Ze Committee ZeMC

Ze Committee Ze Math Competition

1st Annual

ZIME

Tuesday, January 16th, 2024

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU TO BEGIN.
2. This is a 15 question, 3 hour examination. All answers are integers ranging from 000 to 999, inclusive. Your score will be the number of correct answers. There is neither partial credit nor penalties for wrong answers.
3. Despite the name, Ze Invitational Mathematics Examination is open for anybody to take. There will not be a 2024 ZeJMO/ZeMO.
4. Submit your answers by PMing them through AoPS to “ihatemath123“, or DMing them through Discord to ”bennywang“. If you use Discord, please specify your AoPS username.
   You may format your answers in any way, as long as it is clear which problem each answer corresponds to.
   If you wish to remain anonymous on the leaderboard, or wish to remain anonymous if your score is below a certain threshold, make sure to specify this in your message.
   DO NOT edit your message; you may be considered for cheating.
5. You should receive a response with your score and distribution within 24 hours, in addition to a link with access to a private discussion forum.
6. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, Dotted Caculators, grid paper and lined paper are NOT allowed. No problems on the contest require the use of a calculator.
7. Figures are not necessarily drawn to scale.

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1. If two palindromes (numbers which read the same backwards and forwards) sum to 2024, find the sum of all possible values for the smaller palindrome.

2. Find the number of ways that the set $\{1, 2, 3, \ldots, 18\}$ can be split into two indistinguishable sets of nine elements, such that one set has a median of 6 and the other set has a median of 12.

3. Evan thinks of two positive integers. Their quotient, which leaves no remainder, divides their sum, and their product is 784,000. Find the remainder when the absolute difference between Evan's two numbers is divided by 1000.

4. Julie picks positive reals b and x with $b \neq 1$ and writes down the logarithm $\log_b x$. If she were to erase b and replace it with $\frac{b}{2}$, the value of the logarithm would increase by 12. *Instead*, if she were to erase b and replace it with $2b$, the value of the logarithm would decrease by 9. Find $\log_2(x)$.

5. Find the number of ways to split an eight by eight square into five rectangles with integer side lengths, such that any two rectangles have at most one vertex in common.

6. Let m and b be real numbers. Points A, B, C and D lie on the line $y = mx + b$ in that order, equally spaced. Given that A and C lie on the parabola $y = x^2 + 13x + 18$ and B and D lie on the parabola $y = x^2 + 5x + 14$, find mb.

7. In $\triangle ABC$, points D, E and F lie on segments $\overline{AB}$, $\overline{AC}$ and $\overline{BC}$ such that $AD = DB = 5$, $\overline{DE}$ bisects $\angle ADC$ and $\overline{DF}$ bisects $\angle ADB$. If $DE = 6$ and $DF = 2$, the length DB can be expressed as $\frac{a+\sqrt{b}}{c}$, where a, b and c are positive integers with $\gcd(a, c) = 1$. Find $a + b + c$.

8. If a, b and c are complex numbers such that

$$\begin{cases} |a| + b + c &= 7i \\ a + |b| + c &= 9i \\ a + b + |c| &= 10i, \end{cases}$$

find $|a + b + c|^2$.

9. Parallelogram $ABCD$ has an area of 350 and satisfies $AB = 35$. Let F and G be points in the interior of the parallelogram such that $FG = 24$ and $FG \parallel AB$. If there exists an ellipse with foci F and G tangent to all four sides of the parallelogram, find BC^2.

10. Alex and Oron are playing a game. They take turns spinning a fair spinner with ten sectors of equal size, numbered $1, 2, \ldots, 10$. Alex goes first. After the first spin, if a player spins a number less than or equal to the number previously spun, the game ends and the other player wins. The probability that Oron loses the game can be expressed as $\frac{m}{n}$, where m and n are coprime integers. Find the remainder when $m + n$ is divided by 1000.

11. Let n and k be positive integers such that the sum of the n smallest perfect powers of k (including 1) is a multiple of 1001. Find the number of possible values of n less than 1001.

12. Points D and E lie on sides AB and AC of $\triangle ABC$, respectively, such that the circumcircles ω_1 and ω_2 of $\triangle ABE$ and $\triangle ADC$, respectively, meet on side $\overline{BC}$. Line DE meets ω_1 and ω_2 at points X and Y, respectively, such that $BC = 14$, $XY = 19$, $BD = 6$ and $CE = 9$. Then, the length DE can be expressed as $\frac{a-\sqrt{b}}{c}$, where a, b and c are positive integers with $\gcd(a, c) = 1$. Find $a + b + c$.

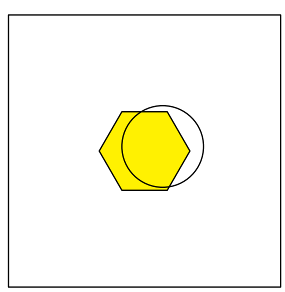
13. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers such that $a_1 = 20$, $b_1 = 23$ and

$$\begin{cases} a_{i+1} & = \sqrt{|a_i \cdot b_i|} + \frac{a_i+b_i}{2} \\ b_{i+1} & = \sqrt{|a_i \cdot b_i|} - \frac{a_i+b_i}{2} \end{cases}$$

for all positive integers i. Find the smallest integer k for which $a_k = b_k$.

14. Alexandre forms a piece of cookie dough in the shape of a regular hexagon with a side length of 8 cm, and places it at the center of a square baking pan with a side length of 50 cm, as shown in the diagram below. He then drops a circular cookie cutter with a radius of 7 cm randomly and uniformly inside the baking pan, such that the entire cutter lies within the pan. The expected number of pieces that the cookie dough gets cut into can be expressed as $\frac{m}{n}$ for coprime positive integers m and n. Find $m + n$.

(For example, in the diagram below, the cookie is cut into two pieces. If the cookie cutter does not touch the dough, the cookie dough is in 1 piece.)



15. Let $\triangle ABC$ be an acute triangle with circumcenter O. Let O_B and O_C be the circumcenters of $\triangle BOA$ and $\triangle COA$, respectively, and let P be the circumcenter of $\triangle OO_BO_C$. If the circumradii of $\triangle ABC$, $\triangle OO_BO_C$ and $\triangle PBC$ are 9, 15 and 11, respectively, find AP^2.

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Administration on an earlier date will probably occur.

- None of the information needed to administer this competition is contained in the ZIME Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT DOES NOT EXIST.
- Answer sheets must be returned to the Ze Committee ZIME office within 2.9 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for all of the answer sheets to get thrown in an incinerator, USPS overnight is strongly recommended.
- The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, friends (if you have them), or digital media of any type during this period is a violation of competition rules.

The ZeMC competition series is made possible by the contributions of the following problem-writers and test-solvers:

asbodke, ayush_agarwal, bissue, Geometry285, ihatemath123, kante314, OronSH, peace09, P_Groudon, Significant and Turtwig113.

Thank you for taking our mock AIME!