

# Simplex

$$\text{maximize } Z = 18x_1 + 12.5x_2$$

$$\text{subject to } x_1 + x_2 \leq 20$$

$$x_1 \leq 12$$

$$x_2 \leq 16$$

$$x_1, x_2 \geq 0$$

Convert to equations:

Add slack variable.

$$x_1 + x_2 + y_1 = 20$$

$$-18x_1 - 12.5x_2 + z = 0$$

Adding slack variables:

$$-18x_1 - 12.5x_2 + z = 0$$

$$x_1 + x_2 + y_1 = 20$$

$$x_1 + y_2 = 12$$

$$x_2 + y_3 = 16$$

$$x_1 \geq 0, x_2 \geq 0$$

# Simplex Tableau

$x_1$     $x_2$     $y_1$     $y_2$     $y_3$     $z$     $C$

1   1   1   0   0   0   20

1   0   0   1   0   0   12

0   1   0   0   1   0   16

-18   -12.5   0   0   0   1   0

# Basic Solution #1

$$\left[ \begin{array}{cccc|c} y_1 & y_2 & y_3 & z & C \\ 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 16 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$y_1 = 20$$

$$y_3 = 16$$

$$y_2 = 12$$

$$z = 0$$

Find the most negative value in bottom row:

This is the **pivot** column.

$x_1$	$x_2$	$y_1$	$y_2$	$y_3$	$z$	$C$
1	1	1	0	0	0	20
1	0	0	1	0	0	12
0	1	0	0	1	0	16
-18	-12.5	0	0	0	1	

# Calculate Quotients

row<sub>1</sub>     $20 \div 1 = 20$      $y_1$

row<sub>2</sub>     $12 \div 1 = 12$      $y_2$

We don't consider row<sub>3</sub>  $\div 0$

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Now we will work with  
entering & departing  
variables.

Entering  $x_1$

Departing  $y_2$

# Perform Pivot

$$R_{\text{new}_1}(\text{new}) = R_1(\text{old}) - R_2(\text{new})$$

$x_1$	$x_2$	$y_1$	$y_2$	$y_3$	$z$	$c$
0	1	1	-1	0	0	8
1	0	0	1	0	0	12
0	1	0	0	1	0	16
0	-12.5	0	18	0	1	216

## Basic Solution #2

$$\left[ \begin{array}{cccc|c} x_1 & y_1 & y_3 & z & C \\ 0 & 1 & 0 & 0 & 8 \\ 1 & 0 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 16 \\ 0 & 0 & 0 & 1 & 216 \end{array} \right]$$

$$x_1 = 12 \quad y_1 = 8$$

$$y_3 = 16$$

$$z = 216$$



# Calculate Quotients

$$\text{Row}_1: 8 \div 1 = 8$$

$$\text{Row}_2: 16 \div 1 = 16$$

$$\text{Row}_3(\text{NEW}) = R_3(OLD) - R_1(\text{NEW})$$

$x_1$	$x_2$	$y_1$	$y_2$	$y_3$	$z$	$C$
0	1	1	-1	0	0	8
1	0	0	1	0	0	12
0	0	-1	1	1	0	8
0	0	12.5	30.5	0	0	316

# Optimal Solution

$$x_1 = 12$$

$$x_2 = 8$$

$$\text{Max } Z = 316$$

