Ben Gurion University of the Negev

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Research proposal

Turbulent Flow course Project

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The problam

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f} \tag{1}$$

Possible solutions and their fault

- DNS.
  - Heavy computational cost, proportional to $Re^{11/4}$ [3].
  - Sensitive to IC.
  - High order schemes are needed, which are not flexible to different geometries (pseudo-spectral methods).
- RANS + some model for Reynolds stresses.
  - Tries to take in wide range of scales. Small scales depend more on ν, while large scale depend more on BC.
  - The constants for this model are sometimes gard to optimize.
- The combination of the two $\rightarrow$ **LES**

About LES

- Invented by Dr. Joseph Smagorinsky (1924-2005), meteorologist and founding director of NOAA's Geophysical Fluid Dynamics Laboratory [2]
- The idea is to solve the large scales and model the small scales.
- The computational cost is proportional to $Re^{13/7}$, one order of magnitude less than DNS [1].

The Model

Instead of the Reynolds decomposition, we use the filter decomposition:

$$\phi(\mathbf{x}, t) = \underbrace{\overline{\phi}(\mathbf{x}, t)}_{\text{resolved scale}} + \underbrace{\phi'(\mathbf{x}, t)}_{\text{subgrid scale}} \tag{2}$$

The filtering operation is defined as:

$$\overline{\phi}(\mathbf{x}, t) = \int_D G(\mathbf{x} - \mathbf{y}, \Delta)\phi(\mathbf{y}, t)\, d\mathbf{y} \tag{3}$$

where G is the filter function, and Δ is the filter width.
Eddies larger than the filter width are computed numerically, while the smaller eddies are calculated after using a model, because they are more homogeneous by nature [3].

The model

There are some possible functions for G, in spatial domain and in Fourier domain.

For example, the Gaussian filter in spatial domain [3]:

$$G_r(\mathbf{x}-\mathbf{y},\Delta) = \left(\frac{6}{\pi\Delta^2}\right)^{3/2} \exp\left[-6\frac{|\mathbf{x}-\mathbf{y}|^2}{\Delta^2}\right] \tag{4}$$

For the velocity:

$$\overline{\mathbf{u}} = \int_D G_r(\mathbf{x}-\mathbf{y},\Delta)\cdot\mathbf{u}(\mathbf{y},t) \tag{5}$$

And similarly for Reynolds decomposition:

$$\mathbf{u}^{'} \equiv \mathbf{u} - \overline{\mathbf{u}} \tag{6}$$

The model

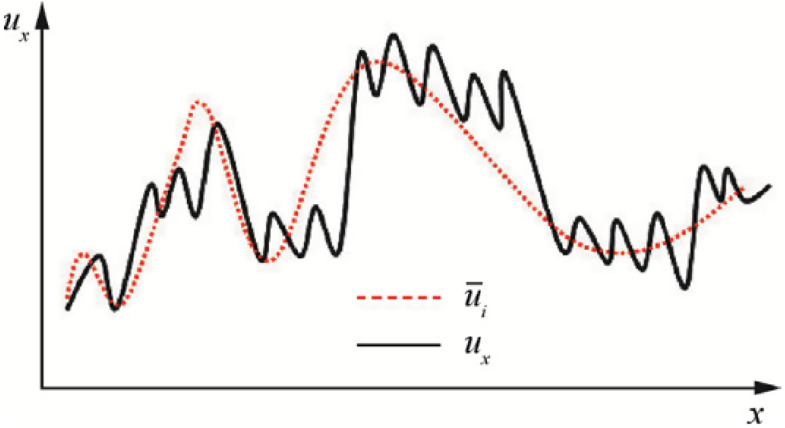

Figure: Velocity and filtered velocity [2]

Filtered Equations

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{7}$$

$$\rho \left(\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} \right) = \rho f_i - \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} + \frac{\partial (2\mu \overline{S}_{ij})}{\partial x_j} + \frac{\partial \tau_{ij,\mathrm{SGS}}}{\partial x_j} \tag{8}$$

$$\rho \left(\frac{\partial \overline{h}}{\partial t} + \frac{\partial \overline{h}\overline{u}_j}{\partial x_j} \right) = \frac{D\overline{p}}{Dt} + \rho f_j \overline{u}_j - \frac{\partial \overline{q}_j}{\partial x_j} + \dot{q} + \frac{\partial H_j}{\partial x_j} \tag{9}$$

where

$$\tau_{ij,\mathrm{SGS}} = -\rho(\overline{u_i u_j} - \overline{u}_i \overline{u}_j) \tag{10}$$

$$H_j = -\rho((\overline{h u_j} - \overline{h}\overline{u}_j)) \tag{11}$$

Modeling the SGS terms

Both $\tau_{ij,\mathrm{SGS}}$ and H_j require modeling.

Eddy viscosity is modeled using Boussinesq hypothesis to calculate eddy viscosity [3]:

$$\tau_{ij,\mathrm{SGS}} - \frac{1}{3}\delta_{ij}\tau_{kk,SGS} = 2\rho v_T \overline{S}_{ij} \tag{12}$$

The SGS heat flux is modeled with gradient-diffusion model [3]:

$$H_j = \frac{\mu_T}{\mathrm{Pr}_T}\frac{\partial \overline{h}}{\partial x_j} \tag{13}$$

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