Substitution Method

$$T(n) = \begin{cases} nT(n-1) & \text{if } n > 1 \\ 1, & \text{if } n = 1 \end{cases}$$

$$T(n-1) = (n-1) \cdot T(n-2)$$

$$T(n-2) = (n-2) \cdot T(n-3)$$

$$T(n) = n \cdot (n-1) \cdot T(n-2)$$

$$T(n) = n \cdot (n-1) \cdot (n-2) \cdot T(n-3)$$

$$T(n) = T(n-k) \cdot (n-(k-1)) \cdot (n-(k-2)) \ldots$$

$$\ldots (n-1) \cdot n$$

Set $n-k$ equal to 1.

$$T(n) = T(1) \cdot k-(k-1) \cdot k-(k-2) \dots$$

$$\dots \cdot k-(k-1) \cdot k$$

$$T(n)\ 1 \cdot 1 \cdot 2 \cdot k-1 \cdot k$$

$$T(n) = 1 + k!$$

$$O(n!)$$

$$T(n) = \begin{cases} T(n-1) + \log n & n > 1 \\ 1, & n = 1 \end{cases}$$

$$T(n-1) = T(n-2) + \log(n-1)$$

$$T(n-2) = T(n-3) + \log(n-2)$$

$$T(n) = T(n-2) + \log(n-1) + \log n$$

$$T(n) = T(n-3) + \log(n-2) + \log(n-1) \ldots$$

$$\ldots + \log(n)$$

$$T(n) = T(n-K) + \log(n-(K-1) \ldots$$

$$\ldots + \log(n-(K-2) \ldots \log(K-1) \ldots$$

$$\ldots + \log n$$

Sub $K = n$

$T(n) = T(1) + \log(K - (K-1) \ldots$

$\ldots + \log(K(K-2)) \ldots \log(K-1) \ldots$

$\ldots + \log K$

$= T(K) + \log(1) + \log(2) \ldots$

$\ldots + \log K$

$= 1 + \log(1 \cdot 2 \cdot 3 \cdot K)$

$= 1 + \log(K!)$ ← Bound by $\log(K^K)$

$= 1 + \log(K^K)$

$= 1 + K \log K$

$= 1 + n \log n \Rightarrow O(n \log n)$

$$T(n) = \begin{cases} 3T(n/2) + n, & n > 1 \\ 1, & n = 1 \end{cases}$$

$$T(n) = 3T(n/2) + n$$

$$T(n/2) = 3T(n/4) + n/2$$

$$T(n/4) = 3T(n/8) + n/4$$

Sub $T(n/2)$ into $T(n)$

& $T(n/4)$ into $T(n/2)$

$$T(n) = 3(3T(n/4) + n) + n$$

$$= 9T(n/4) + 2n$$

$$T(n) = 3(3T(n/8) + n) + n$$

$$= 9T(n/8) + 2n$$

$$= 9(3T(n/8) + n/4)$$

$$= 27(n/8) + 3n$$

Pattern:

$$T(n) = 3^k T\left(\frac{n}{2^k}\right) + kn.$$

$$m = \frac{n}{2^k} \Rightarrow 3^k T(m) + kn$$

$k = \log_2(n)$ becomes zero.

$$T(n) = 3^{\log_2 n} T(1) + kn$$

$$T(n) = n^{\log_2 3} T(1) + kn.$$

Solution:

$$T(n) = C n^{\log_2 3} + kn$$

$$= O\left(n^{\log_2 3}\right)$$