



Ze Committee ZeMC

Ze Committee Ze Math Competition

3rd Annual

ZeMC 10

Tuesday, October 17th, 2023



INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU TO BEGIN.
2. This is a 25 question multiple choice contest. For each question, only one answer choice is correct.
3. Submit your answers by PMing them through AoPS to “ihatemath123”, or DMing them through Discord to “bennywang”. If you use Discord, please specify your AoPS username.
You may format your answers in any way, as long as it is clear which problem each answer corresponds to. If you wish to remain anonymous on the leaderboard, or wish to remain anonymous if your score is below a certain threshold, make sure to specify this in your message.
DO NOT edit your message; you may be considered for cheating.
4. You should receive a response with your score and distribution within 24 hours, in addition to a link with access to a private discussion forum.
5. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
6. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, Dotted Calculators, grid paper and lined paper are NOT allowed. No problems on the contest require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. You will have 75 minutes to complete the contest once you tell you to begin.

The Ze Committee ZeMC Office reserves the right to disqualify scores from an individual if it determines that the rules or the nonexistent required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this ZeMC 10 will be invited to take Ze Invitational Mathematics Competition. More details are on the back page of the test booklet.

1. What is the value of

$$3.1 - 3.14 + 3.141 - 3.1415 + 3.14159?$$

- (A) -0.10109 (B) 0.10109 (C) 0.14159 (D) 3.10109 (E) 3.14159

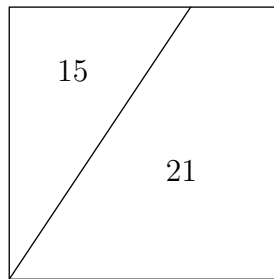
2. The captain of a ship has no coins. After he steals one coin from each of his ten crewmates, everybody on board has the same number of coins. How many coins are on the ship in total?

- (A) 90 (B) 99 (C) 100 (D) 110 (E) 111

3. Alexa evaluates the sum of a positive integer and its reciprocal. She then writes the result down as a simplified fraction. If the numerator of the fraction is 50, what is its denominator?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

4. A segment drawn in a square splits it into a triangle with an area of 15 and a trapezoid with an area of 21. What is the length of this segment?



- (A) $\sqrt{34}$ (B) 6 (C) $2\sqrt{13}$ (D) $\sqrt{61}$ (E) 8

5. Two fifths of the marbles in a bag are cyan and the rest are navy. Sara adds 30 navy marbles to the bag, so that now one fifth of the marbles in the bag are cyan. How many cyan marbles are in the bag?

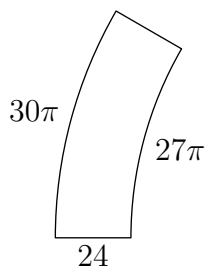
- (A) 10 (B) 12 (C) 15 (D) 18 (E) 20

6. How many of the following statements are true?

- (a) Statement (b) is false.
- (b) Statement (c) is false.
- (c) Statement (d) is false.
- (d) Statement (a) is false.

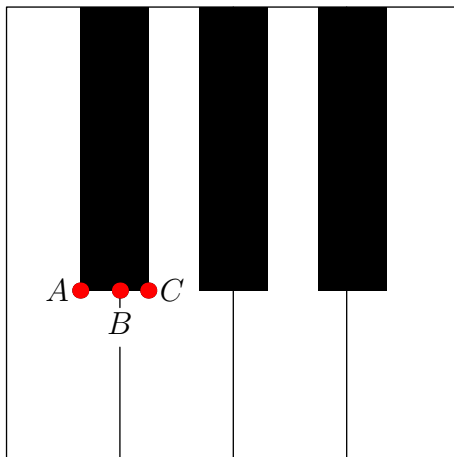
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. The square of Jason's favorite integer begins with a 4. What is the sum of all possible digits that Jason's favorite integer could begin with?
- (A) 9 (B) 11 (C) 13 (D) 15 (E) 17
8. The sum of an integer N and the remainder when N is divided by 23 is 300. What is the sum of the digits of N ?
- (A) 16 (B) 17 (C) 18 (D) 19 (E) 20
9. Victor's model train rolls on tracks, which are constructed from pieces. Each piece is shaped like an annulus sector with a width of 24 cm, an inner circumference of 27π cm and an outer circumference of 30π cm. How many pieces must Victor join together to form a full circle?

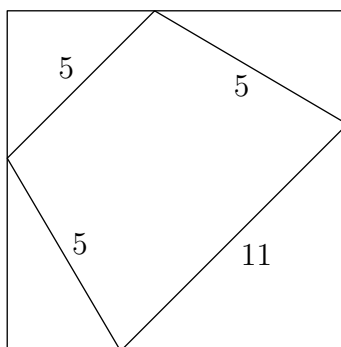


- (A) 15 (B) 16 (C) 18 (D) 20 (E) 24
10. At a math summer camp, 75% of the students have exactly two roommates and 25% of the students have exactly three roommates. What is the average number of students in each room?
- (A) 3.2 (B) 3.25 (C) 3.3 (D) 3.35 (E) 3.4

11. In the section of the keyboard below, each of the four white keys has the same width and area; similarly, each of the three black keys has the same width and area. What is the ratio $AB : BC$?



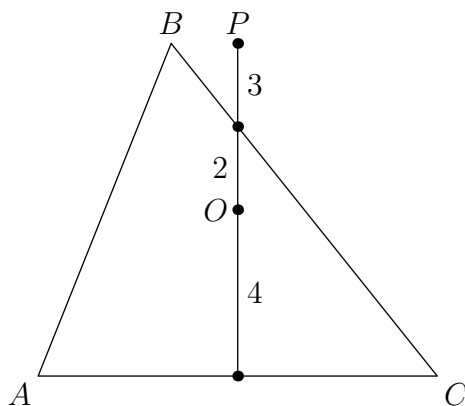
- (A) 1 : 1 (B) 3 : 2 (C) 2 : 1 (D) 3 : 1 (E) 4 : 1
12. An isosceles trapezoid, with side lengths shown below, is inscribed in a square. What is the side length of the square?



- (A) 8 (B) $6\sqrt{2}$ (C) 9 (D) $7\sqrt{2}$ (E) 10
13. Call a day *unlucky* if it is a Friday that falls on the 13th day of the month. Which of the following is the closest to the number of *unlucky* days that have occurred after the start of the year 1900 and before the end of the year 1999?
- (A) 110 (B) 130 (C) 150 (D) 170 (E) 190
14. Let $a_1, a_2, a_3, \dots, a_{32}$ be a geometric series. If $a_1 + a_2 + a_3 + \dots + a_{32} = 20$ and $a_2 + a_4 + a_6 + \dots + a_{32} = 6$, what is $a_4 + a_8 + a_{12} + \dots + a_{32}$?

- (A) $\frac{19}{21}$ (B) $\frac{21}{23}$ (C) $\frac{23}{25}$ (D) $\frac{25}{27}$ (E) $\frac{27}{29}$

15. In $\triangle ABC$, the perpendicular bisector of \overline{AC} is drawn from the midpoint of \overline{AC} to the point P with $\overline{BP} \parallel \overline{AC}$. This segment is split into three pieces by the circumcenter and side \overline{BC} , with lengths that measure 4, 2 and 3. What is the area of $\triangle ABC$?



- (A) 27 (B) $9\sqrt{10}$ (C) $18\sqrt{3}$ (D) 36 (E) $18\sqrt{5}$
16. A list of 25 consecutive perfect cubes and a list of k consecutive perfect squares both begin with the same value and end with the same value. What is the largest possible value of k ?
- (A) 219 (B) 254 (C) 263 (D) 284 (E) 342
17. How many positive integers n are there such that n^n is a divisor of 144^{144} ?
- (A) 15 (B) 16 (C) 19 (D) 20 (E) 21
18. Let X be the number of permutations of the first 23 positive integers for which the sum of any two adjacent numbers is no more than 24. What is the sum of the digits of X ?
- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
19. Celine draws a regular hexagon $ABCDEF$ in the Cartesian plane such that \overline{AB} and \overline{DE} are parallel to the x -axis. She then stretches the plane vertically by a factor of some constant k and draws the image of the hexagon under this stretch, $A'B'C'D'E'F'$. If the distance between lines $A'B'$ and $D'E'$ is 5 units and the distance between lines $B'C'$ and $E'F'$ is 2 units, what is k ?

- (A) $\sqrt{7}$ (B) $\frac{8\sqrt{3}}{5}$ (C) $2\sqrt{2}$ (D) $\frac{4\sqrt{5}}{3}$ (E) 3

20. Ruben writes down a system of equations. Serge spills coffee on the paper, covering a number:

$$\begin{cases} x + y + z &= 7 \\ xy + yz + xz &= 8 \\ xyz &= \blacksquare \end{cases}$$

However, Ruben remembers that there were exactly three ordered triples (x, y, z) of *positive* real numbers satisfying the system of equations. The value that should replace the \blacksquare can be expressed as $\frac{m}{n}$ for coprime positive integers m and n . What is $m + n$?

- (A) 91 (B) 93 (C) 95 (D) 97 (E) 99

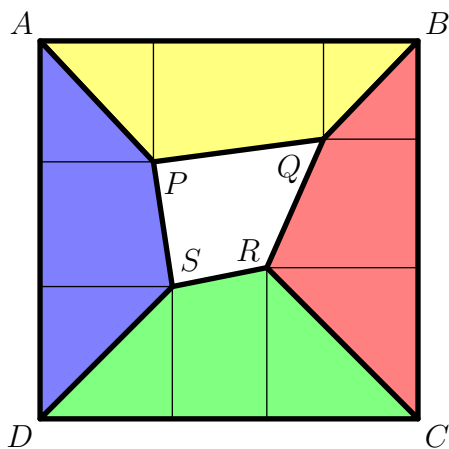
21. Arby the ant is at $(0, 0)$ facing in the positive x direction. On the first day he walks one unit to the point $(1, 0)$. Each day after that, he randomly turns either 60° left or 60° right, and walks one third the distance he travelled the previous day. What is Arby's expected x -coordinate as time goes to infinity?

- (A) $\frac{6}{5}$ (B) $\frac{9}{7}$ (C) $\frac{4}{3}$ (D) $\frac{3}{2}$ (E) $\frac{8}{5}$

22. The fourth largest divisor of an integer is 165. What is the sum of all possible values of its fifth largest divisor?

- (A) 237 (B) 289 (C) 325 (D) 493 (E) 511

23. Four squares are placed in each corner of a larger square, as shown in the diagram below. If the areas of quadrilaterals $APQB$, $BQRC$, $CRSD$ and $DSPA$ are 20, 23, 26 and 25, respectively, what is the area of quadrilateral $PQRS$?



- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

24. How many polynomials $P(x)$ with nonnegative integer coefficients are there such that

$$P(x) \leq 2x^3 + 2x^2 + 2$$

for all real numbers x ?

- (A) 15 (B) 16 (C) 17 (D) 18 (E) 19

25. How many nonempty subsets of $\{1, 2, \dots, 12\}$ have an integer median?

- (A) 2730 (B) 2731 (C) 3070 (D) 3071 (E) 3072

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ZeMC 10

DO NOT OPEN UNTIL Tuesday, October 17th, 2023

****Administration on an earlier date will result in loss of game****

- None of the information needed to administer this competition is contained in the ZeMC 10 Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT DOES NOT EXIST.
 - Answer sheets must be returned to the Ze Committee ZeMC office within 2.9 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for all of the answer sheets to get thrown in an incinerator, USPS overnight is strongly recommended.
 - The first annual Ze Invitational Mathematics Examination will be held on Monday, January 1st, 2024, with no alternate date. It is a 15-question, 3-hour, integer-answer competition. Students who achieve a high score on the ZeMC 10 will be invited to participate. Top-scoring students on the ZeMC 10/12 and ZIME will not be selected to take the Ze (Junior) Mathematical Olympiad. The Ze(J)MO will not be given on Monday and Tuesday, March 32nd and 33rd, 2024. None of these competitions will exist.
 - The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, friends (if you have them), or digital media of any type during this period is a violation of competition rules.
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*The ZeMC competition series is made possible by the contributions of the following
problem-writers and test-solvers:*

asbodke, ayush_agarwal, bissue, Geometry285, ihatemath123,
kante314, peace09, P_Groudon, Significant and Turtwig113

Thank you for taking our mock AMC!