Q8.4.

$$G(s) = \frac{w_n^2}{s^2 + 2w_n\xi s + w_n^2} = \frac{100^2}{s^2 + 2\cdot 100 \cdot 0.5\, s + 100^2}$$

$$= \frac{10000}{s^2 + 100s + 10000}.$$

$$\mathcal{L}(\text{step response}) = G(s)\cdot \mathcal{L}(\text{step input}) = \frac{10000}{s^2+100s+10000}\cdot\frac{1}{s}$$

$$= \frac{As+B}{s^2+100s+10000} + \frac{C}{s}, \quad 10000 = (As+B)s + (s^2+100s+10000)C$$

$$= s^2(A+C) + s(B+100C) + 10000C$$

$C=1$, $A+C=0$, $A=-1$, $B+100C=0$, $B=-100$.

Therefore, $\mathcal{L}(\text{step response}) = -\frac{s+100}{s^2+100s+10000} + \frac{1}{s}$

$$(\text{step response}) = \mathcal{L}^{-1}\left(-\frac{s+100}{s^2+100s+10000} + \frac{1}{s}\right)$$

$$= -\mathcal{L}^{-1}\left(\frac{s+50}{(s+50)^2+7500}\right) - \mathcal{L}^{-1}\left(\frac{50}{(s+50)^2+7500}\right) + \mathcal{L}^{-1}\left(\frac{1}{s}\right)$$

$$= -e^{-50t}\cos(\sqrt{7500}\,t) - \frac{50}{\sqrt{7500}}\cdot e^{-50t}\sin(\sqrt{7500}\,t) + 1$$

$$= \boxed{-e^{-50t}\left(\cos(\sqrt{7500}\,t) + \frac{\sqrt{2}}{3}\sin(\sqrt{7500}\,t)\right) + 1}$$

Q8.5.

(a) $y(s) = \dfrac{s+3}{(s^2+2s+2)(s+1)} = \dfrac{s+3}{(s+1+j)(s+1-j)(s+1)}$

complex poles: $-\zeta w_n \pm j w_n \sqrt{1-\zeta^2} = -1 \pm j$

$\zeta = \frac{\sqrt{2}}{2}, \quad w_n = \sqrt{2}, \quad \zeta w_n = 1$

real pole: $-\alpha_r = -1, \quad \alpha_r = 1$

Since α_r is not much greater than ζw_n, the system cannot be represented as a second order system.

(b) $y(s) = \dfrac{1}{(s^2+2s+2)(s+20)} = \dfrac{1}{(s^2+2\zeta w_n s + w_n^2)(s+20)}$

$w_n = \sqrt{2}, \quad \zeta = \frac{\sqrt{2}}{2}, \quad \alpha_r = +20.$

Since $\alpha_r \gg w_n \zeta$, we can approximate the system to second-order system.

$y(s) \approx \dfrac{1}{20(s^2+2s+2)}$. where $w_n = \sqrt{2}, \ \zeta = \frac{\sqrt{2}}{2}$

$PO = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\% = e^{\frac{-\frac{\sqrt{2}}{2}\cdot\pi}{\sqrt{1-(\frac{\sqrt{2}}{2})^2}}} \times 100\%$

$= e^{\frac{-\sqrt{\frac{1}{2}}\cdot\pi}{\sqrt{\frac{1}{2}}}} \times 100\% = e^{-\pi} \times 100\% = \boxed{4.321\%}$ (PO)

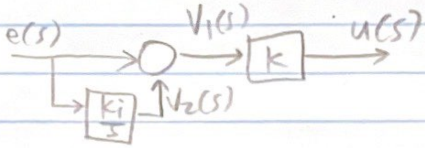
Settling time (2%) $= \dfrac{-\ln(0.02\sqrt{1-\zeta^2})}{\zeta w_n} = \dfrac{-\ln(0.02\sqrt{\frac{1}{2}})}{\frac{\sqrt{2}}{2}\cdot\sqrt{2}}$

$= -\ln(0.02\sqrt{\tfrac{1}{2}}) = \boxed{4.259 s}$ (settling time, 2%)

Rise time: $\dfrac{0.5\pi + \sin^{-1}\zeta}{w_n\sqrt{1-\zeta^2}} = \dfrac{0.5\pi + \sin^{-1}\frac{\sqrt{2}}{2}}{\sqrt{2}\cdot\sqrt{\frac{1}{2}}} = \boxed{2.356 s}$ (rise time)

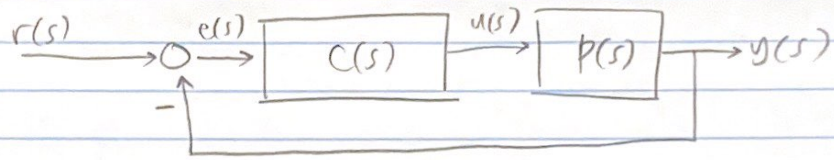
peak time: $\dfrac{\pi}{w_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{\sqrt{2}\cdot\sqrt{\frac{1}{2}}} = \pi = \boxed{3.142 s}$ (peak time)

Q8.8.



$$u(s) = k\cdot V_1(s) = k(e(s)+V_2(s)) = k\left(e(s)+\frac{k_i}{s}e(s)\right)$$
$$= \left(k+\frac{k\cdot k_i}{s}\right)e(s), \quad \frac{u(s)}{e(s)} = k+\frac{k\cdot k_i}{s}.$$

The diagram can be simplified to



where $C(s) = k+\frac{k\cdot k_i}{s}$, $P(s) = \frac{b}{s+a}$

(a) $a_{cl}(s) = (s+3+j2)(s+3-j2) = s^2+6s+13 = s^2+2\zeta w_n s + w_n^2$

$w_n = \sqrt{13}$, $\zeta = \frac{6}{2w_n} = \frac{6}{2\sqrt{13}} = \frac{3}{13}\sqrt{13}$ $\quad \boxed{w_n=\sqrt{13},\ \zeta=\frac{3}{13}\sqrt{13}}$

(b) $$S(s) = \frac{1}{1+P(s)C(s)} = \frac{1}{1+\frac{2}{s+2}\left(k+\frac{k\cdot k_i}{s}\right)} = \frac{1}{1+\frac{2k}{s+2}+\frac{2kk_i}{(s+2)s}}$$
$$= \frac{(s+2)s}{(s+2)s+2ks+2k\cdot k_i} = \frac{s^2+2s}{s^2+(2+2k)s+2k\cdot k_i},$$

$2+2k=6$, $2k\cdot k_i = 13$, $\boxed{k=2,\ k_i=\frac{13}{4}}$

(c) We should check if for C_1 and C_2, $s^2+C_1s+C_2$ always has solution for k and k_i, where $2+2k=C_1$, and $2k\cdot k_i = C_2$. If $C_1=2$ and $C_2\neq 0$, $k=0$, and we cannot find k_i. In other words, if $a_{cl}(s) = s^2+2s+C_2 = (s+1+jm)(s+1-jm)$, where $m\in\mathbb{R}$, there are poles at $-1\pm jm$ yet (k, k_i) cannot be find. Therefore, the statement is false.

(d) If $a_{cl}(s) = (s+C_1+jC_2)(s+C_1-jC_2)$, where $C_1 << 0$, $a_{cl}(s) = s^2+2C_1s+C_1^2+C_2^2$. This means $2+2k=C_1$, $k=\frac{C_1}{2}-1>>0$ and $k_i = \frac{C_1^2+C_2^2}{2k}>>0$. This means the gains are big, having risk that $u(s)$, the output of the controller, is too big. This can be problematic since there is a limit in the magnitude of the current that can flow without damaging the circuit.