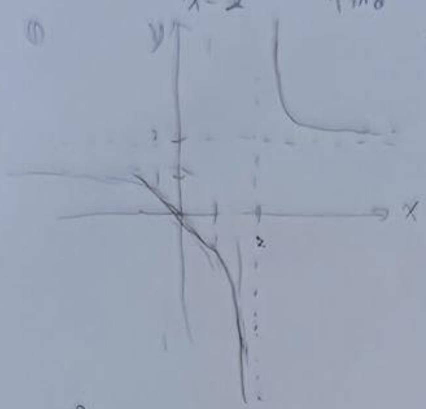
1. ① sketch $\frac{2x+1}{x-2}$ ② find $\lim_{x\to 3} \frac{2x+1}{x-2}$ (已改題目)

①



Vertical Asymptote $x=2$

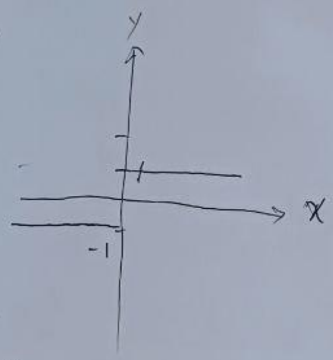
Horizon $y=2$

②

$$\frac{6+1}{3-2} = \frac{7}{1} = 7 \ \#$$

2. $f(x) = \frac{|x|}{x}$ ① ② $\lim_{x\to 0^+} f(x)$ $\lim_{x\to 0^-} f(x)$ $\lim_{x\to 0} f(x)$ (已改)

①



$x>0 \quad y=1$

$x<0 \quad y=-1$

②

$$\lim_{x\to 0^+} f(x) = 1 \cdots ① \quad \lim_{x\to 0^-} f(x) = -1 \cdots ②$$

$\because$ ① $\neq$ ②

$\therefore \lim_{x\to 0} f(x)$ does not exist

3. $f(x) = \sqrt{x}$ find $f'(x)$ (未改)

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \Rightarrow \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h \times (\sqrt{x+h} + \sqrt{x})} \Rightarrow \frac{\cancel{x}+h-\cancel{x}}{h \times (\sqrt{x+h} + \sqrt{x})} \Rightarrow \lim_{h\to 0} \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}} \ \#$$

4. $f(x) = (x^2-5)(x^4-1)$ find $f'(x)$ (未改)

$$f'(x) = (x^2-5)'(x^4-1) + (x^2-5)(x^4-1)'$$

$$= 2x(x^4-1) + 4x^3(x^2-5)$$

$$= 2x^5 - 2x + 4x^5 - 20x^3 = 6x^5 - 20x^3 - 2x \quad \#$$

5. tagent line to $f(x) = \frac{(3x-2)^2}{2}$ at $p(1, 0.5)$ (未改)

$$f(x) = \frac{1}{2}(3x-2)^2$$

$$f'(x) = \frac{1}{2} \times 2(3x-2) \times (3x-2)' = (3x-2) \times 3 = 9x-6$$

$$f'(1) = 3$$

tagent $\Rightarrow$ $y - y_1 = m(x - x_1)$

$$y - 0.5 = 3(x-1)$$

$$y = 3x - 2.5 \quad \#$$

6. $y = \left(\frac{3x-2}{4x^2-5x}\right)$ find $f'(x)$ (未改)

$$f'(x) = \frac{(4x^2-5x)(3x-2)' - (4x^2-5x)'(3x-2)}{(4x^2-5x)^2} = \frac{3(4x^2-5x) - (8x-5)(3x-2)}{(4x^2-5x)^2}$$

$$= \frac{12x^2 - 15x - 24x^2 + 16x + 15x - 10}{(4x^2-5x)^2} = \frac{-12x^2 + 16x - 10}{(4x^2-5x)^2}$$

$\#$

7. $y = \left(\frac{3x}{4x^2-5}\right)^4$

by chain rule $f'(x) = 4\left(\frac{3x}{4x^2-5}\right)^3 \times \left(\frac{3x}{4x^2-5}\right)'$

$$= 4\left(\frac{3x}{4x^2-5}\right)^3 \times \frac{(4x^2-5)(3x)' - (4x^2-5)'(3x)}{(4x^2-5)^2}$$

7. continue

$$= 4\left(\frac{3x}{4x^2-5}\right)^3 \times \frac{3(4x^2-5)-8x(3x)}{(4x^2-5)^2} = 4\left(\frac{3x}{4x^2-5}\right)^3 \times \frac{\overset{-12x^2-15}{\cancel{12x^2}-15-\cancel{24x^2}}}{(4x^2-5)^2}$$

$$= 4 \times \frac{(3x)^3}{(4x^2-5)^3} \times \frac{-12x^2-15}{(4x^2-5)^2} = \frac{4\times 27x^3 \times (-12x^2-15)}{(4x^2-5)^5}$$

$$= \frac{-(1296x^5+1620x^3)}{(4x^2-5)^5}$$

$$\begin{array}{r} 27 \\ 12 \\ \hline 54 \\ 27 \\ \hline 324 \\ \times\ 4 \\ \hline 1296 \end{array} \qquad 108\ \ 15 \qquad 1296x \qquad \begin{array}{r} 108 \\ \times\ 15 \\ \hline 540 \\ 108 \\ \hline 1620 \end{array}$$

8. find the derivative of $y = \frac{\sin(2x+1)\cos(2x^2+1)}{x^2+2}$

$$f'(x) = \frac{(x^2+2)\left(\sin(2x+1)\cos(2x^2+1)\right)' - (x^2+2)'\left(\sin(2x+1)\cos(2x^2+1)\right)}{(x^2+2)^2}$$

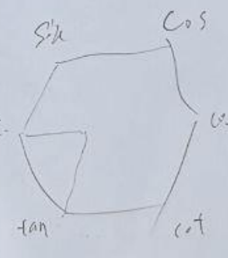
$$\left(\sin(2x+1)\cos(2x^2+1)\right)' = \sin(2x+1)' \times \cos(2x^2+1) + \cos(2x^2+1)'\sin(2x+1)$$

$$= 2x\cos(2x+1) \times \cos(2x^2+1) - 4x\sin(2x^2+1)\ \sin(2x+1)$$

$$f'(x) = \frac{2\cos(2x+1)\cos(2x^2+1)}{x^2+2} - \frac{(4x)\sin(2x^2+1)\sin(2x+1)}{x^2+2} - \frac{(2x)\sin(2x+1)\cos(2x^2+1)}{(x^2+2)^2}$$

#

9. $y = \tan\left(\frac{\pi}{4}(\cos x - \sin x)\right)$ Evulate $y'(0)$



$\tan' = \sec^2$

$$f'(x) = \sec^2\left(\frac{\pi}{4}(\cos x - \sin x)\right) \times \left(-\sin(x) - \cos(x)\right) \times \frac{\pi}{4}$$

$$= -\frac{\pi}{4}\left(\sin(x)+\cos(x)\right)\sec^2\left(\frac{\pi}{4}\left(\cos(x)-\sin(x)\right)\right)$$

$$y'(0) = -\frac{\pi}{4} \times \sec^2\left(\frac{\pi}{4}(1)\right) = -\frac{\pi}{4} \times \sec^2\left(\frac{\pi}{4}\right)$$

$$= -\frac{\pi}{4} \times (\sqrt{2})^2 = -\frac{\pi}{2}$$

10. $\frac{dy}{dx}$ for $x^2+2y^2=50$ point $(0,5)$

$2x+4y\cdot y'=0$

$4yy'=-2x$

$y'=\frac{-2x}{4y}$

$=\frac{-x}{2y}$

y' at $x=0$ is 0

$y-y_1=m(x-x_1)$

$y-5=0(x-0)$

$y=5$ #

11. linear approximation $\cos 32^\circ$

$f(x)=f(a)+f'(a)(x-a)$

$f(x)=\cos(x)$

$f'(x)=\sin(x)$

0.8490

$f'(30^\circ)=\sin 30^\circ=\frac{1}{2}$

$\cos(30^\circ)=\frac{\sqrt{3}}{2}$

$2^\circ=\frac{\pi}{90}$

$f(32^\circ)=f(30^\circ)+f'(30^\circ)\times\frac{\pi}{90}\Rightarrow\cos 32^\circ\approx\frac{\sqrt{3}}{2}+\frac{\pi}{180}$ #

$=\frac{90\sqrt{3}+\pi}{180}$