110321006 蔡秉翰

[11-6] 3

$$\lim_{n\to\infty}\left(\left|\frac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}}\right|\right)=\lim_{n\to\infty}\frac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}}=\frac{5^n\times(n+1)}{5^{n+1}\times n}=\lim_{n\to\infty}\frac{n+1}{5n}=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{5}=\frac{1}{5}$$

$\frac{1}{5}<1$, convergent

7.

$$\lim_{k\to\infty}\left(\left|\frac{\frac{1}{(k+1)!}}{\frac{1}{k!}}\right|\right)=\lim_{k\to\infty}\frac{\frac{1}{(k+1)!}}{\frac{1}{k!}}=\lim_{k\to\infty}\frac{k!}{(k+1)!}=\lim_{k\to\infty}\frac{k!}{(k+1)\times k!}=\lim_{k\to\infty}\frac{1}{k+1}$$

$$=\lim_{k\to\infty}\frac{\frac{1}{k}}{1+\frac{1}{k}}=\frac{0}{1+0}=0$$

$0<1$; convergent

14.

$$\lim_{n\to\infty}\frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}}=\lim_{n\to\infty}\frac{(n+1)!\times n^n}{n!\times(n+1)^{n+1}}=\lim_{n\to\infty}\frac{(n+1)\times n^n}{(n+1)^{n+1}}=\lim_{n\to\infty}\frac{n^n}{(n+1)^n}$$

$$=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n<1$$, convergent

22.

$$\sum_{n=1}^{\infty}\left(\frac{-2}{n}\right)^n \qquad a_n=\left(\frac{-2}{n}\right)^n$$

$$\sqrt[n]{|a_n|}=\frac{-2}{n} \quad n\to\infty \quad \frac{-2}{n}\to 0$$, convergent

25.

$$a_n=\left(1+\frac{1}{n}\right)^{n^2}$$

$$\sqrt[n^2]{\left(1+\frac{1}{n}\right)^{n^2}}=\left(1+\frac{1}{n}\right) \quad n\to\infty \quad >1$$, divegence

[11-7] 9

$$\lim_{n\to\infty} \frac{\frac{n^2-1}{n^3+1}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{(n^2-1)n}{n^3+1} = \lim_{n\to\infty} \frac{(n-1)(n+1)n}{(n+1)\times(n^2-n+1)} = \lim_{n\to\infty}\frac{(n-1)\times n}{n^2-n+1} = \lim_{n\to\infty} \frac{n^2\times(1-\frac{1}{n})}{n^2\times(1-\frac{1}{n}+\frac{1}{n^2})}$$

$$= \lim_{n\to\infty} \frac{1-\frac{1}{n}}{1-\frac{1}{n}+\frac{1}{n^2}} = \frac{1-0}{1-0+0} = 1$$, divergence

[11-7] 13

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2} \qquad a_n = \frac{e^n}{n^2} \Rightarrow \frac{\frac{e^{(n+1)}}{(n+1)^2}}{\frac{e^n}{n^2}} = \frac{e^{(n+1)}\times n^2}{e^n\times(n+1)^2} = \frac{e\times n^2}{(n+1)^2} = \frac{en^2}{n^2+2n+1}$$

$$\lim_{n\to\infty} \frac{en^2}{n^2+2n+1} = \lim_{n\to\infty} \frac{en^2}{n^2\times(1+\frac{2}{n}+\frac{1}{n^2})} = \lim_{n\to\infty}\frac{e}{1+\frac{2}{n}+\frac{1}{n^2}} = \frac{e}{1+0+0} = e = 2.7 > 1$$

$e > 1$, divergence

18. $f(x) = x^2 e^{-x^3}$

$$\int_1^\infty x^2 e^{-x^3}\,dx$$

$$\lim_{n\to\infty} \frac{-1}{3e^{(a^3)}} + \frac{1}{3e} = \frac{1}{3e}$$, convergent

[11-8] 3 $\sum_{n=1}^{\infty} \frac{x^n}{n}$

$$\lim_{n\to\infty} \frac{\frac{x^{n+1}}{n+1}}{\frac{x^n}{n}} = \lim_{n\to\infty}\left(\left|\frac{nx}{n+1}\right|\right) = \lim_{n\to\infty}\left(|x|\times\left|\frac{n}{n+1}\right|\right) = |x| \times \lim_{n\to\infty} \frac{n}{n+1}$$

$$= |x| \times 1$$

$|x| < 1 \quad R = 1 \quad [-1, 1)$ #

[11-8] 13 $\sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow \lim_{n\to\infty} \frac{\frac{x^{(n+1)}}{(n+1)!}}{\frac{x^n}{n!}} = \lim_{n\to\infty} \frac{x\times n!}{(n+1)!} = \lim_{n\to\infty}\left(\left|\frac{x}{n+1}\right|\right)$

$$= |x| \times \lim_{n\to\infty}\left(\left|\frac{1}{n+1}\right|\right)$$

$$= |x| \times 0$$

$R = \infty \quad (-\infty, \infty)$

[11-8] 17

$$\lim_{n\to\infty}\left(\left|\frac{\frac{(-1)^{n+1}\times 4^{n+1}}{\sqrt{n+1}}\times x^{n+1}}{\frac{(-1)^n\times 4^n}{\sqrt{n}}\times x^n}\right|\right) = |4x| \times \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}} = |4x|\times 1 = |4x|$$

$$|4x| < 1$$

$$|x| < \frac{1}{4}. \qquad R = \frac{1}{4} \qquad \left(-\frac{1}{4}, \frac{1}{4}\right]$$

[11-9] 4

$$f(x) = \frac{x}{1+x} = \sum_{n=0}^{\infty} x^{n+1} \times (-1)^n$$

[11-9] 9.

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2^{4n+4}}, \ (-2, 2)$$

[11-9] 17

$$\sum_{n=0}^{\infty} (-1)^n 4^n (n+1) x^{n+1}, \ R = 1$$

[11-10] 5.

$$x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

[11-10] 7

$$2 + \frac{(x-8)}{12} - \frac{(x-8)^2}{288} + \frac{5\times(x-8)^2}{20735}$$

[11-10]21

$$50 + 105 \times (x-2) + 92 \times (x-2)^2 + 42 \times (x-2)^3 + 10 \times (x-2)^4 + (x-2)^5$$

$$R = \infty$$

[11-10]19

$$e^{-x^4}$$