Spectral Risk for Digital Assets

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Abstract

Digital assets (DAs) are a unique asset class that presents investors with opportunities and risks that are contingent upon their particular characteristics such as volatility, type, and profile, among other factors. Among DAs, cryptocurrencies (CCs) have emerged as the most liquid asset class, holding this distinction for almost a decade. However, while CCs offer a high level of liquidity, investors must be aware of the potential risks and rewards associated with investing in this asset class, and should conduct a thorough evaluation before making any investment decisions. Our study examines the risk profile of CCs through portfolio analysis, utilizing Spectral Risk Measures (SRMs) as the commonly applied method. In this study, we investigate the application of SRMs in assessing the risk structure of CC portfolios, and their alignment with investors' risk preferences. We employ SRMs to evaluate the CC index CRIX and portfolios constructed from the most liquid 10 CCs from the Blockchain Research Center (BRC), optimizing different SRMs.Our empirical findings suggest that various optimal portfolio allocations can be formulated to meet the unique risk appetites of individual investors. All Quantlets (macros, code snippets) are available via quantlet.com and instructive educational element are available on quantinar.com.

Keywords: Spectral Risk Measure, Value-at-Risk, Expected Shortfall, Digital Assets, CRIX, Portfolio
JEL classification: G11, G32

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1. Introduction

Cryptocurrencies (CCs) have gained prominence among Digital Assets (DAs) as the most liquid asset class, maintaining this position for nearly ten years (Trimborn and Härdle, 2018). Additionally, CCs have become an increasingly appealing asset class for institutional asset managers and private investors alike in recent years. As more investors add CCs to their portfolios, it has become crucial to understand the potential downside risks associated with this asset class. To address this need, this study employs portfolio selection techniques that minimize quantile-based risk measures, in response to the rapid growth and evolution of the CC market.

The focus of this study is a broad class of quantile-based risk measures that can be expressed as a weighted average of quantiles from the return probability distribution. These measures, known as spectral risk measures (SRMs), are capable of capturing both value-at-risk (VaR) and expected shortfall (ES) as special cases. This approach is based on previous work by Acerbi (2002, 2004), and allows for a comprehensive evaluation of the risk structure of investments. A special case for a probability level of α is $VaR_{1-\alpha}$ the quantile of the P&L (Profit&Loss) that employs a degenerate weighting function: the Dirac delta. By contrast, $ES_{1-\alpha}$ is another SRM that uses a simple step weighting function equal to $1/\alpha$ for all tail quantiles and 0 for all others. VaR has been extensively studied in financial literature and is widely used as a risk management tool in the industry (Weiß, 2013; Mögel and Auer, 2018; Simlai, 2021). Despite its popularity, VaR has been found to have several limitations, with the most significant being that, as per the terminology introduced by Artzner (1997), it is not a coherent measure of risk due to its general lack of subadditivity (Mögel and Auer, 2018). It is worth noting that only a particular subset of risk measures satisfy the axiomatic conditions necessary for a proper risk measure.

The advantage of SRM is that conveniently fits investors' preferences. Examples are exponential and power utilities, as introduced by Dowd and Blake (2006) that link SRMs with investors preferences. However, these models have been found to possess certain properties that can pose challenges when applied to practical risk management. Despite these limitations, it has been demonstrated that the exponential utility function may be a viable option under certain circumstances, as originally proposed by Bühlmann (1980). However, the choice of utility function and the determination of corresponding risk aversion profiles depend on the specific asset at hand.

In this study we concentrate on portfolio allocation of CCs, whereby the weights are determined through the minimization of SRM. Through the exploration of various parameters, this research investigates the characteristics of the portfolios that result from different parameter values. Additionally, the study provides an overview of SRMs and examines their relationship to investors' preferences. To ensure the accuracy and reliability of our results, we adopt a rolling window methodology with rebalancing intervals of 30 days on a monthly basis, which is aligned with the rebalancing frequency of the CRIX index. Additionally, we conduct the same analysis using longer intervals of 90 days on a quarterly basis, as well as shorter intervals of 14 days on a bi-weekly basis, to further strengthen the robustness of our findings. To evaluate the performance of each portfolio, we utilize a range of out-of-sample performance metrics, including Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), Standard Deviation (SD), Sharpe Ratio (SR), and Calmar Ratio (CR). We compare the outcomes of the portfolios based on these measures.

This research paper offers three main contributions to the existing financial literature on risk management. Initially, we propose an approach to determine the optimal weights for a portfolio of CCs by minimizing quantile-based risk measures. To the best of our knowledge, these have not been previously addressed in the published literature. We emphasize the importance of aligning the chosen risk measure with the risk preference of the investor, and suggest that SRMs are particularly suited for this purpose. SRMs incorporate the investor's level of risk aversion and enable practitioners to construct portfolio allocations that cater to the varying degrees of risk aversion among investors. Our empirical findings show that portfolios of risk-averse investors have lower associated risk compared to less risk-averse investors, with risk lovers experiencing considerably higher rewards associated with Fung (2013). As such, various optimal portfolio allocations can be determined to suit the distinctive risk appetites of individual investors.

Secondly, CCs have established themselves as the most liquid asset class among

DAs, maintaining this position for almost a decade. Despite the high liquidity that CCs provide, investors should exercise caution and carefully consider the potential risks and rewards before making any investment decisions. Conducting a comprehensive evaluation is essential in this regard. Thirdly, we examine the variations in the weightings of each CC concerning the degree of risk aversion of each portfolio. Our analysis indicates that as investors become more risk-averse, the weight of Bitcoin dominates over the other CCs. This can be attributed to the fact that the standard deviation of Bitcoin's value is comparatively lower than that of other CCs, as illustrated in Table 2. However, the power utility function with a risk aversion parameter $0 < \gamma < 1$ behaves differently from other risk measures, leading to questions about its practical usefulness. Our study's results offer valuable insights into the characteristics of these risk measures.

The empirical evidence of this study can be summarized as follows. For one thing, when attempting to estimate the total value of portfolios, as we observe, during the pandemic the returns of portfolio experiencing higher volatility reach more extreme values. Additionally, after analyzing the performance of Minimum Exponential SRM portfolios across a range of parameters (from 1 to 25), it is found that the portfolios with a parameter value of $k = 1$ (representing the least risk-averse portfolio) demonstrated higher volatility in terms of their cumulative wealth. Conversely, the portfolio with a parameter value of $k = 25$ (representing the most risk-averse portfolio) exhibits the best performance. What is more, power SRM is estimated by considering two situations, $0 < \gamma < 1$ and $\gamma > 1$. Based on our analysis, it can be inferred that as the SRM values estimating from these two situations decreases, the corresponding Turnover (TO) also decreases. This implies that a strategy with lower turnover is less susceptible to the impact of transaction costs and is more likely to achieve better performance. One final remark, we proceed to compare the portfolios using the selected SRMs in Section 5.1, both with each other and with the benchmarks. These benchmarks include the popular Cryptocurrency Index (CRIX), the Minimum Variance (MinVar) portfolio, and the Equally Weighted strategy, also known as the Naive portfolio. Our analysis reveals that the portfolio that minimizes the Power SRM with a parameter of $\gamma = 0.1$ outperforms the other portfolios significantly.

The paper is structured as follows. Section 2 we have the literature review. In section 3, we discuss the methodology and the construction of the optimization portfolio model. Section 4, we present our dataset of DA. Section 5 contains the details of empirical analysis. Section 6 concludes.

2. Literature Review

The theory of spectral risk measures, introduced by Acerbi (2002), has emerged as a captivating and highly promising development in the field of financial risk. These measures are closely linked to the coherent risk measures proposed by Artzner (1997); Artzner et al. (1999). Prior research has primarily concentrated on estimating the VaR risk measure at high confidence levels. Nonetheless, VaR has faced significant criticism as a risk measure due to its failure to satisfy coherence properties, particularly its lack of subadditivity (Artzner et al., 1999; Acerbi, 2004). Subadditivity is based on the intuitive idea that individual risks tend to diversify, or at the very least, not increase when we combine risky positions. When VaR fails to satisfy subadditivity, it can result in unexpected and unfavorable consequences. ES has emerged as the most prevalent alternative (Acerbi and Tasche, 2002; Rockafellar and Uryasev, 2002) to VaR. ES has been extended to the class of SRMs, see e.g. SFM course.

A notable advantage of SRMs is their ability to connect the risk measure to the risk-aversion of the user. Essentially, the SRM represents a weighted average of the loss distribution's quantiles, with the weights determined by the user's level of risk-aversion. Consequently, SRMs allow us to link the risk measure to the user's risk preference, implying that a more risk-averse user should face a higher level of risk, all other factors being equal, as determined by the SRM value. SRMs have found widespread use in various domains, such as determining capital requirements or achieving optimal trade-offs between risk and expected returns. Overbeck (2004) suggests that they could be useful for capital allocation, while Cotter and Dowd (2006) propose that SRMs could assist futures clearinghouses in setting margin requirements that align with their corporate risk preference. In addition to its application to Artzner et al. (1999) original framework of bank regulation, SRMs have been applied in portfolio selection (Adam et al., 2008; Brandtner, 2013) and optimal insurance contracts (Cai et al., 2008), among others. While this expanded perspective has broadened the scope of SRM application, it has also raised new questions. Adam et al. (2008) explore risk measurement and portfolio optimization under risk constraints using various risk measures, applied to a hedge fund asset management framework, and shows the robustness of optimal portfolios to the choice of risk measure.

The focus of this study is on exploring alternative SRMs that are based on different underlying utility functions. Specifically, the study investigates exponential SRMs that rely on an exponential utility function, which are equivalent to those previously studied by Acerbi, and power SRMs that rely on a power utility function. Even though they reveal that SRM has some properties which cause problems when applying to practical risk management, they show that exponential utility function might be plausible in some circumstances under weak conditions Bühlmann (1980). If the relative risk aversion coefficients (RRA) are less than 1, Dowd and Blake (2006) address that the weighting of lower risk-averse is higher than the higher risk-averse as the loss of portfolio increases. On the other hand, the power SRM proposed by Dowd and Blake (2006) when the relative risk aversion coefficients (RRA) are larger than 1, has also proper features to give a higher weight as loss increase. It is important to note that the choice of utility function and the determination of the risk aversion parameter will depend on the specific financial problem being addressed.

In this study, SRMs are utilized for the purpose of CCs portfolio asset allocation. The optimal weights of CCs are determined by minimizing the SRMs. Specifically, this paper provides estimations of VaR, ES, exponential SRM, and power SRM, for each of the CCs' positions. The different estimates are compared with one another, and alternative methods of estimating precision are examined and compared.

3. Methodology

3.1. Spectral Risk Measures

The Spectral Risk Measure (SRM) is a quantile-based risk measure, like the VaR but with the difference that it involves a weighting function. This weighting function $\omega(p)$ can be individually characterized by investors more general depending on the investor's risk aversion. More precisely the SRM is given by

$$M_\omega(X) = -\int_0^1 \omega(p) F^{-1}(p) dp \tag{1}$$

hence a weighted average of the quantile function. If $\omega(p)$ is the Dirac delta function which gives the outcome at say $p = 0.95$ an infinite weight and gives every other outcome a weight of zero then the SRM boils down to the VaR at the level of 95 % since it simply equals $F^{-1}(0.95)$, where F is a cdf of P&L.

The characteristics of $\omega(p)$ determine the usability and the properties of SRM in practice. A desired property of any risk measure is coherence, simply the property that combining portfolios into a larger portfolio should result in smaller risk. According to Acerbi (2003) an SRM is coherent if it satisfies the properties of non-negativity, increasingness and normalisation. The last condition ensures that the weighting function rises with p, that is, higher losses are assigned with weights that should be no less than the weights assigned to lower losses.

P1. Non-negativity: $$\omega(p) \geq 0 \tag{2a}$$

P2. Increasingness: $$\omega'(p) \geq 0 \tag{2b}$$

P3. Normalisation: $$\int_0^1 \omega(p) dp = 1 \tag{2c}$$

The first property requires that the weights are non-negative, and the second property is intended to reflect user risk aversion. The third one requires that the probability-weighted weights should sum to 1.

Note that VaR_α and ES_α are included in spectral risk measures as special cases. The weighting function of VaR_α is a Dirac delta function which gives the outcome an infinite weight and the others a zero weight. On the other hand, the ES_α gives all tail quantiles exceeding VaR_α the same weight. Both of them are not a suitable weight function for capturing investors' risk attitudes. By setting a 'well-behaved' risk-aversion function which indicates the weights will rise more rapidly when the degree of risk aversion is higher (Dowd et al., 2008), we investigate the behaviors of the users in terms of different weight functions.

3.1.1. Value-at-Risk

Mathematically, the weight function for in terms of SRM can be expressed as follows:

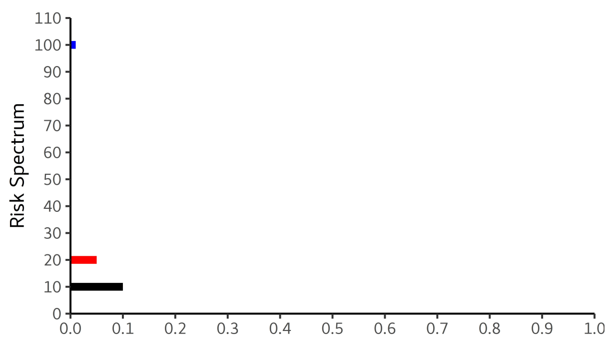
$$\omega_{VaR_\alpha}(p) = \lim_{\delta \longrightarrow 0} \delta^{-1}\, 1(\alpha + \delta > p > \alpha). \tag{3}$$

3.1.2. Expected Shortfall

The weighting function for ES gives the same weight of $(1-\alpha)^{-1}$ to all tail quantiles and gives zero to non-tail quantiles. Accordingly, the weight function for ES_α in terms of SRM can be written as (see Figure 1):

$$\omega_{ES_\alpha}(p) = (1-\alpha)^{-1}\, 1(p > \alpha). \tag{4}$$

Figure 1: Expected Shortfall Risk Spectrum for different α



Notes: ES Risk Spectrum for $\alpha = 10\%$, $\alpha = 5\%$ and $\alpha = 1\%$.

3.1.3. Exponential Risk Measure

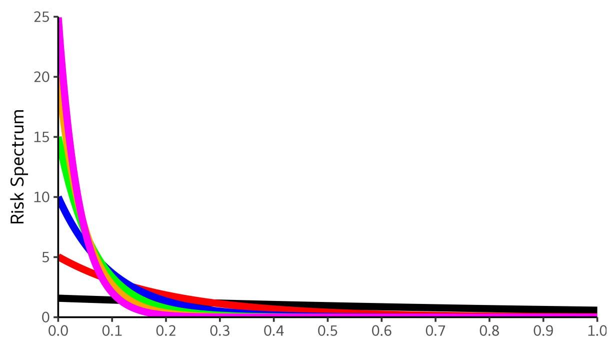
The exponential SRM is specified by only one risk parameter k. To obtain the risk spectrum, we set $\omega(p) = \lambda e^{-kp}$ and $\lambda = \frac{k}{1-e^{-k}}$. Then, the risk spectrum is (see Figure 2)

$$\omega(p) = \frac{ke^{-kp}}{1-e^{-k}}, \tag{5}$$

where $k \in (0, \infty)$, $p \in [0, 1]$. This function depends on only one k. By substituting weight function into (1), the exponential SRM is

$$M_{\omega}(R_p) = -\int_0^1 \frac{ke^{-kp}}{1 - e^{-k}} F^{-1}(p)dp. \tag{6}$$

Figure 2: Exponential SRM Risk Spectrum for different k



Notes: Exponential SRM Risk Spectrum for $k = 1$, $k = 5$, $k = 10$, $k = 15$, $k = 20$ and $k = 25$.

3.1.4. *Power Risk Measure*

The power weighting function only has one parameter γ, which leads to

$$\omega(p) = \frac{\lambda(1 - p)^{\gamma - 1}}{1 - \gamma} \tag{7}$$

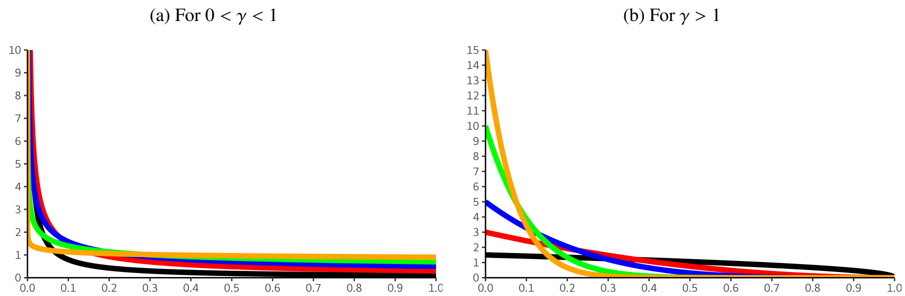
as $\gamma > 1$. Then, by setting $\lambda = \gamma(1 - \gamma)$, the risk spectrum is (see Figure 3)

$$\omega(p) = \gamma(1 - p)^{\gamma - 1}. \tag{8}$$

Plugging the weighting function to (1), the power SRM is obtained

$$M_{\omega}(X) = -\int_0^1 \gamma(1 - p)^{\gamma - 1} F^{-1}(p)dp. \tag{9}$$

Figure 3: Power SRM Risk Spectrum for different γ



Notes: Power SRM Risk Spectrum for (a) $0 < \gamma < 1$ are $\gamma = 0.1$, $\gamma = 0.3$, $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$. Power SRM Risk Spectrum for (b) $\gamma > 1$ are $\gamma = 1.5$, $\gamma = 3$, $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$.

For the case of $0 < \gamma < 1$, we have

$$\omega(p) = \frac{\lambda p^{\gamma-1}}{1-\gamma} \tag{10}$$

with $\lambda = \gamma(1-\gamma)$. The risk spectrum is written as

$$\omega(p) = \gamma p^{\gamma-1}. \tag{11}$$

The power SRM for $\gamma > 1$ then becomes

$$M_\omega(X) = -\int_0^1 \gamma p^{\gamma-1} F^{-1}(p)dp. \tag{12}$$

3.2. Coherent Risk Measures

SRMs have originally been designed to fulfill a set of six axioms. It is noteworthy to state that the axioms of translation-invariance and positive homogeneity together induce the following linearity property. A risk measure satisfies the following properties (H1)-(H4) (Artzner et al., 1999; Bellini and Bignozzi, 2015) is coherent.

(H1) Positive homogeneity: A risk measure is positive homogeneous, if the risk of a portfolio is scalable by the size (defined by λ) of the portfolio.

$$\rho(\lambda X) = \lambda\rho(X), \lambda > 0 \tag{13}$$

(H2) Translation-invariance: A risk measure is translation invariant, if the risk of the portfolio is reduced by exactly the same amount λ when adding this risk-free asset into the portfolio.

$$\rho(X + \lambda) = \rho(X) - \lambda \tag{14}$$

(H3) Monotonicity: A risk measure is monotonic, if a portfolio almost surely generates a higher return than another portfolio, the portfolio with higher return should have less riskiness embedded in it.

$$X \geq Y, \rho(X) \leq \rho(Y) \tag{15}$$

(H4) Sub-additivity: A risk measure is sub-additive, if the risk of one large portfolio comprised of multiple securities is no larger than the sum of the risks of the single positions.

$$\rho(X + Y) \leq \rho(X) + \rho(Y) \tag{16}$$

In addition to these four properties mentioned above Alexandre et al. (2008) also list two other properties which are desired for risk measures in practice. (H5) Law-invariance: A risk measure is law invariant, if the two risk positions X and Y have the same return loss distributions, they will also have the same risk measure value.

$$F_X = F_Y, \rho(X) = \rho(Y) \tag{17}$$

(H6) Comonotonic additivity: A risk measure is comonotonic additive, if the loss distributions of two risk positions are comonotonic, combining these two risk positions will not result in any diversification effects as their losses are influenced by some common risk factor(s).

$$\rho(X + Y) = \rho(X) + \rho(Y) \tag{18}$$

Table 1 gives us a summary of these three risk measures in terms of six axioms.

In addition, it is worth noting that Value-at-Risk (VaR) does not fulfill the property of subadditivity, which means that diversification may not effectively decrease the risk as measured by VaR. This is not an ideal situation. The following example demonstrates how VaR violates subadditivity. Let X and Y be *i.i.d* and $X, Y \in \{0, 100\}$, then

Table 1: Comparison of different risk measures in terms of 6 axioms

	H1	H2	H3	H4	H5	H6
VaR	✓	✓	✓		✓	
ES	✓	✓	✓	✓	✓	✓
SRM	✓	✓	✓	✓	✓	✓

the discrete random variable X or Y has the following probability distribution:

$$P(X = 0) = P(Y = 0) = p; P(X = 100) = P(Y = 100) = 1 - p. \tag{19}$$

For $p = 0.96$, $\alpha = 0.95$, it holds that $VaR_\alpha(X) = VaR_\alpha(Y) = 0$. As the cdf of discrete random variable X (in an analogous manner for random variable Y) is given by:

$$F_X = \begin{cases} 0, X = 0; \\ p, X = 100; \\ 1, X = 100. \end{cases} \tag{20}$$

So when $\alpha = 0.95$, the $VaR_{0.95}(X)$ or $VaR_{0.95}(Y)$ which is $F^{-1}(0.95)$ is still equal to zero, but jump to 100 when $p = 0.96$ as $VaR_{0.96}(X) = VaR_{0.96}(Y) = 100$. Meanwhile, assume we have 3 states of $X + Y$:

State 1: $$P(X + Y = 0) = p^2 \tag{21a}$$

State 2: $$P(X + Y = 100) = 2p(1 - p) \tag{21b}$$

State 3: $$P(X + Y = 200) = (1 - p)^2. \tag{21c}$$

The cdf of is:

$$F_{X+Y} = \begin{cases} 0, X + Y = 0; \\ p^2, X + Y = 100; \\ p^2 + 2p(1 - p), X + Y = 200; \\ 1, X + Y = 200. \end{cases} \tag{22}$$

The cdf of $X + Y$ jumps from 0 to 100 at $p = 0.9216$, followed by jump to 200 at

$p = 0.9984$. Both $p = 0.96$ and $\alpha = 0.95$ are smaller than 0.9984, so the $VaR_{0.95}(X + Y) = 100$. In this case, we have the following result,

$$\text{VaR}_\alpha(X + Y) = 100 > \text{VaR}_\alpha(X) + \text{VaR}_\alpha(Y) = 0 + 0 = 0. \tag{23}$$

VaR_α therefore fails to satisfy the sub-additivity property, and it can lead to unexpected and unfavourable consequences. Since coherent risk measures are discussed intensively in finance and risk management, the recent questions have arisen are more coherent asset allocation (Kroll et al., 2021).

3.3. Portfolio Selection Model

This study examines the allocation of cryptocurrencies by minimising the risk measures, i.e. VaR, ES, exponential SRM and power SRM. The return R_p on a portfolio is given by

$$R_p = \sum_{i=1}^{n} w_i * r_i, \tag{24}$$

where r_i and w_i are the return and weight of constituent CC i, respectively. The problem consists in determining the weights, w_i, by minimising a quantile risk measure for a given confidence level $1 - p$, where p is the cumulative probability. The quantile-based risk measures here serve as risk minimisation objectives which we consider different quantile-based risk measures, $M_\omega(R_p)$,

$$\min \qquad M_\omega(R_p), \tag{25}$$

$$s.t. \qquad \sum_{i=1}^{n} w_i = 1, \tag{26}$$

$$\sum_{i=1}^{n} w_i * r_i = R_p \geq R_{exp}, \tag{27}$$

$$0 \leq w_i \leq 1, \; i = 1, \cdots, n. \tag{28}$$

where R_{exp} indicates the required premium. In our case, the required premium is set to the return of CRIX.

3.4. Performance Measures

The results of the portfolios are compared using the out-of-sample performance measures for each portfolio (Phengpis and Swanson, 2011), which we have chosen as well as Petukhina et al. (2023):

Turnover (TO) is calculated to evaluate the potential trading costs associated with portfolio rebalancing. This measure is defined following DeMiguel et al. (2009) and DeMiguel and Nogales (2009), as the average sum of the absolute value of the rebalancing trades across the N assets of the investment universe and over L rebalancing months.

$$TO = L^{-1} \sum_{l=1}^{L} \sum_{j=1}^{N} |\hat{w}_{j,l+1} - \hat{w}_{j,l+}| \tag{29}$$

where $\hat{w}_{j,l+1}$ is the weight assigned to the asset j for the rebalancing period $l + 1$ and $\hat{w}_{j,l+}$ denotes the weight just before rebalancing at $l + 1$.

To explain further, TO considers the weights assigned to each asset during rebalancing periods and just before rebalancing at time l. This accounts for price changes during the period, as trades need to be executed to rebalance the portfolio towards the target. High turnover results in significant transaction costs. Therefore, a strategy with a lower Turnover (TO) is less affected by non-zero transaction costs and performs better. While market makers do not consider reduced turnover a critical parameter in their decision-making process, we believe that buy-side investors could benefit from a lower turnover.

In addition, we also calculate a ***Target Turnover (TTO)***, following the approach of Petukhina et al. (2021).

$$TTO = L^{-1} \sum_{l=1}^{L} \sum_{j=1}^{N} |\hat{w}_{j,l+1} - \hat{w}_{j,l}| \tag{30}$$

where $\hat{w}_{j,l}$ is the weight assigned to the asset j for the rebalancing period l. This measure is constructed to focus on modifications of the target portfolio weights due to active management decisions, and is cleaned from the influence of assets' price dynamics. Unlike the previous definition of turnover, this target turnover measure is designed to have a value of zero for the equally weighted (EW) portfolio.

The Cumulative Wealth (CW) achieved by each benchmark strategy with an initial investment $CW_0 = 1$ is calculated as follows Petukhina et al. (2021):

$$CW_{l+1} = CW_l + \hat{w}_l^T r_{l+1}. \tag{31}$$

Standard deviation (SD), we calculate the standard deviation of the out-of-sample daily returns series.

Sharpe Ratio (SR) is a measure of risk-adjusted returns. This version of the ratio takes account of the difference in returns and the associated risk measured by Standard deviation SD, and therefore it can be very useful for performance evaluation Leng and Noronha (2019). The Sharpe ratio is given as

$$SR = \frac{\overline{R_p}}{SD} \tag{32}$$

where $\overline{R_p}$ is an average out-of-sample return of the portfolio and SD is standard deviation.

The Calmar Ratio (CR) is a metric for risk-adjusted return that is popular among practitioners because of its unique focus on maximum drawdown instead of volatility, as in the Sharpe Ratio Petukhina et al. (2021). CR is calculated as follows:

$$CR = \frac{365 \times \overline{R_p}}{MD_p} \tag{33}$$

where $\overline{R_p}$ is the average out-of sample daily return multiplied by 365 to annualized, as cryptocurrencies are traded every day and MD_p is maximum drawdown.

4. Data

Data on digital assets are abundant, but here we concentrate on crypto currencies, since they are the most popular DA as of today. More DA data can be found on the BRC Blockchain-Research-center.com where one can download e.g. NFTs for digital art pieces or high frequency order books and options for the most dominant CCs. The CRIX, the most well known CC Index can be also found on SP Global. CRIX is calculated via a Laspeyre mechanism but with varying number of constituents and a slick model choice procedure based on a modified AIC technique, called the CTDC.

Our dataset consists of the daily values of the CRIX index since its beginning, i.e., from March 16, 2018 to March 19, 2023. This data will be used for the initial analysis of spectral risk measures, and we will use the average daily prices of the ten most liquid cryptocurrencies available on the BRC in the subsequent construction of the portfolios. Specifically, we used the following cryptocurrencies: Bitcoin (BTC), Litecoin (LTC), Ripple (XRP), Dogecoin (DOGE), Dash (DASH), Ethereum (ETH), Ethereum Classic (ETC), Zcash (ZEC), Bitcoin Cash (BCH), Bitcoin SV (BSV). The dataset for these cryptocurrencies is from 10 November 2018, which is the data availability date for all selected cryptocurrencies on BRC until 19 March 2023.

Out of the cryptocurrencies selected by us, four of them (BTC, ETH, XRP, DOGE) are also part of the CRIX index, which currently (March 2023) consists of a total of six cryptocurrencies, including Cardano and Binance Coin in addition to the previously mentioned ones.

Table 2: Descriptive statistics of chosen crypto currencies and CRIX

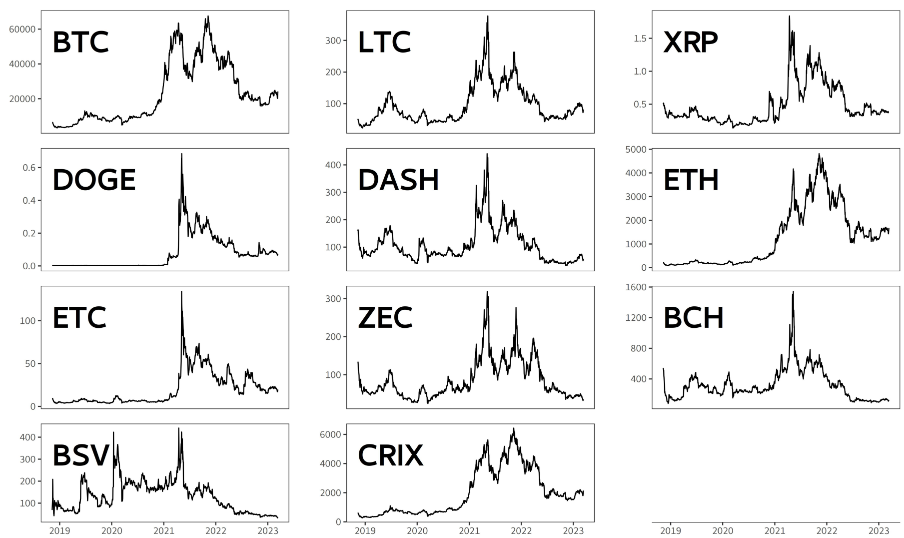
	Mean	S.D.	Min.	Median	Max.	$\rho(1)$	$\rho(7)$
BTC	0.0016	0.0372	-0.3717	0.0009	0.1875	-0.0476	-0.0273
LTC	0.0015	0.0512	-0.3618	0.0004	0.3083	-0.0413	-0.0276
XRP	0.0014	0.0565	-0.4233	-0.0005	0.5601	-0.0204	0.0200
DOGE	0.0051	0.1130	-0.4026	-0.0011	3.5557	0.0507	0.0441
DASH	0.0009	0.0576	-0.3722	0.0007	0.5704	-0.0349	-0.0226
ETH	0.0025	0.0482	-0.4235	0.0011	0.2595	-0.0631	-0.0247
ETC	0.0022	0.0604	-0.3973	0.0004	0.4226	0.0235	-0.0224
ZEC	0.0009	0.0570	-0.4169	-0.0001	0.2876	-0.0663	-0.0329
BCH	0.0008	0.0582	-0.4296	-0.0004	0.5232	-0.0274	-0.0442
BSV	0.0023	0.0790	-0.4640	-0.0011	1.4250	-0.0540	-0.0584
CRIX	0.0016	0.0446	-0.2386	0.0013	0.2085	-0.0039	-0.0207

Notes: S.D. stands for standard deviation, $\rho(1)$ represents the first-order (day) autocorrelation coefficient and $\rho(7)$ denotes the seven-order (week) autocorrelation.

Table 2 presents descriptive statistics of the daily return series for our specific assets. The majority of cryptocurrencies exhibit an average daily return of around 0.1%, with DOGE having the highest average daily return at approximately 0.5%. DOGE also has the highest volatility, as measured by SD, followed by BSV, while BTC with the CRIX index exhibits the lowest volatility.

Upon examining Figure 4, it is evident that most of CCs have been a sudden and significant surge in the data for the begging of the year 2021. It's understandable that during that period, in order to minimize the impact of the COVID-19 pandemic on the economy, the government and central bank injected a massive amount of liquidity into the economy. However, between 2021 and the middle of 2022, the CC market experienced increased volatility and instability, likely attributed to factors such as inflation, wars, and global economic recessions.

Figure 4: Price of CCs and value of CRIX from November 09, 2018 to March 19, 2023



Notes:

5. Empirical results

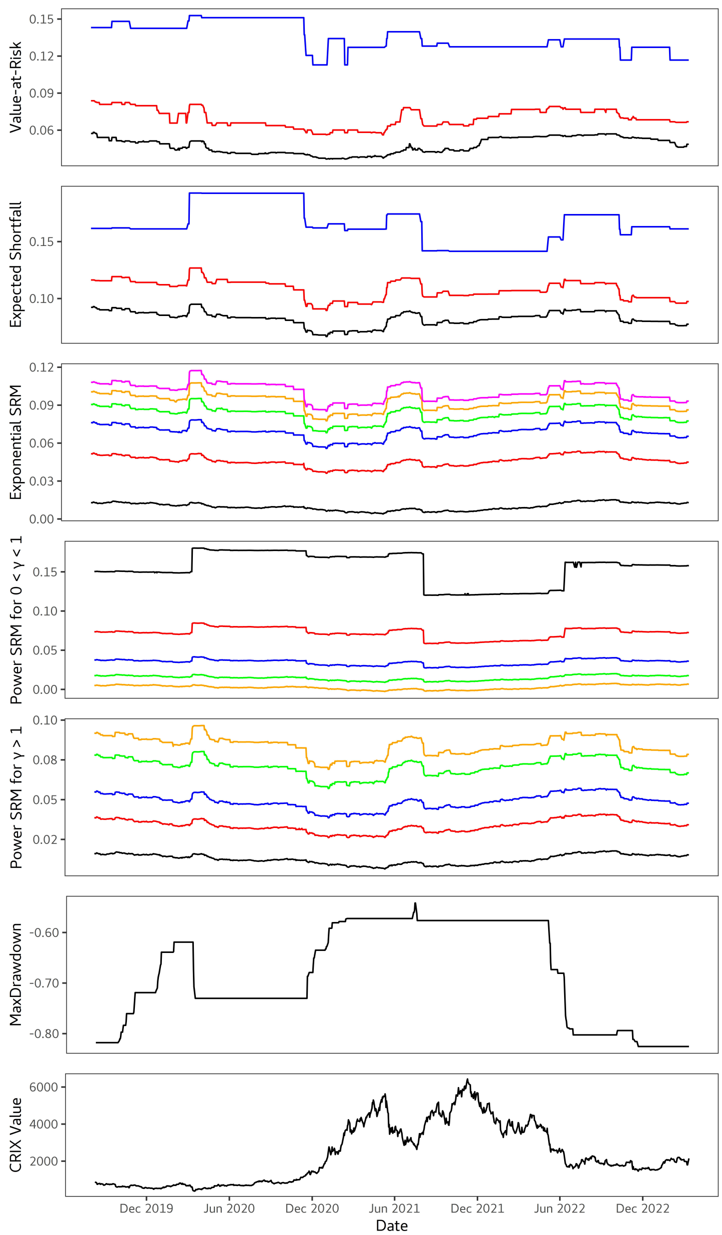
To begin this section, we look at how different SRMs react to movements in the CCs market, which is represented by the CRIX index. Then, in the rest of this section, we examine portfolios that minimise the different SRMs.

Figure 5 shows the daily evolution of the different SRMs and the maximum drawdown using an estimation window of 365 days. The last sub-graph shows the evolution of the value of the CRIX index and therefore we can see the response of the different SRMs to a change in the value of CRIX. From the figure of VaR evolution, we can see that this measure increases as α decreases, while it reacts more sharply to changes in the evolution of the CRIX index. Observing the sub-figure showing the evolution of ES over time for different parameters $alpha$, it can be concluded that this evolution is similar to the one in the sub-figure with VaR.

The next sub-figure in Figure 5 shows the evolution of the Exponential SRM, from which we can observe that as the parameter k increases, the value of this SRM increases and also responds more steeply to changes in the value of the CRIX index. This observation is consistent with Figure 2, where we see that as the parameter k increases, lower values (higher risk) are given a higher weight.

In Figure 5, we present two sub-figures that depict the power SRM for different values of the parameter γ. The fourth sub-figure shows the evolution of the SRM for $\gamma \in (0, 1)$. It can be observed that as the value of γ increases, the value of the SRM decreases. Furthermore, the peaks and troughs in the SRM trajectory become smaller in magnitude as γ increases. These findings are consistent with Figure 3. The second sub-figure depicting the Power SRM corresponds to values of $\gamma > 1$. Here, we can observe opposite results compared to the previous case. As the parameter γ increases, the value of the Power SRM also increases. Moreover, we can observe more pronounced reactions to changes in the values of the CRIX index.

Figure 5: VaR, ES, Exponential SRM and Power SRM for different parameters, Maximum Drawdown and value of CRIX from November 09, 2018 to March 19, 2023



Notes: The first and sub-figure shows VaR and second ES of the CRIX for parameters α = (10%, 5%, 1%). The third sub-figure shows Exponential SRM for k = (1, 5, 10, 15, 20, 25). The fourth sub-figure shows Power SRM for γ = (0.1, 0.3, 0.5, 0.7, 0.9) and fifth for γ = (1.5, 3, 5, 10, 15). The sixth present maximum drawdown and the last CRIX value.

5.1. The spectral risk minimising portfolio models

In this section, we find portfolios that have a minimum SRM based on the model presented in section 3.3. In the portfolio construction, we use a rolling window approach with monthly (30-day) rebalancing as the CRIX index is rebalanced, which we also extend to quarterly (90-day) and bi-weekly (14-day) rebalancing for robustness of the results. Due to the high volatility of our chosen asset, cryptocurrencies, we opted for only a one-year (365-day) estimation window when we calculate the weights.

To compare our results, we chose two classical approaches of the benchmark portfolio theory and the best known crypto index CRIX. The first benchmark approach is the naive allocation strategy or equally weighted portfolio approach, which, according to DeMiguel et al. (2009), is difficult to outperform in practical applications. This approach is typically used as a benchmark for comparative analyses.The second principle we chose as a benchmark is the Markowitz (1952) Mean-Variance or more practically used Minimum-Variance (MinVar) portfolio. This model is a basic model of portfolio theory and is also often used as a benchmark in various studies.

The results of portfolios minimizing different spectral risk measures for different parameters will now be considered. Specifically, we used the Value at Risk, Expected Shortfall, Exponential, and Power SRM.

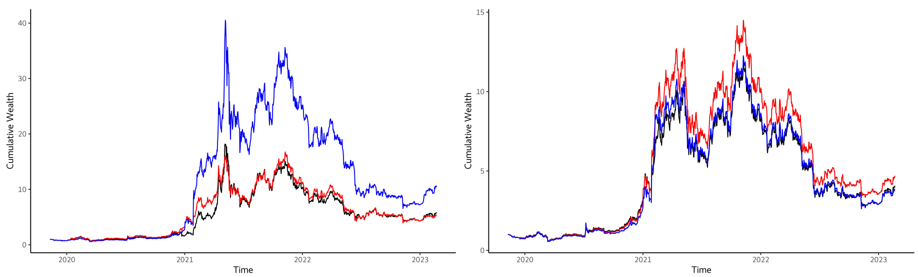
5.1.1. Minimum Value at Risk and Expected Shortfall portfolios for different α

We start the analysis with the spectral risk measures Value at Risk and Expected Shortfall, which were defined in Sections 3.1.1 and 3.1.2. The results of portfolios minimizing these SRMs using a rolling window of 1 month (30days) can be seen in Figure 6 and Table 3. As a robustness check we produce also the rolling window 14days and 90days, these results you can find in Appendix A.1 and Appendix A.2. In Figure 6 we can see the evolution of the cumulative wealth of Minimum ES and VaR portfolios for different parameter, specifically we used values of 1%, 5% and 10%.

When we look at the evolution of the cumulative wealth of the portfolios in Figure 6 we can see that all the portfolios except the Minimum VaR on the 1% aspect have a similar evolution. A given portfolio reaches more extreme values and is more volatile. We can also observe that all the VaR minimizing portfolios achieve better cumulative

Figure 6: Minimum VaR and ES portfolios for different α

(a) Minimum VaR portfolios (b) Minimum ES portfolios



Notes: Minimum VaR and ES portfolio for $\alpha = 10\%$, $\alpha = 5\%$ and $\alpha = 1\%$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing.

wealth values than the ES minimizing portfolios. Very similar results can also be found when rebalancing, every 14 days in Figure A.1 and 90 days in Figure A.2. with one exception, which is that the portfolio Minimizing VaR at $\alpha = 1\%$ does not dominate so strongly at the 14 days rolling window and if we look at the results at the 90 days rolling window this portfolio is even the worst, which implies that this portfolio is very volatile and unstable in results.

Table 3: Performance of Minimum VaR and ES portfolios

	TO	TTO	CW	SD	SR	CR
VaR $\alpha = 10\%$	24.5504	24.7571	5.7313	0.0481	0.0542	1.2291
VaR $\alpha = 5\%$	23.2028	22.2104	5.3099	0.0433	0.0537	1.1090
VaR $\alpha = 1\%$	16.0428	15.0262	**10.4464**	0.0605	**0.0561**	**1.4791**
ES $\alpha = 10\%$	**4.5276**	**3.6731**	3.9980	**0.0405**	0.0485	0.9308
ES $\alpha = 5\%$	4.9173	4.2890	4.6298	0.0438	0.0498	1.0345
ES $\alpha = 1\%$	6.6132	5.9901	3.8210	0.0456	0.0457	0.9665

Notes: Out-of-sample performance measures of minimum Var and ES portfolios using 10 most liquid CC from BRC with monthly (30 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

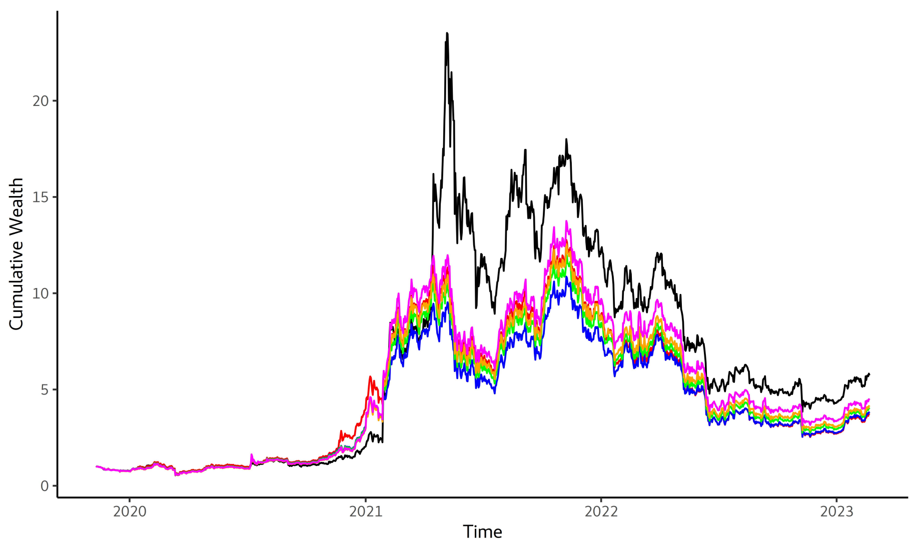
We can compare the performance of the individual portfolios in Table 3, which shows Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), Standard Deviation (SD), Sharpe Ratio and Calmar Ratio (CR). VaR minimizing portfolios have reached several times higher TO and TTO values, indicating that there are significant changes in positions when rebalancing, from this point of view, ES minimizing portfolios are more attractive. Similarly, portfolios minimizing ES perform a bit better in terms of risk measured by SD. In terms of SR and CR, VaR-minimising portfolios again perform better, with the best VaR portfolio on the level $\alpha = 1\%$ reaching 0.056 in SR and 1.479 in CR. Based on the results, we select the VaR-minimizing portfolio with the $\alpha = 5\%$ and the ES portfolio at the 5% level for the final comparison in Section 5.1.5. The $\alpha = 1\%$ VaR portfolio was not selected because, while it significantly outperformed the other portfolios based on the 30 day rolling window results, we can see that these results are not robust after reviewing the results in Appendix A.1 and Appendix A.2. Portfolio minimizing VaR at 5% achieved the second best SR. The ES portfolio with $\alpha = 5\%$ achieved the second lowest TO and TTO, and also achieved the highest CW, SR and CR from the minimum ES portfolios.

5.1.2. Minimum Exponential Spectral Risk Measure portfolios for different k

Another spectral risk measure for which we constructed portfolios is exponential. The exponential spectral risk measure was defined in Section 3.1.3. The outcomes of portfolios that minimize exponential SRM by using a rolling window of 1 month (30 days) could be found in Figure 7 and Table 4. Figure 7 displays the performance of Minimum Exponential SRM portfolios for different parameters , namely 1, 5, 10, 15, 20 and 25, by showing the cumulative wealth evolution. As a robustness check, we also generate results using rolling windows of 14 days and 90 days, which are presented in Appendix A.3 and Appendix A.4, respectively.

In Figure 7, we can observe that all portfolios perform very similarly, except for the portfolio with parameter $k = 1$, which performs above and is also more volatile. Interestingly, in terms of cumulative wealth, the portfolios with parameter $k = 1$, which by definition is the least risk averse, and portfolio $k = 25$, which in contrast is the most risk averse, perform the best. Very similar results can be found when rebalancing, every 14 days in Figure A.3 and 90 days in Figure A.4 with one exception, namely that the Minimizing Exponential SRM portfolio with $k = 1$ does not dominate as strongly in the 14-day rolling window and if we look at the results in the 90-day rolling window, this portfolio is not in the top two, indicating that it is more volatile and less stable

Figure 7: Minimum Exponential Spectral Risk Measure portfolios for different k



Notes: Minimum Exponential SRMs portfolio for $k = 1$, $k = 5$, $k = 10$, $k = 15$, $k = 20$ and $k = 25$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing.

Table 4: Performance of Minimum Exponential SRM portfolios

	TO	TTO	CW	SD	SR	CR
EXP $k = 1$	18.6888	19.3862	**5.7374**	0.0526	**0.0513**	**1.1784**
EXP $k = 5$	12.9574	12.1562	3.6903	**0.0389**	0.0476	0.8425
EXP $k = 10$	5.8037	5.1116	3.7737	0.0398	0.0476	0.9011
EXP $k = 15$	5.1046	4.3240	3.9872	0.0408	0.0482	0.9331
EXP $k = 20$	**4.5224**	**3.7088**	4.1032	0.0423	0.0482	0.9644
EXP $k = 25$	4.6920	3.8173	4.4551	0.0435	0.0492	1.0133

Notes: Out-of-sample performance measures of Minimum Exponential SRM portfolios using 10 most liquid CC from BRC with monthly (30 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

In Table 4, we can assess the effectiveness of each portfolio by examining various metrics such as Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), Standard Deviation (SD), Sharpe Ratio (SR) and Calmar Ratio (CR). According to Table 4, there is an inverse relationship between k and TO or TTO, where a increase in parameter k leads to a decrease in TO or TTO. Additionally, an increase in k typically results in an increase in CW, with the exception of the case where $k = 1$. When measuring risk using SD, the results for most parameters of k are similar, except for the portfolio where $k = 1$. Interestingly, the portfolio with an exponential SRM and a parameter of $k = 1$ showed the best SR and CR, while higher values of k generally led to an increase in the ratio, except the case $k = 1$.

Based on results in Figure 7 and Table 4, we select the portfolio minimising Exponential SRM portfolio for parameter $k = 25$ for final comparison in Section 5.1.5. Our decision to select this particular portfolio was based on several factors. Firstly, the cumulative wealth of this portfolio was found to be the second highest, after the portfolio with parameter $k = 1$ which is less stable as we show in Appendix A.3 and Appendix A.4. Additionally, the Exponential SRM portfolio with parameter $k = 25$ demonstrated less risk based on SD, and exhibited a more favorable TO and TTO.

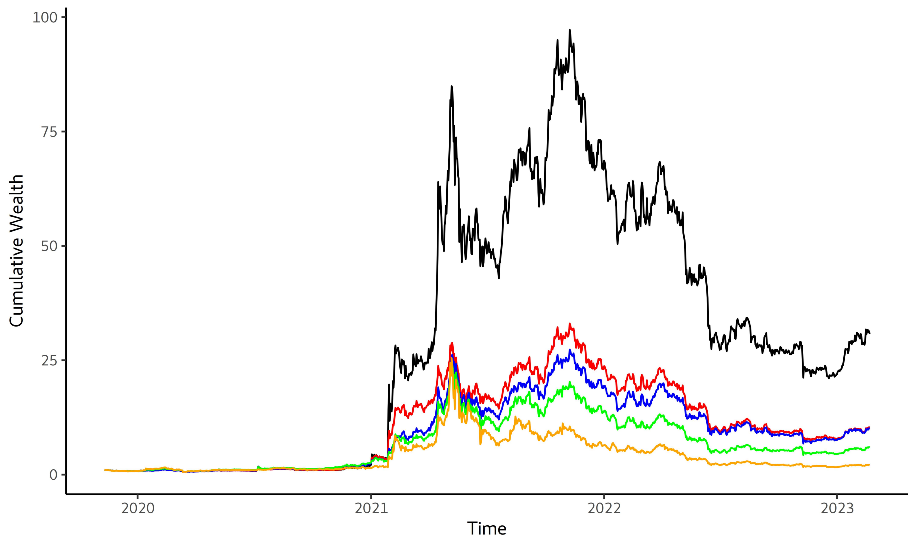
5.1.3. Minimum Power Spectral Risk Measure portfolios for different $0 < \gamma < 1$

In Section 3.1.4, we introduced the power spectral risk measure. In this section, we present the results of constructing portfolios that minimize this risk measure. Specifically, we consider the power spectral risk measure and use a rolling window approach of 30 days (1 month) and the minimum power spectral risk portfolios are computed from the previous 365 days (1 year) of data. We report the results in Table 5 and display the cumulative wealth evolution in Figure 8 for various values of the parameter $0 < \gamma < 1$ $(0.1, 0.3, 0.5, 0.7, 0.9)$. As a robustness check, we have also computed results using rolling windows of 14 days and 90 days, which can be found in Appendix A.5 and Appendix A.6, respectively.

In Figure 8 we can observe that again several portfolios evolve similarly except for one, which in this case is the portfolio with parameter $\gamma = 0.1$. Further, we can observe from the figure that as the parameter γ increases, our cumulative wealth decreases. Very similar results also found when rebalancing, every 14 days in Figure A.5 and 90 days in Figure A.6 with one exception, namely that the Minimizing Power SRM portfolio with $\gamma = 0.1$ does not dominate so strongly in the 14-day rolling window respectively. it is beaten by the portfolio with $\gamma = 0.3$.

Table 5 provides an evaluation of the effectiveness of each portfolio based on multiple metrics, including Turnover (TO), Target Turnover (TTO), Cumulative Wealth

Figure 8: Minimum Power Spectral Risk Measure portfolios for different $0 < \gamma < 1$



Notes: Minimum Power SRMs portfolio for $\gamma = 0.1$, $\gamma = 0.3$, $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing.

Table 5: Performance of Minimum Power SRM portfolios with $\gamma < 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 0.1$	**8.3784**	**7.0337**	**30.7839**	0.1175	0.0515	**2.8202**
PWR $\gamma = 0.3$	15.3953	14.2989	10.2969	0.0644	0.0540	1.6223
PWR $\gamma = 0.5$	17.9112	17.4035	10.0713	0.0503	**0.0611**	1.4917
PWR $\gamma = 0.7$	17.7675	18.8776	5.9595	**0.0492**	0.0538	1.1722
PWR $\gamma = 0.9$	23.3522	25.9639	2.1081	0.0666	0.0389	1.0107

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma < 1$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

(CW), Standard deviation (SD), Sharpe Ratio (SR) and Calmar Ratio (CR). The data presented in Table 5 indicates that as the parameter γ increases, there is a corresponding increase in TO and TTO. We can also see from Table 5 and Figure 8 that as our parameter γ increases, our cumulative wealth (CW) decreases. In terms of risk measured by SD, we can see that the risk decreases from $\gamma = 0.1$ to $\gamma = 0.7$ and then the value of risk increase again. The last two indicators are the SR and CR, in which case we notice again that as parameter γ increases, its value decreases.

Based on the results in Table 5 and Figure 8, we select the portfolio minimizing the power SRM with parameter $\gamma = 0.1$ for the final comparison in Section 5.1.5, since this portfolio has the lowest TO and TTO and also the highest CW. In terms of risk, this portfolio has a increased risk compared to the portfolio with parameter $\gamma = 0.7$ but as we can see in the SR and CR, this risk is significantly compensated by the return and also by the lowest TO or TTO.

5.1.4. Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$

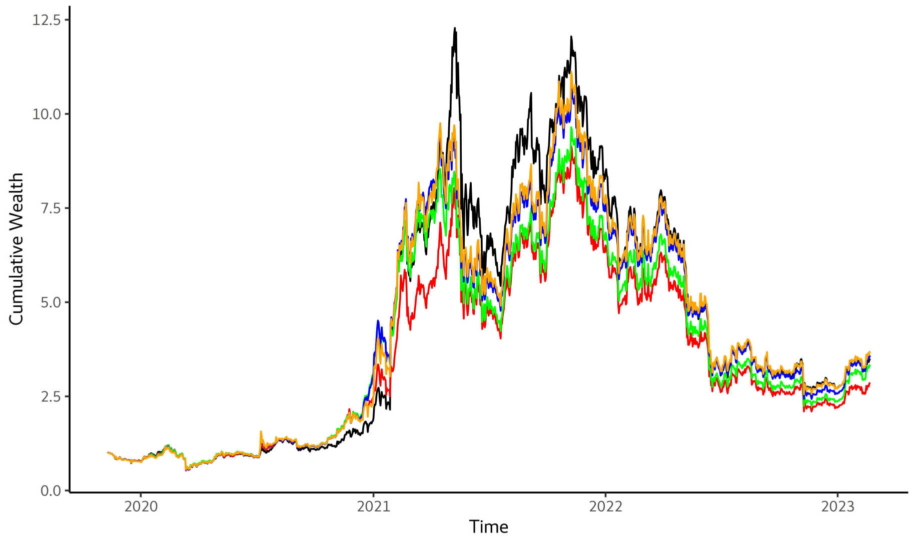
In this section we continue with the Power Spectral Risk Measure for values $\gamma > 1$, results for values $\gamma < 1$ can be found in Section 5.1.3. Our analysis focuses on the power spectral risk measure, where we adopt a rolling window method spanning 30 days (1 month). To determine the minimum power spectral risk portfolios, we compute data from the preceding 365 days (1 year). Our findings are presented in Table 6, and we illustrate the cumulative wealth evolution for different parameter $\gamma > 1$ values $(1.5, 3, 5, 10, 15)$ in Figure 9. As a robustness check, we also generate results using rolling windows of 14 days and 90 days, which are presented in Appendix A.7 and Appendix A.8, respectively.

In Figure 9 we can see that all portfolios that minimise the Power SRM with parameter $\gamma > 1$ evolve almost exactly the same. When we look in more detail, we can see that the cumulative wealth increases with parameter γ, which is opposite to the case of portfolios minimising Power SRM with parameter $\gamma < 1$. However, we can find one exception and that is the portfolio with parameter $\gamma = 1.5$, which is last at the beginning (year 2020) but in 2021 it starts to outperform the other portfolios and finally ends up third in terms of cumulative wealth.

Different results can be found when rebalancing, every 14 days in Figure A.7 and 90 days in Figure A.8. When rebalancing every 14 days, the portfolio with $\gamma = 1.5$ dominates, but when rolling the 90 days window, on the contrary, this portfolio is the worst and the portfolio with $\gamma = 3$ dominates. The portfolios for the rest of the γ parameters i.e. $5, 10$ and 15 are quite stable.

Table 6 provides a comprehensive evaluation of the effectiveness of each portfolio based on multiple metrics, including Turnover (TO), Target Turnover (TTO), Cumula-

Figure 9: Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$



Notes: Minimum Power SRMs portfolio for $\gamma = 1.5$, $\gamma = 3$, $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing.

Table 6: Performance of Minimum Power SRM portfolios with $\gamma > 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 1.5$	19.7550	20.2728	3.4556	0.0479	0.0433	**0.9556**
PWR $\gamma = 3$	16.7563	17.3794	2.8262	0.0395	0.0419	0.7837
PWR $\gamma = 5$	**6.5494**	6.4898	3.2880	**0.0390**	0.0450	0.8330
PWR $\gamma = 10$	6.9074	6.3847	3.5373	0.0398	**0.0462**	0.8672
PWR $\gamma = 15$	6.9621	**6.2826**	**3.6433**	0.0418	**0.0462**	0.9141

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma > 1$ using 10 most liquid CC from BRC with monthly (30 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

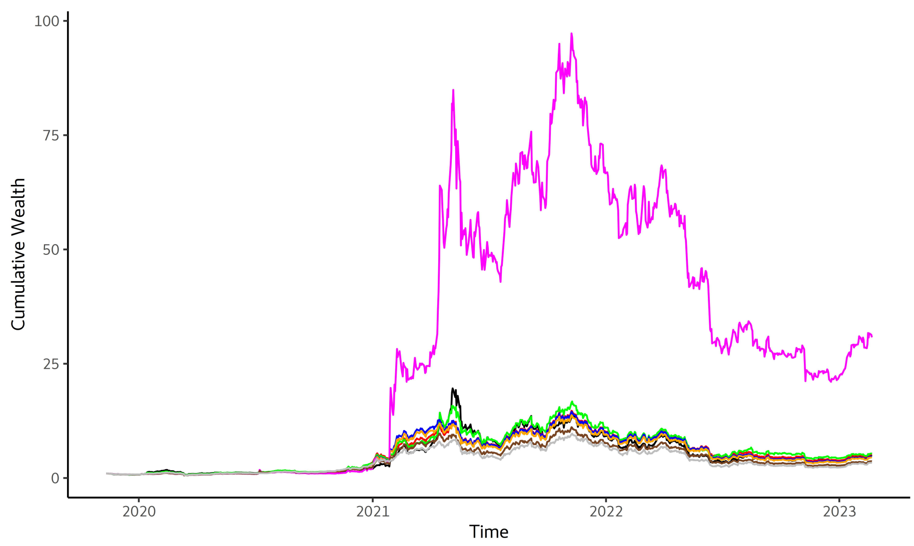
tive Wealth (CW),Standard Deviation (SD), Sharpe Ratio (SR) and Calmar ratio (CR). The data presented in Table 6 demonstrates that as the parameter γ increases, there is a corresponding decrease in TO and TTO. Moreover, our analysis reveals that the CW generally increases as the γ parameter increases, with the exception of $\gamma = 1.5$. Regarding the risk measurement, we observe that the SD is at a similar level except in the case for $\gamma = 1.5$. Finally, we analyze the SR and CR, where we observe that its value generally decreases as the γ parameter increases, except for the case of $\gamma = 1.5$.

Based on the results in Table 6, and in Appendix A.7 or Appendix A.8 we select the portfolio that minimizes the Power Spectral Risk Measure with parameter for the final comparison in section 4.5. This portfolio achieved the best results in TTO, CW, SR, and fell behind the portfolio with parameter by only 0.003 in the SD.

5.1.5. Comparison of Best Minimum SRM portfolios against benchmark portfolios

In the previous sections, we compared portfolios minimising four SRM (VaR, ES, Exponential, Power) using different parameters to find out the properties of the portfolios when varying the parameters. We now compare the portfolios using the different SRMs we selected in Sections 5.1.1 to 5.1.4 with each other and also with the benchmarks, which are the well-known cryptocurrency index CRIX, the Minimum Variance (MinVar) portfolio, and the Equally Weighted strategy, which is also often referred to as the Naive portfolio. The results using a 30-day rolling window are in Figure 10 and Table 7.

Figure 10: Final comparison of portfolios minimising SRM with benchmarks



Notes: Naive portfolio, Minimum Variance, Minimum VaR with $\alpha = 5\%$, Minimum ES with $\alpha = 5\%$, Minimum Exponential SRM with $k = 25$, Minimum Power SRM with $\gamma = 0.1$, Minimum Power SRM with $\gamma = 1.5$ and CRIX using 10 most liquid CC from BRC with monthly (30 days) rebalancing.

In Figure 10, where the evolution of CW is shown, we can observe that all portfolios perform very similarly, except for the portfolio minimizing the Power SRM with parameter $\gamma = 0.1$, which outperforms significantly. The second best performing portfolio is the VaR minimising portfolio at 5% and the worst performing is the CRIX index, which is the only one that has different assets in its composition.

Table 7: Final comparison of portfolios minimising SRM with benchmarks

	TO	TTO	CW	SD	SR	CR
Naive	4.9512	0.0000	4.8195	0.0487	0.0516	1.0972
MinVar	6.3532	5.1078	5.1094	**0.0397**	**0.0541**	1.0355
CRIX	-	-	3.2666	0.0427	**0.0541**	1.0904
VaR $\alpha = 5\%$	23.2028	22.2104	5.3099	0.0433	0.0537	1.1090
ES $\alpha = 5\%$	4.9173	4.2890	4.6298	0.0438	0.0498	1.0345
EXP $k = 25$	**4.6920**	**3.8173**	4.4551	0.0435	0.0492	1.0133
PWR $\gamma = 0.1$	8.3784	7.0337	**30.7839**	0.1175	0.0515	**2.8202**
PWR $\gamma = 15$	6.9621	6.2826	3.6433	0.0418	0.0462	0.9141

Notes: Out-of-sample performance measures of chosen portfolios to final comparison with benchmark portfolios Minimum Variance (MinVar), Naive using 10 most liquid CC from BRC with monthly (30 days) rebalancing and index CRIX: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Table 7 provides an evaluation of the effectiveness of each portfolio and index CRIX based on multiple metrics, including Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), Standard deviation (SD), Sharpe Ratio (SR) and Calmar Ratio (CR). In Table 7, we can see that in terms of TO (we don't calculate it for CRIX), it is most convenient to invest in a portfolio that minimizes the Exponential SRM with $k = 25$. In this criteria, the minimum VaR portfolio with $\alpha = 5\%$ performed the worst and is immediately followed by the Minimum Power SRM portfolio with $\gamma = 0.1$, however, these portfolios achieved the best performance in terms of CW.

In terms of risk as measured by SD, almost all portfolios are the similar level except for the Minimum Power SRM with $\gamma = 0.1$, which has this value multiple times higher than the other portfolios, and the MinVar portfolio performed the best as expected. When examining the risk-return ratios, specifically the SR and CR, it can be observed that the Minimum Power SRM portfolio with a parameter value of $\gamma = 0.1$ outperforms other portfolios significantly in CR, followed by the Minimum VaR portfolio at a significance level of $\alpha = 5\%$ and the Naive portfolio. In terms of SR, the best ratio had CRIX and Minimum Variance.

In summary, the results from Figure 10 and Table 7 indicate that the portfolio minimizing the Power SRM with parameter $\gamma = 0.1$ performs significantly better than other portfolios in terms of CW and risk-return ratios. However, this portfolio exhibits significantly higher volatility compared to other portfolios, as well as higher TO, which could potentially lower the overall performance of this portfolio.

5.1.6. Comparison of the weight allocations of CCs by minimizing different SRMs

Additionally, this study examines the optimal weight variation of CCs by minimizing different SRMs. Specifically, we report portfolio weights of minimized Variance, Value at Risk (VaR) with a confidence level of $\alpha = 5\%$, Exponential SRMs with $k = 5$ and $k = 25$, and Power SRMs with $\gamma = 0.1$, $\gamma = 0.5$, $\gamma = 10$, and $\gamma = 15$. The optimal weight distributions for these risk measures are presented in Figures B.1-B.8 in Appendix B. From a first look at Figures B.1-B.8, which show the weights of different portfolios, we can see that the portfolios invest in smaller number of assets, with one exception, the VaR minimizing portfolio. Figure B.2 shows that the allocation of CCs differs from period to period when optimizing for minimum VaR at $\alpha = 5\%$. Interestingly, this suggests that a particular portfolio may perform more extreme and exhibit greater volatility of the weight changes. Investors can thus invest in a portfolio of different CCs but we can see from Table 7 that this comes at the cost of higher transactions costs, measured by TO and TTO.

The portfolio shown in Figure B.4 is designed for investors who are more risk-averse, as measured by Exponential SRM with $k = 25$, compared to those in Figure B.3. The weight of Bitcoin in the portfolio increases after 2021 because more risk-averse investors seek assets with lower risk. Although the data after 2021 indicates that many CCs have become more volatile, which we can observe in the Figure 4. Table 2 reveals that Bitcoin has a lower standard deviation than other CC assets, making it a comparatively stable asset in the market. Furthermore, as investors become more risk-averse, the weight of Bitcoin in their portfolios increases, as demonstrated in Figures B.7 and B.8.

In Figure B.5 and B.6, we can observe that the power utility function with behaves differently from other risk measures, leading to distinct outcomes. According to the power weighting function, higher risk aversion results in more weight being placed on lower losses and less weight on higher losses (Dowd et al., 2008). This contradicts our intuition and leads to questions about the practical usefulness of some SRMs, particularly those based on power utility functions with $0 < \gamma < 1$, as argued by Dowd and Blake (2006).The key takeaway from our study is that those utilizing spectral risk measures should exercise caution and choose utility functions that match the specific characteristics of the problem they are addressing. In particular, when using power SRMs, users must be especially careful. Nevertheless, our findings provide valuable insights into the properties of these risk measures.

6. Conclusion

To sum up, cryptocurrencies have become a highly attractive investment opportunity, but also come with inherent risks that need to be evaluated before making any investment decisions. This study focuses on cryptocurrency portfolio selection techniques that minimize quantile risk measures, specifically SRMs, which are capable of capturing both VaR and ES as special cases. We provide an introduction to SRMs, explores their links to investors' preferences, and compute SRMs for cryptocurrency portfolios. Overall, investors need to conduct thorough evaluations of the risks and rewards associated with investing in cryptocurrencies before making any investment decisions. Our study also presents a parsimonious and intuitive measure for practitioners, the resulting weights of each CCs can be a representation of the risk profile for different level of risk-averse investors.

The results of the portfolios are compared using out-of-sample performance measures such as turnover, target turnover, cumulative wealth, standard deviation, Sharpe ratio and Calmar Ratio. Data on digital assets, particularly cryptocurrencies, are abundant, and the dataset used in this study includes the CRIX index and the ten most liquid cryptocurrencies available on the BRC. The use of rolling window approach with monthly, quarterly, and bi-weekly rebalancing is done for the construction of the portfolio. The study also highlights the high volatility of cryptocurrencies, which necessitates the need for effective risk measures to better manage digital asset allocation.

There are four primary areas in which the analysis can be expanded. The first area concerns optimizing the portfolio for each SRM in order to consider other values of their parameters. This extension can provide greater insights into the performance of investment strategies in relation to different parameters. The second area involves expanding the performance measures used to evaluate investment strategies. This approach could lead to a more comprehensive assessment of the effectiveness of the investment strategies. The third area entails including additional portfolio-allocation strategies in the comparison. This extension can provide a more detailed understanding of the strengths and weaknesses of various portfolio-allocation strategies. The last area could be to constrain the maximum weight of an asset in the portfolio to avoid allocating weights to only one asset as we have shown in Section 5.1.6.

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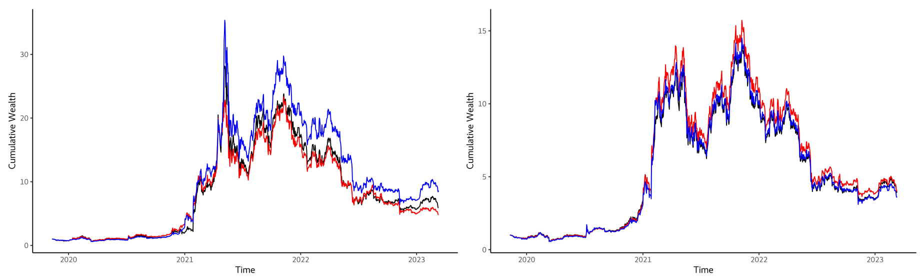
Appendix A. Robustness check

Appendix A.1. Minimum VaR and ES for 14 days rebalancing window

Figure A.1: Minimum VaR and ES portfolios for different α

(a) Minimum VaR portfolios (b) Minimum ES portfolios



Notes: Minimum VaR and ES portfolio for $\alpha = 10\%$, $\alpha = 5\%$ and $\alpha = 1\%$ using 10 most liquid CC from BRC with monthly bi-weekly (14-day).

Table A.1: Performance of Minimum VaR and ES portfolios

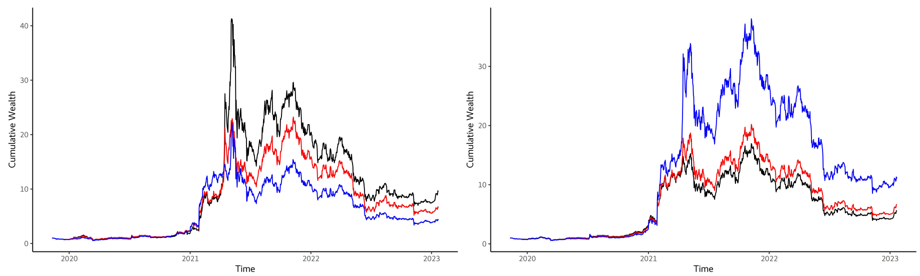
	TO	TTO	CW	SD	SR	CR
VaR $\alpha = 10\%$	102.1451	100.0324	5.8964	0.0570	0.0498	1.2716
VaR $\alpha = 5\%$	90.2496	86.4243	4.7856	0.0461	0.0501	1.0592
VaR $\alpha = 1\%$	43.3868	38.6929	**8.6057**	0.0515	**0.0574**	**1.3170**
ES $\alpha = 10\%$	**12.3886**	**9.9061**	3.9881	**0.0417**	0.0473	0.9356
ES $\alpha = 5\%$	14.0029	10.2190	4.0784	0.0438	0.0468	0.9730
ES $\alpha = 1\%$	15.6496	12.8441	3.6362	0.0452	0.0445	0.9338

Notes: Out-of-sample performance measures of minimum Var and ES portfolios using 10 most liquid CC from BRC with bi-weekly (14 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.2. Minimum VaR and ES for 90 days rebalancing window

Figure A.2: Minimum VaR and ES portfolios for different α

(a) Minimum VaR portfolios (b) Minimum ES portfolios



Notes: Minimum VaR and ES portfolio for $\alpha = 10\%$, $\alpha = 5\%$ and $\alpha = 1\%$ using 10 most liquid CC from BRC with monthly quarterly (90-day) rebalancing.

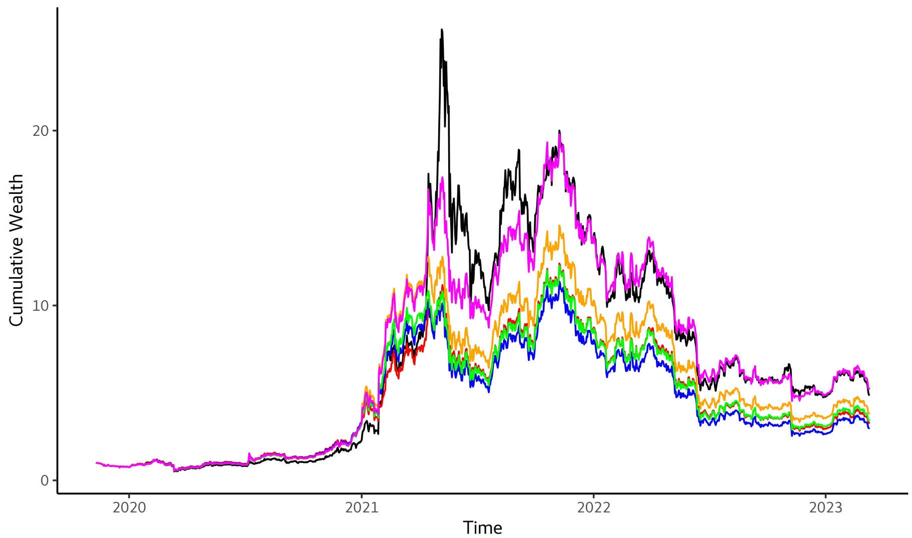
Table A.2: Performance of Minimum VaR and ES portfolios

	TO	TTO	CW	SD	SR	CR
VaR $\alpha = 10\%$	2.8002	2.7902	9.6455	0.0522	**0.0619**	1.4170
VaR $\alpha = 5\%$	3.0127	2.9974	6.6365	0.0484	0.0562	1.2778
VaR $\alpha = 1\%$	3.1306	3.0248	4.3879	0.0549	0.0466	1.1024
ES $\alpha = 10\%$	**1.2770**	**0.8279**	5.6023	**0.0454**	0.0534	1.1537
ES $\alpha = 5\%$	1.3745	0.9044	6.6262	0.0478	0.0551	1.2548
ES $\alpha = 1\%$	1.9656	1.3980	**11.2038**	0.0612	0.0572	**1.6467**

Notes: Out-of-sample performance measures of minimum Var and ES portfolios using 10 most liquid CC from BRC with quarterly (90-day) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.3. Minimum Exponential SRM portfolios for 14 days rebalancing window

Figure A.3: Minimum Exponential Spectral Risk Measure portfolios for different k



Notes: Minimum Exponential SRMs portfolio for k = 1, k = 5, k = 10, k = 15, k = 20 and k = 25 using 10 most liquid CC from BRC with bi-weekly (14-day) rebalancing.

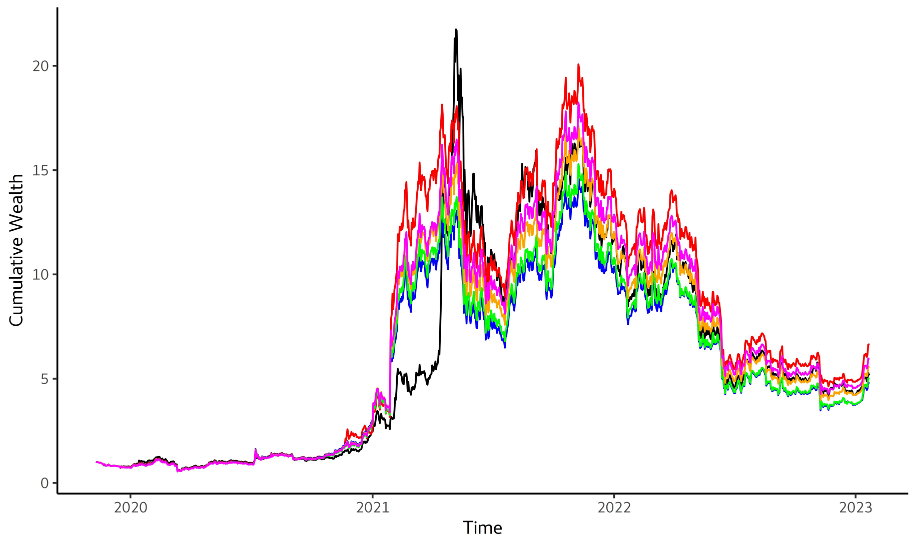
Table A.3: Performance of Minimum Exponential SRM portfolios

	TO	TTO	CW	SD	SR	CR
EXP k = 1	88.2501	89.7872	4.8520	0.0464	0.0508	1.0555
EXP k = 5	40.8039	39.1985	3.3323	**0.0387**	0.0450	0.8230
EXP k = 10	19.8798	18.8635	3.0159	0.0396	0.0425	0.7913
EXP k = 15	17.9017	15.7459	3.4801	0.0406	0.0451	0.8748
EXP k = 20	16.9683	14.5126	3.8770	0.0421	0.0467	0.9286
EXP k = 25	**14.7762**	**10.8768**	**5.3044**	0.0428	**0.0524**	**1.0628**

Notes: Out-of-sample performance measures of Minimum Exponential SRM portfolios using 10 most liquid CC from BRC with bi-weekly (14 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.4. Minimum Exponential SRM portfolios for 90 days rebalancing window

Figure A.4: Minimum Exponential Spectral Risk Measure portfolios for different k



Notes: Minimum Exponential SRMs portfolio for $k = 1$, $k = 5$, $k = 10$, $k = 15$, $k = 20$ and $k = 25$ using 10 most liquid CC from BRC with quarterly (90-day) rebalancing.

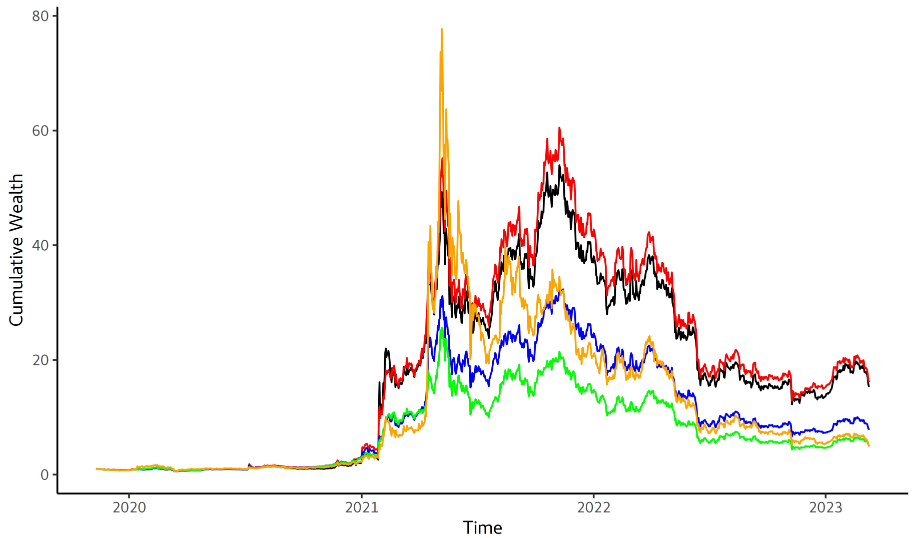
Table A.4: Performance of Minimum Exponential SRM portfolios

	TO	TTO	CW	SD	SR	CR
EXP $k = 1$	3.0889	3.0706	5.2265	0.0469	**0.0536**	1.1242
EXP $k = 5$	1.8134	1.4309	**6.6294**	0.0537	0.0523	**1.3320**
EXP $k = 10$	1.7636	1.3829	4.8185	**0.0448**	0.0509	1.0919
EXP $k = 15$	**1.2735**	**0.8804**	5.0074	0.0468	0.0505	1.1246
EXP $k = 20$	1.3603	0.9170	5.5646	0.0480	0.0519	1.1832
EXP $k = 25$	1.4629	0.9754	5.9624	0.0487	0.0527	1.2235

Notes: Out-of-sample performance measures of Minimum Exponential SRM portfolios using 10 most liquid CC from BRC with quarterly (90 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.5. Minimum Power Spectral Risk Measure portfolios for different $\gamma < 1$ for 14 days rebalancing window

Figure A.5: Minimum Power Spectral Risk Measure portfolios for different $\gamma < 1$ with 14 days rebalancing window



Notes: Minimum Power SRMs portfolio for $\gamma = 0.1$, $\gamma = 0.3$, $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$ using 10 most liquid CC from BRC with bi-weekly (14-day) rebalancing.

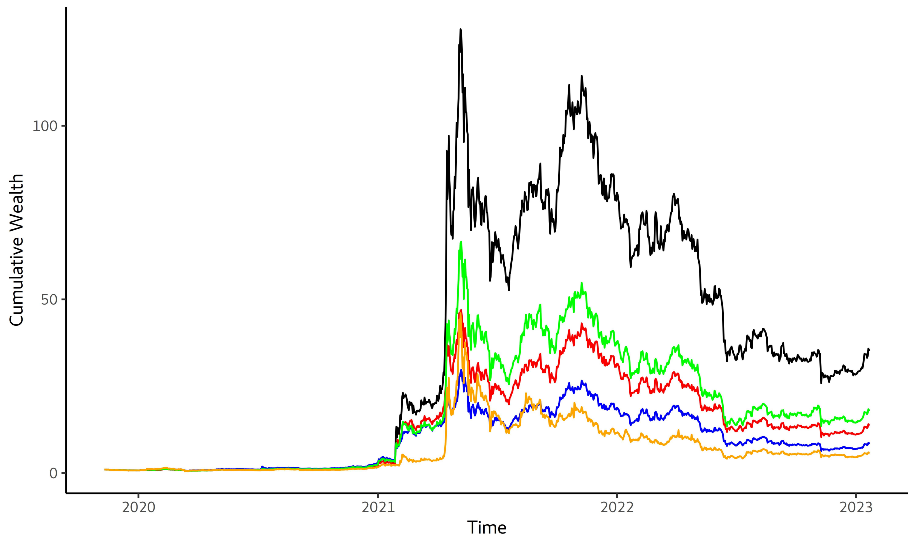
Table A.5: Performance of Minimum Power SRM portfolios with $\gamma < 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 0.1$	**38.1945**	**36.2857**	15.4365	0.1133	0.0460	**2.4622**
PWR $\gamma = 0.3$	52.6548	48.6516	**16.3405**	0.0633	**0.0601**	1.7644
PWR $\gamma = 0.5$	86.9030	85.7957	8.0386	0.0482	0.0578	1.2703
PWR $\gamma = 0.7$	76.0743	77.8070	5.2217	**0.0475**	0.0508	1.0606
PWR $\gamma = 0.9$	114.5320	119.2257	4.9121	0.0646	0.0506	1.2751

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma < 1$ using 10 most liquid CC from BRC with bi-weekly (14 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.6. Minimum Power Spectral Risk Measure portfolios for different $\gamma < 1$ for 90 days rebalancing window

Figure A.6: Minimum Power Spectral Risk Measure portfolios for different $\gamma < 1$ with 90 days rebalancing window



Notes: Minimum Power SRMs portfolio for $\gamma = 0.1$, $\gamma = 0.3$, $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$ using 10 most liquid CC from BRC with quarterly (90-day) rebalancing.

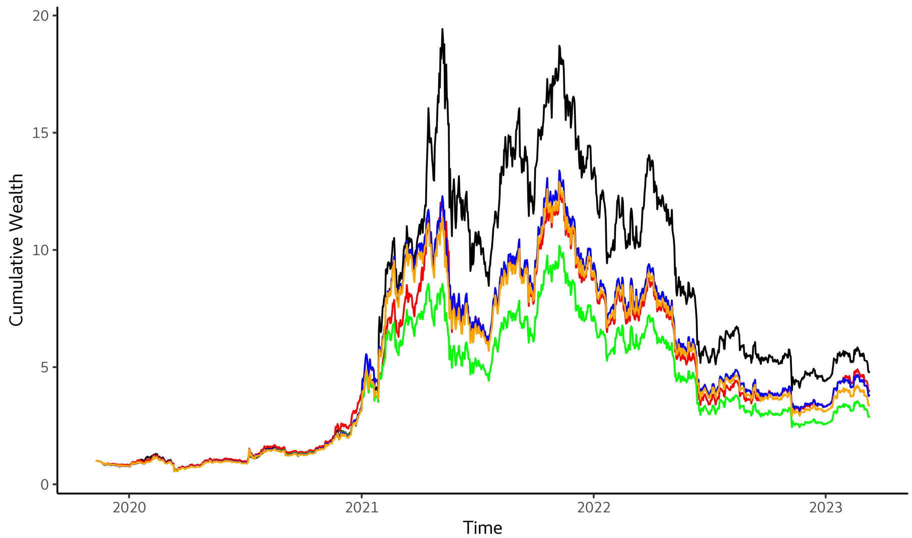
Table A.6: Performance of Minimum Power SRM portfolios with $\gamma < 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 0.1$	**1.4537**	**1.2911**	**35.5660**	0.1040	0.0573	**2.7290**
PWR $\gamma = 0.3$	2.4935	2.2058	13.9268	0.0713	0.0567	1.8942
PWR $\gamma = 0.5$	2.9904	2.7954	8.6511	**0.0542**	0.0572	1.4389
PWR $\gamma = 0.7$	2.0871	2.0058	18.1807	0.0629	**0.0648**	1.8630
PWR $\gamma = 0.9$	3.4522	3.5947	5.8980	0.0693	0.0535	1.4853

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma < 1$ using 10 most liquid CC from BRC with quarterly (90 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.7. Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$ for 14 days rebalancing window

Figure A.7: Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$ with 14 days rebalancing window



Notes: Minimum Power SRMs portfolio for $\gamma = 1.5$, $\gamma = 3$, $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$ using 10 most liquid CC from BRC with bi-weekly (14-day) rebalancing.

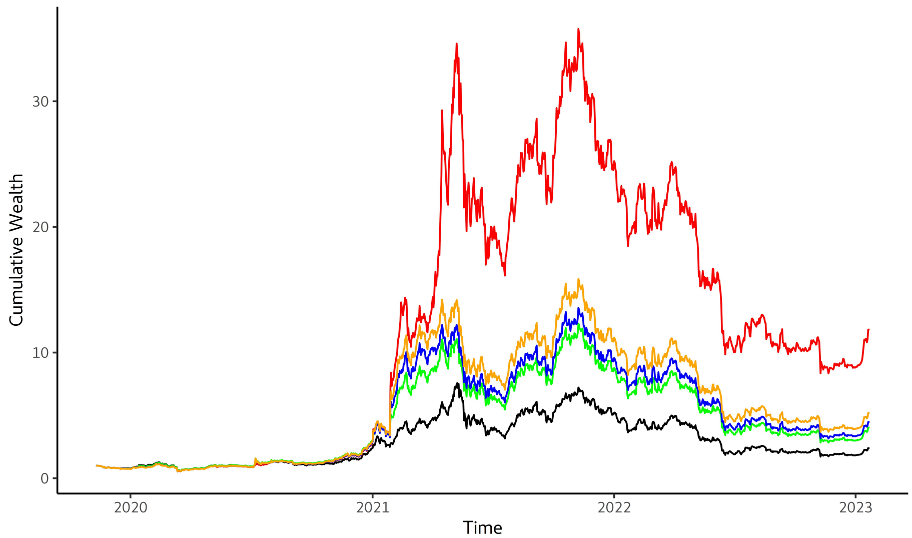
Table A.7: Performance of Minimum Power SRM portfolios with $\gamma > 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 1.5$	80.3406	80.3938	**4.8295**	0.0455	**0.0501**	**1.0488**
PWR $\gamma = 3$	77.3312	75.8724	4.0309	0.0395	0.0490	0.9324
PWR $\gamma = 5$	30.3027	29.1677	2.9243	**0.0392**	0.0421	0.7904
PWR $\gamma = 10$	32.2383	30.3357	3.8313	0.0402	0.0473	0.9041
PWR $\gamma = 15$	**17.8039**	**16.0298**	3.4081	0.0409	0.0445	0.8582

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma > 1$ using 10 most liquid CC from BRC with bi-weekly (14 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

Appendix A.8. Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$ for 90 days rebalancing window

Figure A.8: Minimum Power Spectral Risk Measure portfolios for different $\gamma > 1$ with 90 days rebalancing window



Notes: Minimum Power SRMs portfolio for $\gamma = 1.5$, $\gamma = 3$, $\gamma = 5$, $\gamma = 10$ and $\gamma = 15$ using 10 most liquid CC from BRC with quarterly (90-day) rebalancing.

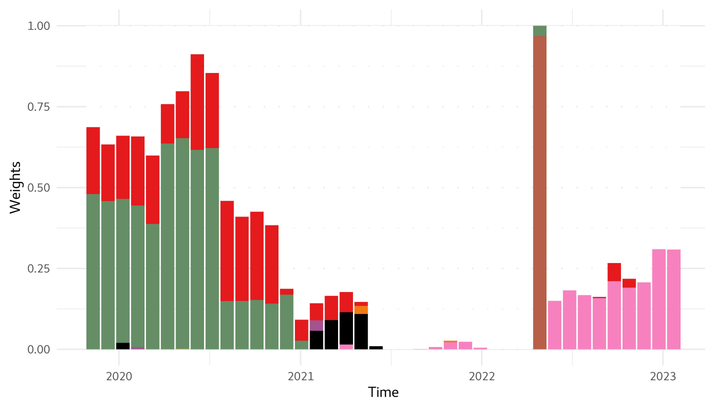
Table A.8: Performance of Minimum Power SRM portfolios with $\gamma > 1$

	TO	TTO	CW	SD	SR	CR
PWR $\gamma = 1.5$	2.2253	2.3435	2.3941	**0.0404**	0.0392	0.7466
PWR $\gamma = 3$	2.2353	2.1377	**11.8211**	0.0539	**0.0617**	**1.5817**
PWR $\gamma = 5$	1.3634	1.0706	4.0228	0.0413	0.0491	0.9618
PWR $\gamma = 10$	**1.1751**	**0.8448**	4.4712	0.0447	0.0494	1.0511
PWR $\gamma = 15$	1.3072	0.8984	5.1907	0.0469	0.0511	1.1404

Notes: Out-of-sample performance measures of Minimum Power SRM portfolios with $\gamma > 1$ using 10 most liquid CC from BRC with quarterly (90 days) rebalancing: Turnover (TO), Target Turnover (TTO), Cumulative Wealth (CW), standard deviation (SD), Sharpe ratio (SR), Calmar ratio (CR).

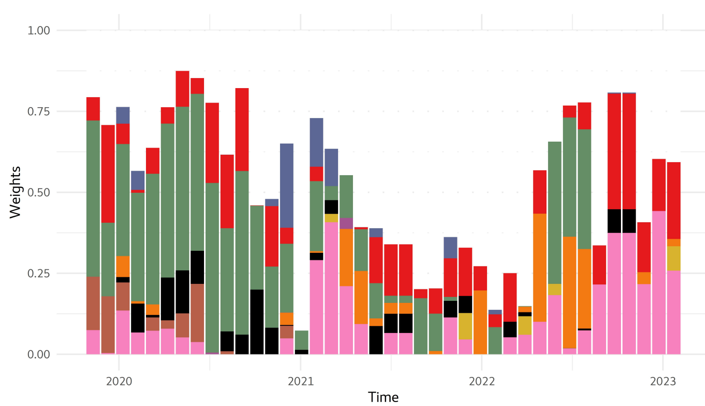
Appendix B. Weights allocation

Figure B.1: The weights of CCs in portfolio minimizing variance using a 30-day rolling window



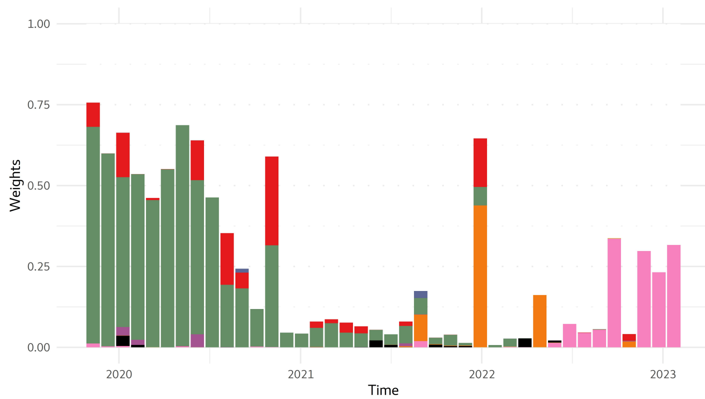
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.2: The weights of CCs in portfolio minimizing VaR using a 30-day rolling window



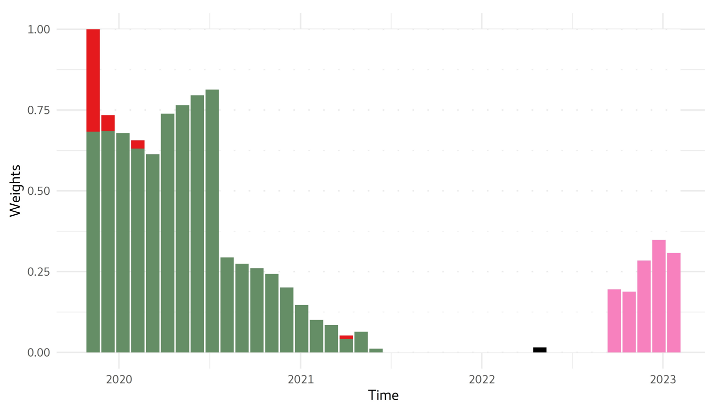
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.3: The weights of CCs in portfolio minimizing Exponential SRM with k = 5 using a 30-day rolling window



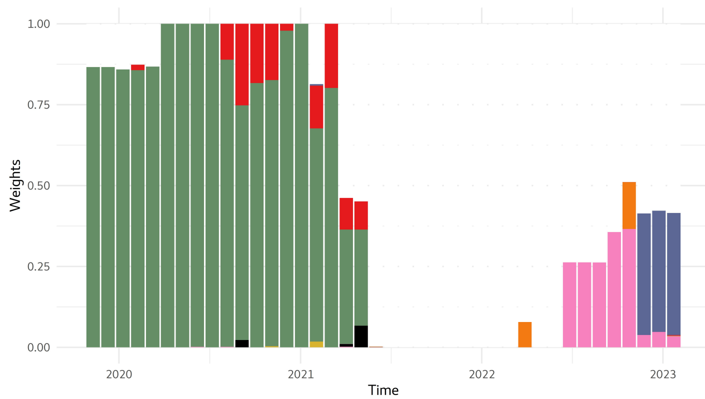
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.4: The weights of CCs in portfolio minimizing Exponential SRM with k = 25 using a 30-day rolling window



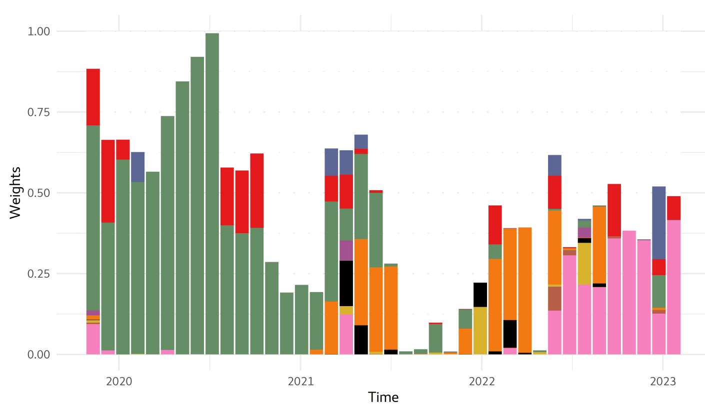
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.5: The weights of CCs in portfolio minimizing Power SRM with γ = 0.1 using a 30-day rolling window



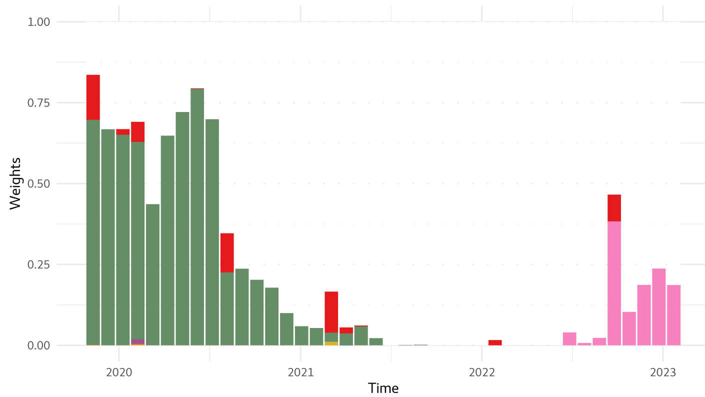
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.6: The weights of CCs in portfolio minimizing Power SRM with γ = 0.5 using a 30-day rolling window



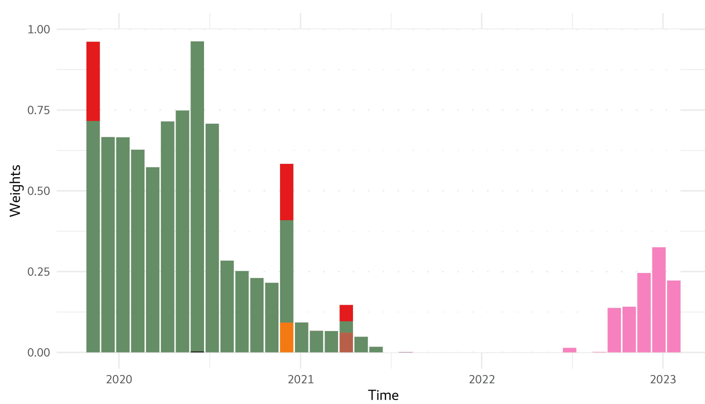
Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.7: The weights of CCs in portfolio minimizing Power SRM with γ = 10 using a 30-day rolling window



Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.

Figure B.8: The weights of CCs in portfolio minimizing Power SRM with γ = 15 using a 30-day rolling window



Notes: CC used in portfolio: BTC (white), LTC, XRP, DOGE, DASH, ETH, ETC, ZEC, BCH and BSV.