Code-Division Multiple Access (CDMA) Fundamentals (2-3 hours)

Introduction

- Direct sequence (DS) CDMA- cellular 3G, WLAN 802.11b, GPS Internet of Things (IoT) LoRa and Sigfox (asynchronous, so can sleep for as little or as long as desire)
- PD NOMA (power domain non-orthogonal multiple access) for 5G downlink/uplink: special case of DS CDMA with correlation 1
- Spreading sequence are commonly used as synchronization method in many communication systems
- Multi-Carrier (MC) CDMA and CD (code-domain) NOMA - Sparse Code Multiple Access (SCMA) - candidate for 5G uplink grant-free: special case of MC CDMA with signatures chips in some subcarriers

Direct Sequence Code-division Multiple Access (DS CDMA)

Spread spectrum +multiple access=> CDMA

- Use different spreading codes for each user (code-division), we accomplish code division multiple access (CDMA).
- Spread spectrum + multiple access (SSMA) = CDMA
- ***CDMA就像在一個吵雜的環境中利用個人聲音的特質，還是可以辨別出他說了什麼。***

DS CDMA Spreading and Despreading

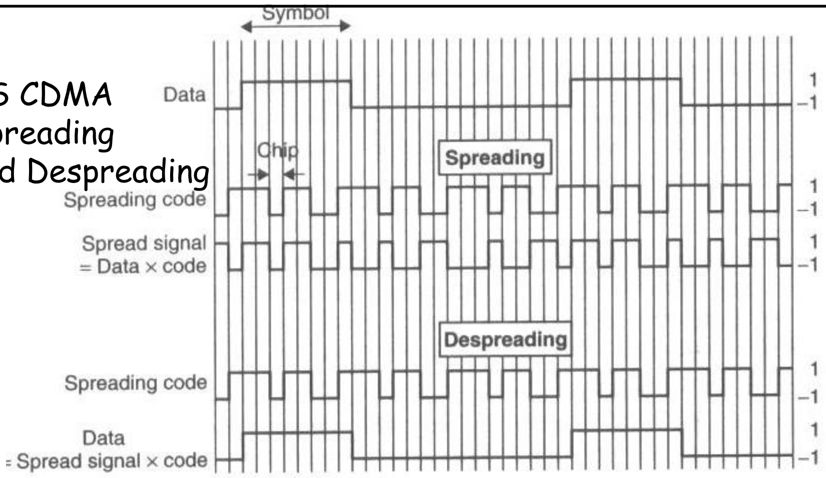
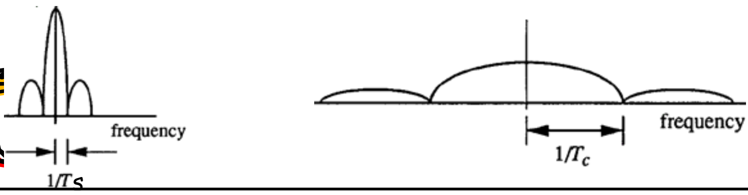


Figure 3.2. Spreading and despreading in DS-CDMA



Why DS CDMA?

- Resistant to interference (crosscorrelation of spreading sequences)
- Resistant to multipath fading (narrow autocorrelation mainlobe, see next page) – Using Rake receiver
- **Soft capacity (flexible number of users)** : if interference is reduced, capacity is increased

Therefore, all 3G cellular standards use DS CDMA (IS-95:GSM=2:1 in capacity), GPS too (narrow autocorrelation mainlobe=> accurate possitioning) .

Multipath Radio Channel

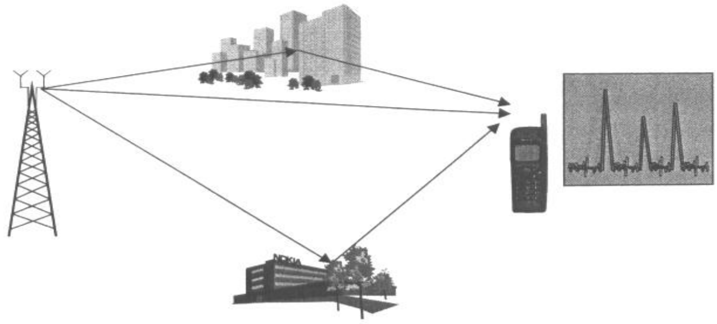
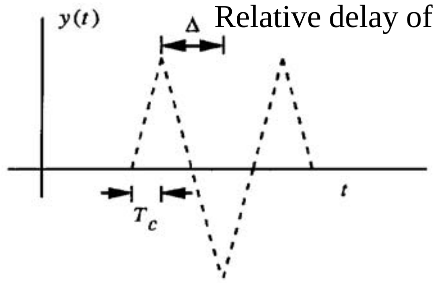


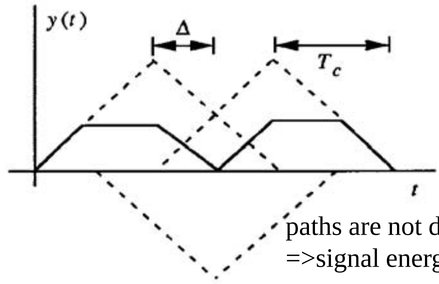
Figure 3.4. Multipath propagation leads to a multipath delay profile

Relative delay of adjacent paths



Matched filter output for high chip rate.

Figure
Multipath fading



paths are not distinguishable
=>signal energy decreases

Matched filter output for low chip rate.

- If the paths (each may contain many physical paths) are resolvable, we can compensate channel phase response of different paths and recombine the divided energy in different paths=> Rake receiver.
- Rake receiver doesn't work with TDMA because TDMA received signal doesn't have very narrow mainlobe to resolve paths.

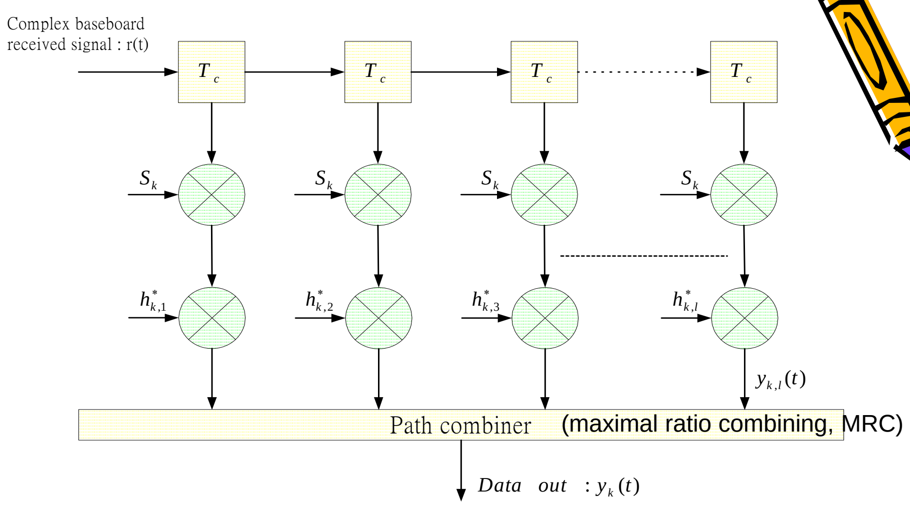


Fig 2. The Rake receiver structure.

S_k : the kth user's code. (with proper delays)

$h^*_{k,l}$: the estimated complex conjugate chanel impulse response.

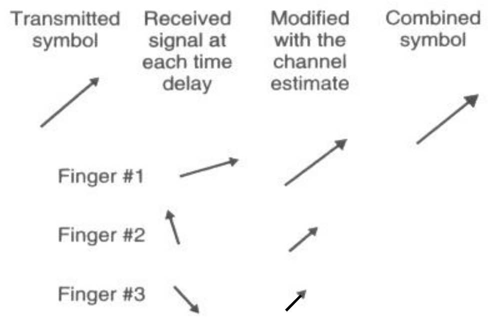
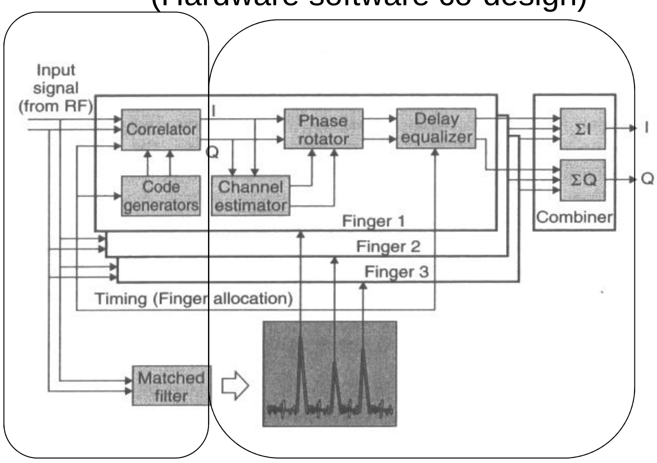


Figure 3.6. The principle of maximal ratio combining within the CDMA Rake receiver

<u>MRC is discussed in NWTT2</u>

Fig. 3.7 Rake receiver
(Hardware software co-design)

I&Q complex data



ASIC (chip rate)

DSP (symbol rate)

Spreading Sequences

Desired Properties of Spreading Sequences

- Pseudo-random or Pseudo-noise=> look like white noise
- Autocorrelation function closer to delta function: distinguish a spreading signal from its time-shifted version=>for sequence acquisition and multipath combining (usually used for timing recovery)
- Crosscorrelation function closer to all-zero: distinguish a spreading signal from other spreading signals including time-shifted versions of them. =>for reducing CDMA multiple access interference
- Easy to generate => linear feedback shift register

DS CDMA with ***deterministic spreading sequences***

Consider 2 - users communication system.
The hatted receiver listens to the hatted transmitter
The unhatted transmitter produces interference

suppose $b(t)=\sum_{n=\infty}^{\infty} b_n P_T(t-nT)$, where b_n is i.i.d. random variables

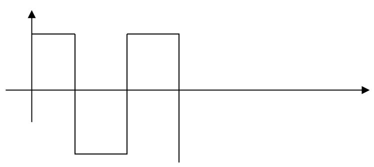
such that $p_r\{b_n=1\}=p_r\{b_n=-1\}=\frac{1}{2}$, $\hat{b}(t)$ is defined similarly.

$P_T(t)$ is a unit - amplitude rectangular pulse during [0, T]

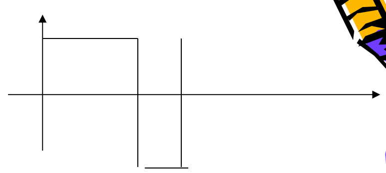
$a(t)$ and $\hat{a}(t)$ are periodic with period T = 3

n(t) is AWGN with power spectral density $N_0/2$

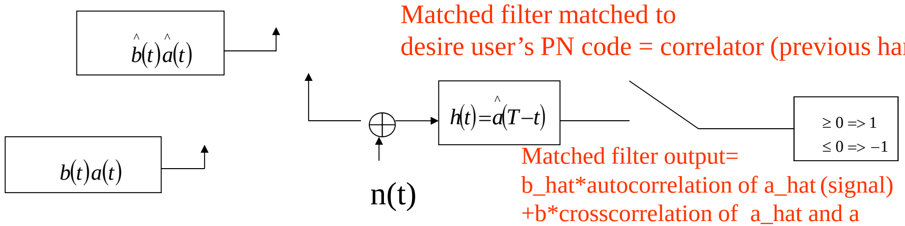
$\hat{a}(t)=+-+$



$a(t)=++-$



Matched filter matched to
desire user's PN code = correlator (previous handout)



Matched filter output=
b_hat*autocorrelation of a_hat (signal)
+b*crosscorrelation of a_hat and a
(interference)
+noise

The received signal is

$$r(t) = \hat{b}(t)\hat{a}(t) + b(t-\tau)a(t-\tau) + n(t)$$

(a) If $b_0 = b_{-1} = 1$ sketch the interference from the unhatted transmitter at t = T as a function of τ for $0 \le \tau \le T = 3$

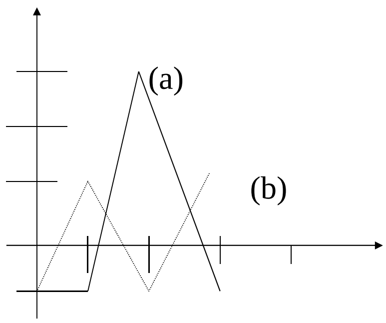
(b) Repeat (a) if $b_0 = -b_{-1} = 1$

(c) computer the variance of interference where τ is uniformly distributed on [0, T)

(d) computer SINR of this system

Solution to (a)(b)

τ	$\theta_{k,i}(\tau)\ (b_0 = b_{-1})$	$\hat{\theta}_{k,i}(\tau)\ (b_0 = -b_{-1})$
0	-1	-1
1	-1	1
2	3	-1
3	-1	1



In case τ =1 (other τ is skipped)

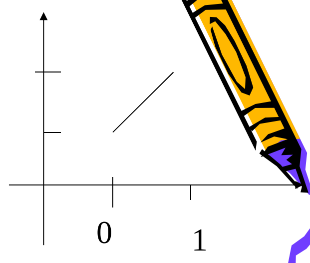
1 1 -1 1 1 –1
1 -1 1
=-1

-1 -1 1 1 1 –1
1-1 1
=1

solution to (c)

$$E\left[c^2(\tau)\right]=\int_0^1 \left[A(1-x)+Bx\right]^2 dx=\frac{A^2+AB+B^2}{3}$$

$$=\frac{1}{6}\frac{1}{3}\left[\begin{matrix}(1+1+1)+(1-3+9)+(9-3+1)\\+(1-1+1)+(1-1+1)+(1-1+1)\end{matrix}\right]$$



solution to (d)

desired signal contribution = 3

power of thermal noise $=\int \frac{N_0}{2}|H|^2 df$ (ref NWTT1)

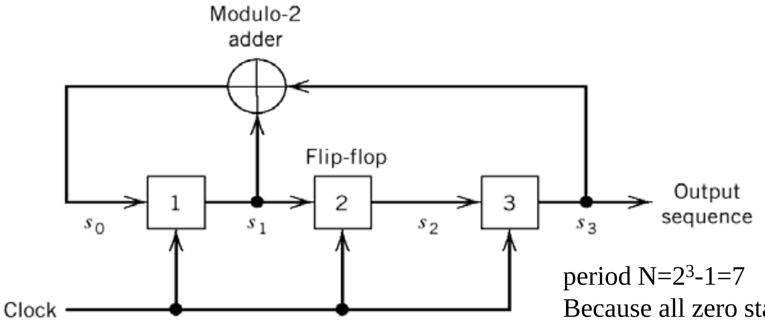
$=\int \frac{N_0}{2}|h|^2 dt$ (Parseval theorem) $=\frac{3N_0}{2}$

signal-to-interference-plus-noise ratio (SINR) $=\dfrac{3^2}{\frac{10}{9}+\frac{3}{2}N_0}$

Popular Spreading Sequences

- Maximal-length sequences (m-sequences): pseudo noise (PN) sequences with maximal length 2^m-1, where m is the shift register length
- Gold sequences
- Hadamard sequences

Figure
A m-sequence generator for shift register length $m = 3$ and feedback tap [3 1]



period $N=2^3-1=7$
Because all zero state is not allowed

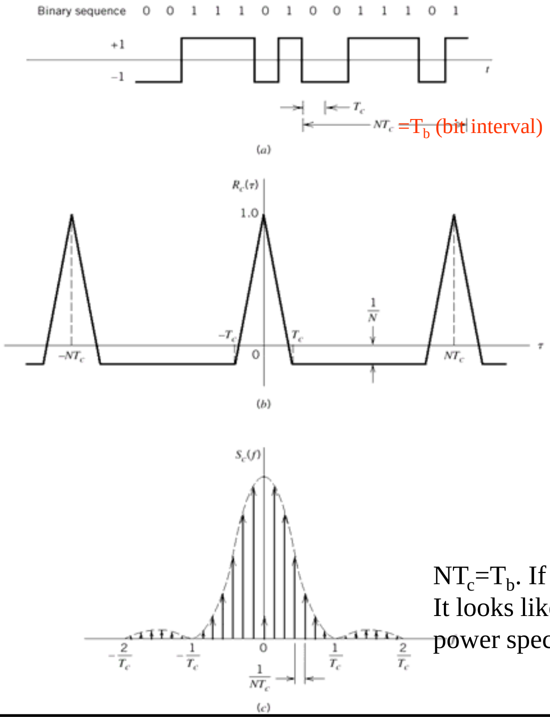
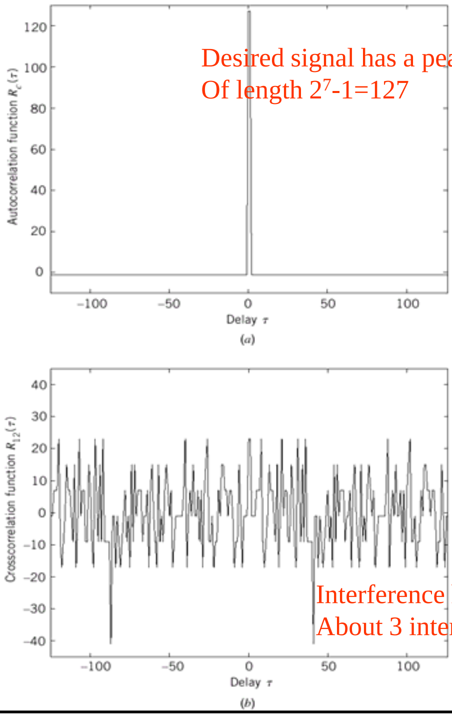


Figure
(*a*) Waveform of m-sequence for period *N* = 7, c(t). (*b*) Autocorrelation =∫c(t)c(t+τ)dt. (*c*) Power spectral density=Fourier transform of autocorrelation.
All three parts refer to the output of the feedback shift register of Figure 1.

$NT_c=T_b$. If Tc->0 (N->∞)
It looks like white noise (autocorrelation and power spectrum density)-thus called pseudo noise code



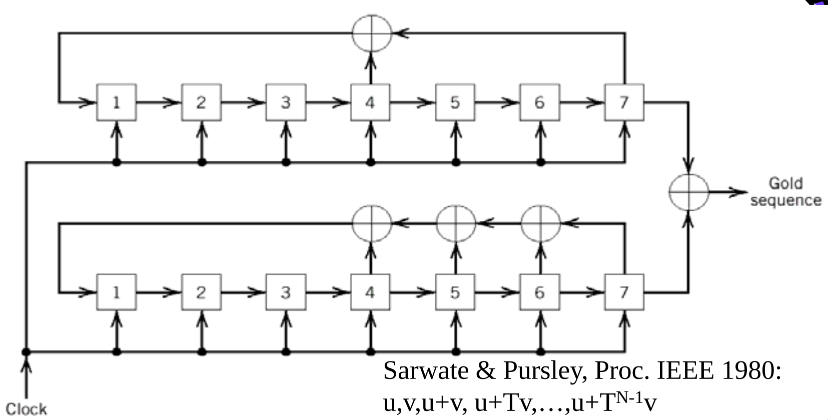
Desired signal has a peak (127) at Rx matched output matched to c(t)
Of length 2^7-1=127

Figure
(*a*) Autocorrelation function $R_c(\tau)$ function =∫c(t)c(t+τ)dt, (*b*) cross-correlation function $R_{12}(\tau)$ function= ∫c1(t)c2(t+τ)dt of the two sequences [7, 1] and [7, 6, 5, 4].

Interference has energy spread all over (max is 40)
About 3 interfering users for SIR=0dB

Figure

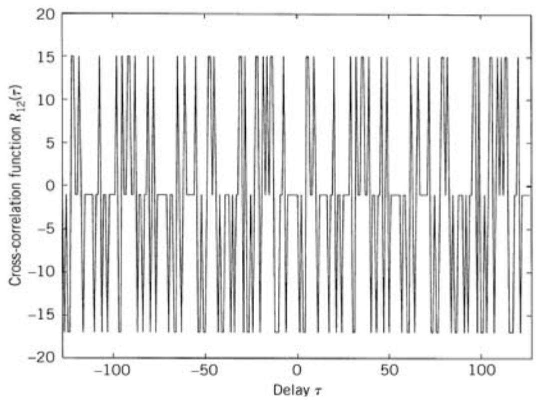
Generator for a Gold sequence of period $N=2^7-1=127$ by [7,4] and [7,6,5,4]



Sarwate & Pursley, Proc. IEEE 1980:
$u, v, u+v, u+Tv, \ldots, u+T^{N-1}v$
are N+2 Gold sequences,
where T denotes a shift

Figure

Cross-correlation function $R_{12}(\tau)$ of a pair of Gold sequences based on the two sequences [7, 4] and [7, 6, 5, 4]



Max of absolute value is 17<40 of PN code cross correlation
(7 vs. 3 interfering users for SIR about 0 dB)

Walsh Hadamard matrix

$$\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$

$$H_1 = [1]$$

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & -\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

- Gold sequences have large sets of sequences with good crosscorrelation but autocorrelation is not as good as m-sequences.
- Different Gold sequences are for transmitter separation in both uplink and downlink in WCDMA.
- Different Hadamard sequences are for channel separation of the same transmitter.
- Different Gold sequences are for different satellites in GPS.

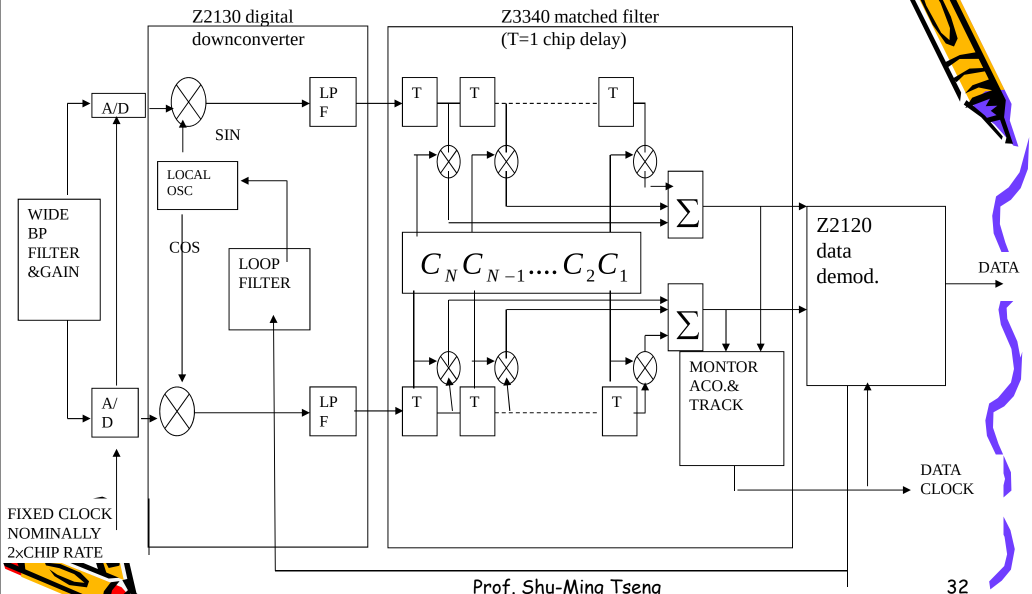
DS CDMA Code Synchronization

code synchronization

- Different initial states of linear feedback shift register results in different cyclic shifts of the same periodic PN codes.
- PN code generators in Tx and Rx must be in sync (the same cyclic shift; the same code *phase*).
- Acquisition and tracking are required.

- Acquisition: accomplish Rx's time reference within Tc in step with incoming signal.
- Tracking: to adjust and maintain local reference signal in step with the desired received signal
- Two ways to implementation: matched filter (high cost) and correlater

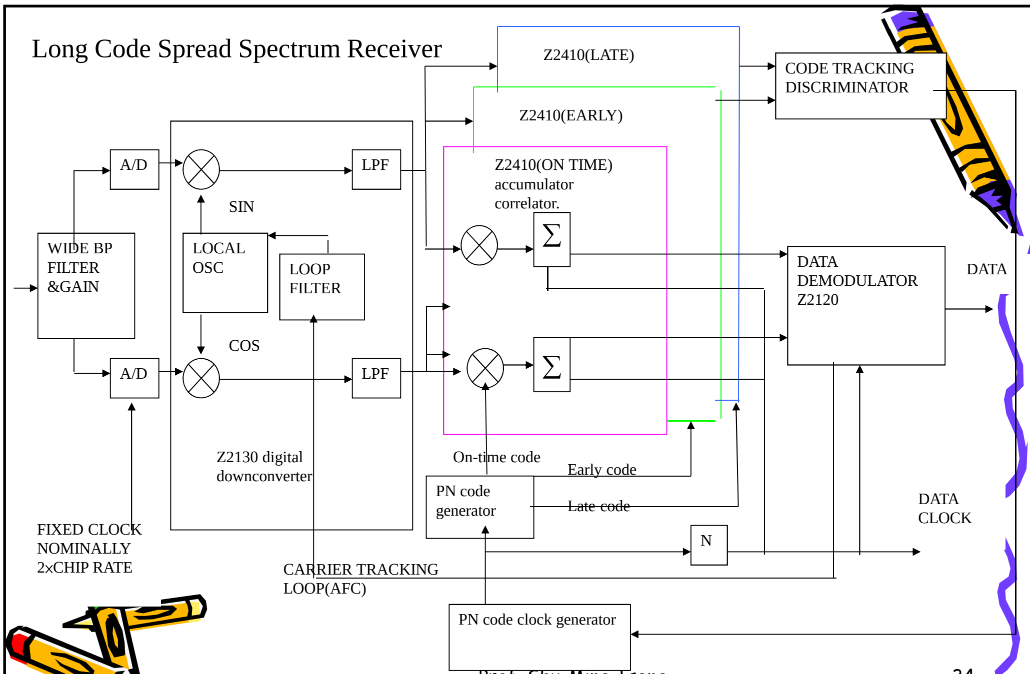
Fast Acquisition Spread Spectrum Receiver (simulate convolution by hand)



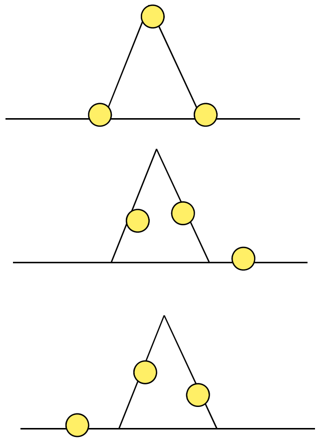
Zilos chip implementation

- PN code correlation is performed once per chip by matched filter receiver using stored local PN sequence
- Acquisition is fast and no explicit code tracking is needed, but more processing power is requited than in correlation receiver designs.

Long Code Spread Spectrum Receiver



Tracking range<2Tchip



Must shift to the left

Must shift to the right

Long Code Spread Spectrum Receiver

- PN code correlation must be performed initially, and local PN sequence must be aligned with received PN sequence for successful demodulation
- Delay locked loop implementation provides code tracking with use of three correlators (early, late, on-time)
- Processing power requirements are low, but acquisition is slower than matched filter receiver
- Can correlate very long sequences, permitting high processing gains

BER of DS CDMA with random spreading sequences

Random vs. deterministic spreading sequences

- There are too many possible deterministic sequences. A probabilistic approach to compute *average* performance over all possible spreading sequences becomes necessary. For specific spreading sequences, they may perform better or worse than the average case derived here.
- A random binary spreading sequence is a sequences of i.i.d binary random variables with equal probability.
- For computer simulation, we need to regenerate spreading sequences many times to get the simulation results close to those of random spreading sequences.

Assumptions

- K: the number of users
- N: the number of chips per data symbol (processing gain)
- E_b: bit energy
- $N_0/2$: two-sided power spectral density of AWGN
- Asynchronous DS CDMA systems with BPSK modulation and binary random spreading codes

Standard Gaussian Approximation (SGA)

- Assume K is large, we can assume MAI as a Gaussian random variables (by central limit theorem).
- BER is given by

$$P_b = Q\left(\left[\frac{K-1}{3N} + \frac{N_0}{2E_b}\right]^{-0.5}\right).$$

Improved Gaussian Approximation (IGA)

Let MAI variance (power) $V = \sum_{k=2}^{K} Var(I_k)$ be a random variable and whose pdf can be computed by convolution of K - 1 terms. IGA gives

$$P_b \approx E_v(P(error \mid V)) = \int_0^{\infty} Q\left(\sqrt{\frac{E_b}{\frac{N_0}{2}+v}}\right) f_V(v)\, dv$$

that is, conditional Gaussian MAI

K-1 folded convolution and numerically integration is required (no close form), so utilility of IGA is limited

Simplified Expression for IGA (SEIGA) using Taylor series [Rappaport] [Holtzman]

$$P_b(K) = \frac{2}{3} Q[(\frac{K-1}{3N} + \frac{N_0}{2E_b})^{-0.5}]$$

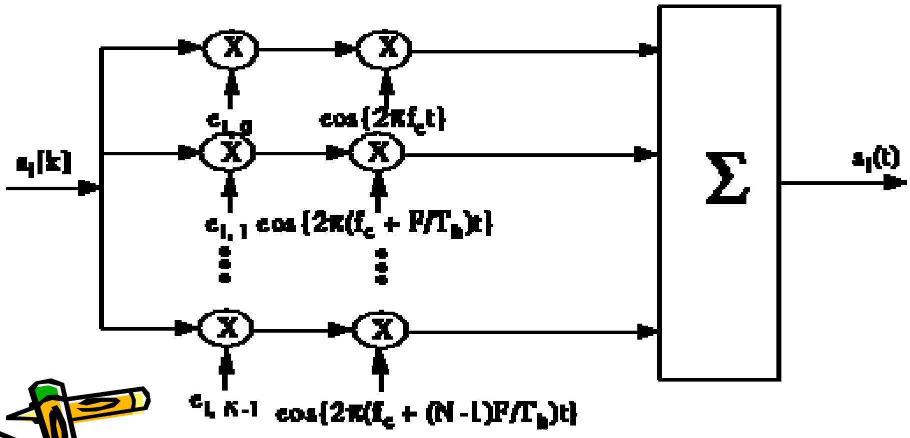
$$+ \frac{1}{6} Q[(\frac{(K-1)(N/3) + \sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b})^{-0.5}]$$

$$+ \frac{1}{6} Q[(\frac{(K-1)(N/3) - \sqrt{3}\sigma}{N^2} + \frac{N_0}{2E_b})^{-0.5}]$$

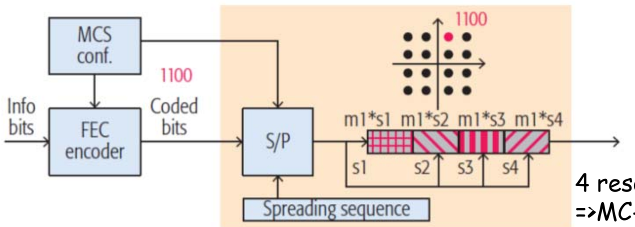
where $\sigma^2 = (K-1)[N^2 \frac{33}{360} + N(\frac{1}{20} + \frac{K-2}{36}) - \frac{1}{20} - \frac{K-2}{36}]$

3rd term is set to zero for small K

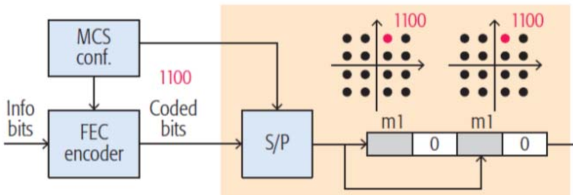
Multi-Carrier (MC) CDMA: signature chips in the frequency domain



Multi-Carrier (MC) CDMA and Sparse Code Multiple Access (SCMA)



4 resource element (RE)=4 subcarrier
=>MC- CDMA spreading sequence in every subcarriers



4 resource element (RE)=4 subcarrier
=>SCMA spreading sequence in Only some (sparse) subcarriers

[Che18] Chen, Yan, et al. "Toward the standardization of non-orthogonal multiple access for next generation wireless networks." *IEEE Communications Magazine* 56.3 (2018): 19-27.