

1.) a.) Fixed points:  $s = f(w(s - \frac{1}{2}))$

$$b = -\frac{w}{2}$$

$$f'(s) = \frac{d}{ds} \left( \frac{e^s}{1+e^s} \right) = \frac{e^s}{(e^s+1)^2}$$

max is at  $x=0$  where  $f'(0) = \frac{1}{4}$

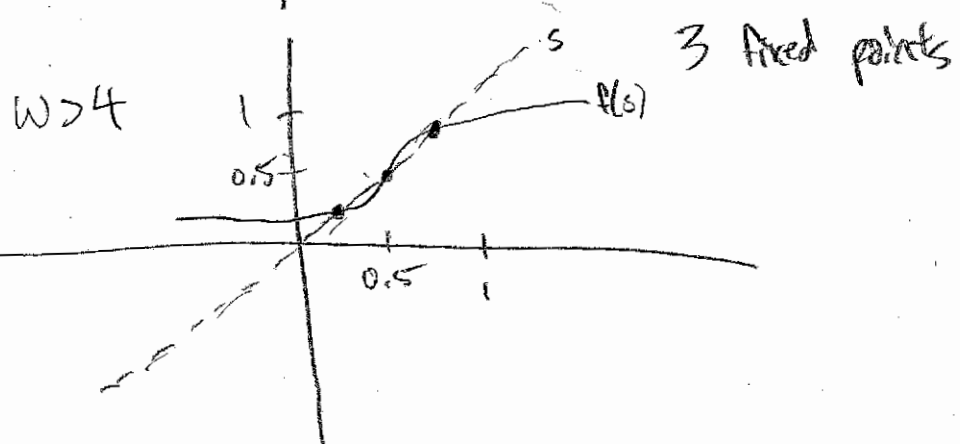
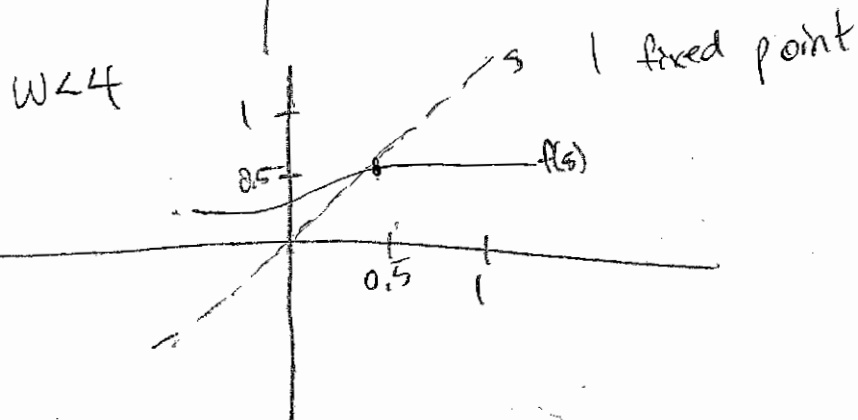
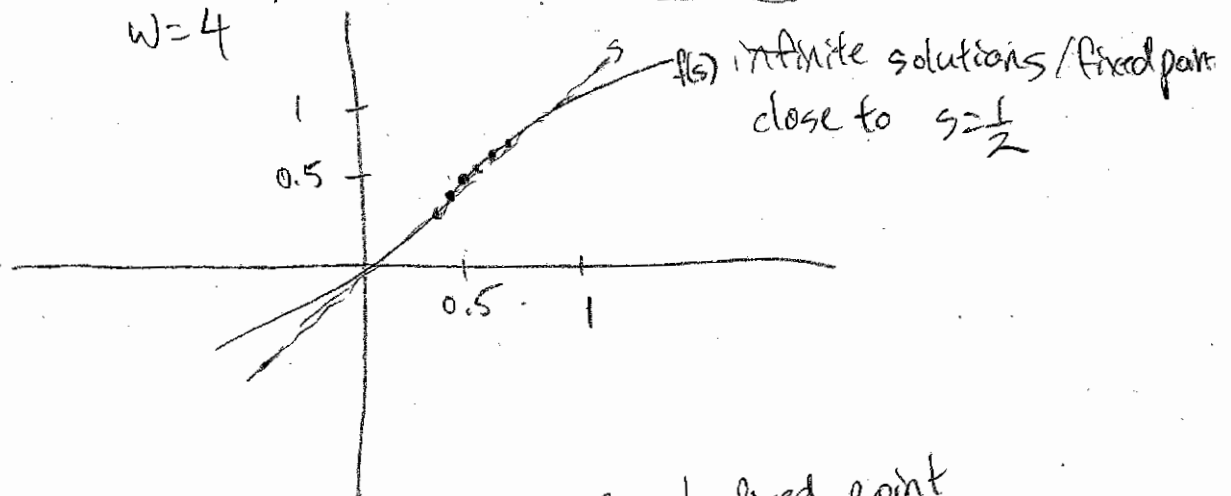
We need to find  $w^*$  such that  $f(w^*(s - \frac{1}{2})) = 1$

at  $s = \frac{1}{2}$

Since  $f'(0) = \frac{1}{4}$  and  $s=0$  is the max of the derivative, we want

$$w^*(\frac{1}{4}) = 1, \text{ so naturally } \boxed{w^* = 4}$$

$$w = 4$$



2.) For  $W=2$ : ( $W < W^*$ )

Fixed points at

$$0 = -s + f(2s-1)$$

$$s = f(2s-1)$$

$$s = \frac{1}{2}$$

$$f'(\frac{1}{2}) = 0.5 \rightarrow \text{stable}$$

For  $W=6$ : ( $W > W^*$ )

Fixed points at  $s = f(6s-3)$

$$s = \frac{e^{6s-3}}{1+e^{6s-3}}$$

$$s = 0.5, 0.0707, 0.9293$$

$$f'(0.0707) < 1 \rightarrow \text{stable}$$

$$f'(\frac{1}{2}) > 1.5 \rightarrow \text{not stable}$$

$$f'(0.9293) < 1 \rightarrow \text{stable}$$

bistable