EE 385V HW#1

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1.) a.) $V(t)$ must reach a certain V_θ to spike, so I^* must be large enough that $V(t) \geq V_\theta$

Substitute $V(t) = V_\theta$ into equation from lecture, $V_\theta = V_m + \frac{I}{g_m}$

$$\therefore V_\theta - V_m = \frac{I}{g_m}$$

$$\boxed{I = g_m(V_\theta - V_m)}$$

b.) $C_m \frac{dV}{dt} = -g_m(V(t) - V_m) + I \rightarrow$ make constant $\qquad \tau_m = \frac{C_m}{g_m} = C_m R_m$

$$V(t) = V_m + \frac{I}{g_m} + \left(V(0) - V_m - \frac{I}{g_m}\right)e^{-\frac{t}{\tau_m}} \quad \text{(5.8 in book)}$$

$V(0) = V_{reset}$ assuming $t = 0$ is when neuron just fired

now set $t = t_{isi}$, the time of next spike

$$V(t_{isi}) = V_\theta = V_m + \frac{I}{g_m} + \left(V_{reset} - V_m - \frac{I}{g_m}\right)e^{-\frac{t_{isi}}{\tau_m}}$$

solve for t_{isi}

$$\frac{V_\theta - V_m - \frac{I}{g_m}}{V_{reset} - V_m - \frac{I}{g_m}} = e^{-\frac{t_{isi}}{\tau_m}}$$

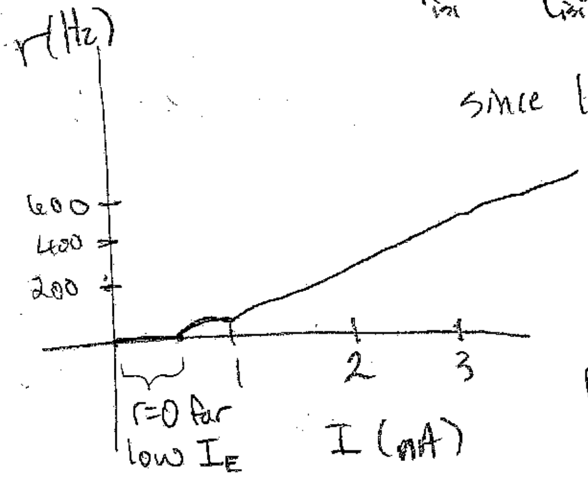
$r = \frac{1}{t}$

$$t_{isi} = -\tau_m \ln\left(\frac{\frac{I}{g_m} + V_m - V_\theta}{\frac{I}{g_m} + V_m - V_{reset}}\right) = \tau_m \ln\left(\frac{\frac{I}{g_m} + V_m - V_{reset}}{\frac{I}{g_m} + V_m - V_\theta}\right)$$

$$r_{isi} = \frac{1}{t_{isi}} = \boxed{\frac{1}{\tau_m \ln\left(\frac{\frac{I}{g_m} + V_m - V_{reset}}{\frac{I}{g_m} + V_m - V_\theta}\right)}}$$

since $\ln(1+x) \approx x$, we can transform the above:

$$\approx \frac{1}{\tau_m \ln\left(1 + \frac{V_\theta - V_{reset}}{\frac{I}{g_m} + V_m - V_\theta}\right)} \approx \boxed{\frac{\frac{I}{g_m} + V_m - V_\theta}{\tau_m(V_\theta - V_{reset})}}$$



For $V_m = -60\,mV$, $g_m = 0.1\,mS/cm^2$, $C_m = 1\,\mu F/cm^2$, $V_\theta = -50\,mV$, $V_{reset} = -55\,mV$

c.) $r_{isi} = \dfrac{1}{\tau_m \ln\left(\boxed{\dfrac{\frac{I}{g_m}+V_m-V_{reset}}{\frac{I}{g_m}+V_m-V_\theta}}\right)} = \dfrac{1}{\tau_m \ln\left(1+\dfrac{V_\theta - V_{reset}}{\frac{I}{g_m}+V_m-V_\theta}\right)}$

Taylor Expansion $\ln(1+x) \doteq x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

converges $\approx x$

now substitute:

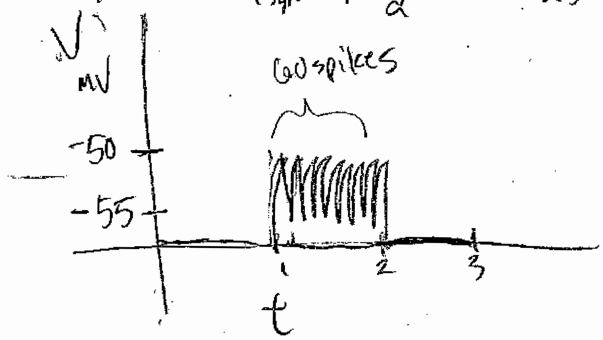
$r_{isi} \approx \dfrac{1}{\tau_m\left(\dfrac{V_\theta - V_{reset}}{\frac{I}{g_m}+V_m-V_\theta}\right)} = \boxed{\dfrac{\frac{I}{g_m}+V_m-V_\theta}{\tau_m(V_\theta - V_{reset})}}$ → linear in terms of I_{ext}

2.) a.) $\dfrac{ds}{dt} = -\dfrac{s}{\tau_{syn}} + \beta \sum_\alpha \delta(t-t_\alpha)$ $\qquad s(0)=0$

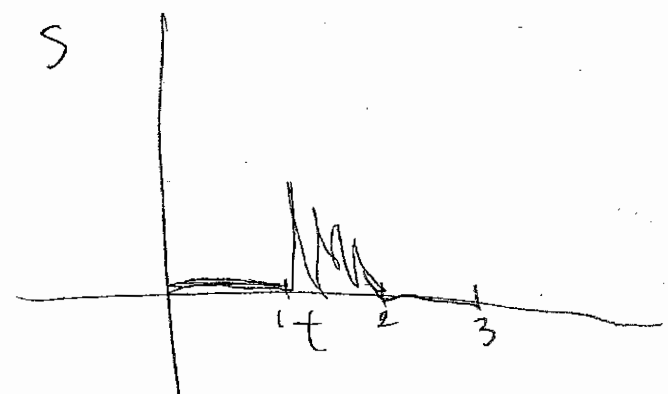
$f(t,s) = -\dfrac{s}{\tau_{syn}} + \beta\sum_\alpha \delta(t-t_\alpha)$

$s_{n+1} = s_n + hf(t_n, s_n)$

n	t_n	s_n	$f(t_n, s_n)$	[illegible]
0	0	0		0
1	0.1	0		



not sure [illegible]
in using [illegible]?



b.) let t_{isi} = one inter-spike-interval

$$\frac{ds}{dt} = -\frac{s}{\tau_{syn}} + \beta \sum_{\alpha} \delta(t - t_{\alpha})$$

$$\frac{1}{t_{isi}} \int_{t}^{t+t_{isi}} \sum_{\alpha} \delta(t - t_{\alpha})\, dt = \frac{1}{t_{isi}} = r_{isi} \leftarrow \text{firing rate from 1.}$$

$$\boxed{\frac{ds}{dt} = -\frac{s}{\tau_{syn}} + \beta r_{isi}}$$