Predicates

predicate_2way
predicate_1way
predicate_checked



Example Programs

■ **Table 3** Example Programs and the core benchmark items they depend on

filter	← positive + predicate_2way (or _1way) + tuple_elements (for lists)
flatten	← positive + negative
TreeNode	← positive + negative + predicate_checked + nesting_body
Rainfall	← positive + object_properties + nesting_body

Datasheet

Sorbet (Ruby)

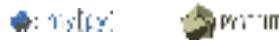
Sorbet adds static types to Ruby.

- Language resources:
 - <https://sorbet.org/docs/overview>
 - <https://github.com/sorbet/sorbet>
 - <https://sorbet.org/docs/gradual>
 - <https://sorbet.org/docs/from-typescript>
- If-T version: 1.0
- Implementation: [./main.rb](#), [./examples.rb](#)
- Raw command to run the benchmark: `srb tc main.rb`, `srb tc examples.rb` (or, using bundler: `bundle exec srb tc main.rb`, `bundle exec srb tc examples.rb`)

Type System Basics

Q. What is the top type in this language? What is the bottom type? What is the dynamic type? If these types do not exist, explain the alternatives.

- Top = `T.anything`
- Bottom = `T.noreturn`



Basic

positive
negative
connectives
nesting_body

Compound Structures

struct_fields
tuple_elements
tuple_length

Advanced Control Flow

alias
nesting_condition
merge_with_union

Custom Predicates

predicate_2way
predicate_1way
predicate_checked

Basic								
positive	o	o	o	o	o	o	o	o
negative	o	o	o	o	o	o	o	o
connectives	o	o	o	o	o	o	o	o
nesting_body	o	o	o	o	o	o	o	o
Compound Structures								
struct_fields	o	o	o	o	x	o	o	o
tuple_elements	o	o	o	o	o	o	o	o
tuple_length	o	o	o	o	x	x	x	o
Advanced Control Flow								
alias	x	o	o	x	o	x	o	o
nesting_condition	x	x	x	x	o	x	o	o
merge_with_union	x	o	o	o	o	x	o	o
Custom Predicates								
predicate_2way	o	o	o	o	x	x	o	o
predicate_1way	o	o	o	o	x	x	o	o
predicate_checked	x	x	x	o	x	x	o	o



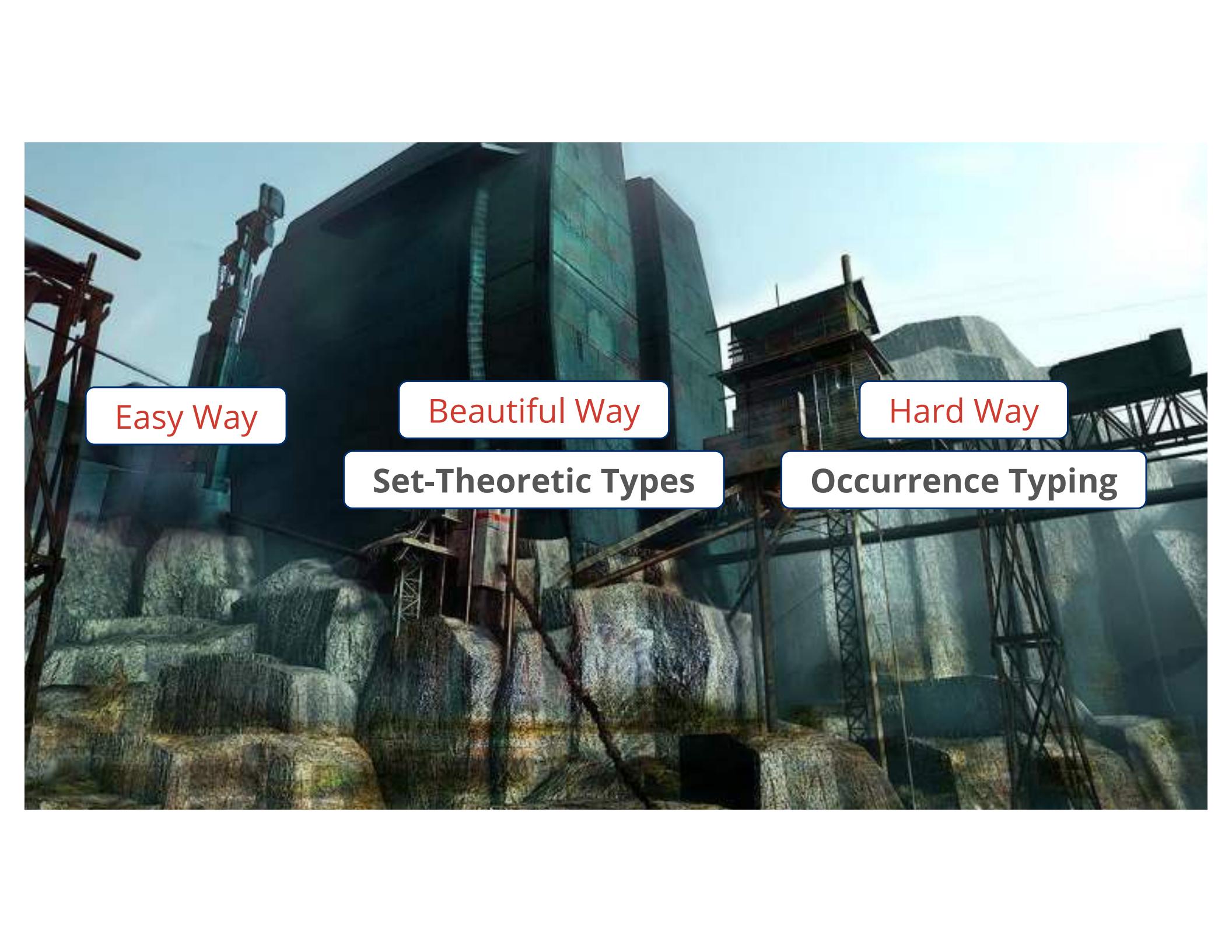
<https://github.com/utahplt/ift-benchmark>



Easy Way

Beautiful Way

Hard Way

The background image shows a dark, industrial setting with large pipes, scaffolding, and a building under construction or renovation. The lighting is dramatic, with strong highlights and deep shadows.

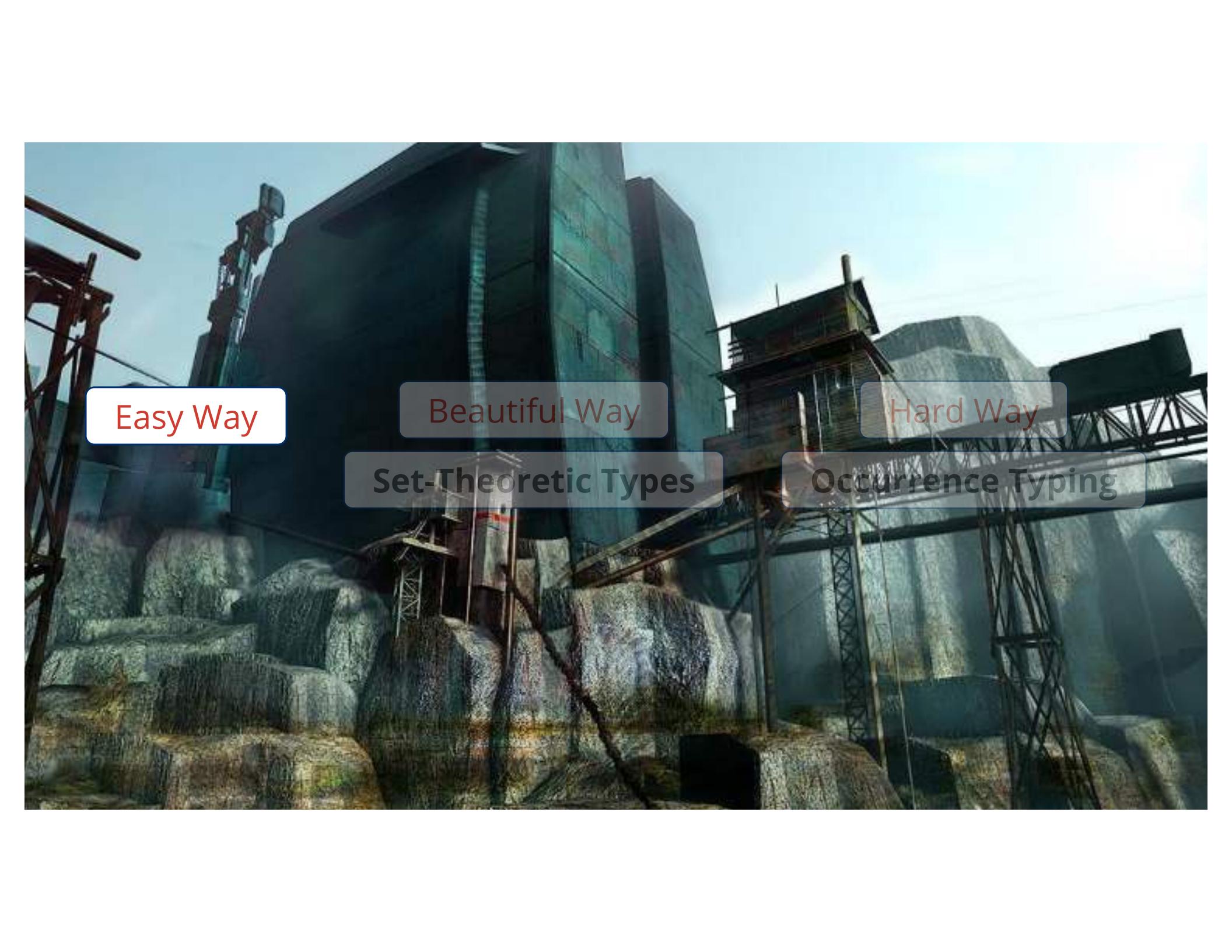
Easy Way

Beautiful Way

Hard Way

Set-Theoretic Types

Occurrence Typing

The background image shows a dark, industrial setting with large pipes, scaffolding, and a building under construction or renovation. The sky is bright and overexposed.

Easy Way

Beautiful Way

Hard Way

Set-Theoretic Types

Occurrence Typing

 my[py]

Easy Way

$$\frac{\Gamma \vdash e_0 : \text{Bool} \quad \Gamma_+, \Gamma_- = \text{analyze}(e_0) \quad \Gamma_+ \cup \Gamma \vdash e_1 : \tau \quad \Gamma_- \cup \Gamma \vdash e_2 : \tau}{\Gamma \vdash \text{if } e_0 \text{ } e_1 \text{ } e_2 : \tau}$$





: my[py]

Easy Way

$$\frac{\Gamma \vdash e_0 : \text{Bool} \quad \Gamma_+ \cup \Gamma \vdash e_1 : \tau \quad \Gamma_- \cup \Gamma \vdash e_2 : \tau}{\Gamma \vdash \text{if } e_0 \; e_1 \; e_2 : \tau}$$

```
class TypeChecker(NodeVisitor[None], TypeCheckerSharedApi):
    def visit_if_stmt(self, s: IfStmt) -> None:
        """Type check an if statement."""
        # This frame records the knowledge from previous if/elif clauses not being taken.
        # Fall-through to the original frame is handled explicitly in each block.
        with self.binder.frame_context(can_skip=False, conditional_frame=True, fall_through=2):
            for e, b in zip(s.expr, s.body):
                t = get_proper_type(self.expr_checker.accept(e))

                if isinstance(t, DeletedType):
                    self.msg.deleted_as_rvalue(t, s)

                if_map, else_map = self.find_isinstance_check(e)
                # XXX Issue a warning if condition is always False?
                with self.binder.frame_context(can_skip=True, fall_through=2):
                    self.push_type_map(if_map, from_assignment=False)
                    self.accept(b)

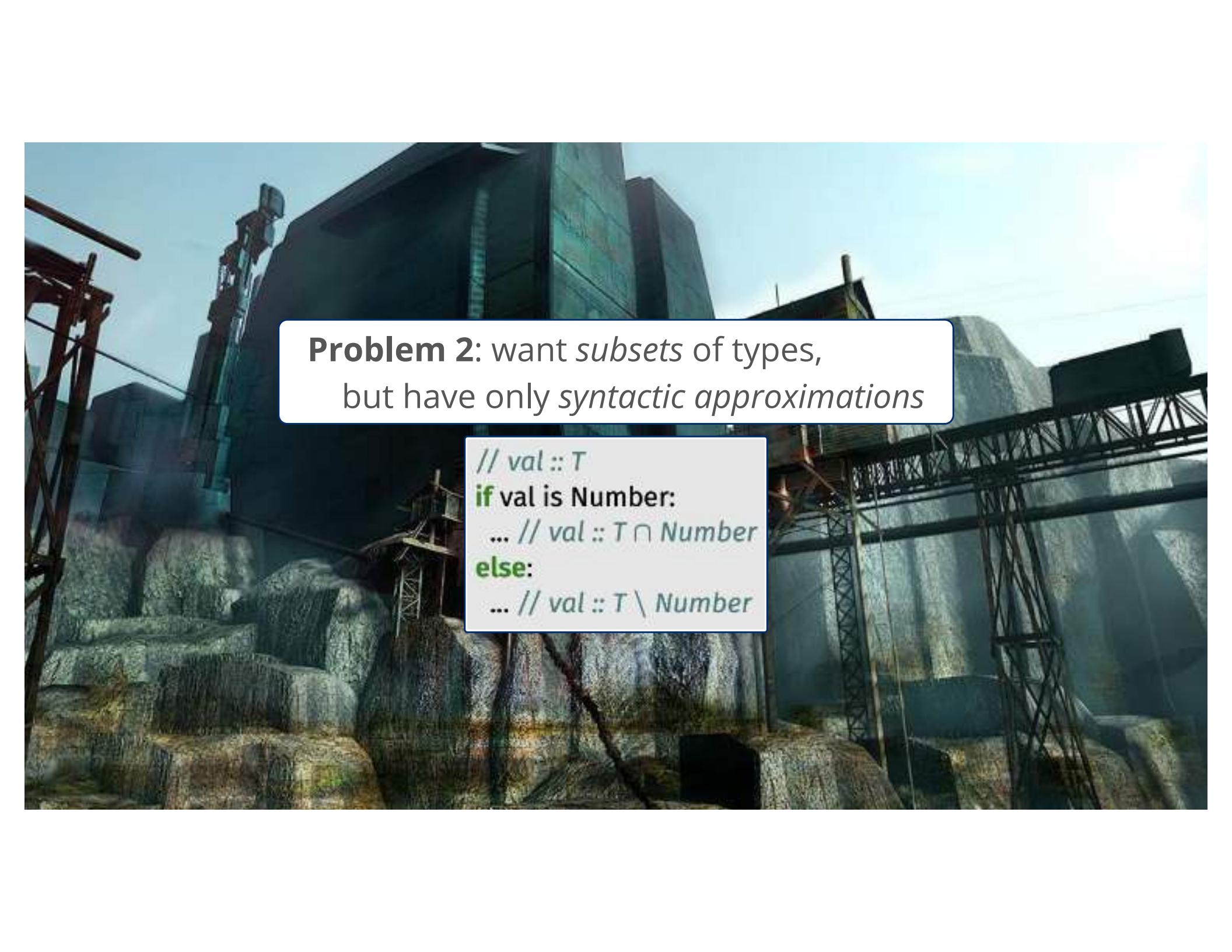
                # XXX Issue a warning if condition is always True?
                self.push_type_map(else_map, from_assignment=False)

                with self.binder.frame_context(can_skip=False, fall_through=2):
                    if s.else_body:
                        self.accept(s.else_body)
```

<https://github.com/python/mypy/blob/master/mypy/checker.py>

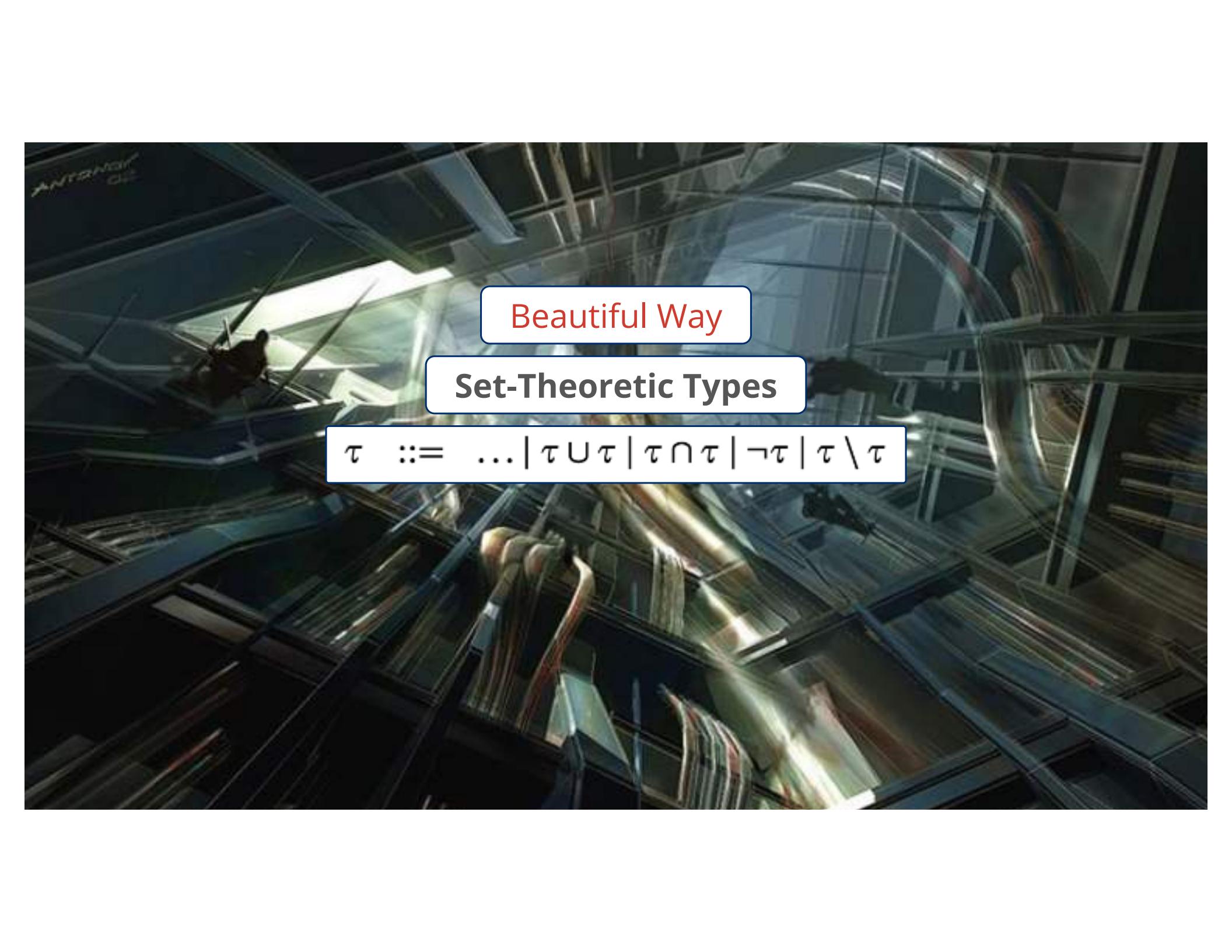
analyze(e_0)

```
class TypeChecker[TypeMap, TypeCheckerSharedApi]:  
    def find_isinstance_check_helper(  
        self, node: Expression, *, in_boolean_context: bool = True  
    ) -> tuple[TypeMap, TypeMap]:  
        if is_true_literal(node):  
            return {}, None  
        if is_false_literal(node):  
            return None, {}  
  
        if isinstance(node, CallExpr) and len(node.args) != 0:  
            expr = collapse_walrus(node.args[0])  
            if refers_to_fullname(node.callee, "builtins.isinstance"):  
                if len(node.args) != 2: # the error will be reported elsewhere  
                    return {}, {}  
                if literal(expr) == LITERAL_TYPE:  
                    return conditional_types_to_typemaps(  
                        expr,  
                        *self.conditional_types_with_intersection(  
                            self.lookup_type(expr), self.get_isinstance_type(node.args[1]), expr  
                        ),  
                    )  
            elif refers_to_fullname(node.callee, "builtins.issubclass"):  
                if len(node.args) != 2: # the error will be reported elsewhere  
                    return {}, {}  
            else:  
                return {}, None
```



Problem 2: want *subsets* of types,
but have only *syntactic approximations*

```
// val :: T
if val is Number:
... // val :: T ∩ Number
else:
... // val :: T \ Number
```

The background is a blurred, abstract image of a modern building's glass and steel structure, creating a sense of motion and complexity.

Beautiful Way

Set-Theoretic Types

$$\tau ::= \dots | \tau \cup \tau | \tau \cap \tau | \neg \tau | \tau \setminus \tau$$

Beautiful Way

Set-Theoretic Types

3 easy rules for type narrowing

$$\frac{\Gamma \vdash e' : t_1 \vee t_2 \quad \Gamma, x:t_1 \vdash e : t \quad \Gamma, x:t_2 \vdash e : t}{\Gamma \vdash e\{e'/x\} : t}$$

$$\frac{\Gamma \vdash e : t \quad \Gamma \vdash e_1 : t_1}{\Gamma \vdash (e \in t) ? e_1 : e_2 : t_1} \quad \frac{\Gamma \vdash e : \neg t \quad \Gamma \vdash e_2 : t_2}{\Gamma \vdash (e \in t) ? e_1 : e_2 : t_2}$$

[Castagna, Laurent, Nguyen, Lutze POPL'22]

Set-Theoretic Types

Remarkable type inference!

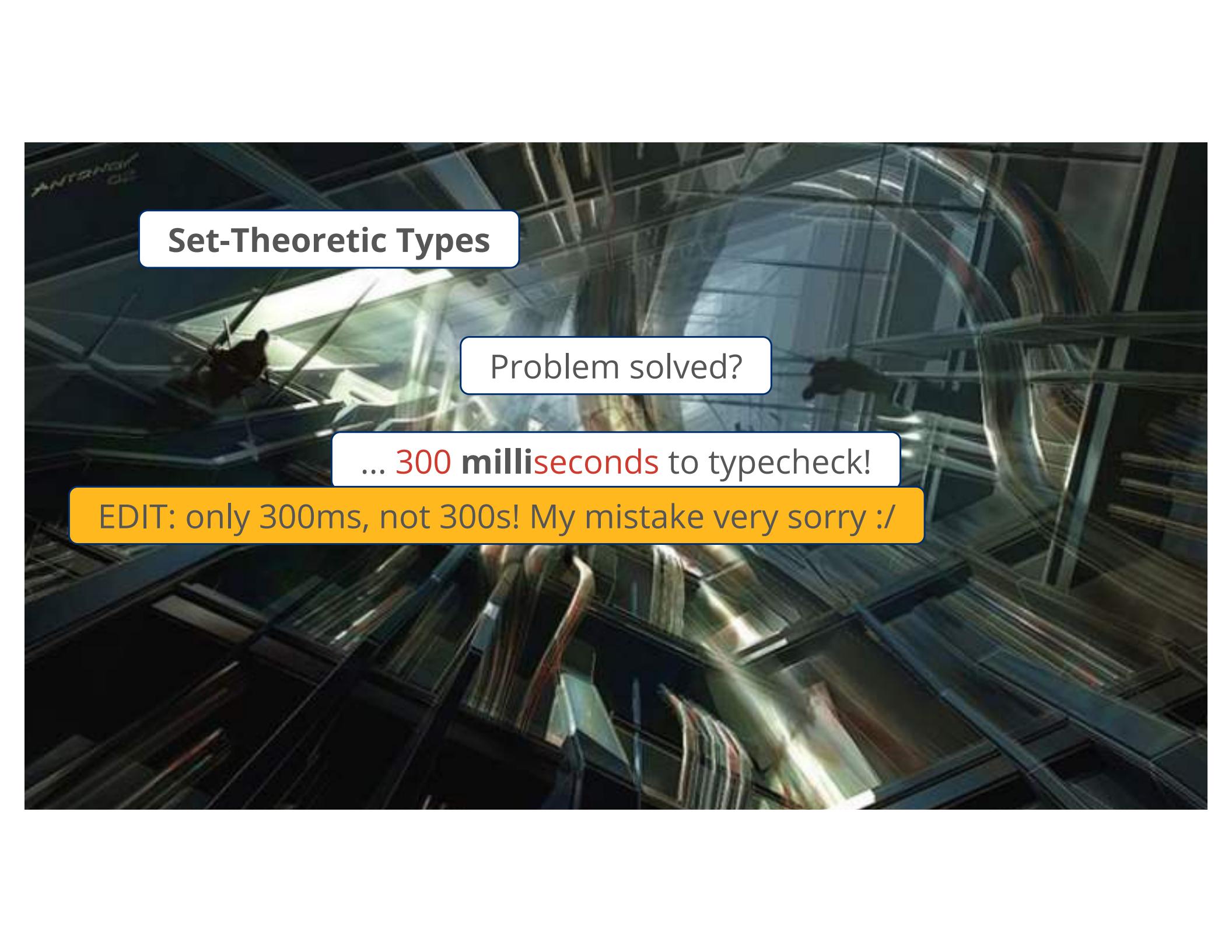
```
flatten([[a], b, [[c, d]]]) = [a, b, c, d]
```

```
let rec flatten t = match t
| [] -> []
| hd::tl -> concat (flatten hd) (flatten tl)
| _ -> [t]
```

```
flatten :: Nested      -> List(NotList)
          /\ NotList -> List(NotList)
```

```
Nested  = List(Nested) \/\ NotList
NotList = A \ List(Top)
```

[Castagna, Laurent, Nguyen POPL'24]



Set-Theoretic Types

Problem solved?

... 300 milliseconds to typecheck!

EDIT: only 300ms, not 300s! My mistake very sorry :/

```

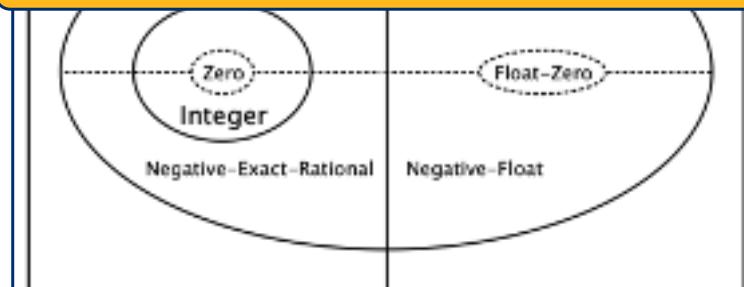
let plus : {byte, byte} -> index
: {index, index} -> nonnegFixnum
: {negFixnum, one} -> nonposFixnum
: {one, negFixnum} -> nonposFixnum
: {nonposFixnum, nonnegFixnum} -> fixnum
: {nonnegFixnum, nonposFixnum} -> fixnum
: {integer, integer} -> integer
: {float, real} -> float
: {real, float} -> float
: {singleFloat, rational | singleFloat} -> singleFloat
: {rational | singleFloat, singleFloat} -> singleFloat
: {posReal, nonnegReal} -> posReal
: {nonnegReal, posReal} -> posReal
: {negReal, nonposReal} -> negReal
: {nonposReal, negReal} -> negReal
: {nonnegReal, nonnegReal} -> nonnegReal
: {nonposReal, nonposReal} -> nonposReal
: {real, real} -> real
: {exactNumber, exactNumber} -> exactNumber
: {floatComplex, number} -> floatComplex
: {number, floatComplex} -> floatComplex
: {float, inexactComplex} -> floatComplex
: {inexactComplex, float} -> floatComplex
: {singleFloatComplex, rational | singleFloat | singleFloatComplex} -> singleFloatComplex
: {rational | singleFloat | singleFloatComplex, singleFloatComplex} -> singleFloatComplex
: {number, number} -> number
}
| (a, b) -> raise (a,b)
;;
let applyToPair (f : ('a , 'b) -> 'c) (p : ('a , 'b)) : 'c = f p;;
(* This line takes an extremely long time to type check (> 15 min); *)
let addPosBytes (b1 : posByte) (b2 : posByte) : posIndex =
  applyToPair plus (b1, b2);;

```

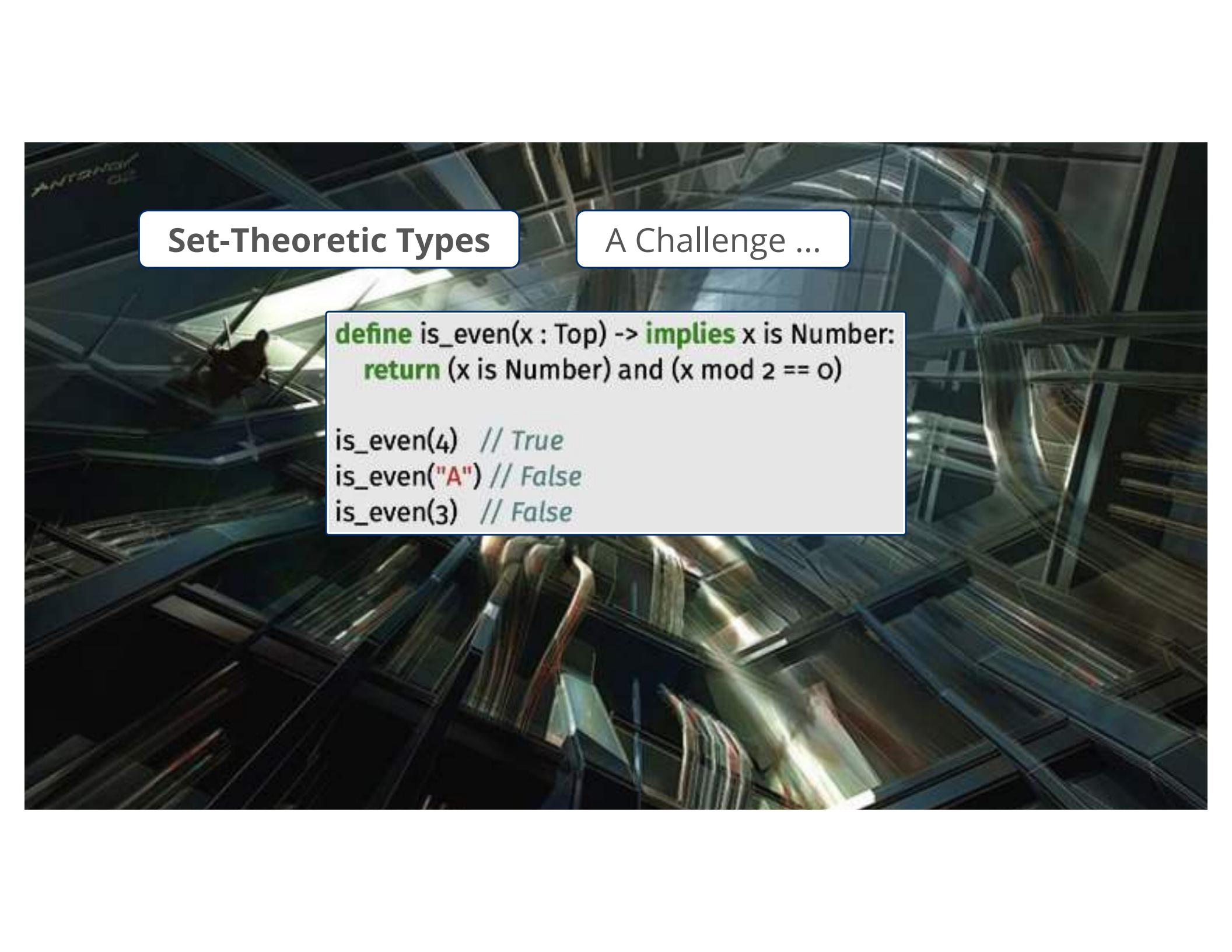
+15 minutes to typecheck!

Andrew Kent, 2019

2026: fixed by M. Laurent,
J. Valim,
G. Duboc



Exact-Rational \cup Float = Real



Set-Theoretic Types

A Challenge ...

```
define is_even(x : Top) -> implies x is Number:  
  return (x is Number) and (x mod 2 == 0)
```

```
is_even(4) // True  
is_even("A") // False  
is_even(3) // False
```

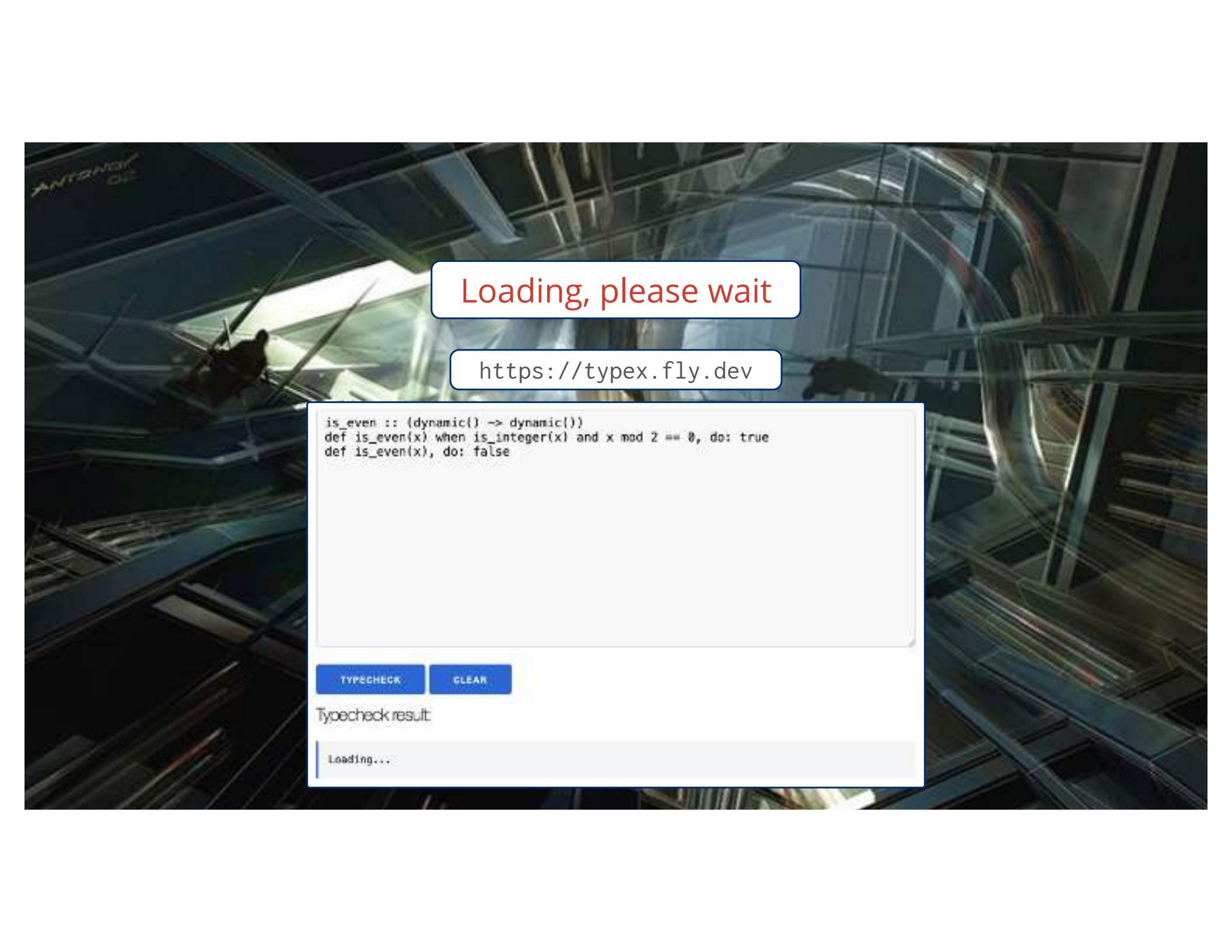
Set-Theoretic Types

A Challenge ...

```
define is_even(x : Top) -> implies x is Number:  
  return (x is Number) and (x mod 2 == 0)  
  
is_even(4) // True  
is_even("A") // False  
is_even(3) // False
```

```
is_even :: Even          -> True  
          /\ (A \ Even) -> False
```

Even = 0, 2, 4, ...



ANTONIO C

Loading, please wait

<https://typex.fly.dev>

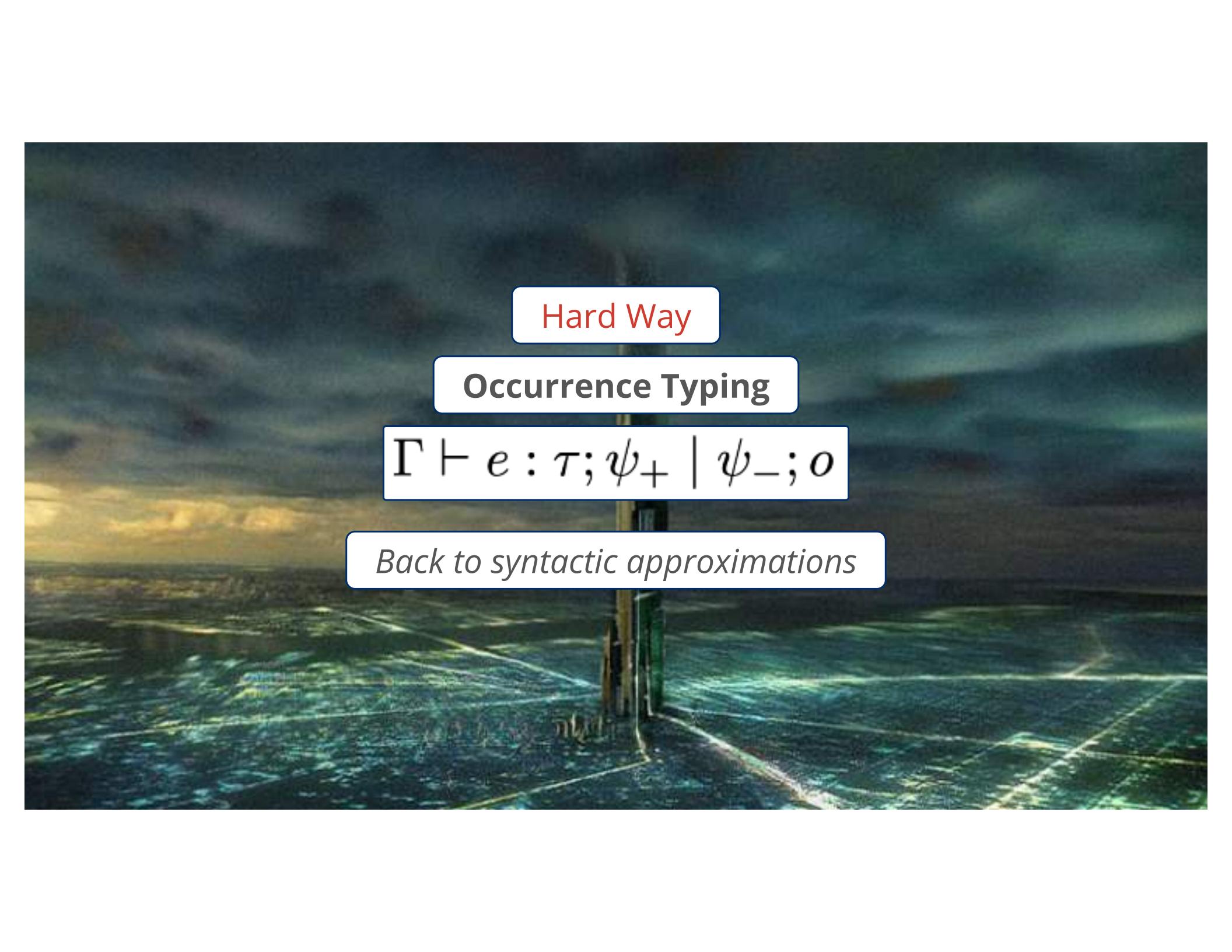
```
is_even :: (dynamic() -> dynamic())
def is_even(x) when is_integer(x) and x mod 2 == 0, do: true
def is_even(x), do: false
```

TYPECHECK

CLEAR

Typecheck result:

Loading...



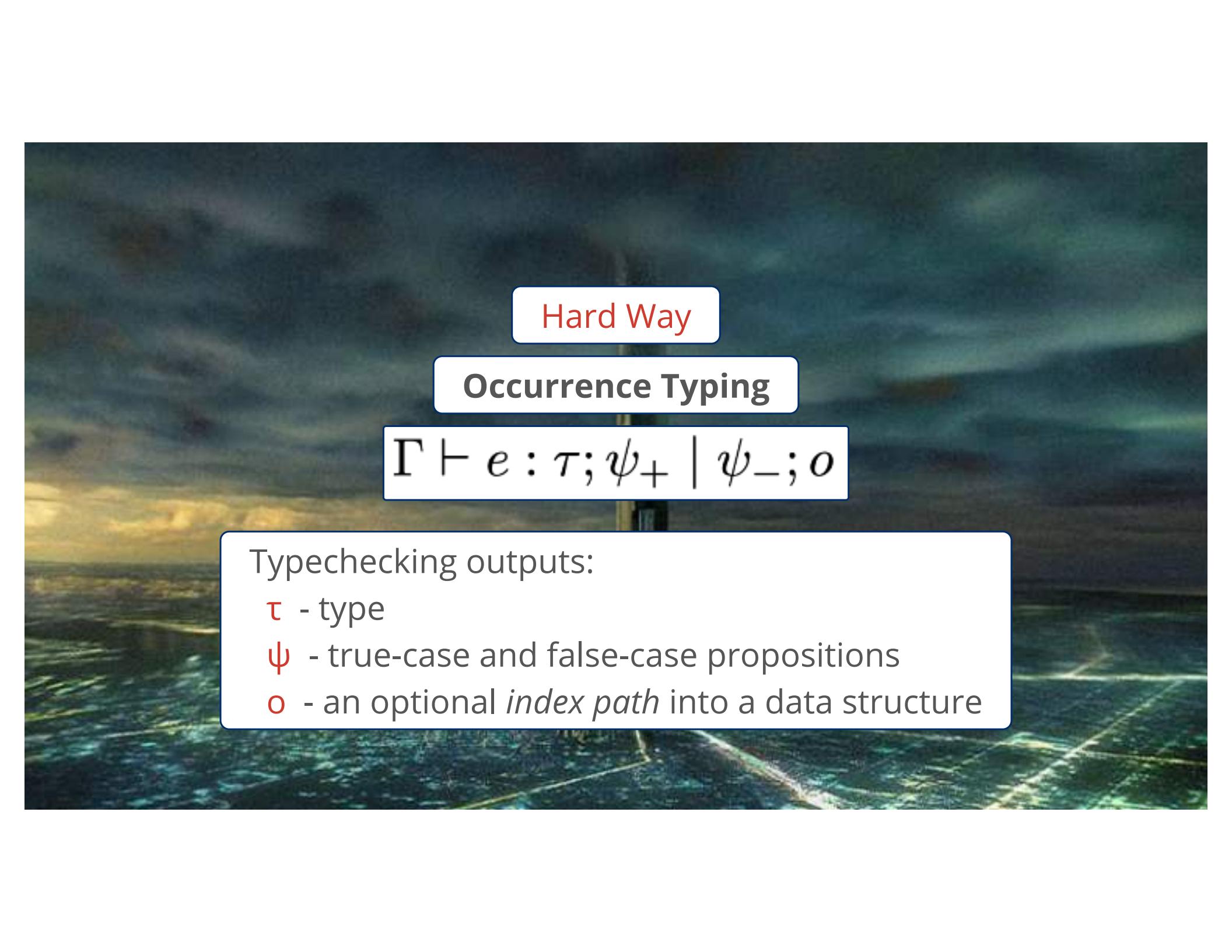
A painting of a bridge over water at sunset. The sky is filled with dramatic, colorful clouds. The water reflects the light, creating a shimmering effect. Overlaid on the image are several white rectangular boxes containing text.

Hard Way

Occurrence Typing

$$\Gamma \vdash e : \tau ; \psi_+ \mid \psi_- ; o$$

Back to syntactic approximations



Hard Way

Occurrence Typing

$$\Gamma \vdash e : \tau ; \psi_+ \mid \psi_- ; o$$

Typechecking outputs:

τ - type

ψ - true-case and false-case propositions

o - an optional *index path* into a data structure

Occurrence Typing

- Complex output

```
; this structure represents the result of typechecking an expression
; fields are #f only when the direct result of parsing or annotations
(def-rep tc-result ([t Type?]
                    [pset (c:or/c PropSet? #f)])
                    [o (c:or/c OptObject? #f)])
                    [exi? boolean?])
```

Occurrence Typing

- Complex output

```
;; this structure represents the result of typechecking an expression
;; fields are #f only when the direct result of parsing or annotations
(define/cond-contract (tc-expr/check/internal form expected)
  (parameterize ([current-orig-stx form])
    (syntax-parse form
      ;; data
      [(:quote #f) (ret (-val #f) -false-propset)]
      [(:quote #t) (ret (-val #t) -true-propset)]
      [(:quote val)
        (ret (match expected
                  [((tc-result1: t) (tc-literal #'val t))
                   (_ (tc-literal #'val))])
             -true-propset)
        ;; sometimes we want integer's symbolic objects
        ;; to be themselves
        (let ([v (syntax-e #'val)])
          (if (and (exact-integer? v)
                    (with-refinements?))
              (-lexp v)
              -empty-obj)))]))
```

```
; this structure represents the result of typechecking an expression
; fields are #f only when the direct result of parsing or annotations
(def-rep tc-result ([t Type?]
  [pset (c:or/c PropSet? #f)])
  [o (c:or/c OptObject? #f)])
  [exi? boolean?])
```

<https://github.com/racket/typed-racket/blob/master/typed-racket-lib/typed-racket/types/tc-result.rkt>
<https://github.com/racket/typed-racket/blob/master/typed-racket-lib/typed-racket/typecheck/tc-expr-unit.rkt>

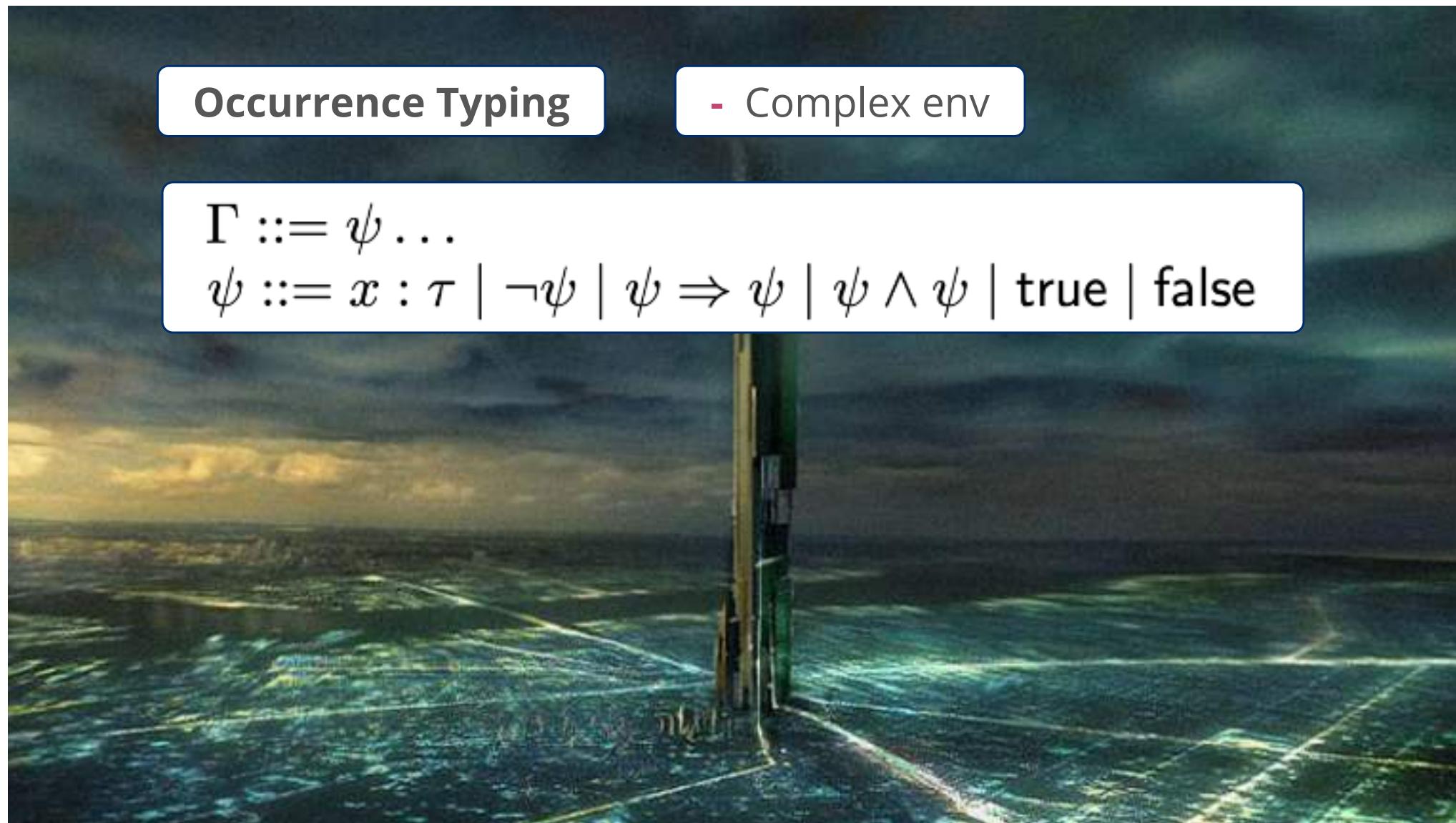


Occurrence Typing

- Complex env

$\Gamma ::= \psi \dots$

$\psi ::= x : \tau \mid \neg\psi \mid \psi \Rightarrow \psi \mid \psi \wedge \psi \mid \text{true} \mid \text{false}$



Occurrence Typing

- Complex env

$\Gamma ::= \psi \dots$

$\psi ::= x : \tau \mid \neg\psi \mid \psi \Rightarrow \psi \mid \psi \wedge \psi \mid \text{true} \mid \text{false}$

```
;; types is a free-id-table of identifiers to types
;; props is a list of known propositions
(define-struct env ([types immutable-free-id-table?]
                    [idx-types (hash/c Index? Type? #:immutable #t)])
  [props (listof Prop?)])
[aliases immutable-free-id-table?])
```



<https://github.com/racket/typed-racket/blob/master/typed-racket-lib/typed-racket/env/type-env-structs.rkt>

Occurrence Typing

+ Free of ad-hoc analysis

```
(define (tc/if-twoarm tst thn els [expected #f])
  (match-define (tc-result1: _ (PropSet: p+ p-) _) (single-value tst))
  (define thn-res
    (with-lexical-env+props (list p+)
      #:expected expected
      #:unreachable (warn-unreachable thn)
      (test-position-add-true tst)
      (tc-expr/check thn expected)))
  (define els-res
    (with-lexical-env+props (list p-)
      #:expected expected
      #:unreachable (warn-unreachable els)
      (test-position-add-false tst)
      (tc-expr/check els expected)))

  (match expected
    ;; if there was not any expected results; then merge the 'then'
    ;; and 'else' results so we propagate the correct info upwards
    [(or #f (tc-any-results: #f)) (merge-tc-results (list thn-res els-res))]
    ;; otherwise, the subcomponents have already been checked and
```

Occurrence Typing

+ Compositional

```
define f(x: Top, y: Top) -> Number:  
  if (if x is Number: y is String else: false):  
    return x + String.length(y)  
  else:  
    return 0
```

Very hard for **Easy Way** checkers!

Occurrence Typing

+ Detect wrong predicates

```
define f(x: String | Number | Boolean) -> x is String:  
  return x is String or x is Number // may return true when predicate is false
```

```
define g(x: String | Number | Boolean) -> x is Number | Boolean:  
  return x is Number // may return false when predicate is true
```

```
; check-below : (/ \ (Results Type -> Result)  
;                  (Results Results -> Result)  
;                  (Type Type -> Type))  
(define (check-below tr1 expected)  
  (define (prop-set-better? p1 p2)  
    (match* (p1 p2)  
            [(p p) #t]  
            [(p #f) #t]  
            [((PropSet: p1+ p1-) (PropSet: p2+ p2-))  
             (define positive-implies?  
               (or (TrueProp? p2+)  
                   (FalseProp? p1+)  
                   (implies-in-env? (lexical-env) p1+ p2+)))  
             (and positive-implies?  
                  (or (TrueProp? p2-)  
                      (FalseProp? p1-)  
                      (implies-in-env? (lexical-env) p1- p2-)))])  
            [(_ _) #f])  
  (define (object-better? o1 o2)
```

Occurrence Typing

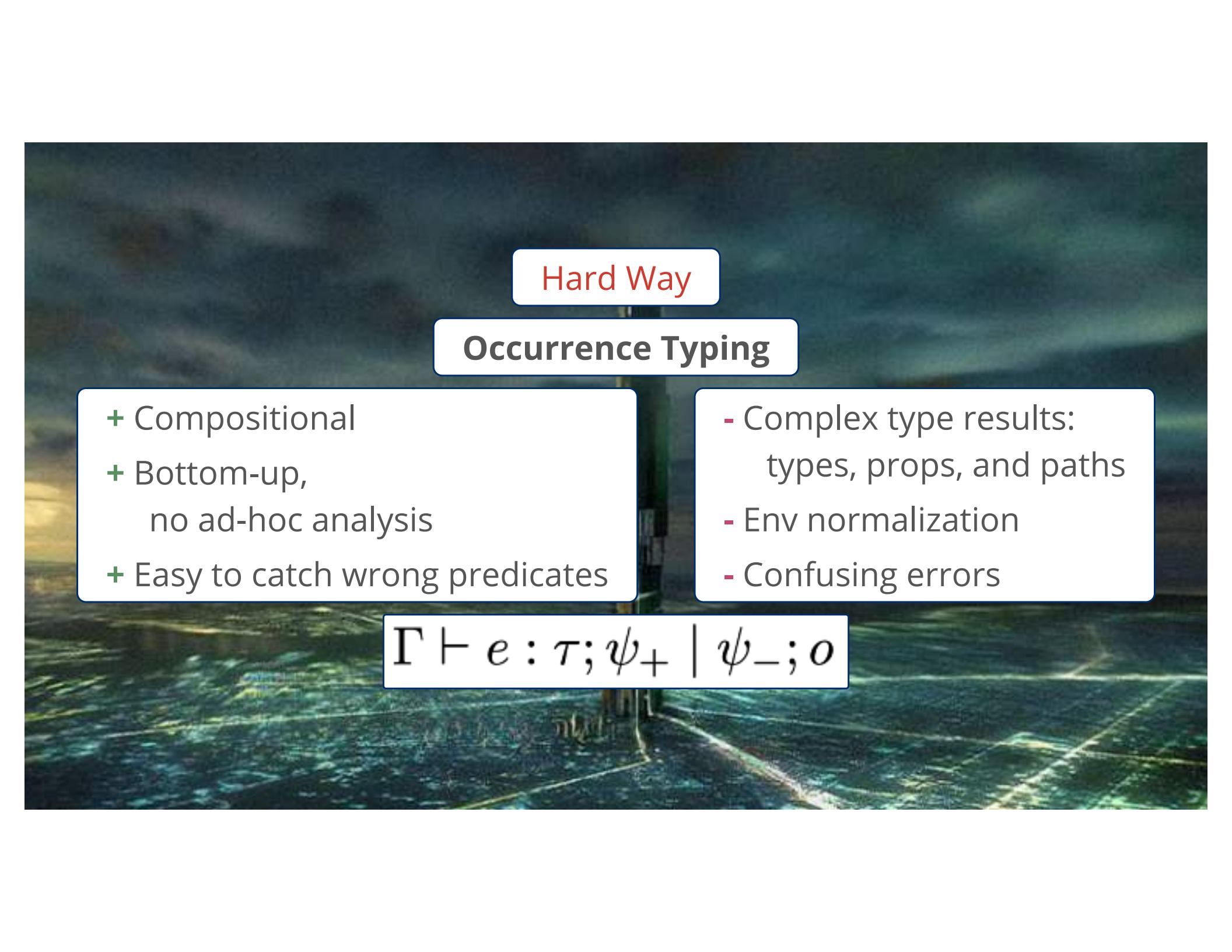
+ Detect wrong predicates

```
define f(x: String | Number | Boolean) -> x is String:  
  return x is String or x is Number // may return true when predicate is false
```

```
define g(x: String | Number | Boolean) -> x is Number | Boolean:  
  return x is Number // may return false when predicate is true
```

Type Checker: type mismatch;
mismatch in proposition
expected: ((: x String) | (! x String))
given: ((: x Number) | (! x Number))
in: (number? x)

Type Checker: type mismatch;
mismatch in proposition
expected: ((: x (U Boolean Number)) | (! x (U Boolean Number)))
given: ((: x Number) | (! x Number))
in: (number? x)



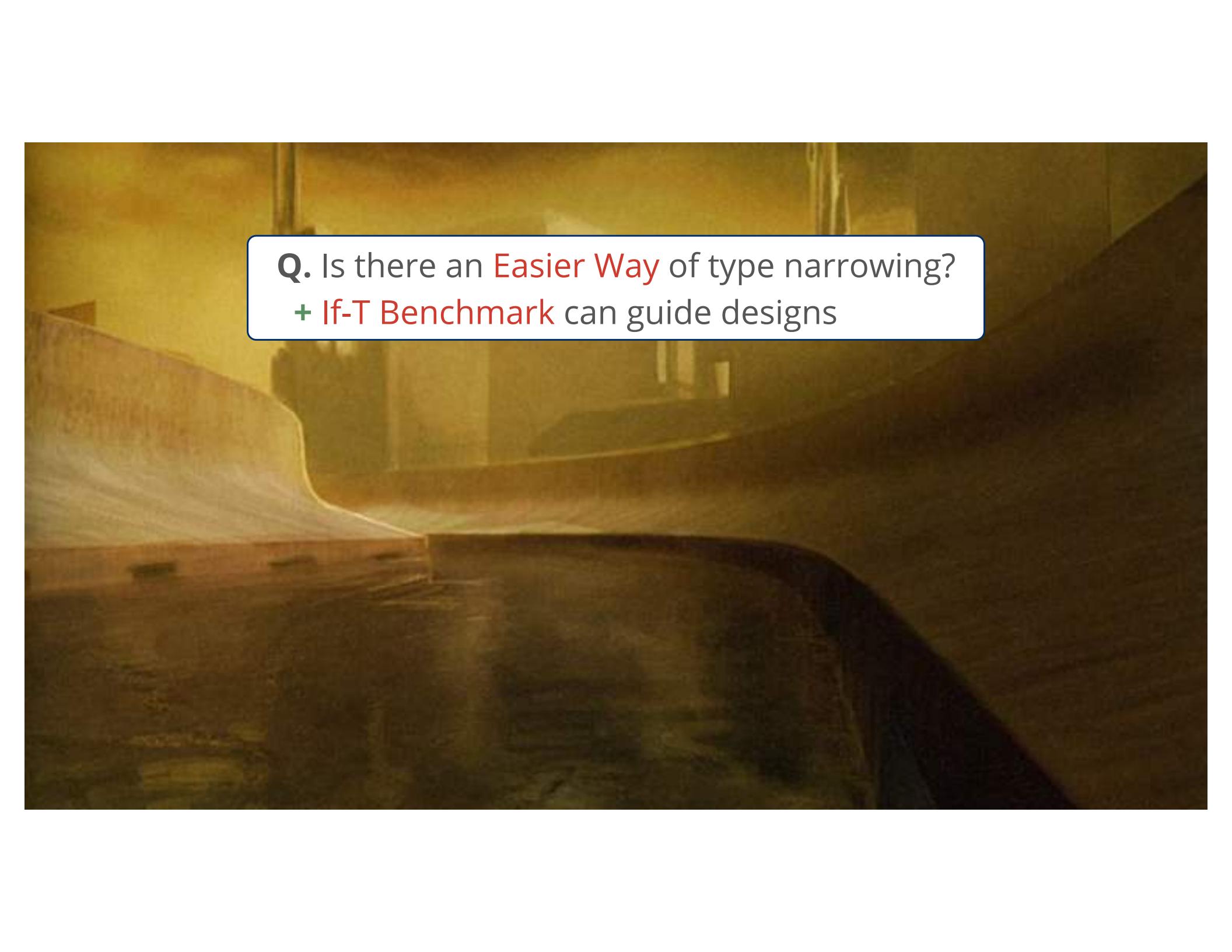
Hard Way

Occurrence Typing

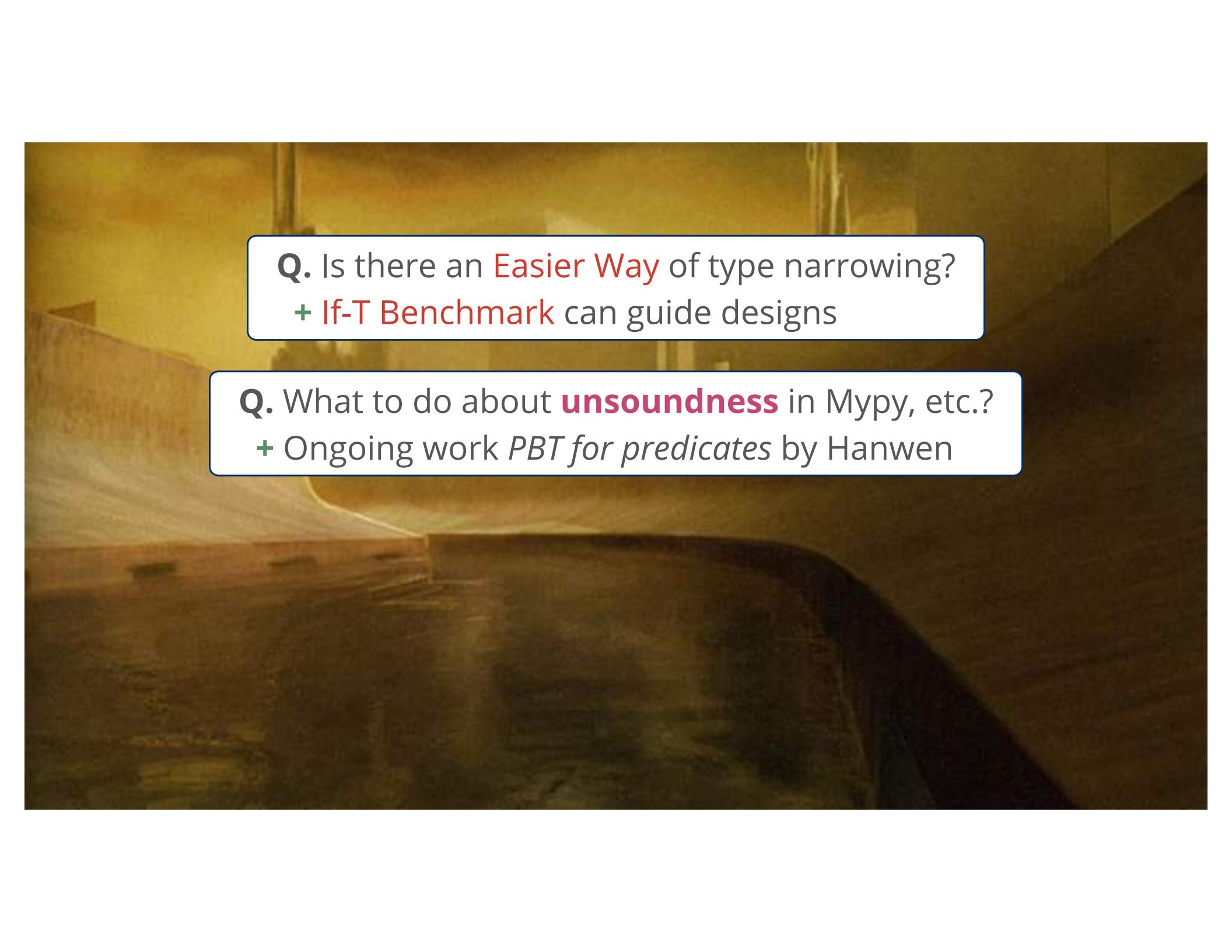
- + Compositional
- + Bottom-up,
no ad-hoc analysis
- + Easy to catch wrong predicates

- Complex type results:
types, props, and paths
- Env normalization
- Confusing errors

$$\Gamma \vdash e : \tau; \psi_+ \mid \psi_-; o$$

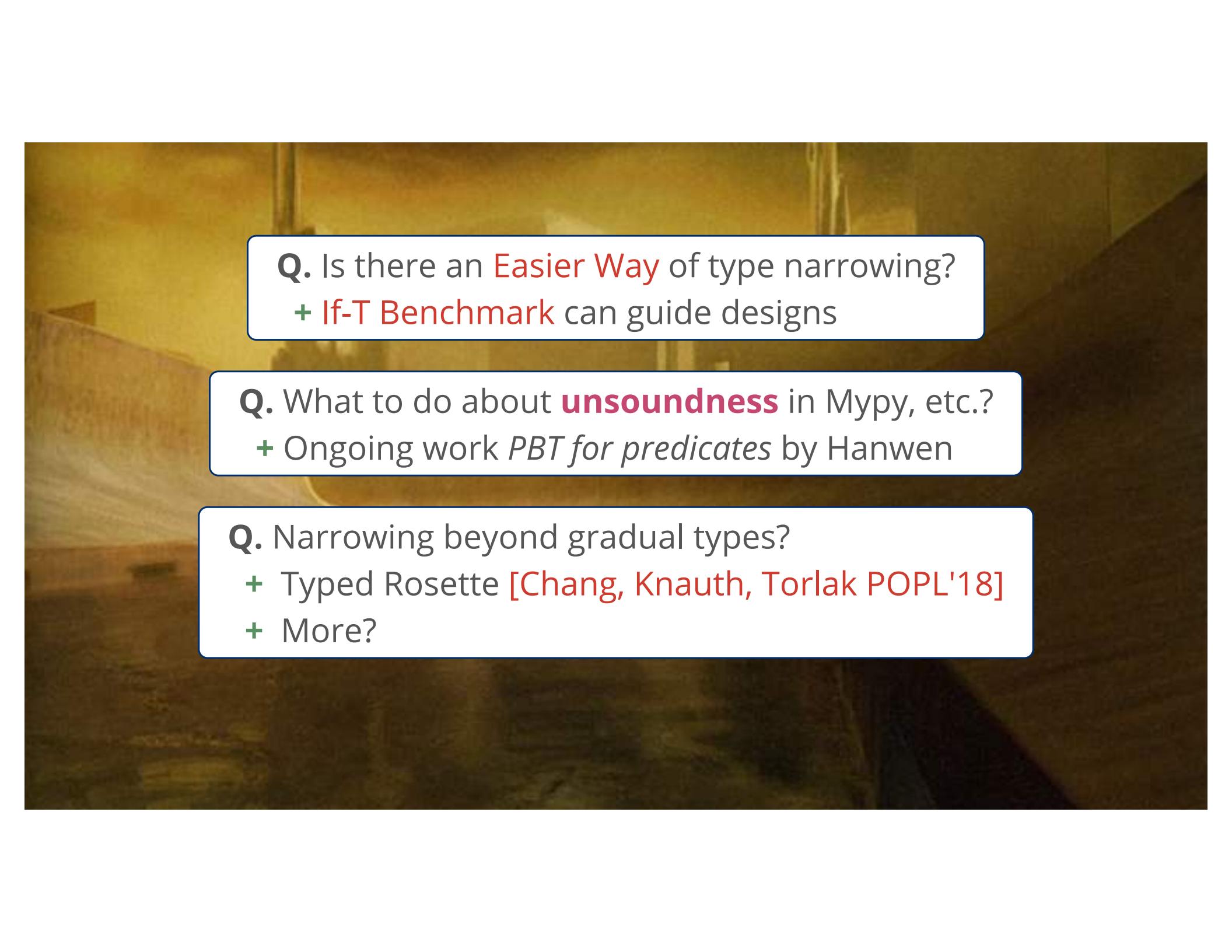


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The background of the slide shows a person from behind, sitting at a desk and looking at a laptop screen. The scene is dimly lit, with warm tones.

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Q. Narrowing beyond gradual types?
+ Typed Rosette [Chang, Knauth, Torlak POPL'18]
+ More?

