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3-16

[11-1]5

$2^2+2=6$

$2^3+3=11$

$2^4+4=20$

$2^5+5=37$

$2^6+6=70$

9. $a_n=\cos n\pi$

$a_1=\cos(1\pi)=-1$

$a_2=\cos(2\pi)=1$

$a_3=\cos(3\pi)=-1$

$a_4=\cos(4\pi)=1$

$a_5=\cos(5\pi)=-1$

[11-1]27

$a_n=\frac{5}{n+2}$

$\lim_{n\to\infty}\frac{5}{n+2}=0$, converges

[11-1]35.

$a_n=e^{\frac{-1}{\sqrt{n}}} \to 0$

$n\to\infty$

converges, $e^0=1$

[11-1]58

$n-\sqrt{n+1}\sqrt{n+3}$

$=n-\sqrt{n^2+4n+3}$

$a_n \approx n-\sqrt{n}\times\sqrt{n}=0$

converges.

[11-2]17

$$\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n}\right)$$

$$\cancel{\frac{1}{3}}-\frac{1}{1}+\cancel{\frac{1}{4}}-\frac{1}{2}+\cancel{\frac{1}{5}}-\cancel{\frac{1}{3}}+\cancel{\frac{1}{6}}-\cancel{\frac{1}{4}}+\frac{1}{7}-\cancel{\frac{1}{5}}+\frac{1}{8}-\cancel{\frac{1}{6}}$$

$=-1-\frac{1}{2}=\frac{-3}{2}$ #

20. $\sum_{n=1}^{\infty}\ln\frac{n}{n+1}$

$\ln\frac{1}{2}+\ln\frac{2}{3}+\ln\frac{3}{4}+\ln\frac{4}{5}=\ln\left(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\right)=\ln\left(\frac{1}{5}\right)$

$=-\ln(5)$

∞: $-\ln(\infty)$ diverges

2) 29

$$\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{4^n}$$

$a_n=\frac{a_1}{1-r}$ $=\frac{(-3)^0}{4}=\frac{1}{4}$

$r=\frac{-3}{4}$

$\frac{\frac{1}{4}}{1+\frac{3}{4}}=\frac{\frac{1}{4}}{\frac{7}{4}}=\frac{1}{7}$ #

32.

$$\sum_{n=1}^{\infty} \frac{6 \times 2^{(2n-1)}}{3^n} \qquad r = \frac{(2^2)^n}{3^n} = \frac{4^n}{3^n} = \frac{4}{3} > 1$$

divergents

37.

$$\sum_{n=1}^{\infty} \frac{\frac{2}{n}+1}{\frac{1}{n}-2} = \frac{\lim_{n\to\infty}\frac{2}{n} + \lim_{n\to\infty} 1}{\lim_{n\to\infty}\frac{1}{n} - \lim_{n\to\infty} 2} = \frac{2\times 0+1}{0-2} = \frac{1}{-2}$$

divergents

$$\frac{2+1}{1-2} + \frac{1+1}{\frac{1}{2}-2} + \frac{\frac{2}{3}+1}{\frac{1}{3}-2}$$

$$\frac{3}{-1} \qquad \frac{2}{-\frac{3}{2}} \qquad \frac{\frac{5}{3}}{-\frac{5}{3}}$$

[11-3] 5 $\quad -3 \quad \frac{-4}{3} \quad -1$

$$\frac{2}{5n-1} \geq \frac{2}{5n}$$

$$\sum_{n=1}^{\infty} \frac{2}{5n}$$

divegent

[11-3] 9

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} = \frac{1}{2\ln(2)^2}$$ convergent

[11-3] 17

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}+4}{n^2} = \lim_{a\to\infty}\left(\frac{-2}{\sqrt{a}} - \frac{4}{a} + 6\right) = 6$$

convergent

25 $$\sum_{k=1}^{\infty} ke^{-k} = \int_1^{\infty} xe^{-x}\,dx$$

$$= \lim_{a\to\infty}\left(\int_1^a xe^{-x}dx\right)$$

$$= \lim_{a\to\infty}\left(-\frac{a+1}{e^a} + \frac{2}{e}\right)$$

$$= \frac{2}{e}$$ convergent

[11-4] 11

$$\sum_{n=1}^{\infty} \frac{9^n}{3+10^n} \leq \frac{9^n}{10^n}$$ ⇒ convergent

[11-4] 31

$$\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}$$

convergent

[11-5] 22

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$$

$$\lim_{n\to\infty} \frac{1}{n^4} = 0 \quad \frac{1}{n^4} \geq \frac{1}{(n+1)^4}$$

abs convergent

[11-5] 29 Absolutely converg

31 conditionaly conve

[11-5] 41

−0.4597