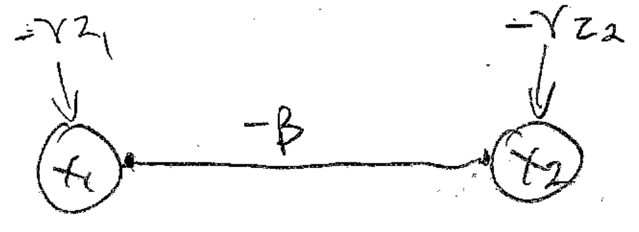
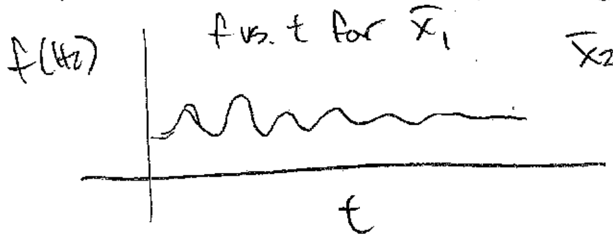
1.)



For $\frac{b_1}{b_2} > \frac{1+\gamma}{\beta}$,

$\tau >> 0$
$\beta > 1$
$b_1 > b_2 > 0$
$x_1(0) = 0$
$x_2(0) = 0$

the system is Lyapunov stable, because the fixed points near $\bar{x}_1$ and $\bar{x}_2$ stay near them:



$\bar{x}_2$ is suppressed, so $\lim_{t \to \infty} \bar{x}_2 = 0$

this is because $b_1 > b_2$.

For $\frac{b_1}{b_2} \leq \frac{1+\gamma}{\beta}$, the system is unstable because the nearby points are pushed away from the fixed points, and x_1 and x_2 take turns switching on and off

2.) a.i) Suppose s_i changes (updates) to s_i' and E' represents the energy function for s_i'.

$\Rightarrow \Delta E = E' - E =$

$$= -\frac{1}{2}\sum_{i,j=1} w_{ij} s_i' s_j' + \frac{1}{2}\sum_{i,j=1} w_{ij} s_i s_j$$

* $w_{ij} = w_{ji}$
* $w_{ii} = 0$

$$\Rightarrow \Delta E = (s_i - s_i') \sum_{j \neq i} w_{ij} s_j$$

case 1: $s_i = s_i' \Rightarrow \Delta E = 0$

case 2: $s_i = 1, s_i' = -1 \Rightarrow \Delta E = 2\boxed{\sum_{j \neq i} w_{ij} s_j} < 0$ — < 0 b.c. $\mathrm{sign}(u) = -1$

case 3: $s_i = -1, s_i' = 1 \Rightarrow \Delta E = -2\boxed{\sum_{j \neq i} w_{ij} s_j} \leq 0$ — ≥ 0 b.c. $\mathrm{sign}(u) = 1$

$\therefore$ $\Delta E \leq 0$ in all cases, which means E is non-increasing

AND E is lower bounded by $E(s_i)$ for some s_i because there are only a finite number of states.

$\boxed{\text{This proves (i) and (ii) in the problem set.}}$

b.) If $s_i = \mathrm{sign}\left(\sum_j w_{ij} s_j + b_i\right)$, then we change the energy function such that the bias b_i makes E the sum of all the states with their biases: $\sum_i s_i b_i$ in addition to the [illegible] connection weight. Thus;

$$\boxed{E = \sum_i s_i b_i - \frac{1}{2}\sum_{i,j=0} w_{ij} s_i s_j}$$

c.) For $H(u)$, we need to find E such that E is non-increasing for all $s_i, s_i' \in \{0,1\}$. Intuitively, it should be the same as E for $\text{sign}(u)$ because the cases still hold:

$$\Delta E = -(s_i - s_i')\left(\sum_{j\neq i} w_{ij}s_j + b_i\right)$$

case 1: $s_i = s_i' \Rightarrow \Delta E = 0$

case 2: $s_i = 1,\ s_i' = 0 \Rightarrow \Delta E = \sum_{j\neq i} w_{ij}s_j \leq 0$ ≤0 because $H(u)=0$

case 3: $s_i = 0,\ s_i' = 1 \Rightarrow \Delta E = -\sum_{j\neq i} w_{ij}s_j < 0$ >0 b.c. $H(u)=1$

$\Delta E \leq 0$ in all cases

$$\boxed{\therefore\ E = -\frac{1}{2}\sum_{ij\neq 0} w_{ij}s_is_j + \sum_i s_ib_i \text{ is still valid}}$$

3.) a.) As p increases, there are more crosstalk terms $C_i^v > 1$, and this leads to memory corruption. The shape of the normal curve becomes flatter.

b.) i.) $$\sum_j w_{ij}\xi_j^v = \frac{1}{Nf(1-f)}\sum_j\sum_\mu(\xi_i^\mu - f)(\xi_j^\mu - f)\xi_j^v$$

$$= \frac{1}{Nf(1-f)}\sum_j(\xi_i^v - \overset{\to 0}{f})(\xi_j^v - \overset{\to 0}{f})\xi_j^v + \frac{1}{Nf(1-f)}\sum_j\sum_{\mu\neq v}(\xi_i^\mu - f)(\xi_j^\mu - f)\xi_j^v$$

for Large N and small f

$$\approx \boxed{\xi_i^v + \underbrace{\frac{1}{Nf(1-f)}\sum_j^N\sum_{\mu\neq v}^p(\xi_i^\mu - f)(\xi_j^\mu - f)\xi_j^v}_{\text{crosstalk}}}$$

ii.) Since N is large and f is small, the term C_i^v is a random variable that is $\frac{1}{N(f(1-f))}\cdot \sim NP$ random variables.
Since $x_i \in \{0,1\}$, C_i^v is a normal distribution with 0 mean

iv.) $Q_i = 0$

v.) With $f = 0.05$, $P \approx -6.676\, N$. My results showed that for varying P and f, following the formula $P = \frac{N}{2f|\log(f)|}$ gave me $P(C_i^v > 1) \approx 0.014$, which is close to the Perr target.

c.) The capacity of the sparse network is greater