1. $f(x) = x^4 - 2x^2 \qquad [-2, 2]$

$f'(x) = 4x^3 - 4x = 4x(x^2-1)$
$= 4x(x+1)(x-1)$

critical $= x=0, x=1, x=-1$

$f(0) = 0$
$f(1) = -1$
$f(-1) = -1$

check End point : $f(2) = 8 \quad f(-2) = 8$

absolute minimum $= (1,-1), (-1,-1)$
absolute maximum $= (-2,8), (2,8)$
Local Maximum $= (0,0)$

#

2. $f(x) = x^3 + 24x^2$

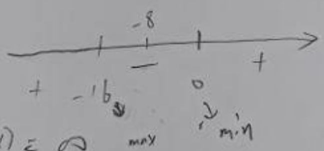
$f'(x) = 3x^2 + 48x = x(3x+48) = 3x(x+16)$ critical: $x = 0, -16$

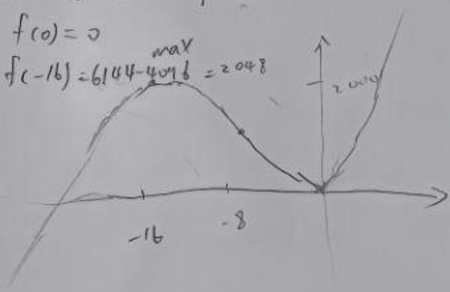
$f''(x) = 6x + 48 = 6(x+8)$ Inflection point: $x = -8$

$f(0) = 0$

max

$f(-16) = 6144 - 4096 = 2048$





$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

#

3. $x - y = 100$ $\quad x \times y = \text{minimum}$

$y = x - 100$

$P(x) = x \times y = x(x-100) = x^2 - 100x$

$P'(x) = 2x - 100$ critical point $x = 50$

$-$ 50 $+$

local ↓ minimal

$y = x - 100$

$y = -50$

$x = 50$

$y = -50$ #

4.

$$\int \frac{x^2+6}{x^2}\,dx$$

$$= \int \left(1 + \frac{6}{x^2}\right) dx$$

$$= \int (1 + 6x^{-2})\,dx$$

$$= x + 6x(-x^{-1}) + C$$

$$= x - \frac{6}{x} + C$$

5. $\int_0^1 x\sqrt{1-x^2}\,dx$

$u = 1 - x^2$

$\frac{du}{dx} = -2x \quad du = -2x\,dx \quad x = 0 \to u = 1$

$-\frac{1}{2}du = x\,dx \quad x = 1 \to u = 0$

$$\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 -\frac{1}{2}\sqrt{u}\,du$$

$$= -\frac{1}{2}\int_1^0 \sqrt{u}\,du$$

$$= -\frac{1}{2} \times \frac{2}{3}u^{\frac{3}{2}} \Big|_{u=1}^{u=0}$$

$$= 0 - \left(-\frac{1}{2} \times \frac{2}{3} \times 1^{\frac{3}{2}}\right) = \frac{1}{3}$$

6. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \int_{u=0}^{u=\pi} \frac{1}{2}\cos(u)\,du$$

$$= \frac{1}{2}\int_{u=0}^{u=\pi} \cos(u)\,du$$

$$= \frac{1}{2}\sin(u)\Big|_{u=0}^{u=\pi}$$

$$= \frac{1}{2}\times 0 - \frac{1}{2}\times 0$$

$$= 0$$ #

7. $\int_0^3 |2x^2-8|\,dx \quad [0,3]$

$$|2x^2-8| = \begin{cases} 8-2x^2 & \text{for } x\in[0,2] \\ 2x^2-8 & \text{for } x\in[2,3] \end{cases}$$

$$\int_0^3 |2x^2-8|\,dx = \int_0^2 8-2x^2\,dx + \int_2^3 2x^2-8\,dx$$

$$= \left[8x - 2\times\frac{1}{3}x^3\right]\Big|_{x=0}^{x=2} + \left[2\times\frac{1}{3}x^3 - 8x\right]\Big|_{x=2}^{x=3}$$

$$= 16-\frac{16}{3} + \left[(18-24) - \left(\frac{16}{3}-16\right)\right]$$

$$-6 + \frac{32}{3}$$

$$= \frac{32}{3} + \frac{14}{3}$$

$$= \frac{46}{3}$$ #

compute $f(x) = 4 - x^2$ $g(x) = 2 - x$

$4 - x^2 = 2 - x \Rightarrow x = -1,\ x = 2$

$f(x) > g(x)$ $f(x)$ is above $g(x)$

$$\int_{-1}^{2} (f(x) - g(x))\, dx = \int_{-1}^{2} 2 + x - x^2\, dx$$

$$= -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \Big|_{-1}^{2}$$

$$= \left(\frac{-8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$

$$= \frac{-8}{3} + 6 - \frac{1}{3} - \frac{1}{2} + 2$$

$$= 8 - 3 - \frac{1}{2} = 4\frac{1}{2} \ \#$$

$$= 4.5$$

9. $V = \frac{4}{3}\pi r^3$

$$\int_{-r}^{r} A(x)\, dx \Rightarrow A(x) = \pi \times (r^2 - x^2)$$

$$\Rightarrow \int_{-r}^{r} \pi \times (r^2 - x^2)\, dx = 2\pi \int_{0}^{r} (r^2 - x^2)\, dx$$

$$= 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

$$= 2\pi \left(r^3 - \frac{r^3}{3}\right)$$

$$= 2\pi \times \frac{2r^3}{3}$$

$$= \frac{4}{3}\pi r^3$$

10、 $x = 2\sqrt{y}$ 　面： $\pi(2\sqrt{y})^2$

$$\int_0^9 \pi(2\sqrt{y})^2 dy = \int_0^9 4\pi y\, dy = 4\pi \int_0^9 y\, dy$$

$$= 4\pi \left[\frac{1}{2}y^2\right]_0^9$$

$$= 4\pi \frac{81}{2}$$

$$= 162\pi \quad \#$$

11、 $f(x) = 4x - x^2 \quad [0,4]$

$$\frac{1}{4-0}\int_0^4 (4x - x^2)\,dx$$

$$= \frac{1}{4}\left[-\frac{1}{3}x^3 + 2x^2\right]_0^4$$

$$= \frac{1}{4}\left(32 - \frac{64}{3}\right) = \frac{32}{12} = \frac{8}{3} \quad \#$$

12. $f(x) = \ln(e^{2x} + 3x)$

$f(x) = \ln(u) \qquad u = e^{2x} + 3x$

$f'(u) = \frac{1}{u}$

$u' = (e^{2x} + 3x)' = 2e^{2x} + 3$

$f'(x) = f'(u) \cdot u' = \frac{2e^{2x} + 3}{e^{2x} + 3x}$

$f'(0) = \frac{2 \times 1 + 3}{1 + 0} = 5 \quad \#$

$$\int \frac{dx}{x(\ln x)^2} \qquad u = \ln x \qquad du = \frac{1}{x}dx$$

$$\int \frac{dx}{x(\ln x)^2} = \int \frac{1}{u^2}du$$

$$= \int u^{-2}du$$

$$= \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{u} + C \Rightarrow -\frac{1}{\ln x} + C \quad \#$$

14 set $y = \arcsin(x)$, $\sin(y) = x$

$$\cos(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} \qquad \overset{x^2}{\sin^2(y)} + \cos^2(y) = 1$$

$$\sin(y) = x \Rightarrow \cos(y) = \sqrt{1-x^2}$$

$$\frac{dy}{dx} : \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

15 $\lim\limits_{x\to\infty} x^{1/x}$

$$y = x^{\frac{1}{x}}$$

$$\ln(y) = \frac{1}{x}\ln(x) \qquad \frac{d}{dx}(\ln(x)) = \frac{1}{x} \qquad e^0 = 1$$

$$\lim_{x\to\infty}\ln(y) = \lim_{x\to\infty}\frac{\ln(x)}{x} \Rightarrow \frac{d}{dx}(x) = 1 \Rightarrow \lim_{x\to\infty} x^{\frac{1}{x}} = 1 \quad \#$$

$$= \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = \lim_{x\to\infty}\frac{1}{x} = 0$$