HW3

[2-3] 3

3. $f(x) = x^{75} - x + 3 \;\Rightarrow\; f'(x) = 75x^{75-1} - 1(1) + 0 = 75x^{74} - 1$

[2-3] 7

7. $f(x) = x^{3/2} + x^{-3} \;\Rightarrow\; f'(x) = \frac{3}{2}x^{1/2} + (-3x^{-4}) = \frac{3}{2}x^{1/2} - 3x^{-4}$

[2-3] 13

13. $g(x) = \dfrac{1}{\sqrt{x}} + \sqrt[4]{x} = x^{-1/2} + x^{1/4} \;\Rightarrow\; g'(x) = -\frac{1}{2}x^{-3/2} + \frac{1}{4}x^{-3/4}$ or $-\dfrac{1}{2x\sqrt{x}} + \dfrac{1}{4\sqrt[4]{x^3}}$

[2-3] 26

26. $y = \dfrac{t}{x^2} + \dfrac{x}{t} = tx^{-2} + xt^{-1}.$

To find dy/dx, we treat t as a constant and x as a variable to get $dy/dx = t(-2x^{-3}) + (1)t^{-1} = -2tx^{-3} + t^{-1}$ or

$-\dfrac{2t}{x^3} + \dfrac{1}{t}.$

To find dy/dt, we treat x as a constant and t as a variable to get $dy/dt = (1)x^{-2} + x(-1t^{-2}) = x^{-2} - xt^{-2}$ or $\dfrac{1}{x^2} - \dfrac{x}{t^2}.$

[2-3] 30

30. By the Product Rule, $y = (10x^2 + 7x - 2)(2 - x^2) \;\Rightarrow$

$y' = (10x^2 + 7x - 2)(2 - x^2)' + (2 - x^2)(10x^2 + 7x - 2)' = (10x^2 + 7x - 2)(-2x) + (2 - x^2)(20x + 7)$

$= -20x^3 - 14x^2 + 4x + 40x + 14 - 20x^3 - 7x^2 = -40x^3 - 21x^2 + 44x + 14$

[2-3] 33

33. By the Quotient Rule, $y = \dfrac{5x}{1+x}$ $\Rightarrow$

$$y' = \frac{(1+x)(5x)' - 5x(1+x)'}{(1+x)^2} = \frac{(1+x)5 - 5x(1)}{(1+x)^2} = \frac{5+5x-5x}{(1+x)^2} = \frac{5}{(1+x)^2}.$$

[2-3] 45

45. $J(u) = \left(\dfrac{1}{u} + \dfrac{1}{u^2}\right)\left(u + \dfrac{1}{u}\right) = (u^{-1} + u^{-2})(u + u^{-1})$ $\overset{\text{PR}}{\Rightarrow}$

$$J'(u) = (u^{-1} + u^{-2})(u + u^{-1})' + (u + u^{-1})(u^{-1} + u^{-2})' = (u^{-1} + u^{-2})(1 - u^{-2}) + (u + u^{-1})(-u^{-2} - 2u^{-3})$$

$$= u^{-1} - u^{-3} + u^{-2} - u^{-4} - u^{-1} - 2u^{-2} - u^{-3} - 2u^{-4} = -u^{-2} - 2u^{-3} - 3u^{-4} = -\left(\frac{1}{u^2} + \frac{2}{u^3} + \frac{3}{u^4}\right)$$

[2-3] 59

59. $y = \dfrac{2x}{x+1}$ $\Rightarrow$ $y' = \dfrac{(x+1)(2) - (2x)(1)}{(x+1)^2} = \dfrac{2}{(x+1)^2}.$

At $(1,1)$, $y' = \frac{1}{2}$, and an equation of the tangent line is $y - 1 = \frac{1}{2}(x - 1)$, or $y = \frac{1}{2}x + \frac{1}{2}$.

[2-4] 1

1. $f(x) = 3\sin x - 2\cos x$ $\Rightarrow$ $f'(x) = 3(\cos x) - 2(-\sin x) = 3\cos x + 2\sin x$

[2-4] 7

7. $y = \sec\theta \tan\theta$ $\Rightarrow$ $y' = \sec\theta\,(\sec^2\theta) + \tan\theta\,(\sec\theta\tan\theta) = \sec\theta\,(\sec^2\theta + \tan^2\theta)$. Using the identity

$1 + \tan^2\theta = \sec^2\theta$, we can write alternative forms of the answer as $\sec\theta\,(1 + 2\tan^2\theta)$ or $\sec\theta\,(2\sec^2\theta - 1)$.

[2-4] 15

15. $y = \dfrac{x}{2 - \tan x} \quad \Rightarrow \quad y' = \dfrac{(2 - \tan x)(1) - x(-\sec^2 x)}{(2 - \tan x)^2} = \dfrac{2 - \tan x + x\sec^2 x}{(2 - \tan x)^2}$

[2-4] 24

24. $\dfrac{d}{dx}(\sec x) = \dfrac{d}{dx}\left(\dfrac{1}{\cos x}\right) = \dfrac{(\cos x)(0) - 1(-\sin x)}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x} = \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x} = \sec x \tan x$

[2-4] 35

35. $g(\theta) = \dfrac{\sin\theta}{\theta} \overset{\text{QR}}{\Rightarrow} g'(\theta) = \dfrac{\theta(\cos\theta) - (\sin\theta)(1)}{\theta^2} = \dfrac{\theta\cos\theta - \sin\theta}{\theta^2}$

Using the Quotient Rule and $g'(\theta) = \dfrac{\theta\cos\theta - \sin\theta}{\theta^2}$, we get

$$g''(\theta) = \frac{\theta^2\{[\theta(-\sin\theta) + (\cos\theta)(1)] - \cos\theta\} - (\theta\cos\theta - \sin\theta)(2\theta)}{(\theta^2)^2}$$

$$= \frac{-\theta^3\sin\theta + \theta^2\cos\theta - \theta^2\cos\theta - 2\theta^2\cos\theta + 2\theta\sin\theta}{\theta^4} = \frac{\theta(-\theta^2\sin\theta - 2\theta\cos\theta + 2\sin\theta)}{\theta \cdot \theta^3}$$

$$= \frac{-\theta^2\sin\theta - 2\theta\cos\theta + 2\sin\theta}{\theta^3}$$

[2-4] 45

45. $\displaystyle\lim_{x\to 0}\frac{\sin 5x}{3x} = \lim_{x\to 0}\frac{5}{3}\left(\frac{\sin 5x}{5x}\right) = \frac{5}{3}\lim_{x\to 0}\frac{\sin 5x}{5x} = \frac{5}{3}\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\ \begin{bmatrix}\text{where } \theta = 5x, \\ \text{using Equation 5}\end{bmatrix} = \frac{5}{3}\cdot 1 = \frac{5}{3}$

[2-4] 51

51. $\displaystyle\lim_{x\to 0}\frac{\tan 2x}{x} = \lim_{x\to 0}\frac{\frac{\sin 2x}{\cos 2x}}{x} = \lim_{x\to 0}\frac{\sin 2x}{x\cos 2x} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{2}{\cos 2x}$

$\displaystyle\qquad = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{2}{\cos 2x} = \lim_{x\to 0}\frac{\sin\theta}{\theta}\cdot\lim_{x\to 0}\frac{2}{\cos 2x}\qquad [\theta = 2x]$

$\displaystyle\qquad = 1\cdot\frac{2}{1} = 2$

[2-4] 62

62. Let $f(x) = x\sin x$ and $h(x) = \sin x$, so $f(x) = xh(x)$. Then $f'(x) = h(x) + xh'(x)$,

$f''(x) = h'(x) + h'(x) + xh''(x) = 2h'(x) + xh''(x)$,

$f'''(x) = 2h''(x) + h''(x) + xh'''(x) = 3h''(x) + xh'''(x), \cdots , f^{(n)}(x) = nh^{(n-1)}(x) + xh^{(n)}(x)$.

Since $34 = 4(8) + 2$, we have $h^{(34)}(x) = h^{(2)}(x) = \dfrac{d^2}{dx^2}(\sin x) = -\sin x$ and $h^{(35)}(x) = -\cos x$.

Thus, $\dfrac{d^{35}}{dx^{35}}(x\sin x) = 35h^{(34)}(x) + xh^{(35)}(x) = -35\sin x - x\cos x$.

[2-5] 3

3. Let $u = g(x) = \cos x$ and $y = f(u) = \sin u$. Then

$\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)(-\sin x) = (\cos(\cos x))(-\sin x) = -\sin x\,\cos(\cos x)$.

[2-5] 6

6. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = \sin u$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\cos u)\left(\tfrac{1}{2}x^{-1/2}\right) = \dfrac{\cos u}{2\sqrt{x}} = \dfrac{\cos\sqrt{x}}{2\sqrt{x}}$.

[2-5] 18

18. Using the Product Rule and the Chain Rule, $f(t) = t\sin\pi t \;\Rightarrow\; f'(t) = t(\cos\pi t)\cdot\pi + (\sin\pi t)\cdot 1 = \pi t\cos\pi t + \sin\pi t$.

[2-5] 29

29. $y = \cos(\sec 4x) \quad \Rightarrow$

$$y' = -\sin(\sec 4x)\,\frac{d}{dx}\,\sec 4x = -\sin(\sec 4x) \cdot \sec 4x \tan 4x \cdot 4 = -4\sin(\sec 4x)\sec 4x \tan 4x$$