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On selecting directions for directional distance functions in a non-parametric framework: a review

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Abstract Directional distance function (DDF) has been a commonly used technique for estimating efficiency and productivity over the past two decades, and the directional vector is usually predetermined in the applications of DDF. The most critical issue of using DDF remains that how to appropriately project the inefficient decision-making unit onto the production frontier along with a justified direction. This paper provides a comprehensive literature review on the techniques for selecting directional vector of the directional distance function. It begins with a brief introduction of the existing methods around the inclusion of the exogenous direction techniques and the endogenous direction techniques. The former commonly includes arbitrary direction and conditional direction techniques, while the latter involves the techniques for seeking theoretically optimized directions (i.e., direction towards the closest benchmark or indicating the largest efficiency improvement potential) and market-oriented directions (i.e., directions towards cost minimization, profit maximization, or marginal profit maximization benchmarks). The main advantages and disadvantages of these techniques are summarized, and the limitations inherent in the exogenous direction-selecting techniques are discussed. It also analytically argues the mechanism of each endogenous direction technique. The literature review is end up with a numerical example of efficiency estimation for power plants, in which most of the reviewed directions for DDF are demonstrated and their evaluation performance are compared.

Keywords Data envelopment analysis (DEA) · Least distance · Endogenous mechanism · Cost efficiency · Profit efficiency · Marginal profit maximization

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1 Introduction

A substantial literature has estimated efficiency and productivity by a relatively attractive approach named directional distance function (DDF) (Luenberger 1992; Chambers et al. 1996a). One advantage of this approach is that it can solve multiple-input and multiple-output productivity evaluation problem, where the outputs can be intended or unintended. Besides, unlike the distance function proposed by Shephard (1970), it can extend intended outputs and contract unintended outputs or inputs simultaneously. Within this framework, a large number of studies employed the DDF approach to estimate efficiency and productivity, and to analyze their improvement potentials in various areas such as business administration, energy management and environmental protection.

The usage of directional distance function is usually associated with two paradigms, which are parametric efficiency estimation and non-parametric efficiency estimation. The former is a regression-based model such as Stochastic Frontier Analysis (SFA) technique, which requires a predetermined specific form of production function. Generally, this approach is used to estimate the shadow prices of pollutants, see, for example, Vardanyan and Noh (2006), Färe et al. (2012), Matsushita and Yamane (2012) and Du et al. (2016). The latter is based on a non-parametric mathematical programming model such as data envelopment analysis (DEA), which does not need an assumption on the specific functional form of production function. Usually, this approach is used to measure efficiency and productivity in empirical researches. Examples of its applications can be found in Pathomsiri et al. (2008), Macpherson et al. (2010), Wang et al. (2012) and Arabi et al. (2014). In this paper, we would focus on the direction-selecting technique (DST) for DDF in a non-parametric DEA framework.

In recent years, how to select an appropriate direction of directional distance function has become one of the key issues in the modelling and application of DDF based efficiency and productivity studies. Vardanyan and Noh (2006) who showed that, the choice of direction would have a great impact on the calculation of technical efficiency, scale efficiency, as well as the measures comprising productivity change. Agee et al. (2012) further pointed out that the parameter estimates of all structure parameters (e.g., technical efficiency, productivity change, technical change, and efficiency change) depend on the choice of the directional vectors of DDF. Within this framework, Leleu (2013) further stressed that the choice of direction would substantially affect the estimation of shadow pricing of unintended outputs. Although there have been many literatures suggesting the direction-selecting techniques in DDF, no study, to the best of our knowledge, have presented a comprehensive review on these techniques. This study tries to fill up this gap through providing a comparative analysis of the existing major direction-selecting techniques for DDF.

In Fig. 1, we first illustrate a classification of the major direction-selecting techniques for DDF viewed in this study.

The direction-selecting techniques of DDF can be generally divided into exogenous direction-selecting techniques and endogenous direction-selecting techniques. The former directly determines the direction by researchers in advance, and the latter seeks the direction through a specific endogenous mechanism. Different direction-selecting techniques provide researchers different choices to measure efficiency and productivity, depending on the research motivations and the development of technologies.

First, the exogenous direction-selecting techniques commonly include arbitrary direction and conditional direction-selecting techniques:

(i) With respect to the arbitrary direction-selecting technique, two kinds of directional vectors are widely utilized in literatures: the input/output observation value direction

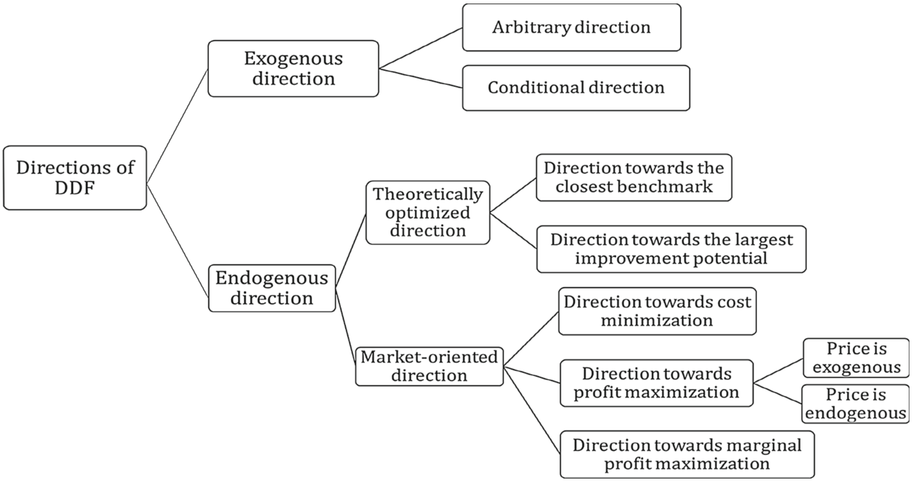


Fig. 1 A classification of the direction-selecting techniques for DDF

(Chambers et al. 1996b) and the unit value direction (Färe et al. 2006). In the former, different DMUs have different directions according to their specific observed input-output values which assumes the DMU keeps moving along the observation vector for expansion and makes the DDF a relatively complex model, while in the latter, the 1 or -1 unit value directions are assigned to all DMUs which simplifies the DDF model. There are some applications of input/output observation value direction, such as Kumar (2006), Oh (2010), Oum et al. (2013), and Hampf and Kruger (2014b); while, the applications of unit value direction can be found in Färe et al. (2005), Picazo-Tadeo et al. (2005), Bellenger and Herlihy (2009), and Halkos and Tzeremes (2013). A main feature of this arbitrary technique is that it does not need any specific assumption and imposition on the selection of directional vector, and hence, it is lack of explanations, such as economic meaning, policy implication and theoretical basis. In addition, as mentioned in Fukuyama and Weber (2009), the arbitrary direction for DDF may underestimate the inefficiency scores of a decision-making unit (DMU) when there are non-zero slacks on inputs and outputs. Another limitation is that the inefficiency scores calculated from the unit value direction would be not unit invariant, i.e., the inefficiency scores derived from the unit value direction would be affected by the units of inputs and outputs.[1]

(ii) The conditional direction-selecting technique generally extends the arbitrary direction-selecting technique through introducing specific evaluating scenarios such as policy implication, shadow price estimation, system improvement and comparability. One kind of the conditional direction-selecting technique calculates the direction by utilizing the annual plans in terms of environment and production (Lee et al. 2002). The aim of this technique is to estimate the shadow price for pollutant with the production process and abatement schedule taken into account. Another kind of the conditional direction-selecting technique determines the direction as the average inputs (or outputs) of all DMUs (Dervaux et al. 2009; Simar et al. 2012). This technique is more convincing at an input-oriented (or output-oriented) model for the purpose of the comparability between DMUs (Hailu and Chambers 2012). The free disposable hull (FDH) method is the most

[1] The hyperbolic efficiency measure (Färe et al. 1989) is excluded in this study since the model does not provide a linear direction vector which can be used in DDF and shows a nonlinear optimization problem.

common application of average direction-selecting technique. Furthermore, many scholars proposed another kind of conditional direction according to different policy scenarios (e.g., Färe et al. 2007; Watanabe and Tanaka 2007; Picazo-Tadeo et al. 2012; Zhou et al. 2012; Wang et al. 2013; Njuki and Bravo-Ureta 2015). For example, based on the unit value direction, Njuki and Bravo-Ureta (2015) simultaneously shrink unintended outputs and extend intended outputs under environmental regulation, while only increase intended outputs without decreasing the unintended outputs under the scenario of no regulation. Besides, the direction derived from the improvement of a system is also one kind of conditional direction. For instance, Chen et al. (2013) modified DDF approach by choosing a proper feasible reference bundles based on the constraint conditions, then enterprises can determine a conditional direction in this reference bundles according to its real production situation. This model can tackle the problem of infeasibility in conventional DEA technique. Moreover, Fukuyama and Weber (2009) proved that the efficiency DDF measure associated with the different conditional directions is equivalent to some existing slack-based measures (SBM). Compared with the arbitrary direction-selecting technique, the conditional direction-selecting technique goes one-step forward through introducing some particular scenarios in the modelling and application of DDF. These scenarios include, for example, the simulation of policy implication, the reflection of enterprise's choice, and the improvement of theoretical system, which reduce the arbitrariness in direction selection. Although the conditional direction-selecting techniques help to promote the reasonability of the efficiency estimation, rigorously speaking, they are still lack of the support from economic implications or mathematical theories.

Second, the endogenous direction-selecting techniques involve two categories, namely, theoretically optimized direction and market-oriented direction-selecting techniques:

(i) Theoretical optimized direction-selecting technique tends to capture a specific projection from an inefficient DMU to the frontier. With respect to its development, Frei and Harker (1999) first proposed a least-norm model to measure the distance between an inefficient DMU and the supporting hyperplane. However, this model cannot directly provide a favorable efficiency measure. Then, Baek and Lee (2009) introduced a least-distance model to illustrate the shortest distance from the inefficient DMUs to the production frontier. From a completely contrast perspective, Färe and Grosskopf (2010) and Adler and Voltab (2016) introduced an additive structure of directional distance function to seek the furthest distance from an inefficient DMU to the frontier. Their methods maximize the sum of the moving distance of all inputs and outputs. Meanwhile, Färe et al. (2013) and Hampf and Krüger (2014a) construct an endogenous mechanism, which could identify the largest improvement potentials by space traversal. In contrast to the previous researches, the objective of the theoretical optimized direction-selecting technique is to obtain the theoretically optimal state. It can be considered that these techniques are more reasonable than arbitrary and conditional direction-selecting technique, because the efficiency scores derived from an endogenous mechanism have the support of theoretical basis. In addition, the direction-selecting technique, which is to seek the furthest distance, is more convincing than the least-norm or least-distance model, because the evaluated DMU can capture its largest efficiency improvement potentials associated with this technique. However, a major disadvantage of this technique is that the identified largest improvement potentials may be difficult to achieve in the real production process.

(ii) The market-oriented direction-selecting technique is combined with the theoretically optimized direction-selecting technique and some specific economic meanings such as the corporate strategy of cost minimization, profit maximization and marginal profit maximization. Taking the example of cost minimization, Ray and Mukherjee (2000) built up a cost production frontier to choose direction towards cost minimization that takes into account of observed information on input prices. Under a priori assumption that each DMU has a profit maximizing behavior in a long term, Zofio et al. (2013) extended an endogenous model to select direction that rely on the concept of profit efficiency. These directions can project the inefficient DMUs to a closest profit maximizing benchmark on the production frontier. From a similar perspective, Lee (2016) provided a model to obtain the direction towards the Nash equilibrium benchmark in the case of imperfectly competitive markets. In this technique, the prices of inputs and outputs can be endogenous. In addition, Lee (2014) introduced a directional marginal productivity and then, given input-output price, identified the direction toward the marginal profit maximization. This technique is based on the dual program of the primal envelop DEA model. In general, the price information of cost minimization and marginal profit maximization direction-selecting technique is exogenous in the existing literature, whereas the price information of profit maximization direction-selecting technique can be exogenous or endogenous according to the different market situations. Specifically, the aim of the market-oriented direction-selecting technique is not to select a theoretical optimized direction, but to select the most suitable direction that is consistent with economic implications such as cost minimization, profit maximization, or marginal profit maximization. Hence, it can be considered that the direction-selecting technique combined with price information is more valuable for an enterprise's decision-making.

This article mainly focuses on the development of direction-selecting techniques for directional distance function. Firstly, we provide an introduction about the major recent seven direction-selecting techniques. Secondly, we argue some advantages and disadvantages inherent in each technique. Finally, we present a comparative analysis based on the inefficiency evaluation results from these seven direction-selecting techniques. For comparability, we translate all these seven models into their equivalent non-oriented DDF formulations, where the outputs can be either intended or unintended. Therefore, the main contribution of this paper is that it provides a critical review of literatures on the direction-selecting techniques for DDF, and provides a quantitative comparison of seven specific techniques.

The remainder of this article is organized as follows. In Sect. 2, we illustrate some fundamental concepts of directional distance function. Section 3 firstly exhibits several commonly used arbitrary direction-selecting techniques and conditional direction-selecting techniques, and then discusses their limitations. Sections 4 and 5 sequentially introduce and discuss five endogenous direction-selecting techniques, i.e., the closest benchmark selecting technique, the largest improvement potential selecting technique, cost minimization selecting technique, profit maximization selecting technique and marginal profit maximization selecting technique. Section 6 presents a comparison of the efficiency measurements of seven direction-selecting techniques. Finally, we provide a summary of these techniques and discuss the future trends of research in Sect. 7.

2 Fundamental concepts of DDF

By considering a sample of $j = 1, 2, \ldots, n$ observed DMU$_j$, we denote inputs by $\boldsymbol{x} \in \boldsymbol{R}_+^m$, intended outputs by $\boldsymbol{y} \in \boldsymbol{R}_+^s$, and unintended outputs by $\boldsymbol{u} \in \boldsymbol{R}_+^h$. Let i, r and f represent the index of input, intended output and unintended outputs, respectively. Thus, observations $x_{ij} (i = 1, 2, \ldots, m)$ represents the ith input level, $y_{rj} (r = 1, 2, \ldots, s)$ the rth intend output level, and u_{fj} $(f = 1, 2, \ldots, h)$ the fth unintended output level of DMU$_j$. The corresponding production possibility set is denoted by:

$$\boldsymbol{T} = \left\{ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in \boldsymbol{R}_{+}^{m+s+h} : \boldsymbol{x} \text{ can produce } (\boldsymbol{y},\boldsymbol{u}) \right\} \tag{1}$$

In words, for each input vector $\boldsymbol{x}$, the outputs vector $(\boldsymbol{y}, \boldsymbol{u})$ can be produced in the feasible region. More formally, the output set $P(\boldsymbol{x})$ represents the combinations of intended and unintended outputs $(\boldsymbol{y}, \boldsymbol{u})$ that are generated by the input vector $\boldsymbol{x}$. It can be denoted by:

$$P\left(\boldsymbol{x}\right) = \left\{ (\boldsymbol{y}, \boldsymbol{u}) : \ \boldsymbol{x} \ \text{ can produce } \ (\boldsymbol{y}, \boldsymbol{u}) \right\} \tag{2}$$

The production possibility set is required to satisfy the following four standard axioms:

(i) Convexity (Shephard 1970);
(ii) Strong disposability of inputs and intended outputs (Färe and Primont 1995), i.e.,

$$\textit{If } (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in \boldsymbol{T} \textit{ and } \boldsymbol{x}^* \geq \boldsymbol{x}, \textit{ then } \left(\boldsymbol{x}^*, \boldsymbol{y}, \boldsymbol{u}\right) \in \boldsymbol{T} \text{ or } \textit{if } (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in \boldsymbol{T} \textit{ and}$$
$$\boldsymbol{y}^* \leq \boldsymbol{y}, \textit{ then } \left(\boldsymbol{x}, \boldsymbol{y}^*, \boldsymbol{u}\right) \in \boldsymbol{T};$$

(iii) Weak disposability of unintended outputs associated with intended outputs (Färe and Grosskopf 2004), i.e.,

$$\textit{If } (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in \boldsymbol{T} \quad \textit{and } 0 \leq \rho \leq 1, \textit{ then } (\boldsymbol{x}, \rho\boldsymbol{y}, \rho\boldsymbol{u}) \in \boldsymbol{T};$$

(iv) Null-jointness of intended outputs and unintended outputs, i.e.,

$$\textit{If } (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}) \in \boldsymbol{T} \quad \textit{and } u = \boldsymbol{0}, \textit{ then } \boldsymbol{y} = \boldsymbol{0}.$$

Unintended outputs would incur a cost for the pollutant abatement in compliance with environmental regulation (Cong and Wei 2010, 2012; Wang and Wei 2016a; Wang et al. 2016b). Therefore, Färe et al. (1986) first proposed the weak disposability assumption in efficiency measurement that includes unintended outputs. In this case, decreasing unintended outputs needs to increase some inputs or reduce some intended outputs by the same factor, implying that there would be some cost in contrasting unintended outputs. More explicitly, Färe and Grosskopf (2004) indicated that the weak disposability assumption is usually accompanied by the null-jointness assumption. It suggests that intended outputs and unintended outputs are always generated at the same time.

Next, we turn to the definition of directional distance function. Shephard (1970) first introduced the concept of distance function, which expands all outputs with the same rate. Then, Luenberger (1992), Chambers et al. (1996a) and Chung et al. (1997) proposed the directional distance function to examine environmental efficiency and productivity. This specification has the advantage to take into account intended and unintended outputs simultaneously. It can be denoted by:

$$\vec{\boldsymbol{D}}\left(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}; \vec{\boldsymbol{g}}\right) = \max\left\{\beta : \left(\boldsymbol{x} - \beta\vec{\boldsymbol{g}}_x, \boldsymbol{y} + \beta\vec{\boldsymbol{g}}_y, \boldsymbol{u} - \beta\vec{\boldsymbol{g}}_u\right) \in \boldsymbol{T}\right\} \tag{3}$$

In Eq. (3), $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right)$ is the so-called directional vector; β (≥ 0) is the inefficiency score. A DMU is evaluated as efficient if $\beta = 0$ and as inefficient if $\beta > 0$. It is noteworthy that the directional distance function can be considered as a general form of the distance function.

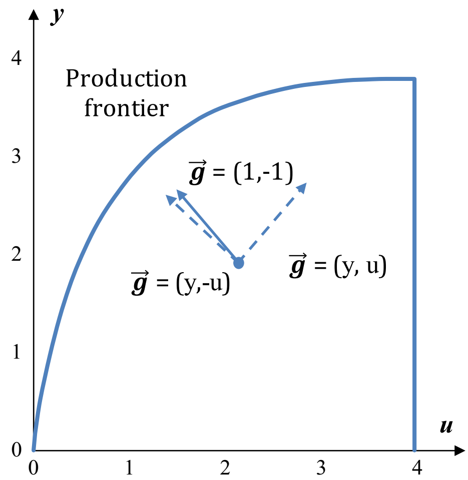


Fig. 2 Output observation value direction and unit value direction

3 Techniques for exogenous direction-selecting

3.1 Arbitrary direction and conditional direction

As mentioned above, the arbitrary direction-selecting techniques rely on researchers' pre-assigned directions in which, two widely used directions are (i) the input/output observation value direction $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = (-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{u})$, and (ii) the unit value direction $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = (-\mathbf{1}, \mathbf{1}, -\mathbf{1})$. It is noteworthy that another input/output observation value direction $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = (-\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u})$, which is introduced by Coggins and Swinton (1996) without contracting the unintended outputs, is not commonly used in practice. Figure 2 illustrates these three kinds of directions in an output-oriented efficiency evaluation framework.

Meanwhile, the associated DEA models for each DMU$_{j0}$ to evaluate the inefficiency score β with common observation value direction and unit value direction are presented as follows in Models (4) and (5), respectively:

$$
\begin{aligned}
&\max_{\beta,\lambda} \ \beta \\
&s.t. \quad x_{ij_0} - \beta x_{ij_0} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{ij_0} + \beta y_{ij_0} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad u_{ij_0} - \beta u_{ij_0} = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
&\qquad \beta, \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
& \max_{\beta,\lambda} \ \beta \\
& s.t. \ \ x_{ij_0} - \beta \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
& \qquad y_{ij_0} + \beta \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
& \qquad u_{ij_0} - \beta = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
& \qquad \beta, \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{5}
$$

where λ_j denotes the intensity variable representing convex combination among the DMUs. The first two inequality constraints indicate the strong disposability property of inputs and intended outputs, while the equality constraint represents the weak disposability assumption of unintended outputs.

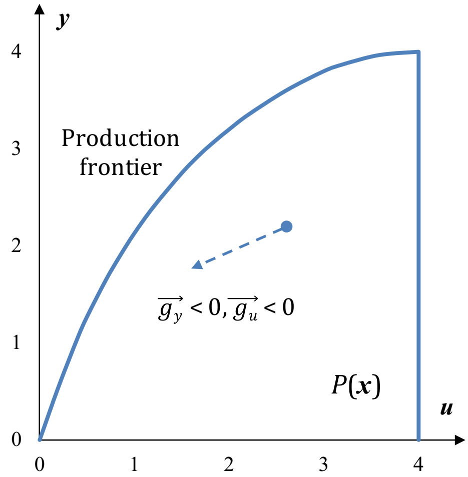
Conditional direction-selecting techniques extend the observation value direction and unit value direction-selecting techniques through introducing specific scenarios, such as policy implication, shadow system improvement and comparability, in DDF. Taking the scenario of policy implication as an example: in the case of only one input (e.g., energy utilization), one intended output (e.g., Gross Domestic Product, GDP) and one unintended output (e.g., pollutant emission), firstly, the directions of $\vec{\boldsymbol{g}} = (-\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{0})$ (or $\vec{\boldsymbol{g}} = (-\boldsymbol{1}, \boldsymbol{0}, \boldsymbol{0})$), $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{y}, \boldsymbol{0})$ (or $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{0})$) and $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{0}, -\boldsymbol{u})$ (or $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{0}, -\boldsymbol{1})$) can be utilized to represent the production scenario of energy conservation, economic growth, and emissions abatement, respectively. Secondly, the direction of $\vec{\boldsymbol{g}} = (-\boldsymbol{x}, \boldsymbol{0}, -\boldsymbol{u})$ (or $\vec{\boldsymbol{g}} = (-\boldsymbol{1}, \boldsymbol{0}, -\boldsymbol{1})$) indicates the joint production scenario of emissions abatement and energy conservation; the direction of $\vec{\boldsymbol{g}} = (-\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{0})$ (or $\vec{\boldsymbol{g}} = (-\boldsymbol{1}, \boldsymbol{1}, \boldsymbol{0})$) denotes the joint production scenario of economic growth and energy conservation; whereas the direction of $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{y}, -\boldsymbol{u})$ (or $\vec{\boldsymbol{g}} = (\boldsymbol{0}, \boldsymbol{1}, -\boldsymbol{1})$) implies the joint production scenario of emissions abatement and economic growth. Thirdly, the direction of $\vec{\boldsymbol{g}} = (-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{u})$ (or $\vec{\boldsymbol{g}} = (-\boldsymbol{1}, \boldsymbol{1}, -\boldsymbol{1})$) stands for the original arbitrary direction, which represents the joint production scenario of emissions abatement, economic growth and energy conservation at the same time.

Another typical example of conditional direction-selecting technique is Lee et al. (2002)'s model. For purpose of the estimation of shadow price for pollutant with environment and production inefficiency taken into account, they computed the directional vector in an output-oriented model by using the production plans of intended output and the annual abatement schedules of unintended outputs as proxy variables, respectively. They calculated the differences in intended an unintended outputs between the base year and the goal year to obtain the components of direction of each individual DMU. Then, they calculated the aggregate direction by weighting individual directions according to the average intended outputs in these two years. In their numerical example, intended and unintended outputs both increased in the goal year, and thus, $\vec{\boldsymbol{g}}_y$ and $\vec{\boldsymbol{g}}_u$ are both negative. Figure 3 illustrates this negative direction in an output-oriented efficiency evaluation framework.

3.2 The limitations of arbitrary direction and conditional direction

The arbitrary direction and conditional direction-selecting techniques usually predetermine the directions by the researchers in advance and, thus, some studies have already argued the objectivity or reasonability of these exogenous directions of DDF.

Fig. 3 Negative direction



In practice, these two techniques, which are lack of policy implication or economic meaning, are considered somewhat arbitrarily with the same importance for inputs and outputs. Within these two categories, inputs, intended outputs and unintended outputs cannot balance the weight automatically, i.e., there would be no trade-offs between inputs, intended outputs and unintended outputs in efficiency evaluation. Another weakness is that in particular, the efficiency scores derived from unit value direction are not unit invariant. Moreover, as shown by Fukuyama and Weber (2009), these techniques may overestimate the efficiency scores along with the predetermined direction when there present some non-zero slacks on inputs and outputs.

It is worth mentioning here that there exists one disadvantage when using the weak disposability formulation mentioned above. Based on the idea of Picazo-Tadeo and Prior (2009), Murty et al. (2012) and Chen and Delmas (2012), the weak disposability assumption of unintended outputs could lead to a downward-sloping segment where the frontier presents a negative slope. In this situation, some inefficient observations, which are located on this segment, would be considered as efficient along with the predetermined directions. Figure 4 illustrates this misclassification problem, in which three DMUs (A, B, C) display a production set with a single intended output and a single unintended output via the same level of inputs. Through utilizing the weak disposability assumption of unintended output in directional distance function, there is a downward-sloping segment BC on production frontier. Thereby, DMU C would be classified efficient along the unit value direction (i.e., the arrow direction in Fig. 4). However, DMU C is inefficient, because it can additionally reduce its unindent output and increase its intended output along the down-sloping segment of the frontier from C to B.

As a conclusion to this sub-section, despite their convenience, the arbitrary direction and conditional direction-selecting techniques still have many obstacles on efficiency and productivity measurement. Given these all drawbacks, some alternative direction-selecting techniques have been proposed in recent years.

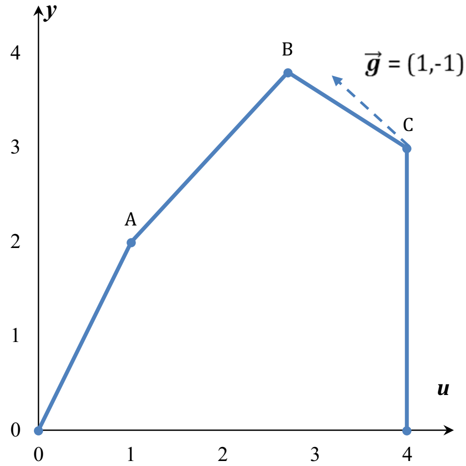


Fig. 4 Misclassification derived from unit value direction

4 Techniques for theoretically optimized direction-selecting

4.1 Direction towards the closest benchmark

Efficiency and productivity analysis is implemented not only to evaluate the current efficiency level, but also to offer details on how to drive productivity, that is, to capture the best practice (i.e., efficient benchmark). From the perspective of searching for the shortest distance to production frontier, it is easy to generate a closest reference set in common. In recent years, scholars have studied on finding the easiest way to improve the efficiency score of an evaluated DMU, i.e., to identify the shortest distance to production frontier. Thus, this sub-section firstly describes a direction-selecting technique for identifying the closest benchmark without unintended outputs.

Frei and Harker (1999) once introduced an additive DEA model to seek the least-norm projection (i.e., the closest efficient point) on the supporting hyperplane (but not on the production frontier). If the production frontier and supporting hyperplane are separated, this model may not able to obtain the accurate efficiency scores. Hence, although this technique is extremely interesting, it is not much appropriate to be used for efficiency estimation.

Hereafter, Baek and Lee (2009) developed a least-distance model to illustrate the minimum distance from an inefficient DMU to the production frontier. This study focuses on the closeness or similarity between the evaluated DMU and its benchmarks, and the associated algorithm and linearization for DMU_j is introduced as follows:

(i) Dividing all DMUs into efficient and inefficient by solving the Range Adjusted Measure (RAM) based model (Cooper et al. 1999) as follows:

$$
\begin{aligned}
\min_{\lambda_j, s_{ij_0}^-, s_{rj_0}^+} \quad & \varphi = 1 - \tfrac{1}{m+s}\left(\sum_{i=1}^{m} \frac{s_{ij_0}^-}{R_i^-} + \sum_{r=1}^{s} \frac{s_{rj_0}^+}{R_r^+}\right) \\
s.t. \quad & x_{ij_0} - s_{ij_0}^- = \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
& y_{rj_0} + s_{rj_0}^+ = \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
& \lambda_j, s_{ij_0}^-, s_{rj_0}^+ \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{6}
$$

where $R_i^- = \max_j \{x_{ij}\} - \min_j \{x_{ij}\}$, and $R_r^+ = \max_j \{y_{rj}\} - \min_j \{y_{rj}\}$. In Model (6), $s_{ij_0}^-$ and $s_{rj_0}^+$ respectively indicate the slacks of ith input and rth output for DMU$_{j0}$. A DMU is evaluated as efficient if $\phi = 1$ and as inefficient if $0 < \phi < 1$. Then, E is used to denote the set of the efficient DMUs as:

$$\begin{aligned} &E = \{(\boldsymbol{x}, \boldsymbol{y}), \quad \max\left(\boldsymbol{e}^T \boldsymbol{s}^+ + \boldsymbol{e}^T \boldsymbol{s}^-\right) = 0 \\ &s.t. \left(\boldsymbol{s}^+, \boldsymbol{s}^-\right) = (\boldsymbol{x} - \boldsymbol{X\lambda}, \boldsymbol{Y\lambda} - \boldsymbol{y}), \quad \boldsymbol{e}^T \boldsymbol{\lambda} = 1, \quad \boldsymbol{\lambda} \geq 0\} \end{aligned} \tag{7}$$

where $\boldsymbol{e}^T$ is a vector of ones. The efficient DMUs belong to the production frontier by definition and thus, their least-distance benchmarks are themselves.

(ii) Calculating the number of the elements of set E (denoted by l) and the number of the reference plane (denoted by $\binom{I}{m+s}$). Since each $m + s$ efficient points can form a reference plane, where the numbers of inputs and outputs are m and s, respectively, then, ($\binom{I}{m+s}$) reference planes are available for each inefficient DMU.

(iii) Designating an optimal solution as $(\boldsymbol{x}', \boldsymbol{y}')$ in Model (8), and this solution is the shortest projection from inefficient DMU$_{j0}$ to the kth reference plane.

$$\begin{aligned} &\min_{\lambda, x_i, y_r} \sum_{i=1}^{m} \left(\frac{x_i - x_{ij_0}}{R_i^-}\right)^2 + \sum_{r=1}^{s} \left(\frac{y_r - y_{rj_0}}{R_r^+}\right)^2 \\ &s.t. \quad x_i = \sum_{DMU_j \in E} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\ &\qquad\ \ y_r = \sum_{DMU_j \in E} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\ &\qquad\ \ \sum_{DMU_j \in E} \lambda_j = 1 \\ &\qquad\ \ \lambda_j \geq 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{8}$$

(iv) For $k = 1, 2, \ldots, \binom{I}{m+s}$, computing an array of $(\boldsymbol{x}', \boldsymbol{y}')$ and corresponding distance d_k of each reference plane one by one for DMU$_{j0}$ in an increasing order. Then, we consider $(\boldsymbol{x}^*, \boldsymbol{y}^*)$ as the closest benchmark if it satisfies $d^* = min\{d_k\}$, and then $\varphi = 1 - \frac{1}{m+s}\left(\sum_{i=1}^{m}\left(\frac{x_i^* - x_{ij_0}}{R_i^-}\right)^2 + \sum_{r=1}^{s}\left(\frac{y_r^* - y_{rj_0}}{R_r^+}\right)^2\right)^{1/2}$ is the efficiency score.

Figure 5 provides a graphic illustration of the above steps. This measure firstly defines the strongly efficient set, and then calculates the shortest distance and the corresponding benchmark from the evaluated DMU to each reference plane.

The original model of Baek and Lee (2009) does not address the unintended outputs. When considering the unintended outputs, model (8) can be developed as follows:

$$\begin{aligned} &\min_{\lambda, x_i, y_r, u_f} \sum_{i=1}^{m} \left(\frac{x_i - x_{ij_0}}{R_i^-}\right)^2 + \sum_{r=1}^{s} \left(\frac{y_r - y_{rj_0}}{R_r^+}\right)^2 + \sum_{f=1}^{h} \left(\frac{u_f - u_{fj_0}}{R_f^+}\right)^2 \\ &s.t. \quad x_i = \sum_{DMU_j \in E} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\ &\qquad\ \ y_r = \sum_{DMU_j \in E} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\ &\qquad\ \ u_f = \sum_{DMU_j \in E} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\ &\qquad\ \ \sum_{DMU_j \in E} \lambda_j = 1 \\ &\qquad\ \ \lambda_j \geq 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{9}$$

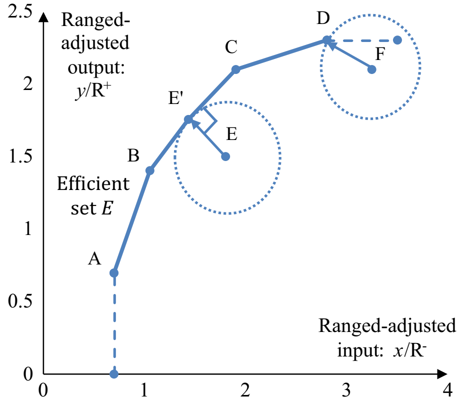


Fig. 5 Direction towards to closest benchmark

where an optimal solution $(\boldsymbol{x}', \boldsymbol{y}')$ and corresponding distance d_k of Model (9) is the shortest projection from inefficient DMU_{j0} to the kth reference plane.

We then consider $(\boldsymbol{x}^*, \boldsymbol{y}^*, \boldsymbol{u}^*)$ as the closest benchmark if it satisfies $d^* = min\{d_k\}$, and $\varphi = 1 - \frac{1}{m+s+f}\left(\sum_{i=1}^{m}\left(\frac{x_i^* - x_{ij0}}{R_i^-}\right)^2 + \sum_{r=1}^{s}\left(\frac{y_r^* - y_{rj0}}{R_r^+}\right)^2 + \sum_{f=1}^{h}\left(\frac{u_f^* - u_{fj0}}{R_f^+}\right)^2\right)^{1/2}$ is the efficiency score.

Moreover, Model (9) can be translated into its equivalent model with in the DDF formulation as follows:

$$
\begin{aligned}
&\min_{\lambda_j, \beta, g_{x_i}, g_{y_r}, g_{u_f}} \sum_{i=1}^{m}\left(\frac{\beta g_{x_i}}{R_i^-}\right)^2 + \sum_{r=1}^{s}\left(\frac{\beta g_{y_r}}{R_r^+}\right)^2 + \sum_{f=1}^{h}\left(\frac{\beta g_{u_f}}{R_f^+}\right)^2 \\
&s.t.\ x_{ij0} - \beta g_{x_i} = \sum_{DMU_j \in E} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\quad y_{rj0} + \beta g_{y_r} = \sum_{DMU_j \in E} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\quad u_{fj0} - \beta g_{u_f} = \sum_{DMU_j \in E} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
&\quad \sum_{DMU_j \in E} \lambda_j = 1 \\
&\quad \lambda_j, \beta \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{10}
$$

The corresponding directional vector derived from the above technique is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = \left(X^E\boldsymbol{\lambda}^* - \boldsymbol{x}_0, Y^E\boldsymbol{\lambda}^* - \boldsymbol{y}_0, U^E\boldsymbol{\lambda}^* - \boldsymbol{u}_0\right)/\beta^*$, where X^E, Y^E, and U^E are the efficient inputs, intended and unintended outputs matrix.

In contrast to the method in Frei and Harker (1999), this technique is a well-defined efficiency measure with the least-norm model, and the benchmarking information and efficiency score can both be obtained. The purpose of this technique is to select the most easily attainable and relevant benchmarks. It can be solved through the standard linear programming. Moreover, the most significant contribution of this technique is that it is the combination of the slack maximization model, which provide the farthest benchmark, and the radial model, which assume the same proportion of input contraction or output expansion. However, though it focuses on the attribute of closeness or similarity between the evaluated DMU and its benchmarks, the other important attributes of benchmarks need to be considered in a specific production progress are the vision of the enterprise and the preference of evaluator. It can be explained that the primary goal of enterprises is to maximize profit or make an efficiency progress rather than find the easiest way to the benchmarks.

4.2 Direction towards the largest improvement potential

This sub-section mainly focus on another theoretically optimized direction-selecting technique, i.e., technique for identifying the direction towards the largest improvement potentials, which is obtained through an endogenous optimization mechanism.

In recent studies, from a theoretical perspective, Färe and Grosskopf (2010) and Adler and Voltab (2016) developed a new DDF within the SBM framework to capture the longest distance to production frontier, i.e., using different directional vectors to change inputs and outputs from their respective inequalities rather than equalities. Meanwhile, Färe et al. (2013) first proposed a technique to endogenously determine the directional vectors, i.e., the furthest distance direction by maximizing the inefficiency score of DMU under evaluation. The associated output-oriented DEA model based on the exogenous normalization constrains for each DMU$_{j0}$ is presented as follows:

$$\begin{aligned} &\max_{\beta,\lambda_j,g_{y_{rj_0}},g_{u_{fj_0}}} \quad \beta \\ &s.t. \quad x_{ij_0} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i=1,2,\ldots,m \\ &\qquad y_{rj_0} + \beta g_{y_{rj_0}} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r=1,2,\ldots,s \\ &\qquad u_{fj_0} - \beta g_{u_{fj_0}} = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f=1,2,\ldots,h \\ &\qquad \sum_{r=1}^{s} g_{y_{rj_0}} + \sum_{f=1}^{f} g_{u_{fj_0}} = 1 \\ &\qquad \beta,\lambda_j,g_{y_{rj_0}},g_{u_{fj_0}} \geq 0, \quad j=1,2,\ldots,n, \quad r=1,2,\ldots,s, \quad f=1,2,\ldots,h. \end{aligned} \tag{11}$$

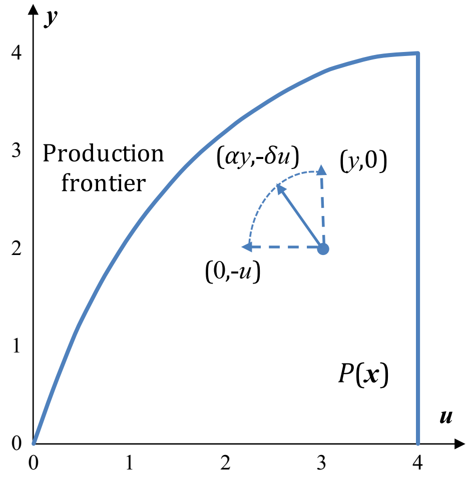
In Model (11), it is assumed that the components $\vec{\boldsymbol{g}}_y$ and $\vec{\boldsymbol{g}}_u$ of $\vec{\boldsymbol{g}} = \left(\vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right)$ is non-negative and belong to the unit simplex, which is denoted by the fourth normalization equation constraint.

Then, Hampf and Krüger (2014a) also developed an output-oriented endogenous mechanism to find the directions along which each decision making unit can capture the furthest distance to the production frontier. The associated DEA model for each DMU$_{j0}$ is presented as follows:

$$\begin{aligned} &\max_{\beta,\lambda_j,\alpha_{rj_0},\delta_{fj_0}} \quad \beta \\ &s.t. \quad x_{ij_0} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i=1,2,\ldots,m \\ &\qquad y_{rj_0} + \beta\alpha_{rj_0} \otimes y_{rj_0} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r=1,2,\ldots,s \\ &\qquad u_{fj_0} - \beta\delta_{fj_0} \otimes u_{fj_0} = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f=1,2,\ldots,h \\ &\qquad \sum_{r=1}^{s} \alpha_{rj_0} + \sum_{f=1}^{f} \delta_{fj_0} = 1 \\ &\qquad \beta,\lambda_j,\alpha_{rj_0},\delta_{fj_0} \geq 0, \quad j=1,2,\ldots,n, \quad r=1,2,\ldots,s, \quad f=1,2,\ldots,h. \end{aligned} \tag{12}$$

In Model (12), the elements of the vectors α and δ denote the different weights for intended outputs and unintended outputs, respectively, whereas "$\otimes$" represents the Hadamard product

Fig. 6 Direction towards largest improvement potential



for two vectors. The non-negative constraints on α and δ suggest that only the directions that do not decrease intended outputs or increase unintended outputs are chosen. Moreover, α and δ belong to the unit simplex, which is denoted by the fourth normalization equation constraint. Figure 6 provides a graphic illustration of direction towards the largest improvement potential, which shows that starting from the observation value direction $(0, -u)$ or $(y, 0)$, each DMU could search an optimal direction associated with its intended and unintended outputs according to adjust their weights α and δ.

Compared with Model (11), this Model (12) can provide a more clearly clarification of the trade-off between intended outputs and unintended outputs when measuring efficiency, because the chosen of their weights only relies on the improvement potential of the evaluated DMU but not on the unit of measurement. We can easily compare the insufficient of each intended output and the excess of each unintended output by inspecting the optimized weight results. It can be considered that Model (12) is the general form of Model (11). It is noteworthy that these two models are output-oriented model, and if the inputs are also taking into consideration, the associated DEA model for each DMU$_{j0}$ can be presented as follows:

$$
\begin{aligned}
&\max_{\beta,\lambda_j,\phi_{ij_0},\alpha_{rj_0},\delta_{fj_0}} \beta \\
&s.t.\quad x_{ij_0} - \beta\phi_{ij_0} \otimes x_{ij_0} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{rj_0} + \beta\alpha_{rj_0} \otimes y_{rj_0} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad u_{fj_0} - \beta\delta_{fj_0} \otimes u_{fj_0} = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
&\qquad \sum_{i=1}^{m}\phi_{ij_0} + \sum_{r=1}^{s}\alpha_{rj_0} + \sum_{f=1}^{f}\delta_{fj_0} = 1 \\
&\qquad \beta, \lambda_j, \phi_{ij_0}, \alpha_{rj_0}, \delta_{fj_0} \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \\
&\qquad\quad r = 1, 2, \ldots, s, \quad f = 1, 2, \ldots, h.
\end{aligned}
\tag{13}
$$

Moreover, the corresponding directional vector of Model (13) is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = (-\phi \otimes \boldsymbol{x}, \alpha \otimes \boldsymbol{y}, -\delta \otimes \boldsymbol{u})$ in DDF formulation.

To summarize, both of the above two techniques are extended models of non-radial DDF, which adjust different outputs with different rates. One interesting thing is that they could obtain an optimal direction which identifies the largest efficiency improvement potential of each DMU. In a typical technique of environmental efficiency evaluation, as mentioned above, usually the intended and unintended outputs cannot balance their weights automatically. The technique, which is proposed for selecting the direction toward the largest improvement potential, can help to overcome this problem. The directional vectors represented by the normalization constraint in Model (13) can be considered as the weights for each input and output. It indicates that this direction gives different importance to inputs, intended outputs and unintended outputs, and it can be sought by space traversal.

However, the primary endogenous mechanisms of directional distance function are addressed without any consideration of enterprise behavior or policy implication. In the next section, we will present three typical techniques for selecting direction associated with different enterprise behaviors.

5 Techniques for market-oriented direction-selecting

5.1 Direction towards cost minimization

More recently, numerous studies have been devoted to evaluate the cost efficiency and Malmquist cost productivity (MCP) in a cost minimization framework (or based on a cost frontier), see, for example, Ray and Mukherjee (2000), Ball et al. (2005), Ray et al. (2008) and Granderson and Prior (2013). Taking account of the cost of inputs and the differences in input prices, this technique employs the input-oriented model.

For the specific output level of $(\boldsymbol{y}_0, \boldsymbol{u}_0)$, the input set can be defined as:

$$V(\boldsymbol{y}_0, \boldsymbol{u}_0) = \{\boldsymbol{x}, (\boldsymbol{y}_0, \boldsymbol{u}_0) \text{ can be generated by } \boldsymbol{x}\} \tag{14}$$

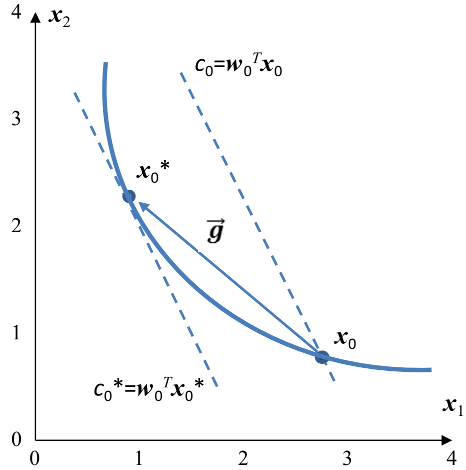
Suppose that the evaluated DMU faces the positive input price vector $\boldsymbol{w}_0 = \left(w_{10}, \mathrm{w}_{20}, \ldots, w_{m0}\right)^T \in \boldsymbol{R}_+^m$, then, its actual cost is $c_0 = \boldsymbol{w}_0^T \boldsymbol{x}_0$. Here, the associated DEA model based on cost minimum assumption for DMU$_{j0}$ is presented as follows:

$$\begin{aligned}
\min_{x_i}\ & c = \sum_{i=1}^{m} w_{i0} x_i \\
s.t.\ & x_i \geq \sum_{j=1}^{n} x_{ij} \lambda_j, \quad i = 1, 2, \ldots, m \\
& y_{rj_0} \leq \sum_{j=1}^{n} y_{rj} \lambda_j, \quad r = 1, 2, \ldots, s \\
& u_{fj_0} = \sum_{j=1}^{n} u_{fj} \lambda_j, \quad f = 1, 2, \ldots, h \\
& \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{15}$$

where $c = \sum_{i=1}^{m} w_{i0} x_i, \boldsymbol{x} \in V(\boldsymbol{y}_0, \boldsymbol{u}_0)$ is the cost function of the cost minimizing benchmark, and the input vector $\boldsymbol{x}$ is a variable.

Figure 7 illustrates that $\boldsymbol{x}_0^*$ is the benchmark of $\boldsymbol{x}_0$ on the cost frontier, i.e., $c_0^*(\boldsymbol{w}_0, \boldsymbol{x}_0^*) = \min \boldsymbol{w}_0^T \boldsymbol{x} : \boldsymbol{x} \in V(\boldsymbol{y}_0, \boldsymbol{u}_0)$. Moreover, its actual cost is $c_0 = \boldsymbol{w}_0^T \boldsymbol{x}_0$, so that this evaluated DMU is projected onto $\boldsymbol{x}_0^*$ along a special direction g.

Fig. 7 Direction towards cost minimization



This direction-selecting technique illustrates one kind of market-oriented direction towards the cost minimization benchmark that takes account of observed information in input prices. The efficiency scores derived from this technique could be helpful for enterprises to plan production scale and make investment decisions so as to optimize input resource reallocation. The results would suggest regulators in measuring the impacts of repealing or imposing regulation on the cost efficiency and productivity growth of enterprises. It is noteworthy that there would be a similar model in revenue maximization framework. It would employ the output-oriented model to seek the direction toward the revenue maximization benchmark that takes account of observed information in output prices.

Moreover, it is warranted that the above cost minimization technique only considers the production cost. However, unintended outputs would also incur costs in production process since the reduction of unintended outputs in compliance with environmental regulation usually needs to divide some of the inputs, which are supposed to be used in the intended output producing activity, to be used in the unintended output reducing activity, and this part of inputs are known as abatement inputs. Therefore, under such condition, there might be a substitution relationship between the abatement inputs and unintended outputs, which can be denoted as reallocation efficiency. Thereby, when the specific information in pollutants prices $\boldsymbol{k}_0 = (k_{10}, k_{20}, \ldots, k_{h0}) \in \boldsymbol{R}_+^h$ is available, both production costs and environmental costs should be incorporated as follows:

$$
\begin{aligned}
\min_{x_i, u_f} \; & c = \sum_{i=1}^{m} w_{i0} x_i + \sum_{f=1}^{h} k_{f0} u_f \\
s.t. \;\; & x_i \geq \sum_{j=1}^{n} x_{ij} \lambda_j, \quad i = 1, 2, \ldots, m \\
& y_{rj_0} \leq \sum_{j=1}^{n} y_{rj} \lambda_j, \quad r = 1, 2, \ldots, s \\
& u_f = \sum_{j=1}^{n} u_{fj} \lambda_j, \quad f = 1, 2, \ldots, h \\
& \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{16}
$$

Model (16) can be translated into its equivalent model with in the DDF formulation as follows:

$$\begin{aligned}
\min_{\beta, g_{xi}, g_{uf}} \quad & c = \sum_{i=1}^{m} w_{i0}\left(x_{ij_0} - \beta g_{xi}\right) + \sum_{f=1}^{h} k_{f0}\left(u_{fj_0} - \beta g_{uf}\right) \\
s.t. \quad & x_{ij_0} - \beta g_{xi} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
& y_{rj_0} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
& u_{fj_0} - \beta g_{uf} = \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
& \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{17}$$

The corresponding directional vector derived from the cost minimization technique is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = \left(X\boldsymbol{\lambda}^* - \boldsymbol{x}_0, 0, U\boldsymbol{\lambda}^* - \boldsymbol{u}_0\right)/\beta^*$, where X and U are the inputs and unintended outputs matrix.

5.2 Direction towards profit maximization

In this sub-section, we assume that the enterprises have a market behavior of profit maximizing. We further illustrate two new techniques to seek direction towards profit maximizing benchmarks based on DEA model when the market prices are exogenous and endogenous.

5.2.1 Price is exogenous

Zofio et al. (2013) supposed that the market prices of inputs and outputs are observed. Thus, they introduced the concept of profit inefficiency to directional distance function. This new technique yields overall foregone profit inefficiency due to allocative inefficiency and technical inefficiency. The allocative inefficiency measures the deviation from an observation to the optimal mix of inputs-outputs, whereas the technical inefficiency measures the distance from an observation to the production frontier (Färe and Logan 1992).

Given the price vectors of inputs and outputs by $\boldsymbol{w} = (w_1, w_2, \ldots, w_m) \in \boldsymbol{R}_+^m$ an $\boldsymbol{p} = (p_1, p_2, \ldots, p_s) \in \boldsymbol{R}_+^s$. Hereafter, the profit function is defined as $\pi(\boldsymbol{w}, \boldsymbol{p}) = \max\limits_{x,y} \{\boldsymbol{py} - \boldsymbol{wx}, (\boldsymbol{x}, \boldsymbol{y}) \in T\}$, and the profit maximizing benchmark is determined as $(\boldsymbol{x}^*, \boldsymbol{y}^*) = \arg\max \{\boldsymbol{py} - \boldsymbol{wx}, (\boldsymbol{x}, \boldsymbol{y}) \in T\}$. It can be observed that $\pi(\boldsymbol{w}, \boldsymbol{p}) \geq \boldsymbol{p}\left[\boldsymbol{y} + \vec{\boldsymbol{D}}(\boldsymbol{x}, \boldsymbol{y}; \vec{g})\,\vec{g}_x\right] - \boldsymbol{w}\left[\boldsymbol{x} + \vec{\boldsymbol{D}}(\boldsymbol{x}, \boldsymbol{y}; \vec{g})\,\vec{g}_y\right]$ and thus:

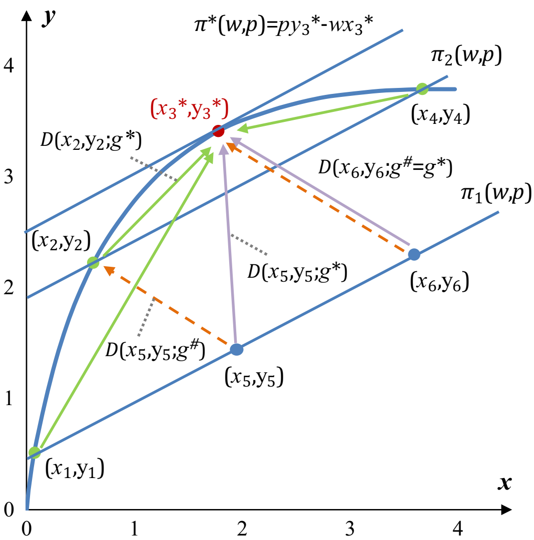
$$\left[\pi(\boldsymbol{w}, \boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] / \left(\boldsymbol{p}\vec{\boldsymbol{g}}_y + \boldsymbol{w}\vec{\boldsymbol{g}}_x\right) \geq \vec{\boldsymbol{D}}(\boldsymbol{x}, \boldsymbol{y}; \vec{\boldsymbol{g}}) \tag{18}$$

Then, the profit inefficiency can be defined as $\mathrm{PE} = \left[\pi(\boldsymbol{w}, \boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] / \left(\boldsymbol{p}\vec{\boldsymbol{g}}_y + \boldsymbol{w}\vec{\boldsymbol{g}}_x\right)$, the allocative inefficiency can be defined as $\mathrm{AE} = \left[\pi(\boldsymbol{w}, \boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] / \left(\boldsymbol{p}\vec{\boldsymbol{g}}_y + \boldsymbol{w}\vec{\boldsymbol{g}}_x\right) - \vec{\boldsymbol{D}}(\boldsymbol{x}, \boldsymbol{y}; \vec{\boldsymbol{g}})$, and the technical inefficiency can be defined as $\mathrm{TE} = \vec{\boldsymbol{D}}(\boldsymbol{x}, \boldsymbol{y}; \vec{\boldsymbol{g}})$. Accordingly, the decomposition of profit efficiency can be expressed as PE=AE+TE (Farrell 1957).

The above technique assumes that the most sensible choice for each inefficient DMU is to project itself to the profit maximizing benchmark, i.e., it seeks a direction that projects DMU $(\boldsymbol{x}, \boldsymbol{y})$ onto $(\boldsymbol{x}^*, \boldsymbol{y}^*)$.

For the inefficiency evaluation with the consideration of input and output prices, we can choose a normalization restriction such that $\boldsymbol{w}\vec{\boldsymbol{g}}_x + \boldsymbol{p}\vec{\boldsymbol{g}}_y = 1$, and thus, the directional vector can be defined as:

Fig. 8 Direction towards profit maximization



$$\left(\vec{g}_x^*, \vec{g}_y^*\right) = (\boldsymbol{x} - \boldsymbol{x}^*, \boldsymbol{y}^* - \boldsymbol{y}) / [\pi\ (\boldsymbol{w}, \boldsymbol{p}) - (\boldsymbol{p}\boldsymbol{y} - \boldsymbol{w}\boldsymbol{x})] \tag{19}$$

In which $\left(\vec{g}_x^*, \vec{g}_y^*\right)$ satisfies $\boldsymbol{w}\vec{g}_x^* + \boldsymbol{p}\vec{g}_y^* = 1$. Consequently, supposing that a specific DMU $(\boldsymbol{x}, \boldsymbol{y})$ still has improvement potentials under the assumption of profit maximization, and then, its directional profit inefficiency can be defined as:

$$\vec{\boldsymbol{D}}^* (\boldsymbol{x}, \boldsymbol{y}; \boldsymbol{w}, \boldsymbol{p}) = \vec{\boldsymbol{D}} \left(\boldsymbol{x}, \boldsymbol{y}; \vec{g}_x^*, \vec{g}_y^*\right) = max \left\{\beta : \left(\boldsymbol{x} - \beta\vec{g}_x^*, \boldsymbol{y} + \beta\vec{g}_y^*\right) \in \boldsymbol{T}\right\} \tag{20}$$

Figure 8 illustrates this process of profit efficiency measurement based on DDF. We suppose that, on the production frontier, only (x_3^*, y_3^*) maximizes profit, i.e., $\pi^*(p, w) = py_3^* - wx_3^* = \max\{\boldsymbol{py} - \boldsymbol{wx} : (\boldsymbol{x}, \boldsymbol{y}) \in \mathrm{T}\}$. In the case of DMUs lying on the production frontier such as (x_1, y_1), (x_2, y_2) and (x_4, y_4), they are technical efficient but allocative inefficient, and thus further adjustments are possible for these DMU to increase their allocative efficiency scores as well as their profit efficiency scores through approaching (x_3^*, y_3^*) along the production frontier. In the case of DMUs lying inside the production frontier such as (x_5, y_5) and (x_6, y_6), they could firstly increase their technical efficiency scores along the input–output observation value direction (e.g., $(g_x^\#, g_y^\#)$) and reach the production frontier. Then they could additionally approach the profit maximizing benchmark along the production frontier. Through this approach, (x_5, y_5) is firstly projected onto (x_2, y_2) which is only a technical efficient benchmark, and then further projected onto (x_3^*, y_3^*) which is an allocative efficient as well as profit efficient benchmark. Similarly, (x_6, y_6) can be directly projected onto the profit maximizing benchmark (x_3^*, y_3^*). Therefore, we could have $D(x_2, y_2; g^*) + D(x_5, y_5; g^\#) = D(x_5, y_5; g^*)$ and $D(x_6, y_6; g^\#) = D(x_6, y_6; g^*)$.

Rigorously speaking, there is no guarantee that along a pre-assigned direction that the evaluated DMU can be projected onto the profit-maximized benchmark. Direction toward profit maximization requires some special techniques to contract inputs and expend outputs from the theoretical perspective. Thus, the associated endogenous program, which calculates the directional profit inefficiency score based on price normalization restriction for each

DMU_{j0}, is presented as follows:

$$\begin{aligned}
&\max_{\beta,\lambda_j,g^*_{y_{rj_0}},g^*_{x_{ij_0}}} \quad \beta \\
&s.t. \;\; x_{ij_0} - \beta g^*_{x_{ij_0}} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{rj_0} + \beta g^*_{y_{rj_0}} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad \textstyle\sum_{r=1}^{s} p_r g^*_{y_{rj_0}} + \sum_{i=1}^{m} w_i g^*_{x_{ij_0}} = 1 \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \beta, \lambda_j, g^*_{y_{rj_0}}, g^*_{x_{ij_0}} \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{21}$$

The above discussions are all based on the situation that there is only one profit-maximized benchmark on the production frontier. However, in practice, the DEA technique forms a piece-wise linear frontier and, thereby, there is no guarantee that the production technology is strictly convex. In the situation that a set of profit-maximized benchmarks arises, Model (21) still generates a directional vector. However, it is uncertainly that it is the smallest adjustment for an inefficient DMU along with this direction. In other words, Model (21) cannot guarantee that the directional vector projects the evaluated DMU to the closest benchmark (Portela et al. 2003; Aparicio et al. 2007). Hence, for addressing this issue, Zofio et al. (2013) also proposed a model to calculate the minimizing Euclidian distance between each DMU_{j0} and its closest profit maximizing benchmark as follows:

$$\begin{aligned}
&\min_{\beta,\lambda_j,g^*_{y_{rj_0}},g^*_{x_{ij_0}}} \quad \left[\pi(\boldsymbol{w},\boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] \times \sqrt{\sum_{i=1}^{m}\left(g^*_{x_{ij_0}}\right)^2 + \sum_{r=1}^{s}\left(g^*_{y_{rj_0}}\right)^2} \\
&s.t. \;\; x_{ij_0} - \left[\pi(\boldsymbol{w},\boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] g^*_{x_{ij_0}} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{rj_0} + \left[\pi(\boldsymbol{w},\boldsymbol{p}) - (\boldsymbol{py} - \boldsymbol{wx})\right] g^*_{y_{rj_0}} \leq \textstyle\sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad \sum_{r=1}^{s} p_r g^*_{y_{rj_0}} + \sum_{i=1}^{m} w_i g^*_{x_{ij_0}} = 1 \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \beta, \lambda_j, g^*_{y_{rj_0}}, g^*_{x_{ij_0}} \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{22}$$

$(\boldsymbol{g}^*_{\boldsymbol{x}}, \boldsymbol{g}^*_{\boldsymbol{y}})$ is the optimal solution of Model (22) which returns a directional profit inefficiency measure. This directional vector can identify the profit maximization and the closest benchmark simultaneously.

It is obvious that the original model of the above direction-selecting technique does not address the unintended outputs. Taking account of the unintended outputs, the associated model that calculate the directional profit inefficiency score under variable returns to scale (VRS) can be presented as follows:

$$
\begin{aligned}
&\max_{\beta,\lambda_j,g^*_{y_{rj_0}},g^*_{x_{ij_0}},g^*_{u_{fj_0}}} \quad \beta \\
&s.t. \quad x_{ij_0} - \beta g^*_{x_{ij_0}} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{rj_0} + \beta g^*_{y_{rj_0}} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad u_{fj_0} - \beta g^*_{u_{fj_0}} \geq \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
&\qquad \sum_{r=1}^{s} p_r g^*_{y_{rj_0}} + \sum_{i=1}^{m} w_i g^*_{x_{ij_0}} + \sum_{f=1}^{h} k_i g^*_{u_{fj_0}} = 1 \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \beta, \lambda_j, g^*_{y_{rj_0}}, g^*_{x_{ij_0}}, g^*_{u_{fj_0}} \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \\
&\qquad\quad r = 1, 2, \ldots, s, \quad f = 1, 2, \ldots, h.
\end{aligned} \tag{23}
$$

Moreover, the associated model that calculate the minimizing Euclidian distance between each DMU$_{j0}$ and its closest profit maximizing can be presented as follows:

$$
\begin{aligned}
&\min_{\beta,\lambda_j,g^*_{y_{rj_0}},g^*_{x_{ij_0}},g^*_{u_{fj_0}}} \quad [\pi\,(w, p, k) - (py - wx - ku)] \\
&\qquad \times \sqrt{\sum_{i=1}^{m}\left(g^*_{x_{ij_0}}\right)^2 + \sum_{r=1}^{s}\left(g^*_{y_{rj_0}}\right)^2 + \sum_{f=1}^{h}\left(g^*_{u_{fj_0}}\right)^2} \\
&s.t. \quad x_{ij_0} - [\pi\,(w, p, k) - (py - wx - ku)]\, g^*_{x_{ij_0}} \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
&\qquad y_{rj_0} + [\pi\,(w, p, k) - (py - wx - ku)]\, g^*_{y_{rj_0}} \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
&\qquad u_{fj_0} - [\pi\,(w, p, k) - (py - wx - ku)]\, g^*_{u_{fj_0}} \geq \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
&\qquad \sum_{r=1}^{s} p_r g^*_{y_{rj_0}} + \sum_{i=1}^{m} w_i g^*_{x_{ij_0}} + \sum_{f=1}^{h} k_f g^*_{u_{fj_0}} = 1 \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \beta, \lambda_j, g^*_{y_{rj_0}}, g^*_{x_{ij_0}}, g^*_{u_{fj_0}} \geq 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m, \\
&\qquad r = 1, 2, \ldots, s, \quad f = 1, 2, \ldots, h.
\end{aligned} \tag{24}
$$

The corresponding directional vector derived from the above technique is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = (\boldsymbol{x}^* - \boldsymbol{x}, \boldsymbol{y}^* - \boldsymbol{y}, \boldsymbol{u}^* - \boldsymbol{u}) \,/\, \left[\pi\,(\boldsymbol{w}, \boldsymbol{p}, \boldsymbol{k}) - (\boldsymbol{py} - \boldsymbol{wx} - \boldsymbol{ku})\right]$.

From a conceptual perspective, Models (23) and (24) endogenously obtain a market-oriented direction that captures the profit maximization. In this concept, profit inefficiency can be decomposed into technical inefficiency and allocative inefficiency, and these two compositions are based on the endogenous choice of directions. From a practical managerial perspective, if there is no assumption of profit maximizing behavior, the evaluated DMUs would only measure technical inefficiency score along with a theoretically optimized direction in Model (23). Conversely, the evaluated DMUs would project to its profit maximizing benchmarks associated with this market-oriented direction in Models (23) and (24).

In the light of this analytical proposal, the profit maximization model presents an ideal goal and provides a long-term efficiency promotion direction. However, a particularly interesting feature of this model is that it also fails to satisfy the consistence property in practice, because the profit maximizing benchmark would dynamically change over time and thus the goal generated by Model (24) might not be fixed. In addition, the target of profit maximization may restrict the adjustable flexibility of the input-output mix and may be too idealist in production.

5.2.2 Price is endogenous

In the real production progress, some markets are usually imperfectly competitive. In this case, enterprises may try to determine the market price by changing their input and output level. To address this issue on the efficiency estimation, Lee and Johnson (2015) and Lee (2016) developed a mixed complementarity problem (MiCP) to identify the Nash equilibrium and seek the direction towards this equilibrium benchmark when the market prices of inputs and outputs are endogenous.

Let $\boldsymbol{\theta}$ be the utility (or profit) function, $\boldsymbol{T}_j$ be the production possibility set of DMU$_j$, and the Nash equilibrium benchmark $(\boldsymbol{x}^*, \boldsymbol{y}^*) = \left(\left(\boldsymbol{x}_1^*, \boldsymbol{y}_1^*\right), \left(\boldsymbol{x}_2^*, \boldsymbol{y}_2^*\right), \ldots, \left(\boldsymbol{x}_n^*, \boldsymbol{y}_n^*\right)\right) \in \boldsymbol{T}_1 \times \boldsymbol{T}_2 \times \cdots \times \boldsymbol{T}_n$ be determined as

$$\theta\left(\boldsymbol{x}^*, \boldsymbol{y}^*\right) \geq \theta\left(\boldsymbol{x}_j, \hat{\boldsymbol{x}}^*_{(-j)}, \boldsymbol{y}_j, \hat{\boldsymbol{y}}^*_{(-j)}\right), \quad \forall \left(\boldsymbol{x}_j, \boldsymbol{y}_j\right) \in \boldsymbol{T}_j, \tag{25}$$

where $\hat{\boldsymbol{x}}^*_{(-j)} = \left(\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_{j-1}^*, \boldsymbol{x}_{j+1}^*, \ldots, \boldsymbol{x}_n^*\right)$ and $\hat{\boldsymbol{y}}^*_{(-j)} = \left(\boldsymbol{y}_1^*, \ldots, \boldsymbol{y}_{j-1}^*, \boldsymbol{y}_{j+1}^*, \ldots, \boldsymbol{y}_n^*\right)$, $j = 1, 2, \ldots, n$.

This technique assumed that the inverse demand function, which should be non-increasing, and the inverse supply function, which should be non-decreasing, are linear and continuously differentiable. Hereafter, the associated direction towards Nash equilibrium for DMU$_{j0}$ can be obtained as follows:

(i) The linear price function of input i and output r can be fitted as $P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) = P_i^{X_0} + \beta_{ii}\bar{X}_i + \sum_{l\neq i}\beta_{il}\bar{X}_l$ and $P_r^Y\left(\bar{Y}_r, \bar{\boldsymbol{Y}}_{(-r)}\right) = P_r^{Y_0} + \alpha_{rr}\bar{Y}_r + \sum_{h\neq r}\alpha_{rh}\bar{Y}_h$, where $\bar{X}_i = \sum_{j\neq j_0} x_{ij} + x_{ij_0}$, $\bar{Y}_r = \sum_{j\neq j_0} y_{rj} + y_{rj_0}$ $\bar{\boldsymbol{X}}_{(-i)} = \left\{\bar{X}_1, \ldots, \bar{X}_{i-1}, \bar{X}_{i+1}, \ldots, \bar{X}_m\right\}$, and $\bar{\boldsymbol{Y}}_{(-r)} = \left\{\bar{Y}_1, \ldots, \bar{Y}_{r-1}, \bar{Y}_{r-1}, \ldots, \bar{Y}_s\right\}$.

(ii) Defining the Nash profit function (NFP) and the associated profit maximization model for DMU$_{j0}$ as

$$\begin{aligned}
NFP^*_{j0} = & \max_{\lambda_j, x_i, y_r} \textstyle\sum_r P_r^Y\left(\bar{Y}_r, \bar{\boldsymbol{Y}}_{(-r)}\right) y_{rj} - \sum_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ij} \\
s.t. \quad & x_i \geq \sum_{j=1}^n x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
& y_r \leq \sum_{j=1}^n y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
& \sum_{j=1}^n \lambda_j = 1 \\
& \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{26}$$

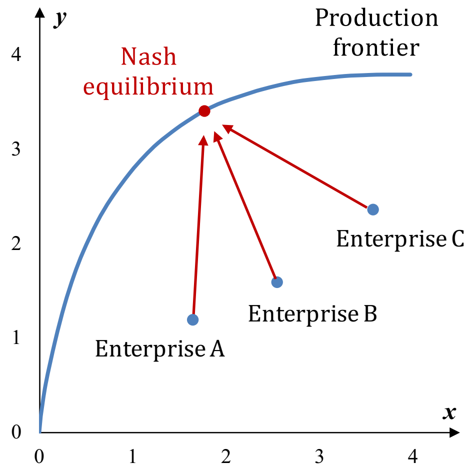


Fig. 9 Direction towards Nash equilibrium benchmark

Thus, the solution generated from Model (26) is the Nash equilibrium benchmark $\left(\boldsymbol{x}^{N*}, \boldsymbol{y}^{N*}\right) = \left(\left(\boldsymbol{x}_1^{N*}, \boldsymbol{y}_1^{N*}\right), \left(\boldsymbol{x}_2^{N*}, \boldsymbol{y}_2^{N*}\right), \ldots, \left(\boldsymbol{x}_n^{N*}, \boldsymbol{y}_n^{N*}\right)\right)$

(iii) From the above steps, the corresponding directional vector for DMU$_{j0}$ is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y\right) = \left(\boldsymbol{x}_{j0}^{N*} - \boldsymbol{x}_{j0}, \boldsymbol{y}_{j0}^{N*} - \boldsymbol{y}_{j0}\right) / \left[\left(\boldsymbol{p}_{y_{j0}}^{N*}\boldsymbol{y}_{j0}^{N*} - \boldsymbol{p}_{x_{j0}}^{N*}\boldsymbol{x}_{j0}^{N*}\right) - \left(\boldsymbol{p}_{y_{j0}}^{N}\boldsymbol{y}_{j0}^{N} - \boldsymbol{p}_{x_{j0}}^{N}\boldsymbol{x}_{j0}^{N}\right)\right]$, where $\left(\boldsymbol{p}_{y_{j0}}^{N*}, \boldsymbol{p}_{x_{j0}}^{N*}\right)$ and $\left(\boldsymbol{p}_{y_{j0}}^{N}, \boldsymbol{p}_{x_{j0}}^{N}\right)$ are the market price computed by the price function of Nash equilibrium $\left(\boldsymbol{y}_{j0}^{N*}, \boldsymbol{x}_{j0}^{N*}\right)$ and observation $\left(\boldsymbol{y}_{j0}^{N}, \boldsymbol{x}_{j0}^{N}\right)$.

Figure 9 illustrates this process of Nash profit efficiency measurement based on DDF.

Obviously, there is no unintended outputs in the above direction-selecting technique. Taking account of the unintended outputs with respect to environmental regulation, we limit the quantity of unintended outputs $\hat{U}_f \leq \tilde{U}_f$, where $\hat{U}_f = \sum_j u_{fj}$ and $\tilde{U}_f$ represents environmental regulation for unintended output f. Therefore, NFP and the associated profit maximization model for DMU$_{j0}$ can be presented as follows:

$$
\begin{aligned}
\mathrm{NFP}_{j0}^{*} = \max_{\lambda_j, x_i, y_r, u_f} & \sum\nolimits_r P_r^Y\left(\bar{Y}_r, \bar{\boldsymbol{Y}}_{(-r)}\right) y_{rj} - \sum\nolimits_i P_i^X\left(\bar{X}_i, \bar{\boldsymbol{X}}_{(-i)}\right) x_{ij} - \sum\nolimits_f P_f^U\left(\bar{U}_f, \bar{\boldsymbol{U}}_{(-f)}\right) u_{fj} \\
s.t. \quad & x_i \geq \sum_{j=1}^{n} x_{ij}\lambda_j, \quad i = 1, 2, \ldots, m \\
& y_r \leq \sum_{j=1}^{n} y_{rj}\lambda_j, \quad r = 1, 2, \ldots, s \\
& u_f \geq \sum_{j=1}^{n} u_{fj}\lambda_j, \quad f = 1, 2, \ldots, h \\
& \hat{U}_f \leq \tilde{U}_f \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{27}
$$

The solution generated from Model (27) is the Nash equilibrium benchmark $\left(\boldsymbol{x}^{N*}, \boldsymbol{y}^{N*}, \boldsymbol{u}^{N*}\right) = \left(\left(\boldsymbol{x}_1^{N*}, \boldsymbol{y}_1^{N*}, \boldsymbol{u}_1^{N*}\right), \left(\boldsymbol{x}_2^{N*}, \boldsymbol{y}_2^{N*}, \boldsymbol{u}_2^{N*}\right), \ldots, \left(\boldsymbol{x}_n^{N*}, \boldsymbol{y}_n^{N*}, \boldsymbol{u}_n^{N*}\right)\right)$, and thus, the corresponding directional vector for DMU$_{j0}$ is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = \left(\boldsymbol{x}_{j0}^{N*} - \boldsymbol{x}_{j0}, \boldsymbol{y}_{j0}^{N*} - \boldsymbol{y}_{j0}, \boldsymbol{u}_{j0}^{N*} - \boldsymbol{u}_{j0}\right) / \left[\left(\boldsymbol{p}_{y_{j0}}^{N*}\boldsymbol{y}_{j0}^{N*} - \boldsymbol{p}_{x_{j0}}^{N*}\boldsymbol{x}_{j0}^{N*} - \boldsymbol{p}_{u_{j0}}^{N*}\boldsymbol{u}_{j0}^{N*}\right) - \left(\boldsymbol{p}_{y_{j0}}^{N}\boldsymbol{y}_{j0}^{N} - \boldsymbol{p}_{x_{j0}}^{N}\boldsymbol{x}_{j0}^{N} - \boldsymbol{p}_{u_{j0}}^{N}\boldsymbol{u}_{j0}^{N}\right)\right]$, where $\left(\boldsymbol{p}_{x_{j0}}^{N*}, \boldsymbol{p}_{y_{j0}}^{N*}, \boldsymbol{p}_{u_{j0}}^{N*}\right)$ and $\left(\boldsymbol{p}_{x_{j0}}^{N}, \boldsymbol{p}_{y_{j0}}^{N}, \boldsymbol{p}_{u_{j0}}^{N}\right)$ are the market price computed by the price function of Nash equilibrium $\left(\boldsymbol{x}_{j0}^{N*}, \boldsymbol{y}_{j0}^{N*}, \boldsymbol{u}_{j0}^{N*}\right)$ and observation $\left(\boldsymbol{x}_{j0}^{N}, \boldsymbol{y}_{j0}^{N}, \boldsymbol{u}_{j0}^{N}\right)$.

Contrary to the previous techniques, this technique uses a MiCP to identify the Nash equilibrium benchmark corresponding to profit maximization production behavior for all enterprises. In this case, a unique Nash equilibrium benchmark can be identified to provide a further insight for improving operational strategies. Thus, the most contribution of this direction-selecting technique is that it describes an efficiency measure in the imperfectly competitive market with endogenous prices of inputs and outputs. However, similar to the model with exogenous prices, this model also lacks of the practical supports. Consequently, another economic meaning has been introduced in the direction-selecting technique, which is illustrates in the next sub-section.

5.3 Direction towards marginal profit maximization

Most studies of DDF focus primarily on seeking the direction for efficiency estimation rather than productivity improvement. This sub-section presents a technique to find a direction towards marginal profit maximization, namely, productivity improvement.

The concept of marginal productivity (MP) represents the amount of an output generated through consuming an additional unit of an input. In other words, MP states the relationship between the changes of a specific output and a specific input. It is usually the differential characteristic of the production frontier. Generally, MP is calculated by the partial derivatives of a smooth production function. However, the DEA technique forms a piece-wise linear frontier and, thereby, it is not everywhere differentiable. Podinovski and Førsund (2010) proposed a definition for differential characteristics of non-differentiable frontier in DEA technique, and pointed out that MP is the dual multiplier linear program of envelopment model. Hereafter, Lee (2014) extended a concept of directional marginal productivity (DMP) based on marginal productivity to efficiency and productivity evaluation. It can identify the marginal profit maximization under DDF and further intimate the trade-off between multiple outputs. In this review, we consider that the DMP derives a direction that indicates productivity improvement.

Let index $i' \in m$ represents a special input, set $S' \in S$ be a subset of intended output, and set $H' \in H$ be a subset of unintended outputs whose MP needed to be investigated. Then, the associated output-oriented DEA model under VRS for each DMU$_{j0}$ can be presented as follows:

$$
\begin{aligned}
&\max_{\beta,\lambda_j,\mu_j} \quad \beta \\
&s.t. \quad x_{i'j_0} \geq \sum\nolimits_{j=1}^{j=n} x_{i'j}(\lambda_j+\mu_j) \\
&\qquad x_{ij_0} \geq \sum\nolimits_{j=1}^{j=n} x_{ij}(\lambda_j+\mu_j), \quad \forall i \neq i' \\
&\qquad y_{r'j_0} + \beta g_{y_{r'j_0}} \leq \sum\nolimits_{j=1}^{j=n} y_{r'j}\lambda_j, \quad \forall r' \in S' \\
&\qquad y_{r'j_0} \leq \sum\nolimits_{j=1}^{j=n} y_{rj}\lambda_j, \quad \forall r \in S \backslash S' \\
&\qquad u_{f'j_0} - \beta g_{u_{f'j_0}} = \sum\nolimits_{j=1}^{j=n} u_{f'j}\lambda_j, \quad \forall f' \in H' \\
&\qquad u_{f'j_0} = \sum\nolimits_{j=1}^{j=n} u_{fj}\lambda_j, \quad \forall f \in H \backslash H' \\
&\qquad 1^T\lambda + 1^T\mu = 1 \\
&\qquad \lambda_j, \mu_j \geq 0, \quad j = 1, 2, \ldots, n.
\end{aligned}
\tag{28}
$$

where μ_j is the decision variable of weak disposability (Kuosmanen 2005), and the last equality constraint presents a situation of VRS. This model generally set $\mathbf{1}^T\boldsymbol{g}_y + \mathbf{1}^T\boldsymbol{g}_u = 1$, which is a normalization constraint, in particular, to ensure compactness (Färe et al. 2013).

Then, let $x_i^{\max} = \max_j \left\{x_{ij}\right\}$, $y_r^{\max} = \max_j \left\{y_{rj}\right\}$, and $u_f^{\max} = \max_j \left\{u_{rj}\right\}$. Model (29) denotes the dual multipliers of inputs constraint, intended outputs constrain, unintended outputs constrain and convex-combination constrain in Model (30) by ϕ_i, γ_r, τ_f and ξ_0 respectively. Here, assuming that DMU$_{j0}$, which is on the frontier, is the efficient benchmark of DMU$_j$. Then, the associated model to calculate DMP of DMU$_{j0}$ can be presented as follows:

$$
\begin{aligned}
&\min_{\phi_i,\gamma_r,\tau_f,\xi_0} \quad \phi_{i'} \\
&s.t. \quad \sum\nolimits_{i=1}^{i=m} \frac{x_{ij_0}}{x_i^{max}}\phi_i - \sum\nolimits_{r=1}^{r=s} \frac{y_{rj_0}}{y_r^{max}}\gamma_r + \sum\nolimits_{f=1}^{f=h} \frac{u_{fj_0}}{u_f^{max}}\tau_f + \xi_0 = 0 \\
&\qquad \sum\nolimits_{i=1}^{i=m} \frac{x_{ij}}{x_i^{max}}\phi_i - \sum\nolimits_{r=1}^{r=s} \frac{y_{rj}}{y_r^{max}}\gamma_r + \sum\nolimits_{f=1}^{f=h} \frac{u_{fj}}{u_f^{max}}\tau_f + \xi_0 \geq 0, \quad \forall j \\
&\qquad \sum\nolimits_{i=1}^{i=m} \frac{x_{ij}}{x_i^{max}}\phi_i + \xi_0 \geq 0, \quad \forall j \\
&\qquad \sum\nolimits_{r' \in S'} \gamma_{r'} g_{y_{r'j_0}} + \sum\nolimits_{f' \in H'} \tau_{r'} g_{u_{f'j_0}} = 1 \\
&\qquad \phi_i, \gamma_r \geq 0, \quad i = 1, 2, \ldots, m, \quad r = 1, 2, \ldots, s.
\end{aligned}
\tag{29}
$$

Model (29) is the dual form of Model (28), and it estimates the DMP of $y_{r'}, r' \in S'$ and $u_{r'}, r' \in R'$with respect to $x_{i'}$. In this case, $\left(\frac{x_{ij}}{x_i^{max}}, \frac{y_{rj}}{y_r^{max}}, \frac{u_{fj}}{u_f^{max}}\right), \forall j$ is a standardization process, indicating that the results would not be affected by the unit of inputs and outputs. Meanwhile, the DMP of DMU$_{j0}$ can be calculated by $\frac{\partial\left(y_{r'j_0}, u_{f'j_0}\right)}{\partial x_{i'j_0}} = \frac{\phi_{i'}}{x_i^{max}}\left(g_{y_{r'}} y_{r'}^{max}, -g_{u_{f'}} u_{f'}^{max}\right), \forall r' \in S', \forall f' \in R'$, which would help the managers to conduct an efficient resource allocation.

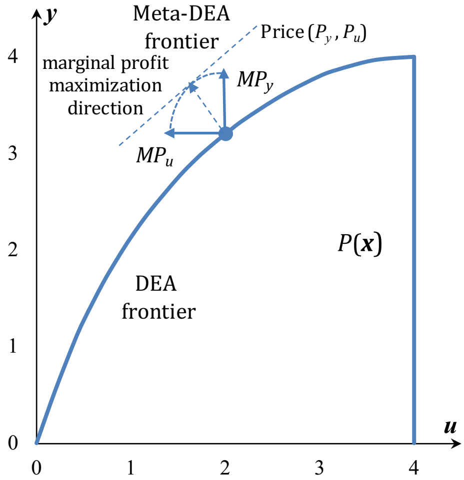


Fig. 10 Direction towards marginal profit maximization

In addition, the directional vectors can be considered as the weights of MPs for each investigated output in this model. It can be found that different directions could generate different DMPs, and the closer the direction to MP of a single output has a higher weight. Furthermore, it can be seen that the MP is a special case of DMP. Therefore, MPs of each output would generate a span like a frontier on the surface of the production frontier. In other words, the associated DEA model could form a meta-DEA frontier (i.e., marginal production possibility set) towards the input $\boldsymbol{x} + \Delta \boldsymbol{x}$, whereas $\Delta \boldsymbol{x}$ is an additional unit for one special input. To visualize this situation, a meta-DEA frontier is presented in Fig. 10, where $P(\mathrm{x})$ is an output-oriented production possibility set.

When the price vector $\left(P_{x_i}, P_{y_r}, P_{u_f}\right)$ of input, intended output and unintended output can be observed, the associated direction towards marginal profits maximization for DMU$_{j0}$ can be obtained as follows:

(i) Given a fixed interval, there would be a series of weighting directions under normalization constraint. For instance, the directional vector of Model (29) can be assigned as $\left(\vec{\boldsymbol{g}}_y, \vec{\boldsymbol{g}}_u\right) = (0, 1), (0.2, 0.8), (0.4, 0.6), (0.6, 0.4), (0.8, 0.2)$, or $(1, 0)$ with a 0.2 interval in a two-output case.

(ii) Supposing that the φth direction satisfies $\sum_{r' \in S} g^{\varphi}_{y_{r'}} + \sum_{f' \in R'} g^{\varphi}_{u_{f'}} = 1$ and $0 \leq g^{\varphi}_{y_{r'}}, g^{\varphi}_{u_{f'}} \leq 1, \forall r' \in S', f' \in R', \forall \varphi$. Then the optimal direction $\left(g^*_{y_{r'}}, g^*_{u_{f'}}\right)$ for identifying marginal profit maximization can be obtained as:

$$\left(g^*_{y_{r'}}, g^*_{u_{f'}}\right) = arg \max_{g^{\varphi}_{y_{r'}}, g^{\varphi}_{u_{f'}}} \left\{ \left(\sum_{r' \in S'} P_{y_{r'}} g^{\varphi}_{y_{r'}} y^{max}_{r'} + \sum_{f' \in R'} P_{u_{f'}} g^{\varphi}_{u_{f'}} u^{max}_{f'} \right) \phi^{\varphi}_{i'} / x^{max}_{i'} \right.$$
$$\left. - P_{x_{i'}} \,\middle|\, \text{Model (29) given } \left(P_{x_i}, P_{y_r}, P_{u_f}\right), \quad \forall \varphi \right\} \tag{30}$$

in which $\left(\sum_{r' \in S'} P_{y_{r'}} g_{y_{r'}} y^{max}_{r'} + \sum_{f' \in R'} P_{u_{f'}} g_{u_{f'}} u^{max}_{f'} \right) \phi_{i'} / x^{max}_{i'}$ represents the marginal revenue, and $P_{x_{i'}}$ represents the marginal cost. The aim is to measure the impact on all investigated outputs while expanding an additional unit of a specific input. Thus, this technique tends to find the output directions towards marginal revenue maximization simultaneously by fixing input price.

(iii) From the above steps, the corresponding directional vector that refers to DMP is $\vec{\boldsymbol{g}} = \left(-\vec{\boldsymbol{g}}_x, \vec{\boldsymbol{g}}_y, -\vec{\boldsymbol{g}}_u\right) = \left(-\boldsymbol{g}_x, \partial\left(\boldsymbol{y}, \boldsymbol{u}\right)/\partial x\right) = \left(1, \phi_{i'^*} g^*_{y_{r'}} y^{max}_{r'}/x^{max}_i, -\phi_{i'^*} g^*_{u_{f'}} u^{max}_{f'}/x^{max}_i\right)$.

The above definitions are proposed for DMUs on production frontier (i.e., efficient DMUs), and MP cannot be measured for DMUs lying inside of the frontier (i.e., inefficient DMUs). Here, it should be noted that the marginal productivity of an inefficient DMU would refer to the MP measured by its reference benchmark via a given direction (e.g. output orientation).

The main feature of DMP is that it can provide helpful information regarding a DMU when it is measured as technical efficient but allocative inefficient. In addition, most previous studies employ the input-oriented (or output-oriented) model by controlling the output (or input) level. However, in practice, there is some linkage between inputs and outputs, for example, enterprises usually expand both inputs and outputs because they are not willing to take the risk to lose market or sale share by contracting some inputs. Thus, the DMP can provide a clear explanation to this linkage. Another advantage is that the meta-DEA frontier intimates the trade-off between multiple outputs, and emphasizes the relationship between inputs and outputs, when expanding one extra unit of a specific input. At last, the DMP model focuses on productivity improvement rather than efficiency measurement. Contrary to the directions towards profit maximization, the directions towards marginal profit maximization focus on a short-term target, and give a dynamic adjustment of resource. This step-by-step productivity improvement approach is consistent with the business practice. In a real setting, the DMUs can select the appropriate efficiency estimating approach according to specific evaluation purpose.

6 Numerical example for comparison

As mentioned above, the most critical issue for DDF is the selection of direction along which to estimate the inefficiency score of a DMU. Obviously, we would obtain different inefficiency scores corresponding to the different selection of direction. Thus, we will present a comparative analysis between the inefficiency evaluation result based on the above reviewed seven direction-selecting techniques: (i) input/output value direction, (ii) unit value direction, (iii) direction towards the closest benchmark, (iv) direction towards the largest improvement potential, (v) direction towards cost minimization, (vi) directions towards profit maximization, and (vii) direction towards marginal profit maximization. Table 1 presents all the variables, parameters and each directional vector for these seven direction-selecting techniques. Because the selecting technique, which is to seek the direction toward profit-maximized benchmark with endogenous prices (i.e., Nash equilibrium benchmark), describes the case in an imperfectly competitive market, we would not compute its inefficiency scores for comparison. In addition, we employ the non-oriented model for all techniques, except for the cost minimization model. In this section, we introduce two criteria (i.e., distinction and inconsistency) to evaluate the performance of the seven direction-selecting techniques.

Following the variables of Färe et al. (2016), we provide the efficiency evaluation consist of the inputs and outputs of 83 power plants operating in the U.S. in 2003. We estimate the inefficiency scores for each plant using three inputs (capital, labor, and energy), one intended output (net electrical generation) and one unintended output (sulfur dioxide, SO_2). Capital

Table 1 Comparison of seven typical direction-selecting techniques

Direction	Study	Parameter	Variable	Directional vector	Price information
Input/output value	Chambers et al. (1996b)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{g}$	$\beta, \boldsymbol{\lambda}$	$(-\boldsymbol{x}, \boldsymbol{y}, -\boldsymbol{u})$	No
Unit value	Färe et al. (2006)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{g}$	$\beta, \boldsymbol{\lambda}$	$(-\mathbf{1}, \mathbf{1}, -\mathbf{1})$	No
Closest benchmark	Baek and Lee (2009)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}$	$\beta, \boldsymbol{\lambda}, \boldsymbol{s}^{+}, \boldsymbol{s}^{-}, \boldsymbol{g}$	$\left(X^{E}\boldsymbol{\lambda}^{*} - \boldsymbol{x}_{0}, Y^{E}\boldsymbol{\lambda}^{*} - \boldsymbol{y}_{0}, U^{E}\boldsymbol{\lambda}^{*} - \boldsymbol{u}_{0}\right)/\beta^{*}$	No
Largest improvement potential	Hampf and Krüger (2014a)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}$	$\beta, \boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\delta}, \boldsymbol{\phi}, \boldsymbol{g}$	$(-\phi \otimes \boldsymbol{x}, \alpha \otimes \boldsymbol{y}, -\delta \otimes \boldsymbol{u})$	No
Cost minimization	Ray and Mukherjee (2000)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{w}, \boldsymbol{k}$	$\beta, \boldsymbol{\lambda}, \boldsymbol{g}$	$\left(X\boldsymbol{\lambda}^{*} - \boldsymbol{x}_{0}, \mathbf{0}, U\boldsymbol{\lambda}^{*} - \boldsymbol{u}_{0}\right)/\beta^{*}$	Exogenous price
Profit maximization	Zofio et al. (2013)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{w}, \boldsymbol{k}, \boldsymbol{p}$	$\beta, \boldsymbol{\lambda}, \boldsymbol{g}$	$\frac{(\boldsymbol{x}^{*}-\boldsymbol{x}, \boldsymbol{y}^{*}-\boldsymbol{y}, \boldsymbol{u}^{*}-\boldsymbol{u})}{\pi(\boldsymbol{w},\boldsymbol{p},\boldsymbol{k})-(\boldsymbol{py}-\boldsymbol{wx}-\boldsymbol{ku})}$	Exogenous price
	Lee (2016)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}$	$\boldsymbol{\lambda}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{P}_{x}, \boldsymbol{P}_{y}, \boldsymbol{P}_{u}, \boldsymbol{x}^{N}, \boldsymbol{y}^{N}, \boldsymbol{u}^{N}$	$\frac{\left(\boldsymbol{x}_{j0}^{N*}-\boldsymbol{x}_{j0}, \boldsymbol{y}_{j0}^{N*}-\boldsymbol{y}_{j0}, \boldsymbol{u}_{j0}^{N*}-\boldsymbol{u}_{j0}\right)}{\left(\boldsymbol{p}_{y_{j0}}^{N*}\boldsymbol{y}_{j0}^{N*}-\boldsymbol{p}_{x_{j0}}^{N*}\boldsymbol{x}_{j0}^{N*}-\boldsymbol{p}_{u_{j0}}^{N*}\boldsymbol{u}_{j0}^{N*}\right)-\left(\boldsymbol{p}_{y_{j0}}^{N}\boldsymbol{y}_{j0}^{N}-\boldsymbol{p}_{x_{j0}}^{N}\boldsymbol{x}_{j0}^{N}-\boldsymbol{p}_{u_{j0}}^{N}\boldsymbol{u}_{j0}^{N}\right)}$	Endogenous price
Marginal profit maximization	Lee (2014)	$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{u}, \boldsymbol{P}_{x}, \boldsymbol{P}_{y}, \boldsymbol{P}_{u}$	$\boldsymbol{\phi}, \boldsymbol{\gamma}, \boldsymbol{\tau}, \boldsymbol{\xi}, \boldsymbol{g}$	$\left(1, \phi_{i'*} g_{y_{r'}}^{*} y_{r'}^{max}/x_{i}^{max}, -\phi_{i'*} g_{u_{f'}}^{*} u_{f'}^{max}/x_{i}^{max}\right)$	Exogenous price

Table 2 Input, output and price values of power plants

US 83 power plants in 2005	Unit	Mean	SD	Maximum	Minimum
Capital	Million Dollars	308.6	889.9	6691.1	2.2
Labor	Number of employers	107.9	136.9	778.0	4.0
Energy	Billion Btu	25,392.3	38,300.1	197,247.3	29.9
Net electrical generation	Million kWh	2309.3	3506.1	17,734.2	2.8
SO_2	Tons	6907.6	9855.4	38,717.0	0.5
Price of capital	Dollars per unit of capital	4.8148	43.3496	394.99	0.0183
Price of labor	Million dollars per employer	0.0166	0.0159	0.0916	0.0008
Price of energy	Million dollars per billion Btu	0.0026	0.0019	0.0062	0.0014
Price of electrical	Dollars per kWh	0.0805	0.0327	0.3208	0.0014
Price of SO_2	Million dollars per ton	0.0224	0.1778	1.6186	0.0000

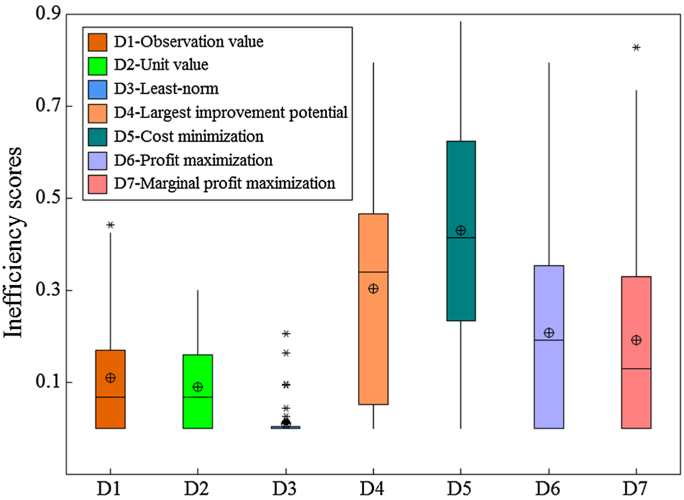


Fig. 11 Box plot of inefficiency scores derived from different direction-selecting techniques

is measured by the capital stock of each plant, and its corresponding price is calculated by the sum of rent, depreciation and interest expenses per dollars in millions of the gross value capital stock. Labor is denoted by the number of employees, and its corresponding price is measured by the sum of operation, maintenance, supervision and engineering expenses per worker. Energy is measured by the total heat content of energy consumption, and its corresponding price is derived from the annual average cost of fossil fuels based on the plant primary fossil fuel category. Meanwhile, the electricity price is the annual average sales price according to the state where the power plant located. Finally, the cost of SO_2 emission could be considered as the marginal abatement costs (i.e., the shadow prices) which are obtained from the estimations of Rezek and Campbell (2007). The descriptive statistics of the data are reported in Table 2.

Figure 11 displays the distribution features of inefficiency scores of the seven direction-selecting techniques and gives a comparison of distinction. Firstly, it can be seen that the inefficiency scores derived from the cost minimization selecting technique present the highest median and mean values than those from the other direction-selecting techniques, implying that there is still much improvement potential in terms of cost reduction. For the least-norm model, the inefficiency scores for most DMUs are close to zero, which can be explained that the purpose of this technique is to identify the shortest distance to production frontier. Conversely, the technique seeking the direction towards the largest improvement potential has the higher median and mean values on inefficiency scores than those of other direction-selecting techniques, except for cost minimization selecting technique. Moreover, the performance of observation value direction-selecting technique and unit value direction-selecting technique are very similar. They are similar with relatively low mean and median values, indicating that the detected efficiency improvement under endogenous direction-selecting technique is more obvious than that under the exogenous direction-selecting technique.

Secondly, the degree of distinction of inefficiency evaluation, which represents the discriminating power of each direction-selecting technique, has a similar result with the inefficiency scores. As can be easily checked that the largest improvement potential selecting technique presents the highest discriminating power with the longest box in Fig. 11, while the least-norm selecting technique presents the lowest discriminating power with the shortest box. The long-term efficiency promotion techniques (i.e., cost minimization and profit maximization), which can identify the larger efficiency improvement potential, has a higher discriminating power than the short-term efficiency promotion techniques (i.e., marginal profit maximization). In addition, the observation value direction and the unit value direction-selecting techniques present a relatively low discriminating power. The above results also indicate that the inefficiency derived from endogenous direction-selecting techniques is more obvious than those from the exogenous direction-selecting techniques.

In order to provide a further insight into the difference between direction-selecting techniques, we testify the Färe’s weak disposability technology and compare the seven directions with respect to the degree of inconsistency, which is defined as the sensitivity of efficiency score in increasing unintended output. Specifically, we construct two scenarios to calculate the inefficiency scores. First, we use the original inputs and outputs to estimate the inefficiency scores for each technique. Second, we simply double the unintended output of each evaluated DMU to estimate its inefficiency score, while keeping the inputs and outputs of all other DMUs unchanged (Chen and Delmas 2012). Intuitively, the inefficiency score would increase when unintended output (i.e., SO_2 emission in this example) presents a massive surge in expanding. Thus, if the inefficiency score of a DMU decreases, the inconsistency occurs which is considered as an estimation error. Then, we record the number of such error, and calculate the error rate as the number of the errors divided by the number of all DMUs.

Table 3 provides the experimental results of these seven direction-selecting techniques. Generally, the exogenous direction-selecting techniques (i.e., observation value direction and unit value direction-selecting technique) have a higher error rate than the endogenous direction-selecting techniques, i.e., more DMUs become more efficient after doubling their unintended outputs. For the exogenous direction-selecting techniques, observation value direction and unit value direction-selecting technique are very sensitive to expansion in unintended outputs with the error rate of 54.2 and 48.2%, respectively. In other words, they have a high degree of inconsistency. With respect to the endogenous direction-selecting technique, the error number and error rate of the largest improvement potential selecting technique are both zero, indicating that it shows a very low degree of inconsistency regarding expansion in unintended outputs. This situation is similar in the cost minimization selecting technique. Conversely, there are 13 errors in the profit maximization selecting technique,

Table 3 Experimental results of each direction-selecting technique

Error	Observation value direction	Unit value direction	Least-norm direction	Largest improvement potential direction	Cost minimization direction	Profit maximization direction	Marginal profit maximization direction
Rate	0.542	0.482	0.410	0.000	0.012	0.157	0.398
Number	45	40	34	0	1	13	33

implying that this technique is insensitive to changes in unintended outputs, but its degree of inconsistency is higher than the above two endogenous direction-selecting techniques. More specifically, the error rate is high in least-norm and marginal profit maximization selecting techniques, which account for 41 and 39.8%, respectively, meaning that these two techniques have a very high degree of inconsistency when expanding unintended outputs.

7 Conclusions and future research

In this paper, we presented a comprehensive review on the selecting techniques of direction for directional distance function. We particularly separate the techniques into two categories: the exogenous direction-selecting techniques, i.e., arbitrary direction and conditional direction-selecting techniques, and the endogenous direction-selecting techniques, i.e., theoretically optimized direction and market-oriented direction-selecting techniques. The theoretically optimized direction includes the directions towards closest benchmark and largest improvement potential, while the market-oriented direction involves the direction leads to cost minimization, profit maximization, or marginal profit maximization.

We pointed out that although the arbitrary direction-selecting technique is the most commonly used approach over the past two decades, it faces a serious criticism that the arbitrary direction may lack of reasonable explanations such as economic meaning, policy implication or theoretical basis. Through introducing some specific scenarios like policy implication, shadow price estimation, system improvement and comparability, the conditional direction-selecting technique presents a clear advantage over arbitrary direction-selecting technique. However, it is still economically and mathematically meaningless. To overcome this limitation, there have been a series of new techniques proposed recently. From a theoretical optimization point of view, the techniques, which consider the shortest or the further distance from an inefficient DMU to the production frontier, are more reasonable resulting in efficiency improvement potentials. In particular, the closest benchmark direction-selecting technique discusses the most easily attainable and relevant benchmark information, whereas the largest improvement potential technique can measure the furthest distance between the inefficient DMU and the production frontier. Despite these two techniques can reach a theoretical optimized state, they are difficult to be realized in a real setting. Thus, from a practical point of view, some techniques are developed to estimate efficiency with price information and enterprise behaviors taken into account. Cost minimization direction-selecting technique illustrates a framework for cost production frontier that takes account for observed information in input prices. It should be helpful for planning production scale and making investment decisions; but it ignores the production goals. In addition, the technique, which is to seek the direction toward profit maximization, attempts to present a long-term target and provide a profit efficiency measure. There are two situations in this technique: the input and output prices are exogenous and endogenous. The former is in the case of competitive market, while the latter is in the case of imperfectly competitive market. However, a disadvantage of the profit maximization direction-selecting technique is that the efficiency estimation results rely on sectional data so as to fails connecting to business practice. In contrast, marginal profit maximization direction-selecting technique pursues a short-term target and emphasizes the step-by-step adjustment of resource, and its purpose is productivity improvement rather than efficiency measurement.

In this review, we present the development of direction-selecting techniques of directional distance function in a non-parametric framework. For the future research, we suggest, first, more efforts should be put forward to address the remaining issues such as how to appropriately introduce enterprise behaviors and policy implications in the development of DDF from the perspective of both short-term and long-term economics. Second, more discussions should be conducted in the estimation of shadow prices for unintended outputs within the DDF framework, since the market prices of unintended outputs are usually not directly available but very useful for policy makers enterprises.

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