Ben Gurion University of the Negev

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Research proposal

Turbulent Flow course Project

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The problam

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [**?**].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.
  - Tries to take in wide range of scales. Small scales depend more on ν, while large scale depend more on BC.
  - The constants for this model are sometimes gard to optimize.
- The combination of the two $\rightarrow$ **LES**

About LES

- Invented by Dr. Joseph Smagorinsky (1924-2005), meteorologist and founding director of NOAA's Geophysical Fluid Dynamics Laboratory [**?**]
- The idea is to solve the large scales and model the small scales.
- The computational cost is proportional to $Re^{9/4}$, one order of magnitude less than DNS.

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\bar{\phi}(\mathbf{x}, t)}_{largescale} + \underbrace{\phi'(\mathbf{x}, t)}_{smallscale} \tag{2}$$

The filtering operation is defined as:

$$\overline{\phi}(\mathbf{x}, t) = \int_D G(\mathbf{x} - \mathbf{x}^*, \Delta)\phi(\mathbf{x}^*, t)\, d\mathbf{x}^* \tag{3}$$

where G is the filter function, and Δ is the filter width.
Eddies larger than the filter width are computed numerically, while the smaller eddies are calculated after using a model, because they are more homogeneous by nature. after using a model.

The model

There are some possible functions for G, in spatial domain and in Fourier domain. $1{=}1{+}x$

Example

Equations