

## OFDM(A) principle/ synchronization/ channel estimation/ power allocation

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3-4 hr

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## Outline

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- Review: FT, DTFT, DFT, and FFT
- OFDM(A) principle
- Synchronization of DVB-T (an OFDM system)
- Channel estimation of DVB-T (an OFDM system)

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## Review: FT, DTFT, DFT, and FFT

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## 離散時間傅立葉轉換 (DTFT, DISCRETE-TIME FOURIER TRANSFORM)

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由於電腦內的資料都是離散 (Discrete) 的形式，因此上面這種連續性積分的公式，並不太適用於電腦資料的運算上，於是便有離散式傅立葉轉換公式的需要。

傅立葉轉換

$$F\{f_{sp}(t)\} = \int_{-\infty}^{\infty} \hat{f}(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} f(nT) \delta(t - nT) e^{-i\omega t} dt$$

取樣序列  $f(nT)$  的DTFT

$$F_{DTFT}(e^{i\omega T}) = \sum_{n=-\infty}^{\infty} f(nT) e^{-in\omega}$$

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## 離散時間傅立葉轉換 (DTFT, DISCRETE-TIME FOURIER TRANSFORM)

通常將採樣間隔 $T=1$ ，得公式：

離散時間傅立葉轉換：

$$F_{DTFT}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} f(n)e^{-jn\omega}$$

離散時間傅立葉逆轉換：

$$f(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_{DTFT}(e^{j\omega}) e^{jn\omega} d\omega$$

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## 離散傅立葉轉換 (DFT, DISCRETE FOURIER TRANSFORM)

公式：

離散傅立葉轉換：

$$X[k] = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}nk} x[n], \text{ where } \omega = \frac{2\pi}{N}k \text{ 其中 } k = 0, 1, \dots, N-1$$

離散傅立葉逆轉換：

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}nk} X[k] \text{ 其中 } n = 0, 1, \dots, N-1$$

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## FT、DTFT、DFT比較

傅立葉轉換	時域	頻域
FT	連續 非週期性	連續 非週期性
DTFT	離散 非週期性	連續 週期性
DFT	離散 週期性	離散 週期性

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## 快速傅立葉轉換 (FFT, FAST FOURIER TRANSFORM)

說明：快速傅立葉轉換是離散傅立葉轉換的快速演算法，也可用於計算離散傅立葉轉換的逆轉換

計算 DFT，所需要的複雜度是  $O(n^2)$ ，但在 1965 年，有兩位學者提出來一套更精簡的演算法，所需的複雜度只有  $O(n \log n)$ ，這一套演算法稱為「快速傅立葉轉換」(Fast Fourier Transform, 簡稱 FFT)，換句話說，FFT 是用來計算 DFT 的快速方法。

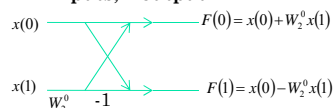
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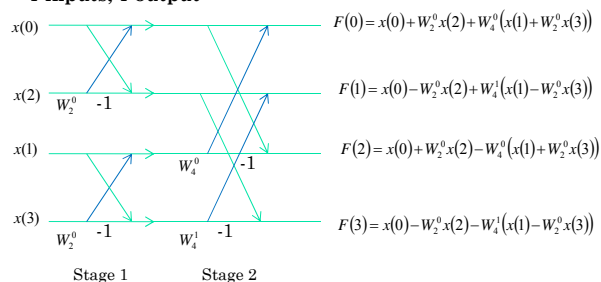
## 快速傅立葉轉換 (FFT, FAST FOURIER TRANSFORM)

### Butterfly diagram

#### 2-inputs, 2-output



#### 4-inputs, 4-output



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### 2. 離散傅氏轉換之性質

#### (a) 線性性質：

$$\alpha x[n] + \beta y[n] \xrightarrow{\text{DFT}} \alpha X[k] + \beta Y[k], \forall \alpha, \beta \in \mathbb{R}$$

$\xleftarrow{\text{IDFT}}$

#### (b) 時間延遲性質：

$$x[n - r_0] \xrightarrow{\text{DFT}} X[k] \exp[-j \frac{2\pi}{N} k n_0]$$

$\xleftarrow{\text{IDFT}}$

#### (c) DFT 與 IDFT 互換性質：

$$x[n] = \frac{1}{N} (\text{DFT}\{X^*[k]\})^*$$

#### (d) 時間領域週期迴旋和性質：

$$x[n] \otimes y[n] = \sum_{\lambda=0}^{N-1} x[\lambda] y[n - \lambda] \xrightleftharpoons[\text{IDFT}]{\text{DFT}} X[k] Y[k]$$

Circular convolution

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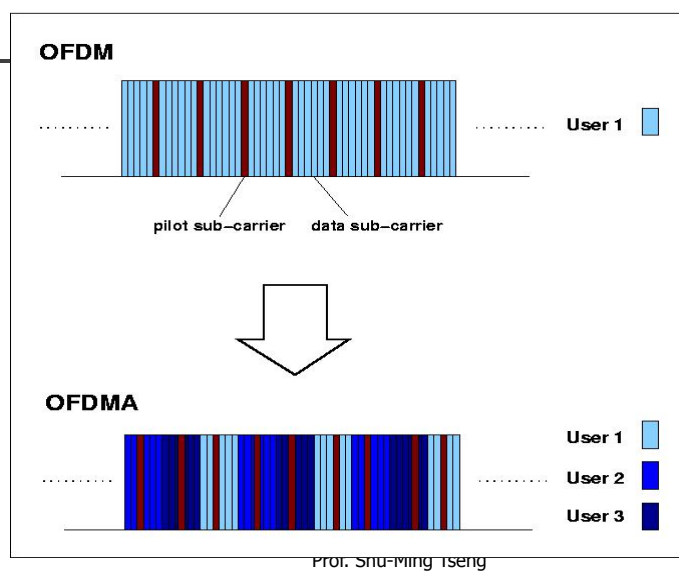
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## OFDM(A) principle

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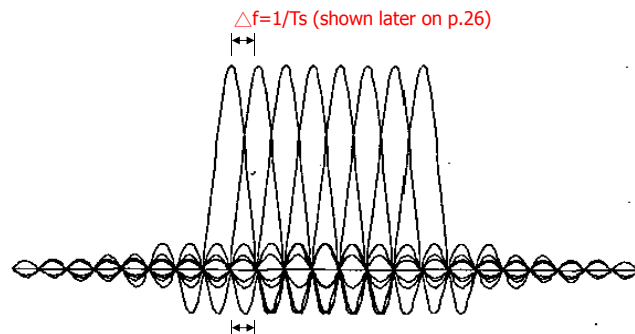
- OFDM: orthogonal frequency division multiplexing
- OFDMA: orthogonal frequency division multiple access (orthogonal +FDMA)



## Introduction

- Because the growth in scientific, military, and entertainment applications, cloud computing, and development of 4G result in high-speed, reliable wireless system.
- thus we need multicarrier transmission method (**for higher data rate**) , such OFDMA, and we make an overview of OFDMA here.

## High spectrum efficiency

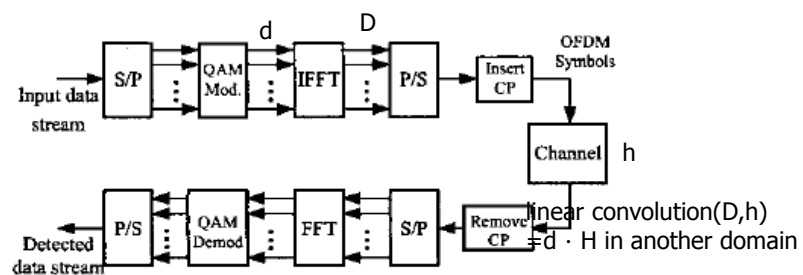


**Figure 1.** An OFDM Symbol in frequency domain (before the subcarriers are summed).

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## OFDM block diagram



**Figure 2.** A simplified block diagram of an OFDM system

QAM Modulation=I/Q mapping (NOT modulated to carrier)

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## OFDMA= FDMA+orthogonal subcarrier+CP

- The symbol rate on each carrier is lower (1/carrier no)
- A cyclic prefix (CP) or zero prefix can be used to avoid the ISI (frequency-selective fading)
- Simple IFFT and FFT circuits for modulation and demodulation=> OFDM become practical since 1990s

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## Cyclic Prefix (CP)

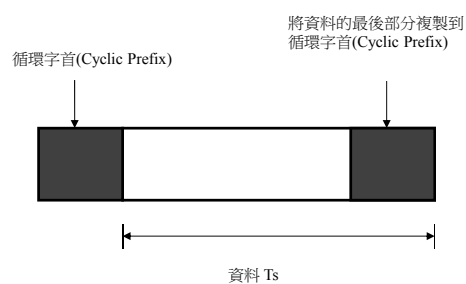


圖 OFDM和OFDMA的循環字首

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## Cyclic Prefix

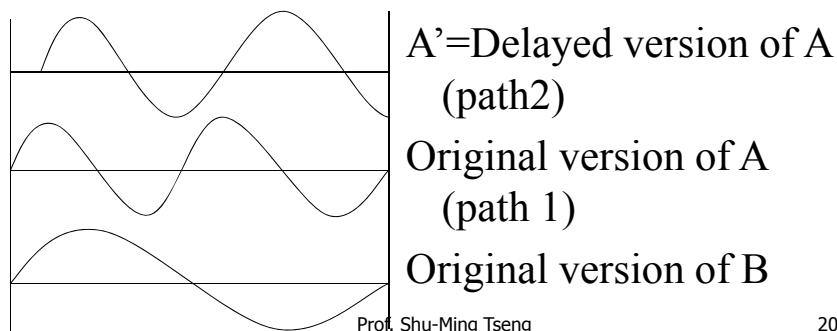
- CP > delay spread to **prevent ISI**.
- CP can convert linear convolution (with channel) into **circular convolution** (periodic nature); thus we get data multiply DFT ( $\Leftrightarrow$  circular convolution, not linear convolution!) of channel.
- CP doesn't contain any information so it reduces throughput. But **CP redundancy ratio is reduced as carrier no increases** (OFDM symbol duration = non-OFDM symbol duration \* carrier no).
- CP can also simplify the synchronization problem (**timing and fractional frequency synchronization**).
- CP is to **prevent inter-carrier interference (ICI) in multipath environment** (next page)

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## Guard interval

- If guard interval = zero padding, A and B are orthogonal, but A' and B are not orthogonal (higher freq components  $\Rightarrow \Delta f \neq 1/T$ )!
- If guard interval = cyclic prefix, A' and B are not orthogonal ( $\Delta f \neq 1/T$ )



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## OFDM in NG-PON2 (next generation passive optical network 2) and PLC (power line communications)

- Anti-dispersion (色散) - In optical fiber, different frequency travel in different speed, so the arrival time is different (dispersion). This is similar to ISI due to multipath in wireless channels, so cyclic prefix can be used.
- Disadvantage: PAPR Source: Networked Communication Program (NCP) newsletter No. 24
- PLC: impedance (50 or 75Ω) mismatch causes EM wave reflections => multipath fading in PLC

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## The criterion of carriers $m\Delta f$ and $n\Delta f$ to be orthogonal

If  $\Delta f = 1/T_s$ , where  $T_s$  is OFDM symbol interval then

$$1/T_s \int_0^{T_s} e^{j2\pi m\Delta f t} e^{-j2\pi n\Delta f t} dt = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

proof : skipped (hint : sinc function)

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## OFDMA advantages

- higher spectrum efficiency(=>higher data rate)
- Transfer frequency selective fading to a large number of flat fading subchannels (easy to equalize, ex 1-tap equalizer).
- two orthogonal sinusoid are orthogonal even in asynchronous operation (orthogonal criterion: frequency difference= $1/T$  async only affect phase diff), not true for CDMA

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- The division of signal will provide flexibility needed for future multimedia services (data rate可微調)

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## OFDMA disadvantages

- OFDMA is sensitive to frequency shifts such as Doppler shift and carrier frequency offsets (CFO)
- P.S. OFDMA users may have different CFO due to tuner inaccuracy etc.
- Peak to Average Power Ratio (PAPR) problem for power amplifiers.

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## DFT IMPLEMENTATION TRANSMITTER

- Transmitted signal can be obtained using a Discrete Fourier Transform

$$x_b(t) = \sum_{k=0}^{N-1} a[k] e^{j2\pi k \Delta f t}, \quad 0 \leq t \leq T_s$$

$$f = k \Delta f$$

$$t = n \Delta t = n \frac{T_s}{N}$$

- If sampled at a rate of  $T_s/N$ ,

$$x_b[n] = x_b\left(\frac{n}{N} T_s\right) = \sum_{k=0}^{N-1} a[k] e^{j2\pi n k \Delta f T_s / N}$$


- For orthogonality,  $\Delta f T_s = 1$ ,

$$x_b[n] = \sum_{k=0}^{N-1} a[k] e^{j2\pi n k / N} = \text{IDFT}\{a[k]\}$$

- Efficient FFT implementation


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## DFT IMPLEMENTATION RECEIVER


$$\begin{aligned}\hat{a}[k] &= \text{DFT}\{x_b[n]\} \\ &= \frac{1}{N} \sum_{n=0}^{N-1} x_b[n] e^{-j2\pi nk/N} \\ &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} a[m] e^{j2\pi n(m-k)/N} \\ &= \frac{1}{N} \sum_{m=0}^{N-1} a[m] \sum_{n=0}^{N-1} e^{j2\pi n(m-k)/N} \\ &= \frac{1}{N} \sum_{m=0}^{N-1} a[m] N \delta[m-k] \\ &= a[k] \quad \text{Restore original signal}\end{aligned}$$

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## OFDM(A) synchronization and channel estimation: PC-based DVB-T software defined radio example

Study spec well before developing products

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# Synchronization of DVB-T (an OFDM system)

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## Functional block diagram

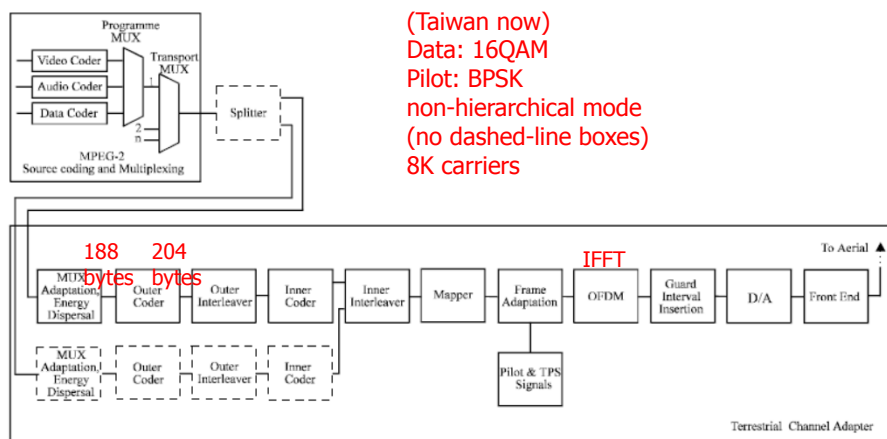
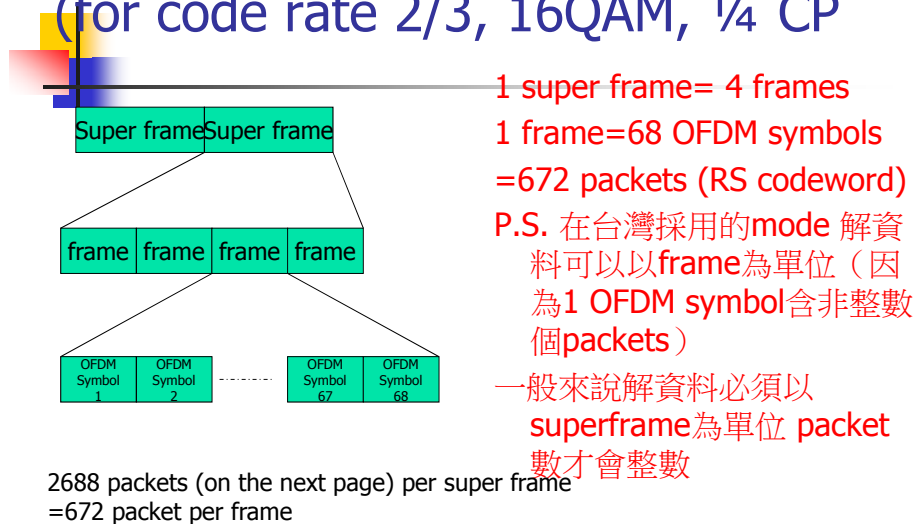


Figure 1: Functional block diagram of the System

## Frame structure (for code rate 2/3, 16QAM, 1/4 CP)



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## OFDM frame structure

The OFDM frame structure allows for an integer number of Reed-Solomon 204 byte packets to be transmitted in an OFDM super-frame (for all code rate and modulation), and therefore avoids the need for any stuffing, whatever the constellation, the guard interval length, the coding rate or the channel bandwidth may be. See table 16.

Table 16: Number of Reed-Solomon packets per OFDM super-frame for all combinations of guard interval, code rates and modulation forms

Code rate	QPSK		16-QAM		64-QAM	
	2K mode	8K mode	2K mode	8K mode	2K mode	8K mode
1/2	252	1008	504	2016	756	3024
2/3	336	1344	672	2688	1008	4032
3/4	378	1512	756	3024	1134	4536
5/6	420	1680	840	3360	1260	5040
7/8	441	1764	882	3528	1323	5292

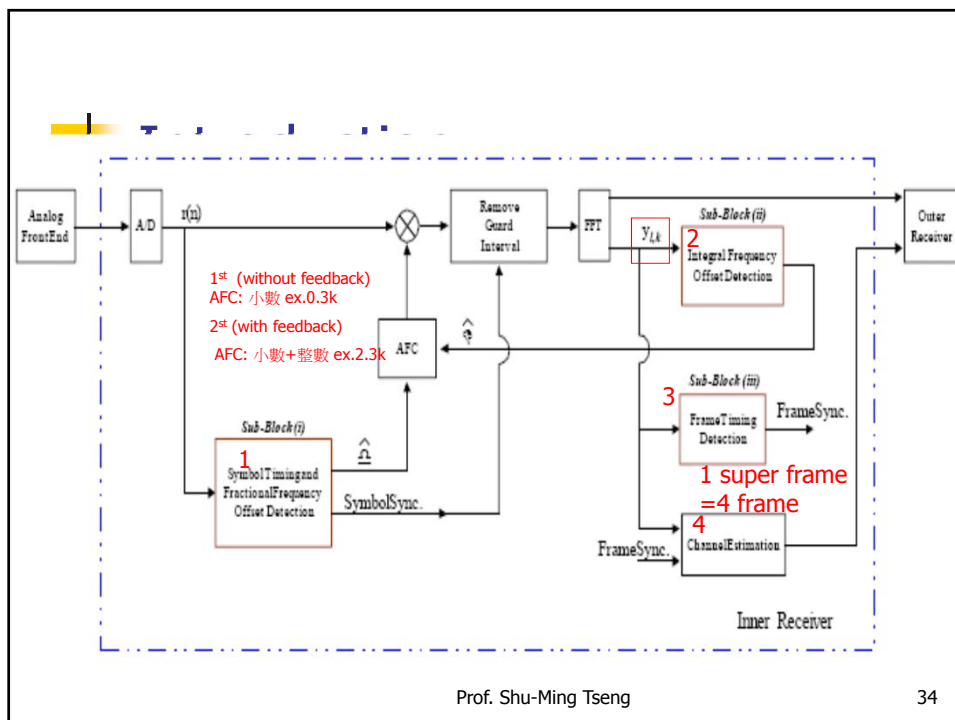


# OUTLINE

- Introduction
- Fractional frequency offset and symbol timing offset: joint ML by cyclic prefix
- Integral frequency: by continual pilot
- Frame timing: by 16 bits of transmission parameter signaling (TPS)- other bits are code rate, modulation etc.
- Scattered pilots are used in channel estimation, not synchronization.

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## 9.953Mbps for real-time DVB-T receiver (at Viterbi decoder input)

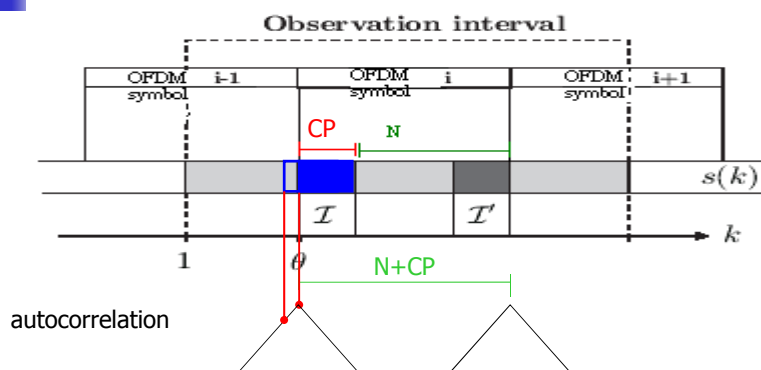
Table E.5: Useful bitrate (Mbit/s) for all combinations of guard interval, constellation and code rate for non-hierarchical systems for 6 MHz channels

Modulation	Code rate	Guard interval			
		1/4	1/8	1/16	1/32
QPSK	1/2	3.732	4.147	4.391	4.524
	2/3	4.976	5.529	5.855	6.032
	3/4	5.599	6.221	6.587	6.786
	5/6	6.221	6.912	7.318	7.540
	7/8	6.532	7.257	7.684	7.917
16-QAM	1/2	7.465	8.294	8.782	9.048
	2/3	9.953	11.059	11.709	12.064
	3/4	11.197	12.441	13.173	13.572
	5/6	12.441	13.824	14.637	15.080
	7/8	13.063	14.515	15.369	15.834
64-QAM	1/2	11.197	12.441	13.173	13.572
	2/3	14.929	16.588	17.564	18.096
	3/4	16.796	18.662	19.760	20.358
	5/6	18.662	20.735	21.955	22.620
	7/8	19.595	21.772	23.053	23.751

NOTE: Figures in *italics* are approximate values.  
 For the hierarchical schemes the useful bit rates can be obtained from table E.5 as follows:  
 HP stream: figures from QPSK columns;  
 LP stream, 16-QAM: figures from QPSK columns;  
 LP stream, 64-QAM: figures from 16-QAM columns.

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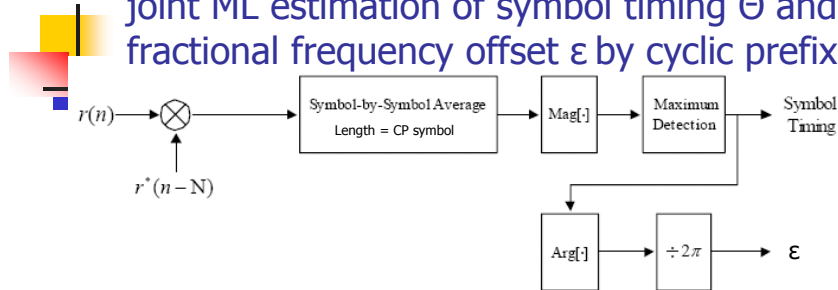
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## joint ML estimation of symbol timing $\Theta$ and fractional frequency offset $\varepsilon$ by cyclic prefix



- Receive signal:  $r(n) = x_t(n)e^{j2\pi\varepsilon n/N}$

- Do auto correlation ;

$$M_r(n) = r(n)r^*(n-N) = x_t(n)x_t^*(n)e^{j2\pi\varepsilon n}$$

$$C(\theta) = \frac{1}{CP} \sum_{n=\theta}^{\theta+CP} M_r[n]$$

- $\Theta' = \text{MAX } |C(\Theta)|$
- $\varepsilon = 1/2\pi \arg |C(\Theta')|$

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## continual pilot (177 carriers) (寫程式時要建表)

Table 7: Carrier indices for continual pilot carriers

Continual pilot carrier positions (index number k)												
2K mode							8K mode					
0	48	54	87	141	156	192	0	48	54	87	141	156
201	255	279	282	333	432	450	201	255	279	282	333	432
483	525	531	618	636	714	759	483	525	531	618	636	714
765	780	804	873	888	918	939	765	780	804	873	888	918
942	969	984	1050	1101	1107	1110	942	969	984	1050	1101	1107
1137	1140	1146	1206	1269	1323	1377	1137	1140	1146	1206	1269	1323
1491	1683	1704					1491	1683	1704			
							1860	1896	1905	1959	1983	1986
							2136	2154	2187	2229	2235	2322
							2418	2463	2489	2484	2508	2577
							2622	2643	2646	2673	2688	2754
							2811	2814	2841	2844	2850	2910
							3027	3081	3195	3387	3408	3456
							3495	3549	3564	3600	3609	3663
							3690	3741	3840	3858	3891	3933
							4026	4044	4122	4167	4173	4188
							4281	4296	4326	4347	4350	4377
							4458	4509	4515	4518	4545	4548
							4614	4677	4731	4785	4899	5091
							5160	5166	5199	5253	5268	5304
							5367	5391	5394	5445	5444	5562
							5637	5643	5730	5748	5826	5871
							5892	5916	5985	6000	6030	6051
							6081	6096	6162	6213	6219	6222
							6252	6258	6318	6381	6435	6489
							6795	6816				6603

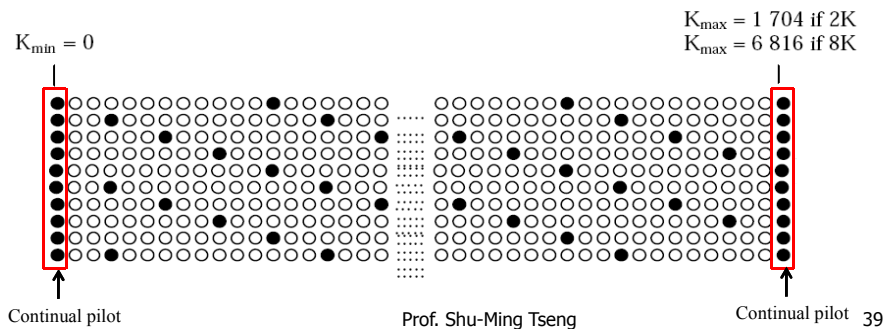
8K mode: 8192 subcarriers, 6817 data+pilot (continual, TPS, scattered pilots),  
1<sup>st</sup> 45 of \*K mode is the same as 2K mode => one table for 2K and 8K

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## Integral frequency synchronization

- The same subcarrier index  $k=0,48,54,\dots$  have the same value of the continual pilot. We can take advantage of this property to find integral frequency offset.



- The value of continual pilot (不同OFDM symbol index 用一樣的):

$$\text{Re}\{C_{m,l,k}\} = \frac{4}{3} \times 2(1/2 - w_k), \text{Im}\{C_{m,l,k}\} = 0$$

$m$ : frame index  $l$ : symbol index  $k$ : subcarrier index

- P.S.  $4/3$ : boosted power;  $2(1/2 - w_k)$ :  $0,1 \Rightarrow -1,+1$
- $w_k$  is generated by generate matrix  $X^{11} + X^9 + 1$

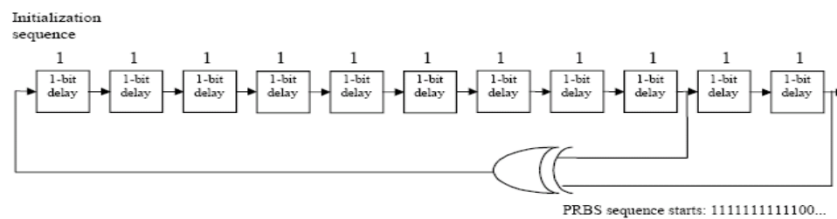
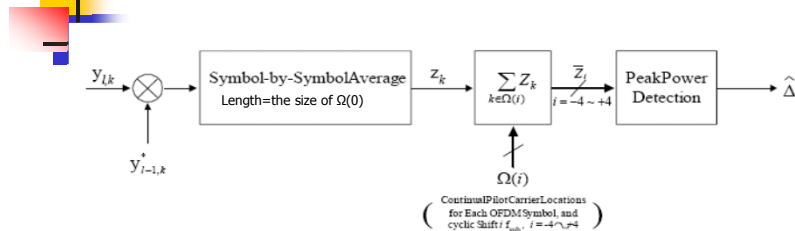


Figure 10: Generation of PRBS sequence

## Adjacent OFDM symbols ( $l, l+1$ ), same carrier index $k$ , same continual pilot



- The set of subcarrier of continue pilot of one symbol is  $\Omega(0)$
- $\Omega(i)$  :  $i$  is circular shift of the number of subcarrier
- $\Omega(0) = \{0, 48, 54, \dots\}$ ,  $\Omega(1) = \{1, 49, 55, \dots\}$  (Table 7)
- $Z_i = \sum_{k \in \Omega(i)} y_{l,k-i} \cdot y_{l-1,k-i}^*$
- The Integral frequency offset is " $i$ " that make  $Z$  is a peak value

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## TPS pilot (68 carriers)

Table 8: Carrier indices for TPS carriers

2K mode					8K mode							
34	50	209	346	413	34	50	209	346	413	569	595	688
569	595	688	790	901	790	901	1073	1219	1262	1286	1469	1594
1073	1219	1262	1286	1469	1687	1738	1754	1913	2050	2117	2273	2299
1594	1687				2392	2494	2605	2777	2923	2966	2990	3173
					3298	3391	3442	3458	3617	3754	3821	3977
					4003	4096	4198	4309	4481	4627	4670	4694
					4877	5002	5095	5146	5162	5321	5458	5525
					5681	5707	5800	5902	6013	6185	6331	6374
					6398	6581	6706	6799				

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## Frame Timing:

### 1 frame=68 OFDM symbols

Bit number	Purpose/Content
$s_0$	Initialization
$s_1 - s_{16}$	Synchronization word
$s_{17} - s_{22}$	Length indicator
$s_{23} - s_{24}$	Frame number
$s_{25} - s_{26}$	Constellation
$s_{27} - s_{28} - s_{29}$	Hierarchy information
$s_{30} - s_{31} - s_{32}$	Code rate, HP stream
$s_{33} - s_{34} - s_{35}$	Code rate, LP stream
$s_{36} - s_{37}$	Guard interval
$s_{38} - s_{39}$	transmission mode
$s_{40} - s_{53}$	Reserved for future use
$s_{54} - s_{67}$	Error protection

- TPS( transmission parameter sequence) is used to record some information of signal.

- 1 frame has 68 OFDM symbol. 1 OFDM symbol has 1 value of TPS. TPS carriers of 1 OFDM symbol are the same(repeat 68 times in 8K mode)

- It use  $s_1 \sim s_{16}$  to do frame synchronization

- DBPSK modulation (resistant to phase errors)

$$\text{BCH code } x_{l,k} = x_{l-1,k} (-1)^{s_l}$$

$k \in \Gamma$   $\Gamma$  is the location of TPS

$l = 0, 1, \dots, 67$  is the OFDM symbol index within a frame

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## Frame Timing:

### 1 superframe=4 frames

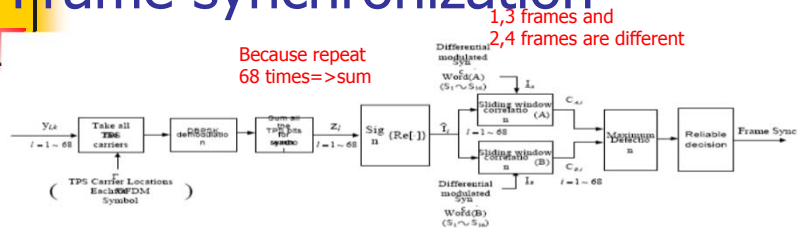


- Bits 1 to 16 of the TPS is a synchronization word.
- The first and third TPS block in each super-frame have the following synchronization word:  
 $s_1 - s_{16} = 0011010111101110.$
- The second and fourth TPS block have the following synchronization word:  
 $s_1 - s_{16} = 1100101000010001.$

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## Frame synchronization



- It use sliding window to find synchronization word.
- The size of sliding window is 16 ,so it must find 16  $s_l$  first.
- Find  $s_l$  :

DPSK modulation

$$y_{l,k} = x_{l,k} \bullet H_{l,k} + n_{l,k} = x_{l-1,k} (-1)^{s_l} \bullet H_{l,k} + n_{l,k}, \quad l=1,2,\dots,16$$

DBPSK demodulation

$$z_{l,k} = y_{l,k} \bullet y_{l-1,k}^* = |x_{l-1,k}|^2 |H_{l,k}|^2 (-1)^{s_l}$$

$$\cong |x_{l-1,k}|^2 |H_{l,k}|^2 (-1)^{s_l}$$

$$z_l = \sum_k z_{l,k} \quad (\text{k68個})$$

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## Channel estimation for DVB-T (an OFDM system)

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## Scattered pilot

- The values of scattered pilot is:  
(continual pilot also 4/3, but TPS not because  
already repeated)

$$\text{Re}\{c_{m,l,k}\} = 4/3 \times 2(1/2 - w_k)$$

$$\text{Im}\{c_{m,l,k}\} = 0$$

- And the scattered pilot position is permuted  
as :

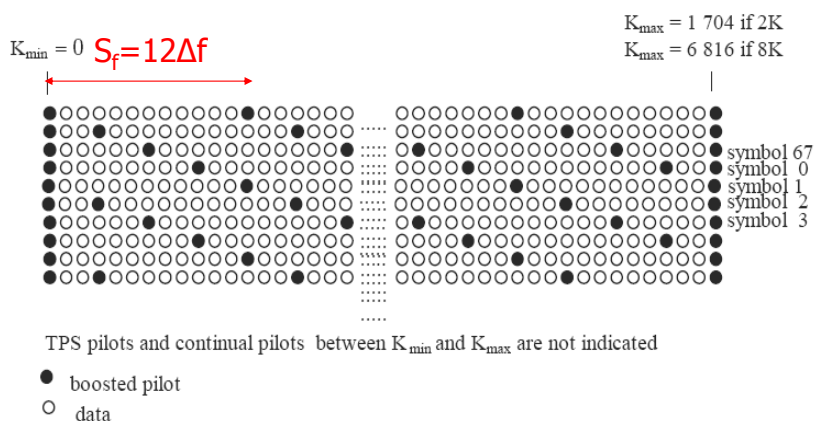
$$\{k = K_{\min} + 3 \times (1 \bmod 4) + 12p \mid p \text{ integer}, p \geq 0, k \in [K_{\min}, K_{\max}]\}$$

M:frame index    l:symbol index    k:carrier index

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## Scattered pilot



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## One-dimensional linear interpolation : inside 1 OFDM symbol

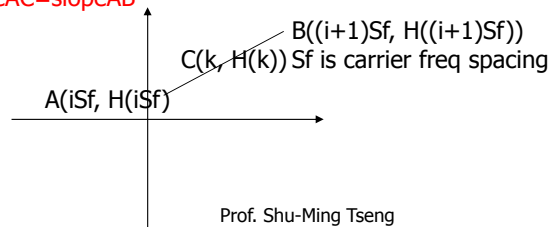
$$H[k] = H[i \cdot S_f] + \{H[(i+1) \cdot S_f] - H[i \cdot S_f]\} \cdot \frac{k - i \cdot S_f}{S_f} \quad \text{for } i \cdot S_f \leq k \leq (i+1) \cdot S_f$$

Simple but low accuracy.

Amplitude and phase should be interpolated separately

And note that  $2\pi$  phase discontinuity

Slope<sub>AC</sub> = slope<sub>AB</sub>



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## One-dimensional linear interpolation + multi OFDM symbol averaging

- very slow-varying channels
- By weak law of large numbers

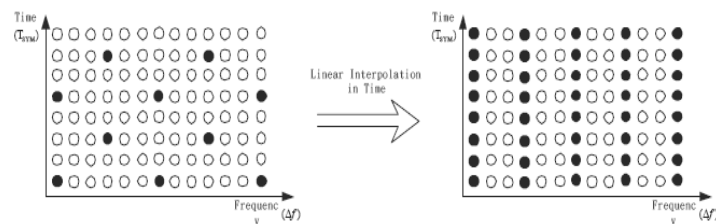
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## Two-dimensional linear interpolation

- Interpolating cross OFDM symbols (vertical) first ( $S_f = 12\Delta f \Rightarrow S_f = 3\Delta f$ )
- Then interpolation inside OFDM symbol (horizontal)



非DVB-T spacing

圖 3.23 時間軸上先做內插法的通道響應取樣訊號示意圖

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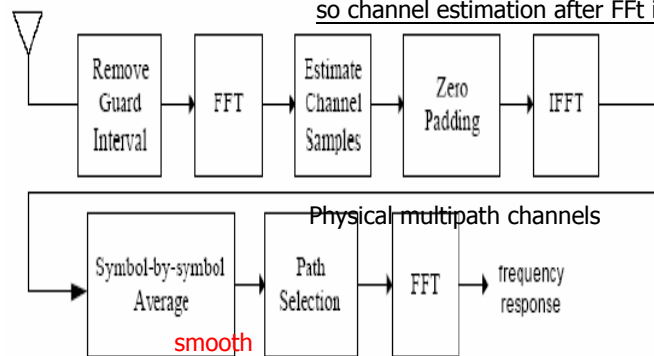
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## One-dimensional FFT interpolation

Periodx12 in one domain  
=> up sampling in the other domain

Pilot (insert pilot before IFFT in TX, (p.30)  
so channel estimation after FFT in RX)



Pick strong paths  
(weak paths dominated by interference and noise)

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## One-dimensional linear interpolation+ One-dimensional FFT interpolation

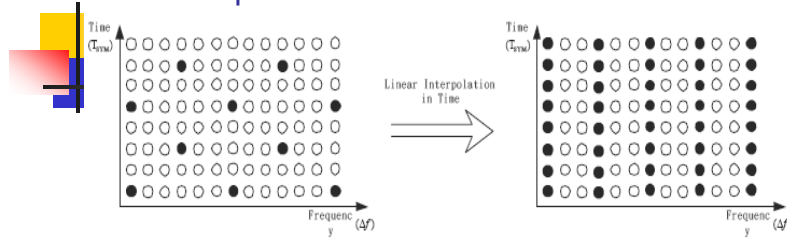
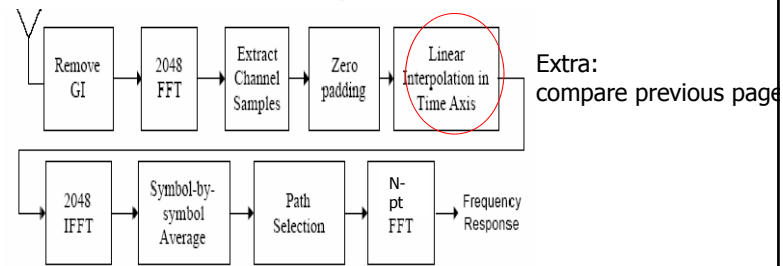


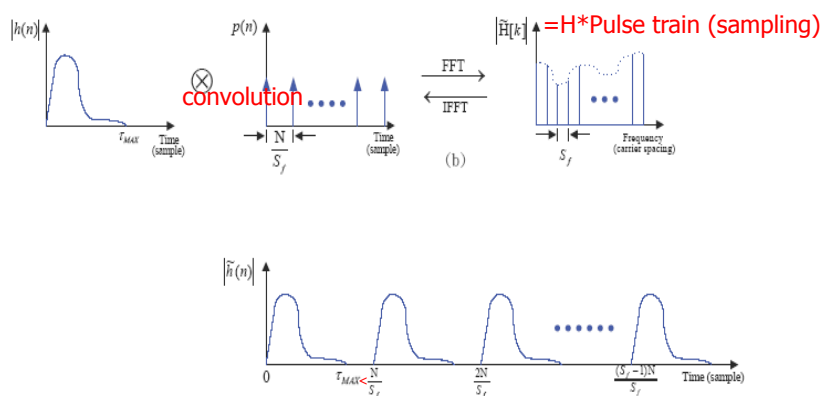
圖 3.23 時間軸上先做內插法的通道響應取樣訊號示意圖



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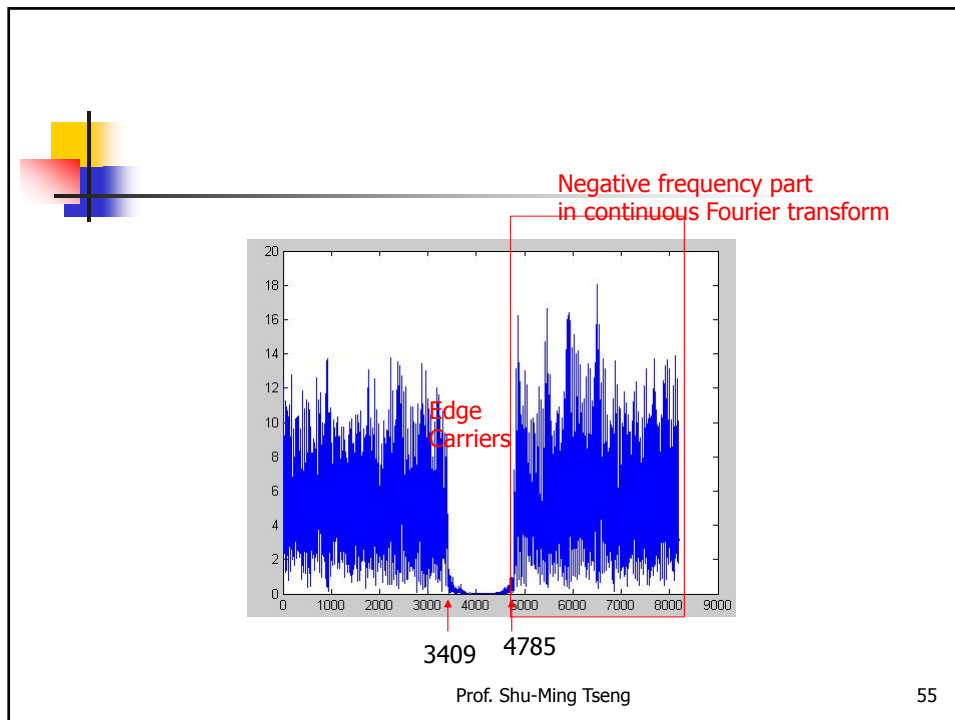
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$S_f$  small  $\Rightarrow$  delay spread  $T_{\max}$  bigger)



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## OFDM power allocation-water filling (WF)

201704索豪聲WF

```

1 % This function calculates the power allocation using water filling
2 % algorithm
3 % The first input is the individual SNR
4 % The second input is the sum of power constraint
5 % The output is the vector of power allocation
6
7 function [P] = WF(SNR,Pt,Qo)
8     n = length(SNR);
9     %Qo=3/((qfuncinv(.01/4))^2);
10
11     SortSNR=sort(SNR); % 由小到大排列
12     for i=1:n
13         Power=1/(SortSNR(i)*Qo)+zeros(1,n);
14         for m=1:n
15             Power(m)=max(0,Power(m)-1/(SNR(m)*Qo));
16         end
17         if sum(Power)>Pt
18             sum(i);
19             break;
20         end
21     end
22 end

```

$\eta = \frac{3}{P_{N_0}} [Q^{-1}(SER_t/4)]^{-2}$   
 In [Wan13] eq(4) 去除  $P_{N_0}$   
 從最大的開始試  
 $(\frac{1}{SortSNR(i)*Qo}) = 1/\lambda$   
 $sum(power) > P_t$   
 試到第i個  
 $sum(power) < P_t$   
 有o個被power allocation到  
 SNR大  
 $\frac{1}{\eta|H_{k,m}|^2} = 1/SNR$   
 (floor)較低 水較深

$P_{k,m}^* = \left[ \frac{1}{\lambda_k} - \frac{1}{\eta|H_{k,m}|^2} \right]^+, \forall m \in A_k^{(i)}$   
 In [Wan13] eq(16)

```

27 for t=n-o+1:n
28     tot=tot+1/(SortSNR(t)*Qo);
29 end
30
31 invth=(tot+Pt)/o;
32
33 P=zeros(1,n);
34 for i=1:n
35     P(i)=max(0, invth-1/(SNR(i)*Qo));
36 end

```

Line 27-36  
 $sum(power) < P_t$   
 $\Rightarrow sum(power) = P_t$   
 $P_{k,m}^* = \left[ \frac{1}{\lambda_k} - \frac{1}{\eta|H_{k,m}|^2} \right]^+, \forall m \in A_k^{(i)}$   
 In [Wan13] eq(16)

$$\begin{aligned}
 & \sum_o P_{k,m}^* \\
 &= inveth * o - \sum_o \frac{1}{\eta|H_{k,m}|^2} \\
 &= (tot + P_t) - tot \\
 &= P_t
 \end{aligned}$$