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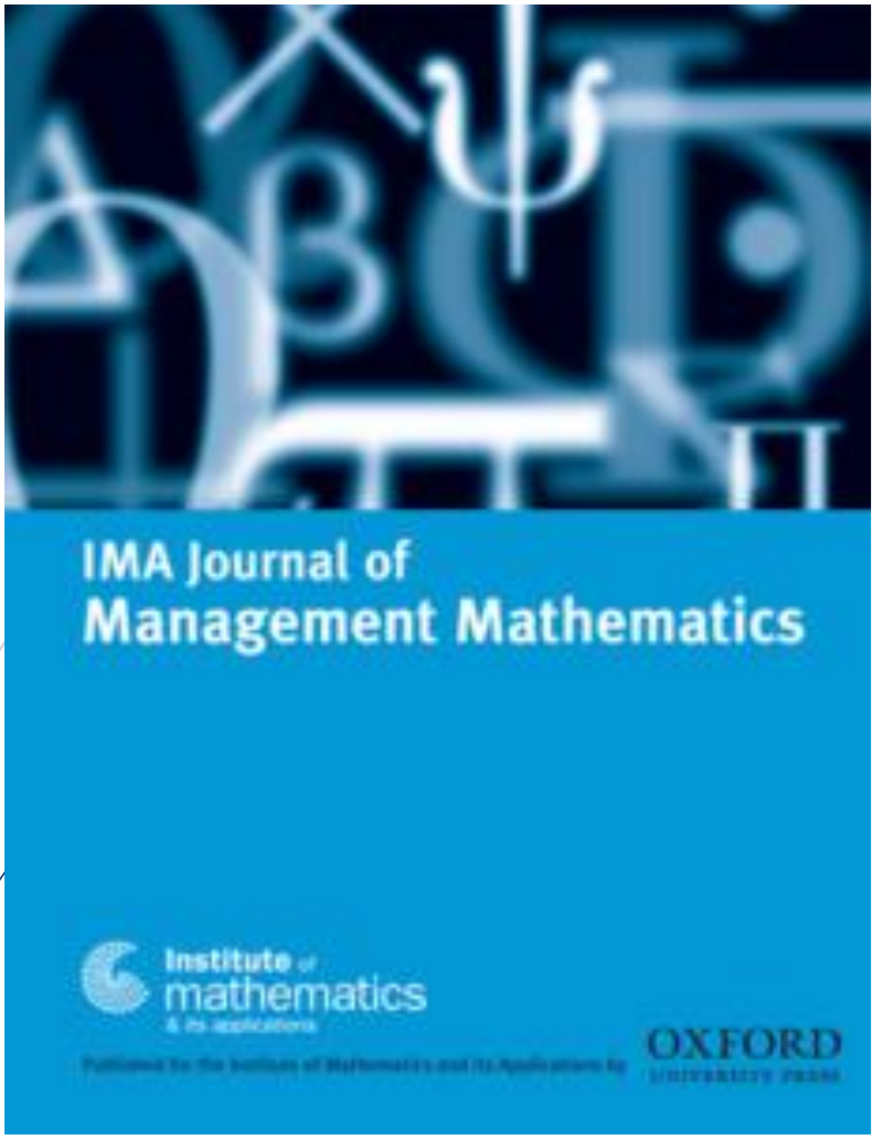

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A Review of Inverse Data Envelopment Analysis: Origins, Development, and Future Directions

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Abstract

Data Envelopment Analysis (DEA) is a widely used mathematical programming approach for assessing the efficiency of decision-making units (DMUs) in various sectors. Inverse DEA is a post-DEA sensitivity analysis approach developed initially for solving resource allocation. The main objective of Inverse DEA is to determine the optimal quantity of inputs and/or outputs for each DMU under input and/or output perturbation(s) that would allow them to reach a given efficiency target. Since the early 2000s, Inverse DEA has been extended theoretically and applied successfully in different areas including banking, energy, education, sustainability, and supply chain management. In recent years, research has demonstrated the potential of Inverse DEA for solving novel inverse problems, such as estimating merger gains, minimizing production pollution, optimizing business partnerships, and more. This paper provides a comprehensive survey of the latest theoretical and practical advancements in Inverse DEA, while also highlighting potential areas for future research and development in this field. One such area is exploring the use of heuristic algorithms and optimization techniques in conjunction with Inverse DEA models to

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address issues of infeasibility and nonlinearity. Moreover, applying Inverse DEA to new sectors such as healthcare, agriculture, and environmental and climate change issues holds great promise for future research. Overall, this paper sets the stage for further advancements in this promising approach.

Keywords: Data envelopment analysis (DEA), Inverse DEA, Resource allocation, Business partnerships, Strategic alliances, Merging, Consolidation

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming tool developed by Charnes et al. (1978) for the efficiency analysis of decision making units (DMUs). Since then, DEA has gained popularity as an approach for evaluating the performance of production units across diverse sectors. Emrouznejad and Yang (2018) provide a comprehensive review of the theory and application of DEA in scholarly literature during the first 40 years.

In recent years, a new research line in DEA has emerged, as Zhang and Cui (1999) raised a research question on the project evaluation system. The question pertains to determining how much additional input can be given to a unit to increase its outputs by a specific amount, such that its current efficiency stays unchanged - if the unit was to continue its operation. Wei et al. (2000) developed the inverse DEA problem in its current format for estimating input and output levels.

Although DEA models have been in use for over four decades, the inverse DEA method was introduced only in the early 2000s. Since then, the method has been extensively applied in multiple sectors, and several methodological extensions of the inverse DEA models have been developed.

2. Theoretical development of inverse DEA

In this section, we offer a concise summary of the recent theoretical advancements in inverse DEA, covering some of the key models that have been developed in the literature. Unlike an optimization problem where the objective is to find the optimal solution(s), inverse optimization involves a feasible solution that may not necessarily be optimal, and the goal is to reduce data perturbation as much as possible to make the given solution is optimal. The concept of inverse optimization and inverse linear programming formed the origin of inverse DEA (Zhang and Liu, 1996; Huang and Liu, 1999; Ahuja and Orlin, 2001). The first introduction of inverse linear programming was by Zhang and Liu (1996), which was further developed by Zhang and Liu (1999) and Huang and Liu (1999). Amin and Emrouznejad (2007) applied the concept of inverse linear programming to estimate forecasting parameters. The first introduction of inverse DEA was by Wei et al. (2000), where radial inverse DEA models were developed to solve inverse problems in input and output orientations.

Inverse DEA solves inverse problems under the umbrella of DEA where the objective is to estimate inputs and/or outputs of the DMU(s) under assessment for given efficiency targets. One typical inverse problem that arises in the context of Inverse DEA is determining how much additional output a DMU can produce if its inputs are increased to a specific level while maintaining at least its pre-perturbation efficiency score. To formulate this inverse problem, assume there are *n* DMUs where DMU-*j* uses *m* inputs $\boldsymbol{x}_j = (x_{1j}, \dots, x_{mj})$ to produce *s* outputs $\boldsymbol{y}_j = (y_{1j}, \dots, y_{rj})$. Wei et al. (2000) supposed that the inputs of $\mathrm{DMU}j_o$ are increased from $\boldsymbol{x}_o$ to $\boldsymbol{\alpha}_o = \boldsymbol{x}_o + \Delta\boldsymbol{x}_o$, where at least one component of $\Delta\boldsymbol{x}_o$ is positive. The inverse problem is to estimate the output vector $\boldsymbol{\beta} = (\beta_1, \dots, \beta_s) = \boldsymbol{y}_o + \Delta\boldsymbol{y}_o$ such that the efficiency score of $\mathrm{DMU}j_o$ remains unchanged after the changes of the inputs and outputs. Wei et al. (2000) presented an output-oriented inverse problem that can be formulated as a multi-objective vector optimization model, as follows:

$$
\begin{aligned}
&\max \Delta \boldsymbol{y}_o = (\Delta y_{1o}, \dots, \Delta y_{so}) \\
&s.t. \\
&\quad \textstyle\sum_{j=1}^{n} \boldsymbol{x}_j \lambda_j \leq \boldsymbol{\alpha}_o \\
&\quad \textstyle\sum_{j=1}^{n} \boldsymbol{y}_j \lambda_j \geq z_o \boldsymbol{\beta} \\
&\quad \boldsymbol{\beta} \geq \boldsymbol{y}_o \\
&\quad \delta_1 (\textstyle\sum_{j=1}^{n} \lambda_j + \delta_2 (-1)^{\delta_3} v) = \delta_1 \\
&\quad v \geq 0, \lambda_j \geq 0,\ j = 1, \dots, n
\end{aligned}
\qquad (1)
$$

Where the parameter z_o is given to make sure the DMU maintains at least its current efficiency score after the perturbation. As explained in Wei et al. (2000), the last constraint in the above model presents type of return to scale with the three binary parameters δ_1, δ_2, and δ_3 When $\delta_1 = 0$, for any binary values of δ_2 and δ_3 the last constraint will be redundant and hence the return to scale will be constant. When $\delta_1 = 1$ and $\delta_2 = 0$, for any binary values of δ_3 the last constraint of model (1) would present variable return to scale. When $\delta_1 = \delta_2 = 1$ and $\delta_3 = 0$ then the constraint would present non-increasing return to scale while $\delta_1 = \delta_2 = \delta_3 = 1$ indicates non-decreasing return to scale.

In the input-orientation, an inverse DEA model can be formulated for situations where outputs must be increased to a certain level. The associated inverse problem involves determining the minimum level of inputs required for the DMU to maintain its pre-perturbation efficiency score. This model can be expressed as follows:

$$
\begin{aligned}
&\min \Delta \boldsymbol{x}_o = (\Delta x_{1o}, \dots, \Delta x_{mo}) \\
&s.t. \\
&\quad \textstyle\sum_{j=1}^{n} \boldsymbol{x}_j \lambda_j \leq \theta_o (\boldsymbol{x}_o + \Delta \boldsymbol{x}_o) \\
&\quad \textstyle\sum_{j=1}^{n} \boldsymbol{y}_j \lambda_j \geq \boldsymbol{y}_o + \Delta \boldsymbol{y}_o \\
&\quad \delta_1 (\textstyle\sum_{j=1}^{n} \lambda_j + \delta_2 (-1)^{\delta_3} v) = \delta_1 \\
&\quad v \geq 0, \lambda_j \geq 0,\ j = 1, \dots, n
\end{aligned}
\qquad (2)
$$

Where θ_o is the pre-perturbation efficiency score of the DMU. Unlike model (1), in the above model $\Delta \boldsymbol{y}_o$ is known prior to solving the model. The literature on inverse DEA has experienced considerable theoretical advancement, with a notable contribution being made by Yan et al. (2002), who extended the inverse DEA framework by incorporating preference cone constraints for input and output estimation in resource allocation.

Hadi-Vencheh et al. (2008) developed a solution procedure for solving multi-objective inverse DEA models. Ghiyasi (2017b) formulated new inverse DEA models based on cost and revenue efficiency. Lim (2016) further extended the inverse DEA by introducing production frontier changes, while Amin and Al-Muharrami (2018) and Ghiyasi and Zhu (2020) extended the inverse DEA models to deal with negative data using the semi-oriented radial measure. Lertworasirikul et al. (2011) dealt with the variable returns to scale model of the inverse DEA which was improved by Ghiyasi (2015). Modhej, et al (2017) further developed this method using both inverse-DEA and neural network to preserve relative efficiency values. Jahanshahloo et al. (2015) proposed an inter-temporal inverse DEA dependence using multiple-objective programming. Zhang and Cui (2016) dealt with the variation in input and output changes preserving the mix of input efficiency and output efficiency. Zhang and Cui (2020) introduced a general non-radial inverse DEA model where slacks play important role. Shiri Daryani (2021) developed inverse DEA models within a two-stage framework that considers allocative efficiency. Ghiyasi (2017a) developed an inverse DEA model to address pollution generated by technologies with undesirable outputs. Chen and Wang (2021) tackled the infeasibility issue in VRS inverse DEA models by introducing a modified input-output range, while, Adimi et al. (2020) explored the effects of efficiency and frontier changes in inverse DEA. To handle imprecise data, Ghomi et al. (2021) proposed an inverse DEA model that incorporates stochastic data. These studies demonstrate the versatility and flexibility of the inverse DEA approach and highlight its potential to address complex and diverse efficiency analysis problems. The inverse DEA has been applied successfully in estimating the potential gains of mergers. Gattoufi et al. (2014) were the pioneers in developing novel inverse DEA models for this purpose. They defined the potential gains of mergers by minimizing inputs and maximizing outputs. Their objective in the input-oriented inverse DEA model was to minimize the quantity of inputs of the merging DMUs required to produce the output levels of the merging units that would enable the merged entity to achieve a given efficiency target. This can be expressed as follows:

$$
\begin{aligned}
&\min\ \textstyle\sum_{i=1}^{m}(\alpha_{ik}+\alpha_{il})\\
&s.t.\\
&\quad \textstyle\sum_{j\in F}\lambda_j x_{ij}+(x_{ik}+x_{il})\lambda_{\mathrm{M}}-(\alpha_{ik}+\alpha_{il})\bar{\theta}\le 0\quad i=1,\cdots,m\\
&\quad \textstyle\sum_{j\in F}\lambda_j y_{rj}+(y_{rk}+y_{rl})\lambda_{\mathrm{M}}\ge y_{rk}+y_{rl}\quad r=1,\cdots,s\\
&\quad \textstyle\sum_{j\in F}\lambda_j+\lambda_M=1\\
&\quad 0\le\alpha_{ik}\le x_{ik},0\le\alpha_{il}\le x_{il}\ \ i=1,\cdots,m\\
&\quad \lambda_j\ge 0,\ j\in F,\lambda_M\ge 0
\end{aligned}
\tag{3}
$$

The above inverse DEA model (3) estimates the gains of the potential merger between two merging DMU$_k$ and DMU_l that would allow the merged entity to realize the given efficiency target $\bar{\theta}$. The decision variables α_{ik} and α_{il} are the quantities of the ith input from the merging DMU$_k$ and DMU_l, respectively, that will be kept by the merged entity M (for each $i=1,\cdots,m$). In fact, the inverse DEA model (3) maximizes the amount of inputs that can be saved from the merging DMUs. It is noteworthy that the coefficient of λ_{M} in the i$^{\text{th}}$ input constraint of the original input-oriented inverse DEA model in Gattoufi et al. (2014) was $(\alpha_{ik}+\alpha_{il})$ which was corrected to $(x_{ik}+x_{il})$ by Amin and Ibn-Boamah (2020). It is also assumed that the inputs have the same measurement units otherwise the objective function of model (3) should be replaced by an appropriate weighted function.

Gattoufi et al. (2014) proposed another application of inverse DEA in estimating the potential gains of mergers under the output-oriented context. Their model aimed to determine the maximum additional outputs that can be produced by the merged entity, subject to maintaining the same input levels as before the merger. The output-oriented inverse DEA model can be expressed as follows:

$$
\begin{aligned}
&\max\ \textstyle\sum_{r=1}^{s}\beta_r\\
&s.t.\\
&\quad \textstyle\sum_{j\in F}\lambda_j x_{ij}+(x_{ik}+x_{il})\lambda_{\mathrm{M}}\le x_{ik}+x_{il}\qquad\qquad i=1,\cdots,m\\
&\quad \textstyle\sum_{j\in F}\lambda_j y_{rj}+(y_{rk}+y_{rl}+\beta_r)\lambda_{\mathrm{M}}\ge(y_{rk}+y_{rl}+\beta_r)\bar{h}\quad r=1,\cdots,s\\
&\quad \textstyle\sum_{j\in F}\lambda_j+\lambda_M=1\\
&\quad \beta_r\ge\ 0\ \ r=1,\cdots,s\\
&\quad \lambda_j\ge 0,\ j\in F,\lambda_M\ge 0
\end{aligned}
\tag{4}
$$

It is worth noting that the above input and output oriented inverse DEA models (3) and (4) are different from the sensitivity analysis DEA models such as the super efficiency DEA sensitivity analysis. The stated inverse DEA models solve inverse problems for estimating mergers gains where a set of at least two pre-merger DMUs combine their resources to make a new entity.

Amin et al. (2017a) contributed to the inverse DEA literature by developing models to identify minor and major consolidations. A merger that leads to a change in the pre-consolidation efficiency frontier is called a major consolidation, while a merger that does not cause such a change is called a minor consolidation (also see Amin et al (2019a) and Amin et al (2019b)). Their work enables practitioners to differentiate between the two types of consolidations and determine their impacts on efficiency scores, providing valuable insights for decision making.

Amin et al. (2017b) extended the concept of merger and acquisitions and introduced a new concept called generalized restructuring, which involves a set of pre-restructuring DMUs that proceed with restructuring to produce a new set of post-restructuring DMUs. They proposed novel inverse DEA models to estimate the potential gains of generalized restructuring. Specifically, they introduced an input-oriented generalized restructuring inverse DEA model, which can be formulated as follows:

Let us define I and R as, respectively, the sets of indices for inputs and outputs. Let $\bar{P}$ be the set of all pre-restructuring DMUs except those belonging to P, where P is the set of selected pre-restructuring DMUs in the generalized restructuring and $\bar{P} = \{1, \dots, n\} - P$. Without loss of generality, Amin et al. (2017b) assumed that all the pre-restructuring DMUs in P disappear after the generalized restructuring. Then, the generalized input-oriented inverse DEA model is as follows.

$$
\begin{aligned}
& min \sum_{\mathrm{i} \in \mathrm{I}} \sum_{\mathrm{q} \in \mathrm{Q}} \sum_{\mathrm{p} \in \mathrm{P}} \alpha_{\mathrm{ip}}^{\mathrm{q}} \\
s.t. \quad & \\
& \sum_{j \in \bar{P}} \lambda_j^q x_{ij} + \sum_{q \in Q} \sum_{p \in P} a_{ip}^q \lambda_q - {}^{-}\bar{\theta}_q \sum_{p \in P} a_{ip}^q \le 0, i \in I, q \in Q \\
& \sum_{j \in \bar{P}} \lambda_j^q y_{rj} + \sum_{q \in Q} \sum_{p \in P} \beta_{rp}^q \lambda_q - \sum_{p \in P} \beta_{rp}^q \ge 0, r \in R, q \in Q \\
& \sum_{j \in \bar{P}} \lambda_j^q + \sum_{q \in Q} \lambda_q = 1, q \in Q \qquad (5)
\end{aligned}
$$

$$\sum_{q\in Q} a_{ip}^{q} \leq x_{ip}, i \in I, p \in P$$
$$\sum_{q\in Q} \beta_{rp}^{q} = y_{rp}, r \in R, p \in P$$
$$\lambda_j^q \geq 0, j \in \bar{P}, q \in Q$$
$$\lambda_q \geq 0, q \in Q$$
$$a_{ip}^{q} \geq 0, i \in I, q \in Q, p \in P$$
$$\beta_{rp}^{q} \geq 0, r \in R, q \in Q, p \in P$$

Where α_{ip}^{q} and β_{rp}^{q} are, respectively, the levels of the ith input and the r^{th} output redistributed from the pre-restructuring DMUp to the post-restructuring DMUq, for all $i \in I, r \in R, q \in Q, p \in P$, and $\bar{\theta}_q$ is the given efficiency target for the post-restructuring DMUq. The objective of the above generalized inverse DEA model (5) is to minimize the sum of inputs required from all pre-restructuring DMUs to allow the post-restructuring DMUs to achieve their predefined efficiency targets. Amin et al. (2017b) also developed output-oriented generalized inverse DEA models where the objective is to maximize the production of additional outputs by the post-restructuring firms. Gerami et al. (2023) suggested a generalized inverse DEA model in the context of firm restructuring on the basis of value efficiency.

Previous research in DEA has primarily focused on estimating potential gains of mergers, but business partnerships and strategic alliances are becoming increasingly important for firms seeking to achieve their goals. Amin and Ibn-Boamah (2023) have extended the application of DEA and inverse DEA to model different types of business partnerships and strategic collaborations between DMUs. They introduce various DEA and inverse DEA models to guide firms on how to redistribute their inputs and outputs to reach target efficiency and enhance their performance.

One of the inverse DEA models proposed by Amin and Ibn-Boamah (2023) focuses on input redistribution in a partnership. This model is particularly useful for decision-makers seeking to set efficiency targets in a partnership. The model identifies the minimum input redistribution between two partners, DMUs k and l, required to achieve their given efficiency targets. The model is formulated as follows.

$$
\begin{aligned}
&\min\ \delta \\
&s.t. \\
&\quad \sum_{j\in F} \lambda_j^k x_{ij} + (x_{ik} - \alpha_i^1)\lambda_k^k + (x_{il} + \alpha_i^1)\lambda_l^k \le \bar{\theta}_k (x_{ik} - \alpha_i^1) \quad i \in I_1 \\
&\quad \sum_{j\in F} \lambda_j^k x_{ij} + (x_{ik} + \alpha_i^2)\lambda_k^k + (x_{il} - \alpha_i^2)\lambda_l^k \le \bar{\theta}_k (x_{ik} + \alpha_i^2) \quad i \in I_2 \\
&\quad \sum_{j\in F} \lambda_j^k x_{ij} + x_{ik}\lambda_k^k + x_{il}\lambda_l^k \le \bar{\theta}_k x_{ik} \quad i \in I_3 \\
&\quad \sum_{j\in F} \lambda_j^k y_{rj} + y_{rk}\lambda_k^k + y_{rl}\lambda_l^k \ge y_{rk} \quad r = 1, \cdots, s \\
&\quad \sum_{j\in F} \lambda_j^k + \lambda_k^k + \lambda_l^k = 1 \\
&\quad \sum_{j\in F} \lambda_j^l x_{ij} + (x_{ik} - \alpha_i^1)\lambda_k^l + (x_{il} + \alpha_i^1)\lambda_l^l \le \bar{\theta}_l (x_{il} + \alpha_i^1) \quad i \in I_1 \\
&\quad \sum_{j\in F} \lambda_j^l x_{ij} + (x_{ik} + \alpha_i^2)\lambda_k^l + (x_{il} - \alpha_i^2)\lambda_l^l \le \bar{\theta}_l (x_{il} - \alpha_i^2) \quad i \in I_2 \\
&\quad \sum_{j\in F} \lambda_j^l x_{ij} + x_{ik}\lambda_k^l + x_{il}\lambda_l^l \le \bar{\theta}_l x_{il} \quad i \in I_3 \\
&\quad \sum_{j\in F} \lambda_j^l y_{rj} + y_{rk}\lambda_k^l + y_{rl}\lambda_l^l \ge y_{rl} \quad r = 1, \cdots, s \\
&\quad \sum_{j\in F} \lambda_j^l + \lambda_k^l + \lambda_l^l = 1 \\
&\qquad \delta \ge \alpha_i^1 \ \forall i \in I_1 \\
&\qquad \delta \ge \alpha_i^2 \ \forall i \in I_2 \\
&\quad \alpha_i^1 \le p_i x_{ik} \quad \forall i \in I_1 \\
&\quad \alpha_i^2 \le q_i x_{il} \quad \forall i \in I_2 \\
&\quad \lambda_j^k \ge 0,\ \lambda_j^l \ge 0,\ j \in F \cup \{k, l\},\ \alpha_i^1 \ge 0, i \in I_1, \alpha_i^2 \ge 0\ i \in I_2
\end{aligned}
\tag{6}
$$

Where:

$$I_1 = \{i: DMU_l \text{ uses some of the } i\text{th input of } DMU_k\}$$
$$I_2 = \{i: DMU_k \text{ uses some of the } i\text{th input of } DMU_l\}$$
$$I_3 = I - (I_1 \cup I_2)$$

In fact, I_3 is the set of other inputs that will not be used by the two partners, and $I = \{1, \ldots, m\}$ is the set of all inputs. Also, α_i^1 as the amount of the ith input that DMU_k gives toDMU_l, for each $i \in I_1$. Similarly, α_i^2 is defined as the quantity of input i that DMU_l gives to DMU_l, for each $i \in I_2$. The objective of model (6) is to minimize the maximum input redistribution between the partners to achieve efficiency targets. This means:

$$\delta = Max\ \{\alpha_i^1: \forall i \in I_1, \alpha_i^2: \forall i \in I_2\}$$

The above inverse DEA models (1 to 6) present some of the developed inverse DEA models in the literature that solve important inverse problems. These models can be classified as radial as their base optimization DEA models are radial DEA models. Nonetheless, if the base optimization DEA model is non-radial then the associated inverse DEA model can be easily written in the no-radial optimization form.

Several other inverse DEA models have been proposed in the literature to estimate potential merger gains. Guijarro et al. (2020) presented inverse DEA models for this purpose. Amin and Ibn-Boamah (2020) focused on cost efficiency and proposed inverse DEA models that optimize the inputs and outputs of merging DMUs to achieve given cost efficiency targets. Ghiyasi et al. (2022) addressed the issue of ratio data in the context of inverse DEA. Younesi et al. (2023) introduced an SBM inverse DEA model that is suitable for the case of integer and continuous interval data. These models extend the application of inverse DEA to different types of data and efficiency targets, providing decision-makers with more options to evaluate potential mergers and acquisitions. Recently, Oukil (2023) applied the concept of inverse DEA to a set of agricultural farms in Oman to estimate the potential gains expected from the merger of two or more farms as well as the redistribution of inputs among the merging farms for given efficiency targets. Soltanifar et al. (2023) developed inverse DEA models for estimating potential merger gains when the data contains ratio and negative values. Mahla et al. (2023) suggested an inverse DEA model for resource allocation and investment analysis with ratio data while Çakır (2017) proposed an integrated Shannon's entropy–inverse DEA model for resource allocation considering fuzzy environment. Kazemi and Galagedera (2023) introduced an inverse DEA model for serially linked two stage constant returns to scale production processes.

3. Recent contributions for applications of inverse DEA

The inverse DEA approach has gained increasing popularity in recent years due to its broad range of applications in sectors such as business, supply chain management, agriculture, education, manufacturing, sustainable production, energy, and environment. In fact, inverse DEA models can complement DEA models, and anywhere DEA models are used, inverse DEA models can be used as well. In addition, inverse DEA models can be used independently for applications such as resource allocation, budgeting, and planning.

One of the benefits of using inverse DEA models is that they can perform sensitivity analysis, which is problematic for DEA models due to their non-linearity. In terms of industries, banking and business, energy and environment, education, sustainability, and supply chain management are among the top industries that utilize inverse DEA. While some applications such as those in banking have been implementing inverse DEA since its early days, other sectors such as healthcare and agriculture are starting to explore the potential of applying inverse DEA in recent years.

Most applied papers that use existing models in the literature have utilized the original inverse DEA models. However, some of the new articles with theoretical developments have implemented their models in real-world applications; this is attracting other researchers to apply these models to different sectors. Some selected applications of inverse DEA in different sectors are provided in Table 1A in the Appendix. Overall, the applications of inverse DEA are expanding to new fields, and it is expected to see more applications in the future.

3.1. *Inverse DEA for Business and banking:*

The inverse DEA approach has been applied in various sectors, but the banking and business sectors are the main areas of application, similar to the classical DEA models. Gattoufi et al. (2014) were the first to use the inverse DEA model for merger analysis in the banking sector. Saen and Nia (2019) proposed a network structured inverse DEA model to assess the performance and sensitivity analysis of after-sale services in a car company. Guijarro et al. (2020) used inverse DEA models and genetic algorithms to address the sector restructuring problem, with applications in banking and higher education sectors. Hosseininia and Saen (2020) developed a slack-based measure for the inverse DEA problem to analyse after-sale services in a car company. An et al. (2019) suggested a two-stage inverse DEA model considering undesirable outputs for resource planning of the Chinese commercial banking system. Yu et al. (2019) proposed an inverse-like DEA model to analyse the operational efficiency of Chinese banks and find potential income gains, taking credit risk into consideration. Amin and Ibn-Boamah (2020) considered cost efficiency measures in the merger analysis of Canadian banks based on the inverse DEA problem. Amin and Ibn-Boamah (2021) developed a two-stage inverse DEA model to investigate potential gains from bank mergers. In a recent paper, Amin and Ibn-Boamah (2023) proposed a strategic business partnership model in an inverse DEA framework to help decision makers consider strategic alliances and partnerships to improve competitiveness. To assess the performance of the banking sector, Ghomi et al. (2021) used an inverse DEA model with stochastic data. Avand et al. (2023) used a multi-objective inverse DEA model to address the units' restructuring problem in the commercial banks of the Persian Gulf Cooperation Council. While the banking and business sectors have extensively used the inverse DEA models, other sectors, such as energy and environment, education, sustainability, and supply chain management, have also seen an increase in the application of inverse DEA models. The current trend shows that the inverse DEA models are expanding into new fields, and other sectors, such as healthcare and agriculture, are expected to be included in the near future.

3.2. *Inverse DEA for Energy and Environmental analysis:*

The energy sector is an integral part of every country's economy, and while energy consumption has slightly declined in recent years, global energy use has significantly increased over the past three decades. However, the dominant share of energy consumption still belongs to non-renewable sources, which affects achieving sustainability goals. Reduction of greenhouse gas emissions is a critical concern in this century, with the Kyoto Protocol setting a target of a 5% reduction of emissions on average between 2008 and 2012 compared to 1990, and the Paris agreement in 2015, setting a goal of a 43% reduction by 2030. Therefore, assessing the performance of the energy and environment sector is of utmost importance. In this review, we focus on some of the most influential inverse DEA models applied to these sectors.

Hu et al. (2020) employed a slack-based inverse DEA model to allocate carbon emission abatement in China, while Emrouznejad et al. (2019) proposed an inverse DEA model for the allocation of CO2 emissions in different regions of China. Gatimbu et al. (2019) used inverse DEA models to improve the quality of life of small-scale tea processors in Kenya, and Wegener and Amin (2019) analysed greenhouse gas emissions in American and Canadian gas companies using inverse DEA models. Ghiyasi (2017a) proposed an inverse DEA model for environmental planning and energy consumption analysis of Iranian provinces. Ghiyasi (2019) developed an inverse DEA method for emission utilization permission while considering environmental efficiencies. Zhang et al. (2021) examined the regional allocation of CO2 emissions from the Chinese construction industry using DEA and inverse DEA methods. Lim (2020) used inverse DEA models to plan operations in the liquefied natural gas industry in Korea from 2009 to 2018. Wegener and Amin (2019) utilized inverse DEA models to analyse the environmental impact of oil and gas companies in North America from 2011 to 2015. Ghiyasi (2022) investigated the environmental efficiency of industrial sectors by proposing a time-series-like inverse DEA approach. Orisaremi et al. (2021) used inverse DEA to analyse the gas-to-wire process of member nations of the Organization of the Petroleum Exporting Countries, and Oukil et al. (2022) detected productive post-merger tomato greenhouse farms in Algeria that potentially have energy gains using an inverse DEA model. Orisaremi et al. (2022) implemented an inverse DEA in a three-stage process to investigate lean production practices in the petroleum industry, and He et al. (2022) used inverse DEA to investigate the optimal allocation of an output indicator related to carbon sequestration in Chinese forests.

3.3. *Inverse DEA for Sustainability and Supply chain management:*

The DEA models have been applied in sustainability and supply chain management, with the inverse DEA model being particularly useful. Hassanzadeh et al. (2018) proposed an inverse DEA model to assess the sustainability of OECD countries, which considered both positive and negative data, and addressed negative data using the SORM model of Emrouznejad et al. (2010). Chen et al.

(2017) investigated sustainable development investment in China using an inverse DEA model that can handle undesirable outputs (See also Lin et al (2019)). In assessing the efficiency of container ports and their resource consumption in the presence of undesirable outputs, Lin et al. (2019) utilized inverse DEA models. Yousefi et al. (2021) proposed a range-adjusted measure to assess and benchmark the sustainability of Islamic countries, considering natural and managerial disposability. Kalantary and Saen (2019) analysed the sustainability of supply chains by using inverse DEA models in a dynamic network structure. In another study, Kalantary et al. (2018) proposed a network dynamic range-adjusted measure (RAM) inverse DEA model to analyse the sustainability of supply chains. Gharibi and Abdollahzadeh (2021) used inverse DEA to perform a sensitivity analysis for the efficiency of disassembly centers in a closed-loop logistics network. Moghaddas et al. (2022) proposed a network inverse DEA-based model to analyse the sustainability of supply chains. These studies demonstrate the applicability of inverse DEA models in sustainability and supply chain management, and the usefulness of these models in addressing issues related to undesirable outputs, network structures, and sensitivity analysis.

3.4. *Inverse DEA for other sectors:*

In addition to the above applications, the inverse DEA model has also found applications in other public sectors such as education and public transportation. For instance, Zeinodin and Ghobadi (2019) developed an inverse DEA model to determine the optimal levels of inputs and outputs for a merge of DMUs while maintaining specific efficiency levels and applied the model to university departments. Le et al. (2021) proposed an inverse mathematical programming model to evaluate the efficiency of the Vietnamese education system and its ability to achieve targets. Meanwhile, Foladi et al. (2020) developed a dynamic version of the inverse DEA model that considers quasi-fixed inputs and applied it to analyse the efficiency of university faculties from 2011-2014. Finally, Chen et al. (2021) proposed a meta-frontier based inverse DEA model to analyse road safety in

China's transportation system. These studies demonstrate the versatility of the inverse DEA model in different public sectors and highlight its potential to improve decision-making processes.

4. Conclusion, future direction and potential applications

In recent years, the inverse data envelopment analysis (DEA) approach has gained popularity as an effective post-DEA tool for resource allocation and sensitivity analysis of decision making units (DMUs) across various sectors. This paper has provided a comprehensive overview of the origin and development of the inverse DEA approach, highlighting its application in sectors such as banking, energy, education, sustainability, and supply chain management. Moreover, it has been predicted that the use of inverse DEA models will continue to expand into new fields, including healthcare, agriculture, environmental and climate change issues leading to a significant increase in publications on inverse DEA models in the coming years. In addition, using different base optimization models including non-radial and fuzzy DEA models will enhance the power of developed inverse DEA models.

Research in recent years has shown the capacity of inverse DEA in solving challenging decision making problems. The inverse DEA can be further extended and developed in various areas where business organizations seek to maximize efficiency by creating strategic alliances and partnerships. Future research could also investigate the potential of inverse DEA in modelling partnerships between DMUs from cross-sectional sectors. Despite the effectiveness of inverse DEA models, performing post-optimality analysis through sensitivity analysis techniques in real-world applications can be challenging due to the nonlinearity and infeasibility of the models. This necessitates approximations of the optimal solution, which can lead to internal uncertainty. Nevertheless, the inverse DEA approach remains useful for performing sensitivity analysis and complementing regular efficiency analysis. The paper has also highlighted the emergence of new theoretical developments in inverse DEA models that focus on estimating inputs and/outputs with minimal perturbation that would allow the DMU(s) under assessment to achieve efficiency targets.

Furthermore, while there are several software programs available for DEA, and some software can solve simple Inverse DEA models, currently there is no user-friendly software designed specifically for policy makers to handle Inverse DEA models.

In conclusion, this paper has provided valuable insights into the current state of inverse DEA and its potential for future research and applications. With the continued development of theoretical models and their application to new sectors, the inverse DEA approach has the potential to become an increasingly powerful tool for solving novel inverse problems of DMUs in a wide range of industries.

Data statement

The data underlying this article are incorporated to the manuscript.

Appendix

Table 1A: Selected applications of inverse DEA in different sectors

Applications in banking						
Author (Year)	**DMUs**	**Inputs**	**Outputs**	**Research field and Data**	**Major Issues Addressed**	**Method**
Lin (2010)	22 chain stores in Taiwan	(i) Manpower, (ii) Store floor area, (iii) operating expense, (iv) number of households in the trade area	(i) Monetary amount of business revenue	Efficiency measurement and revenue setting problems in a Taiwanese home improvement company	Determining the required efficiency level and revenue of a new DMU	Imprecise DEA + Inverse DEA
Gattoufi et al. (2014)	42 GCG banks	(i) amount of deposits, (ii) man hours	(i) interest income, (ii) non-interest income	Mergers and acquisitions in banking	Make decision about the input/output level given a predetermined efficiency target	Inverse DEA model with consolidation constraint
Amin et al. (2017b)	42 GCG banks	(i)Interest expenses, (ii) non-interest expenses	(i) interest income, (ii) non-interest income	Generalised restructuring of firms through consolidation or split	(i)Find the optimal sharing by the post-restructuring DMUs of the inherited inputs and outputs	Generalised Inverse DEA model

					from the pre-restructuring ones, to achieve the desired efficiency targets, (ii) Find the lowest efficiency scores that post-restructuring DMUs can achieve	
Amin et al. (2017a)	42 GCG banks / 10 distributers	(i)Interest expenses, (ii) non-interest expenses/ (i) the staff cost	(i) interest income, (ii) non-interest income / (i)services, (ii) on-time delivery to customers	Determine whether a merger in a market is generating a major or minor consolidation	Determine whether the merge of DMUs affects the frontier or not	Inverse DEA model
Amin et al. (2019a)	42 GCG banks	(i)Interest expenses, (ii) non-interest expenses	(i) interest income, (ii) non-interest income	Incorporation of target setting of mergers for specific input saving/certain output production	To consider the nature of inputs and their degree of complexity in the Inverse DEA model	Goal programming + Inverse DEA

An et al. (2019)	16 Chinese commercial banks	(i) Operation cost, (ii) Interest expense, (iii) Labour/ (Intermediate measure) (i) Deposits	(i) Interest income, (ii) Non-interest income, (iii) Non-performing loan balance	Resource plans formulation for Chinese commercial banks	Meet output targets in the presence of undesirable outputs	Two-stage Inverse DEA models with undesirable outputs
Saen & Nia (2019)	10 representatives of the after-sales service of an Iranian car manufacturing company	(i)Number of stores of spare parts, (ii) Number of staff members, (iii) Number of dealers for providing after-sales services, (iv) Delivery time / (Intermediate measures) (i) Spare parts sales, (ii) Number of accepted cars	(i) Revenue from repairing cars, (ii) Number of accepted cars	Developing an inverse network DEA model for resource allocation	Estimate input or output levels while overall efficiency remains unchanged using an inverse network DEA model	Inverse multistage Network DEA
Yu et al. (2019)	25 Chinese banks	(i) Labour, (ii) Capital, (iii) Deposit	(i) Non-performing loan, (ii) Interest income, (iii) Operational income	Estimate operational efficiencies and potential income gains considering the credit risk for banks	Optimisation of operational income, interest income and non-performing loan amounts, considering operational	Inverse-like DEA model

					capability restriction	
Amin and Ibn-Boamah (2020)	28 Canadian banks	(i) amount of deposits, (ii) man hours	(i) interest income, (ii) non-interest income	Estimate potential merger gains	Determine the required level of inputs and outputs for a merged DMU	Inverse DEA cost efficiency model
Guijarro et al. (2020)	46 GCG banks/32 Colombian state universities	(i)Interest expenses, (ii) non-interest expenses / (i) Administrative Expenses, (ii) Full time equivalent teachers, (iii) Research Staff	(i) interest income, (ii) non-interest income/(i) Number of students, (ii) Employed graduates, (iii) Research papers	Sector restructuring in which all DMUs satisfy a predefined global efficiency level and some affected stakeholders may be resistant to restructuring	Calculate global efficiency target by giving preference to merging DMUs over saving inputs	Inverse DEA + Genetic algorithms
Hosseininia & Saen (2020)	8/12/9 DMUs	2 inputs, x1, x2	1 output y1	–	Maintain the efficiency of DMUs with new input and output and indicate input and output	SBM Inverse DEA

					volumes when efficiency score is increased	
Amin & Ibn-Boamah (2021)	50 US banks for 3 years (2016, 2017, 2019)	(i) Interest expenses, (ii) Labour cost, (iii) Costs of equipment and premises / (Intermediate measures) (i) Cash deposits, (ii) Securities	(i) Loans	Estimating potential gains from bank mergers for the top US commercial banks	Consider intermediate and final outputs at different predefined levels of technical efficiencies	Two-stage Inverse DEA
Applications in Supply Chain Management						
Kalantary et al. (2018)	12 Iranian gear box suppliers	(i) Wage cost, (ii) Energy cost (iii) Meterial cost, (Carry-overs) (i) Green programmes and ISO TS, (ii) Human care programs, (Fixed intermediate measure) (i) Intermediate measure	_	Sustainability of supply chains	Assess sustainability over multiple periods	Inverse network dynamic RAM DEA
Kalantary & Saen (2019)	13 dairy factories in Iran	(i) Wage cost, (ii) Material cost, (Carry-overs) (i) ISO and water recycling programs, (ii) Human care	_	Sustainability of supply chains	Assess sustainability over multiple periods	Network dynamic Inverse DEA

		programs, (Fixed intermediate measure) (i) Raw milk				
Applications in Education						
Le et al. (2021)	63 Vietnamese provincial education systems	(i) Inside expenditure paid by families for their children's education to educational institutions, (ii) Outside expenditure paid by families for their children's education outside educational institutions	(i) Math score, (ii) Vietnamese score	Effectiveness in the efficiency of household expenditure in the case of Vietnamese education system	The university entrance threshold for exam results is incorporated as an effectiveness constraint	Inverse DEA
Foladi et al. (2020)	9 faculties of Urmia University in 4 periods (2011-2014)	(i) The number of academic staff / (quasi-fixed inputs) (i) the number of graduate students, (ii) research funds	(i)the number of published papers in the scientific journals, (ii) income of research projects, (iii) the number of citations to the articles published in the scientific journals	Efficiency evaluation of the research section of universities	Increased the discrimination among DMUs using a dynamic form of the method, distinguished between the faculties and performe	dynamic DEA + Inverse DEA

					d sensitivity analysis of the inputs/outputs	
Hadi Vencheh et al. (2008)	17 University departments	(i) the number of bachelor students, (ii) the number of (full time and part time) faculty members	(i) the number of graduates, (ii) the number of research papers	Estimate input/output levels of a DMU when some or all of its input/output levels are changed, while the efficiency is preserved	Proposed some sufficient conditions for input estimation to overcome a possible failure of the previous method that was using weakly efficient solution of the relevant multiple objective optimization problem	Inverse DEA
Zeinodin & Ghobadi (2019)	14 University departments	(i) Facilities, (ii) Amount of the attention paid to the department by the university	(i) Satisfaction of the students, (ii) Satisfaction of the professors and staff	Estimate input/output levels of merged DMUs, while the efficiency is preserved	Proposed some sufficient conditions for input estimation of the merged DMU	Inverse DEA

Applications in Energy and Environment						
Hu et al. (2020)	30 provinces, autonomous regions, and municipalities in China	(i) Labour, (ii) Asset, (iii) Carbon Emission, (iv) Total Energy Consumption	(i) GDP	Carbon emissions abatement allocation plan that minimizes the GDP loss	Allocation model based on the slack-based model and multiple-objective nonlinear programming and an approximation algorithm to solve it	Inverse SBM DEA
Emrouznejad et al. (2019)	31 two-digit manufacturing industries in China	(i) Labour, (ii) Asset, (iii) Energy	(i) Gross Industrial Output Value, (ii) (Undesirable Output) CO_2 emissions	Allocation of the CO_2 emissions quota set by government goal in Chinese manufacturing industries to different Chinese regions	First, the total amount of CO_2 emission reduction is obtained from the Chinese government goal. Then, this is allocated to different two-digit level manufacturing industries in China. Finally, this is	Directional distance Inverse DEA

					further allocated for each industry into different provinces.	
Gatimbu et al. (2019)	54 small-scale tea processors in Kenya in 5-year period	(i) Green leaf (Kgs), (ii)electricity (Kwts), (iii) firewood (m3), (iv)depreciation (Ksh), (v) number of employees	(i) Process waste (Kgs), (ii) level of GHG emission (Kgs), (iii) wastewater (Kgs)	Environmental efficiency of the small-scale agroprocessors and its determinants	Consider undesirable outputs from tea processing	Inverse DEA with undesirable outputs and panel data +Tobit Regression
Wegener & Amin (2019)	29 North American integrated oil and gas companies with 116 firm-year observations	(i) Wells, (ii) Employees, (iii) Capital Expenditures, (iv) Total Assets	(i) Production, (ii) (Undesirable output) GHG emissions	Lower GHG emissions while increasing production in the oil and gas sector	Minimization of the undesirable output generated by a set of DMUs for producing a certain level of outputs, given that the DMUs maintain at least their existing performance status	Directional distance Inverse DEA with undesirable outputs

Zhang et al. (2021)	30 provinces	(i) the number of employees, (ii) the total power of machinery and equipment owned, (iii) total assets, (iv) energy consumption	(i) Gross output value, (ii) (Undesirable output) CO2 emissions	How to allocate CO2 emissions targets for the Chinese construction industry at the provincial level	Three-stage empirical system to identify the CO2 emissions allocation scheme	Directional distance + Super efficiency DEA + Inverse DEA
Lim (2020)	33 natural gas providers (2007-2018)	(i) pipeline installation, (ii) operating cost	(i) total amount of gas delivered	how observed frontier changes in the Korean natural gas industry can be utilized to provide insights into estimation of the future production frontier	Specification of the maximum allowable output changes that can still yield a target efficiency in VRS Inverse DEA and a new computational procedure to obtain the effective local Rate of Change of the frontier considering environmental factors	Rate of Change + Inverse DEA

Orisaremi et al. (2021)	13 OPEC members	(i) surplus current account, (ii) Wells completed, (iii) Producing wells, (iv) Active rigs, (v) Refining capacity, (i) (Negative input)deficit current account	(i) GDP per capita/ (Undesirable output) (i) routinely flared gas	Estimate potential reductions in global gas flaring for the conversion of flared gas to electricity via the gas-to-wire (GTW) process in member nations of the organization of the petroleum exporting countries (OPEC)	Optimal sizing of the GTW process considering the potential reductions and the annual gas usage requirement of turbines (maintaining the production rate while reducing the undesirable output).	Inverse DEA with negative inputs and undesirable outputs
Oukil et al. (2022)	51 tomato greenhouse (GH) farms from Biskra, Algeria	(i) Human Labour, (ii) Machinery, (iii) Diesel fuel, (iv) Infrastructure, (v) Fertilizers, (vi) Manure, (vii) Pesticides, (viii) Electricity, (ix) Water, (x) Plantlets	(i) Yield	Potential of Mergers & Acquisitions as a novel approach to energy use optimization	Possible mergers among DMUs	Inverse DEA

He et al. (2022)	15 provinces	(i) Forestry practitioners, (ii) Forestry investment completion, (iii) Afforestation area	(i) Forest carbon sink	Carbon sink efficiency assessment from the perspective of resource allocation in China	Estimated the change in the input indicator and the optimal allocation of the output indicator	Inverse DEA + Grey prediction model
Orisaremi et al. (2022)	13 OPEC members	(i) Wells completed, (ii) Producing wells, (iii) Active rigs/ (i) Wells completed, (ii) Producing wells, (iii) Active rigs, (iv) Refining capacity	(i) Crude oil, (ii) (Undesirable output)flared gas/(i) Crude oil, (ii) Refined products	Implementation of lean production practices in the petroleum industry	Three-stage inverse problem involving selected oil-producing nations and a method to rank DMUs	Inverse DEA
Applications in Sustainability						
Hassanzadeh et al. (2018)	20 OECD countries	(i) total material consumption, (ii) labor unemployment	(i) GDP per capita, (ii) (undesirable output) CO2 emission, (iii) employment protection index	Determine resource allocation and investment strategies for assessing sustainability of countries	Handle resource allocation and investment analysis problems given sustainable development aspects in the presence of negative	Inverse semi-oriented DEA

					data	
Yousefi et al. (2021)	15 Islamic countries	(i) Imports of goods and services, (ii) Fossil fuel consumption per capita	(i)GDP per capita, (i) Compulsory education duration, (iii) (undesirable output) CO2 emission	Assess the sustainability of Islamic countries and to present optimal strategies to promote their sustainability	Determined optimal inputs and outputs given natural disposability and managerial disposability so that the sustainability score of DMUs is unchanged.	Range Adjusted Measure Inverse DEA
Lin et al. (2019)	16 container ports	(i) berth length, (ii) equipment asset, (iii) number of employees, (iv) cost	(i) throughput, (ii) profit, (iii) (undesirable output) CO2, (iv) (undesirable output) NOx	Measure container ports' efficiency and analyse their resource consumption in China	Resource optimization of container ports considering undesirable outputs	Inverse DEA with undesirable outputs

Moghaddas et al. (2022)	20 SSCs	(i) Cost of trained personnel on safety and health issues, (ii) Additive material cost, (iii)Labour cost per project, (iv) Warehousing cost / (Intermediate measures) (i) Supplied material, (ii) Supplied products	(i) Number of green products, (ii) Revenue	Evaluated SSCs performance of manufacturing companies considering the network structure	Considered different stages of a network process based on the importance and priority of each stage over the others, considering the relationships between the internal stages and the suggested model guarantees integer values for all parameters	Network Inverse DEA
Applications in other sectors						
Çakır (2017)	16 cement firms	(i) cash ratio, (ii) working capital to assets ratio, (iii) quick ratio, (iv) current ratio	(i) return on assets, (ii) inventory turnover ratio, (iii) operational profit margin, (iv) enterprise value to sales	Evaluate the performance of the Turkish cement firms listed in BIST.	Input/output variable selection using Shannon's entropy, and resource allocation	imprecise Shannon's entropy + Interval Inverse DEA

					in a short run	
Lim (2016)	370 vehicles	(i) Displacement	(i) Power, (ii) Torque	New product target setting practices in vehicle engine development	Integrated the inverse optimization problem with time series to specify the expected changes of the production frontier in the future, as an ex-ante decision support tool	Inverse DEA
Chen et al. (2021)	31 regions	(i) number of employed persons in road transportation, (ii) total length of highway, (iii) number of passenger vehicles, (iv) number of trucks	(i) passenger-kilometres of road transportation, (ii) freight ton-kilometres of road transportation, (iii) (undesirable output) Number of fatalities in road traffic accidents	Determine realisation paths of China's road transportation safety objectives	optimal realization path for achieving a given safety objective with a given safety efficiency.	DEA with undesirable outputs + Meta-frontier + Inverse DEA with undesirable outputs

Ghiyasi et al. (2022)	130 public hospitals	(i) Number of physicians/ Number of hospital beds, (ii) Number of nurses/Number of hospital beds, (iii)Number of special hospital beds/Number of hospital beds	(i) Number of outpatients/ Number of physicians, (ii) Number of hospitalised patients/Number of active hospital beds, (iii)Number of surgical operations/Number of hospital operating rooms, (iv) Number of emergency admission/Number of emergency physicians, (v) Number of surgical operations/Number of surgeons	Efficiency and post-efficiency analysis of public hospitals in Iran	Proposed a novel model that treats ratio data	Inverse DEA with ratio data

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