Ben Gurion University of the Negev

Faculty of Engineering Sciences

Department of Mechanical Engineering

Research proposal

Turbulent Flow course Project

Benny Vradman

8.1.2025

The problam

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

Possible solutions and their fault

- DNS.

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.
  - Tries to take in wide range of scales. Small scales depend more on ν, while large scale depend more on BC.

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.
  - Tries to take in wide range of scales. Small scales depend more on ν, while large scale depend more on BC.
  - The constants for this model are sometimes hard to optimize.

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.
  - Tries to take in wide range of scales. Small scales depend more on ν, while large scale depend more on BC.
  - The constants for this model are sometimes hard to optimize.
- The combination of the two $\rightarrow$ **LES**

About LES

- Invented by Dr. Joseph Smagorinsky (1924-2005), meteorologist and founding director of NOAA's Geophysical Fluid Dynamics Laboratory [2]

About LES

- Invented by Dr. Joseph Smagorinsky (1924-2005), meteorologist and founding director of NOAA's Geophysical Fluid Dynamics Laboratory [2]
- The idea is to solve the large scales and model the small scales.

About LES

- Invented by Dr. Joseph Smagorinsky (1924-2005), meteorologist and founding director of NOAA's Geophysical Fluid Dynamics Laboratory [2]
- The idea is to solve the large scales and model the small scales.
- The computational cost is proportional to $Re^{13/7}$, one order of magnitude less than DNS [1].

The Model

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\overline{\phi}(\mathbf{x}, t)}_{\text{resolved scale}} + \underbrace{\phi'(\mathbf{x}, t)}_{\text{subgrid scale}} \tag{2}$$

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\overline{\phi}(\mathbf{x}, t)}_{\text{resolved scale}} + \underbrace{\phi'(\mathbf{x}, t)}_{\text{subgrid scale}} \tag{2}$$

The filtering operation is defined as:

$$\overline{\phi}(\mathbf{x}, t) = \int_D G(\mathbf{x} - \mathbf{y}, \Delta)\phi(\mathbf{y}, t)\, d\mathbf{y} \tag{3}$$

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\overline{\phi}(\mathbf{x}, t)}_{\text{resolved scale}} + \underbrace{\phi'(\mathbf{x}, t)}_{\text{subgrid scale}} \tag{2}$$

The filtering operation is defined as:

$$\overline{\phi}(\mathbf{x}, t) = \int_D G(\mathbf{x} - \mathbf{y}, \Delta)\phi(\mathbf{y}, t)\, d\mathbf{y} \tag{3}$$

where G is the filter function, and Δ is the filter width.

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\overline{\phi}(\mathbf{x}, t)}_{\text{resolved scale}} + \underbrace{\phi'(\mathbf{x}, t)}_{\text{subgrid scale}} \tag{2}$$

The filtering operation is defined as:

$$\overline{\phi}(\mathbf{x}, t) = \int_D G(\mathbf{x} - \mathbf{y}, \Delta)\phi(\mathbf{y}, t)\, d\mathbf{y} \tag{3}$$

where G is the filter function, and Δ is the filter width.

Eddies larger than the filter width are computed numerically, while the smaller eddies are calculated after using a model, because they are more homogeneous by nature [3].

The model

The model

There are some possible functions for G, in spatial domain and in Fourier domain.

The model

There are some possible functions for G, in spatial domain and in Fourier domain.

The Gaussian filter in spatial domain [3]:

$$G_r(\mathbf{x}-\mathbf{y},\Delta) = \left(\frac{6}{\pi\Delta^2}\right)^{3/2} \exp\left[-6\frac{|\mathbf{x}-\mathbf{y}|^2}{\Delta^2}\right] \qquad (4)$$

The model

There are some possible functions for G, in spatial domain and in Fourier domain.

The Gaussian filter in spatial domain [3]:

$$G_r(\mathbf{x}-\mathbf{y},\Delta) = \left(\frac{6}{\pi\Delta^2}\right)^{3/2} \exp\left[-6\frac{|\mathbf{x}-\mathbf{y}|^2}{\Delta^2}\right] \tag{4}$$

For the velocity:

$$\overline{\mathbf{u}} = \int_D G_r(\mathbf{x}-\mathbf{y},\Delta)\cdot\mathbf{u}(\mathbf{y},t) \tag{5}$$

The model

There are some possible functions for G, in spatial domain and in Fourier domain.

The Gaussian filter in spatial domain [3]:

$$G_r(\mathbf{x}-\mathbf{y},\Delta)=\left(\frac{6}{\pi\Delta^2}\right)^{3/2}\exp\left[-6\frac{|\mathbf{x}-\mathbf{y}|^2}{\Delta^2}\right] \tag{4}$$

For the velocity:

$$\overline{\mathbf{u}}=\int_D G_r(\mathbf{x}-\mathbf{y},\Delta)\cdot\mathbf{u}(\mathbf{y},t) \tag{5}$$

And similarly for Reynolds decomposition:

$$\mathbf{u}^{'}\equiv\mathbf{u}-\overline{\mathbf{u}} \tag{6}$$

The model

The model

Graphically, the filter operation looks like this:

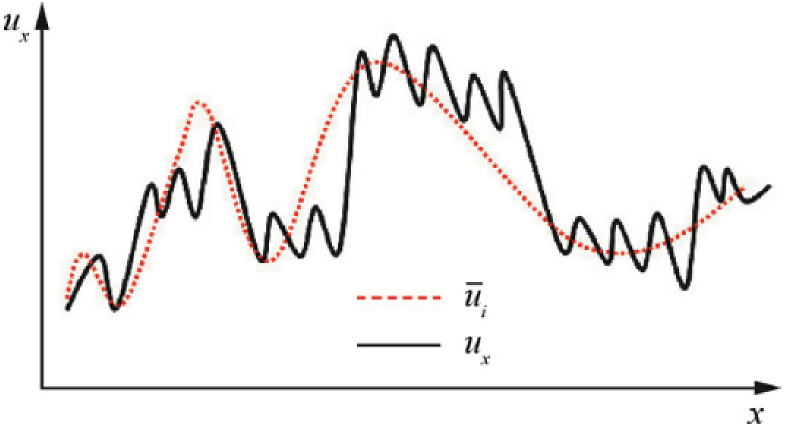

Figure: Velocity and filtered velocity [2]

Filtered Equations

Filtered Equations

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{7}$$

Filtered Equations

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{7}$$

$$\rho \left(\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} \right) = \rho f_i - \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial (2\mu \overline{S}_{ij})}{\partial x_j} + \frac{\partial \tau_{ij,\mathrm{SGS}}}{\partial x_j} \tag{8}$$

Filtered Equations

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{7}$$

$$\rho\left(\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j}\right) = \rho f_i - \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial (2\mu \overline{S}_{ij})}{\partial x_j} + \frac{\partial \tau_{ij,\mathrm{SGS}}}{\partial x_j} \tag{8}$$

$$\rho\left(\frac{\partial \overline{h}}{\partial t} + \frac{\partial \overline{h}\overline{u}_j}{\partial x_j}\right) = \frac{D\overline{p}}{Dt} + \rho f_j \overline{u}_j - \frac{\partial \overline{q}_j}{\partial x_j} + \dot{q} + \frac{\partial H_j}{\partial x_j} \tag{9}$$

Filtered Equations

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{7}$$

$$\rho\left(\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j}\right) = \rho f_i - \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial (2\mu \overline{S}_{ij})}{\partial x_j} + \frac{\partial \tau_{ij,\mathrm{SGS}}}{\partial x_j} \tag{8}$$

$$\rho\left(\frac{\partial \overline{h}}{\partial t} + \frac{\partial \overline{h}\overline{u}_j}{\partial x_j}\right) = \frac{D\overline{p}}{Dt} + \rho f_j \overline{u}_j - \frac{\partial \overline{q}_j}{\partial x_j} + \dot{q} + \frac{\partial H_j}{\partial x_j} \tag{9}$$

Filtered Equations

Filtered Equations

where

$$\tau_{ij,\mathsf{SGS}} = -\rho(\overline{u_i u_j} - \overline{u}_i \overline{u}_j) \tag{10}$$

Filtered Equations

where

$$\tau_{ij,\mathrm{SGS}} = -\rho(\overline{u_i u_j} - \overline{u}_i \overline{u}_j) \tag{10}$$

$$H_j = -\rho((\overline{h u_j} - \overline{h}\overline{u}_j)) \tag{11}$$

Modeling the SGS terms

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.

Eddy viscosity is modeled using Boussinesq hypothesis to calculate eddy viscosity [3]:

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.

Eddy viscosity is modeled using Boussinesq hypothesis to calculate eddy viscosity [3]:

$$\tau_{ij,\mathrm{SGS}} - \frac{1}{3}\delta_{ij}\tau_{kk,SGS} = 2\rho\nu_T\overline{S}_{ij} \tag{12}$$

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.

Eddy viscosity is modeled using Boussinesq hypothesis to calculate eddy viscosity [3]:

$$\tau_{ij,\mathrm{SGS}} - \frac{1}{3}\delta_{ij}\tau_{kk,SGS} = 2\rho\nu_T\overline{S}_{ij} \tag{12}$$

The SGS heat flux is modeled with gradient-diffusion model [3]:

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.
Eddy viscosity is modeled using Boussinesq hypothesis to calculate eddy viscosity [3]:

$$\tau_{ij,\mathrm{SGS}} - \frac{1}{3}\delta_{ij}\tau_{kk,SGS} = 2\rho v_T \overline{S}_{ij} \tag{12}$$

The SGS heat flux is modeled with gradient-diffusion model [3]:

$$H_j = \frac{\mu_T}{\mathrm{Pr}_T}\frac{\partial \overline{h}}{\partial x_j} \tag{13}$$

Haecheon Choi and Parviz Moin.
Grid-point requirements for large eddy simulation: Chapman's estimates revisited.
Physics of Fluids, 24(1), January 2012.
Publisher: AIP Publishing.

Yang Zhiyin.
Large-eddy simulation: Past, present and the future.
Chinese Journal of Aeronautics, 28(1):11–24, February 2015.

G W Zou, S L Liu, W K Chow, and Y Gao.
Large eddy simulation of turbulent flow.
7(1), December 2006.