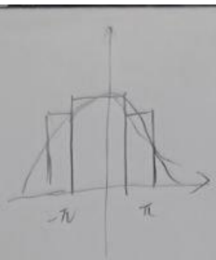
4-1 8. $n=3$



[4-1] 17.

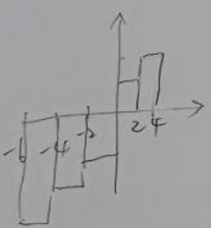
$f(x) = 2+\sin^2 x$, $0 \le x \le \pi$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[2+\sin^2\left(\frac{\pi i}{n}\right)\right] \times \frac{\pi}{n}$$

[4-1] 21

$y = \frac{1}{1+x}$ from 0 to 2

[4-2] 1. $[-6,4]$ $\Delta x = \frac{10}{5} = 2$



$$\sum_{i=1}^{n} f(c_i) \times \Delta x_i$$

$R = f(-4)\times 2 + f(-2)\times 2 + f(0)\times 2 + f(2)\times 2 + f(4)\times 2$

$= -10 - 6 - 2 + 2 + 6 = -10$

R=負數 代表下方面積>上方 #

[4-2] 9.

$n=4$ $\frac{8-0}{4} = 2$

0 2 4 6 8

midpoint = 1 3 5 7

$2\left(f(1)+f(3)+f(5)+f(7)\right) = 2(50+9+25) = 84\times 2 = 168$ #

[4-2] 19

$$\int_0^{\pi} \frac{\sin x}{1+x}\,dx$$

[4-3] 24

$$\int_0^2 3x\,dx = \left[\frac{3}{2}x^2\right]_0^2$$
$$= 6 \quad \#$$

[4-2] 57

$$\int_{-2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx = \int_{-1}^{5} f(x)\,dx$$

#

[4-2] 60

$$\int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx = [3x]_0^3 + \left[\frac{x^2}{2}\right]_3^5$$
$$= 9 + 8$$
$$= 17$$

$\frac{25}{2} - \frac{9}{2}$

[4-2] 63

[4-3] 9.

$$g(x) = \int_0^x \sqrt{t+t^3}\,dt$$

$$g'(x) = \sqrt{x+x^3}$$

[4-3] 15

$$u = \frac{1}{x} \qquad du = -\frac{1}{x^2}\,dx$$

$$(x) = \frac{d}{dx}\int_2^{\frac{1}{x}} \sin^4 t\,dt = \frac{d}{dx}\int_2^{u} \sin^4 t\,dt = \frac{d}{du}\left[\int_2^{u} \sin^4 t\,dt\right]\frac{du}{dx}$$
$$= \sin^4(u)^4 \times \frac{du}{dx} = \sin\left(\frac{1}{x}\right)^4 \times \frac{-1}{x^2}$$

#

]30

$$\int_1^8 x^{-2/3}\,dx$$

$$= 3x^{\frac{1}{3}}\Big|_1^8$$

$$= 6-3=3 \quad \#$$

[4-3] 37

$$\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx$$

$$= \int_1^4 2\times x^{-\frac{1}{2}} + x^2\times x^{-\frac{1}{2}}\,dx$$

$$= \int_1^4 2x^{-\frac{1}{2}} + x^{\frac{3}{2}}\,dx$$

$$= \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\Bigg|_1^4 = 4x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}}\Bigg|_1^4$$

$$= 8 + 32\times\frac{2}{5} - 4 - \frac{2}{5}$$

$$= 4 + \frac{62}{5} = 16 + \frac{2}{5} \quad \#$$

[4-3] 59

$$\frac{d}{dx}\int_{2x}^{3x}\frac{u^2-1}{u^2+1}\,du$$

$$g'(x) = \frac{d}{dx}\int_{2x}^{0}\frac{u^2-1}{u^2+1}\,du + \frac{d}{dx}\int_0^{3x}\frac{u^2-1}{u^2+1}\,du$$

$$= \frac{d}{dx}\int_0^{2x}\frac{1-u^2}{u^2+1}\,du$$

$$\frac{d}{du}\int_0^{u}\frac{1-u^2}{u^2+1}\,du \times \frac{du}{dx}$$

$$\frac{1-u^2}{u^2+1}\times\frac{du}{dx} = \frac{2-8x^2}{4x^2+1}$$

$$\frac{d}{du}\int_0^{u}\frac{u^2-1}{u^2+1}\,du\,\frac{du}{dx}$$

$$= \frac{u^2-1}{u^2+1}\times\frac{d}{dx}(u)$$

$$= 3\times\frac{(3x)^2-1}{(3x)^2+1} = \frac{27x^2-1}{9x^2+1}$$

$$+$$

$$= \frac{2-8x^2}{4x^2+1} + \frac{27x^2-1}{9x^2+1} \quad \#$$

[4-3] 74

$$\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\frac{i}{n}}\right)$$

$$\int_0^1 \sqrt{x}\,dx$$

$$= \int_0^1 x^{\frac{1}{2}}\,dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^1$$

$$= \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1$$

$$= \frac{2}{3} \ \#$$

[4-4] 5

$$\int (3x^2+4x+1)\,dx$$

$$= x^3+2x^2+x+C$$

[4-4] 13.

$$\int (u+4)(2u+1)\,du$$

$$= \int 2u^2+9u+4\ du$$

$$= \frac{2}{3}u^3+\frac{9}{2}u^2+4u+C \quad \#$$

[4-4] 27

$$\int_0^2 (2x-3)(4x^2+1)\,dx$$

$$\int_0^2 8x^3+2x-12x^2-3\ dx$$

$$2\ \frac{8}{4}x^4+x^2-4x^3-3x\Big|_0^2$$

$$= 32+4-32-6$$

$$= -2 \quad \#$$

[4-4] 39.

$$\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2\theta}+1\ d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2\theta}\,d\theta + \int_0^{\frac{\pi}{4}} 1\,d\theta$$

$$= \tan\theta + \theta\Big|_0^{\frac{\pi}{4}}$$

$$= \tan\frac{\pi}{4}+\frac{\pi}{4}-(\tan 0+0)$$

$$= 1+\frac{\pi}{4} \quad \#$$