Fundamentals of Any Mobile Communication Systems: a compact but systematic description (6 hours)

The purposes of computing are insights, not numbers.

P.S. modeling is not very accurate due to approximation, but we know how to adjust variables

Outline

- Introduction to Cellular Systems (ch. 1)
- Propagation Modeling (ch. 2)
- Jakes fading simulation (optional)
- Diversity (ch.6)

Notation

- Ω_t: transmitted power
- Ω_p: received signal power (square of envelope)
- Ω_v: received signal envelope
- L_p: path loss
- β: path loss exponent
- Λ=C/I: carrier-to-interference ratio (CIR) (the desired signal power to the interference power **before** detection)
- Γ=C/N: carrier-to-noise ratio
- base station (BS)
- mobile station (MS)

typical transmitted power value

- WCDMA/LTE BS: 43dBm [Kim13, NOMA]
- MS: 23dBm
- WLAN Access Point: 20dBm
- Bluetooth: 0dBm

P.S. $10*\log_{10}(P/1mW)=dBm$

Cellular Radio Systems

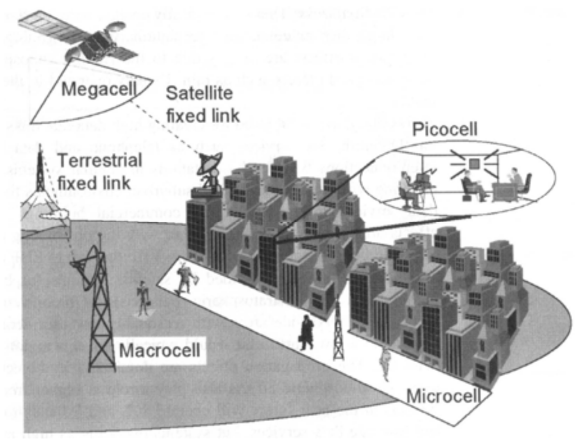
- Basic technological components of cellular telephony are computer technology and ***radio (=wireless) transmission technology***
- Cellular radio system must (i) locate and track moving subscribers, and (ii) continually attempt to connect the moving subscriber to the best available BS=> mobility management
- Current emphasis is toward high spectral efficiency by using

  * microcellular (picocell femtocell, etc.) and macrodiversity (soft handoff etc.) techniques

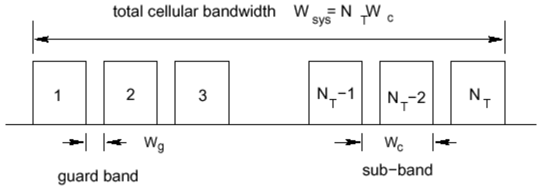
  * rapid radio link quality measurements and control (faster power control, feedback etc.)

  * effective control of co-channel interference (cell sectoring, coordination between BSs, etc)

Cell types



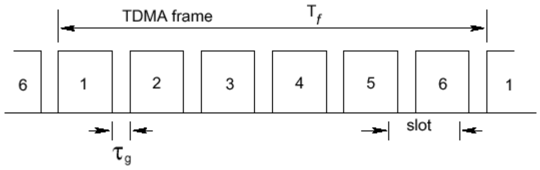
Channel Access Schemes - FDMA



FDMA bandwidth structure

- FDMA = Frequency-Division Multiple-Access
- Single channel per carrier
- Analog FM or digital modulation
- All 1G cellular systems use(d) FDMA only

Channel Access Schemes - TDMA

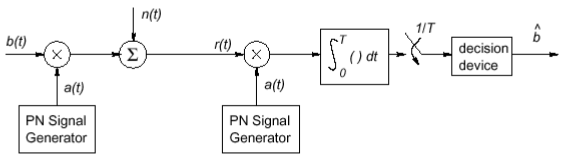


guard interval

TDMA frame structure

- TDMA = Time-Division Multiple-Access
- Multiple channels per carrier
- Must use digital modulation (discrete-time)
- Many 2G systems use TDMA: PDC (Japan), GSM (Europe), IS-54/136 (NA-TDMA in Goodman)(USA)
- Capacity gain: multiple slots on multiple carriers
- IEEE 802.16 (Wi-Max)- OFDMA/TDMA

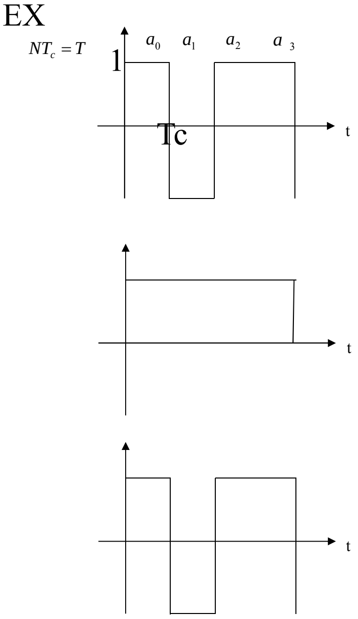
Channel Access Schemes - CDMA



Direct-Sequence CDMA transmitter

- PN=pseudo noise (act like signature to distinguish users)
- CDMA = Code-Division Multiple-Access
- Users share bandwidth by using code sequences that are (ideally) orthogonal to each other
- Based on spread spectrum communications
- A 2G system use CDMA: IS-95 (cdmaOne)
- Performance gradually degradaes as the number of users increases.
- ***Soft capacity***: interference reduction (code design, multiuser detection, smart antenna, etc.) =>capacity increases

EX



Define $a(t) = \sum_{n=-\infty}^{\infty} a_n \psi(t - nT_c)$,

$a_{k+nN} = a_k$, $\psi(t) = p_{T_c}(t)$,

a rectangular pulse with width T_c and height 1

T : symbol duration, T_c : chip duration,

k, n : integer, N : processing gain

Example : N = 4, $a_0 a_1 a_2 a_3 = + - + +$

data bit

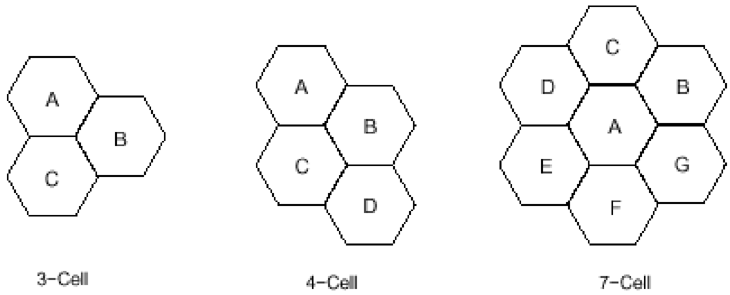
$$b(t) = \sum_{i=-\infty}^{\infty} b_i P_T(t - iT)$$

spread sprectrum signal

$$c(t) = a(t)b(t)$$

- All other multiple access scheme can be regarded as special form of CDMA.
- TDMA: [100000], [010000],[001000],…are spreading sequences
- FDMA (OFDMA): cos(w1t), cos(w2t), …are spreading sequences
- MIMO: “spreading” code in spatial domain (see MIMO handout)

Frequency Re-use => the Cellular Concept (sec. 1.2, p. 16)



Commonly used hexagonal cellular re-use clusters

for macrocellular coverage area
because it approximates a circle
and offer a wide range of reuse cluster size

- WiFi reuse factor=3 => simpler
- 4G/5G reuse factor=1 => inter-cell interference coordination (ICIC) requires extra frequency hopping, more complicated

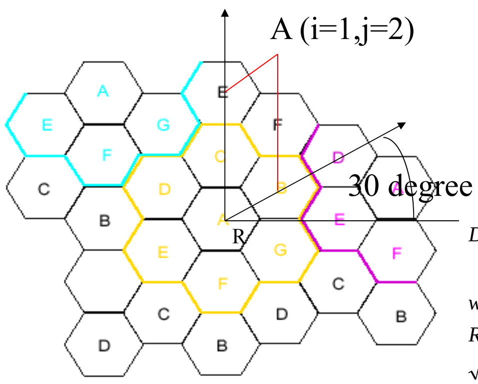
- TDMA & FDMA rely on **frequency re-use** ➔ users in geographically separated cells simultaneously use the same carrier frequency
- Tessellating (formed of small pieces of stone of various colors) hexagonal cluster sizes, N , satisfy

$$N = i^2 + ij + j^2 = 1,3,4,7,9,12, \cdot \ \cdot \ \cdot$$

where i and j non-negative integers

Distance between cells D

(Garg p.81)



Cell layout using 7-cell reuse clusters

$$D^2 = \left(\sqrt{3}R(i\cos 30^\circ)\right)^2 + \left(\sqrt{3}R(j + i\sin 30^\circ)\right)^2$$
$$= (i^2 + j^2 + ij)(\sqrt{3}R)^2$$

where

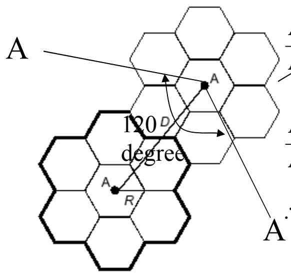
R = center – to – vertex distance

$\sqrt{3}R$ = the distance between two adjacent cells =

(i, j) is the coordinate of non - orthogonal axis of co - channel cell center

unit length = center - to - center distance

Co-channel Reuse Factor



Frequency reuse distance for 7-cell clusters

$$\frac{A_{large}}{A_{small}} = \frac{KD^2}{KR^2} = \frac{D^2}{R^2} \text{(ratio of area of large/small hexagon)}$$

$$\frac{A_{large}}{A_{small}} = \frac{N + 6\left(\frac{N}{3}\right)}{1} = 3N \ (120^\circ/360^\circ = 1/3)$$

$$\therefore \frac{D^2}{R^2} = 3N = 3(i^2 + j^2 + ij)$$

refer to previous page

where N = reuse factor

A_{large} : areas of polygon with outer 6 points A (co-channel cells) as vertex

A_{small} : areas of polygon with bold-faced border (reuse cluster)

(Garg p.83)

Important results p.13,15

- For hexagonal cells, the co-channel reuse factor is

$$N = i^2 + j^2 + ij \quad (1.1)$$

$$\frac{D}{R} = \sqrt{3N} \quad (1.2)$$

- Used in C/I calculation (p.29)

pp.18-19

- For microcellular systems with lower BS antenna heights, regular hexagons are not appropriate.
- 3-cell reuse pattern for highway in north America; 2-cell reuse pattern for urban canyon.
- Frequency re-use introduces **co-channel interference (CCI)**

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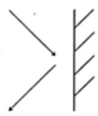
Radio propagation is by three mechanisms:

- Reflection
  - Occurs when a propagating electromagnetic wave impinges upon an object which is flat has very large dimension compared to the wavelength



- Diffraction
  - Occurs when the radio path between the transmitter and receiver is obstructed by an irregular surface or edge

- Scattering
  - Occurs when the size of objects with dimensions being small comparad to the wavelength

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Propagation environment (sec. 1.3, p.19)

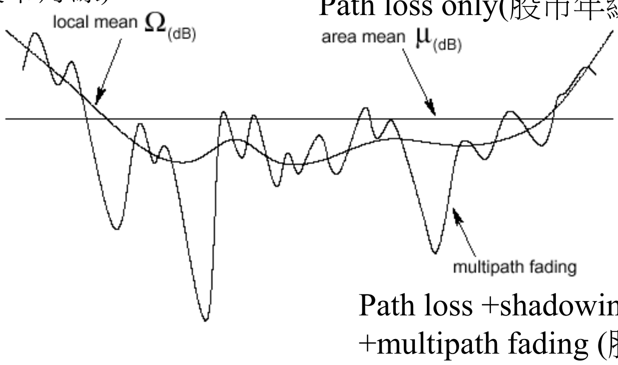
A mobile radio environment is characterized by three nearly independent propagation factors (large-, medium-,and small-scale fading):

1. **Path loss** with distance caused by propagation over long distance
2. **Shadowing** caused by large obstructions such as buildings and hills
3. **Multipath fading** caused by the combination of multipath propagation and subscriber movement

Path Loss + Shadowing + Multipath Fading

Path loss+shadowing only (股市月線)

Path loss only(股市年線)



Path loss +shadowing +multipath fading (股市日線)

Received envelope showing effects of path loss, shadowing, and multipath fading

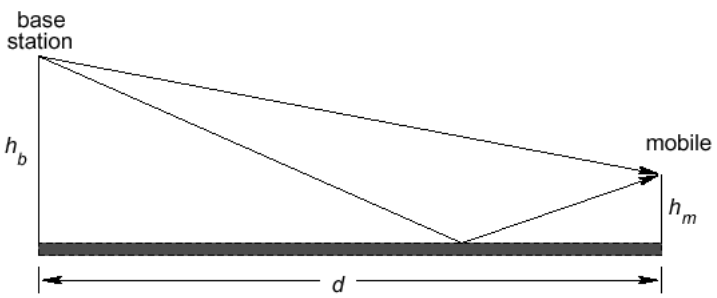
- Macro-diversity can compensate shadowing

Not all base transceiver systems (BTS) are shadowed. The base station controller (BSC) could choose the BTS with best link quality.

- Rake receiver of CDMA systems can combat multipath fading

Rake receiver can separate different paths, compensate their phases, and combine the signals of different paths.

Path Loss (ground reflection, two-ray, see [Rappaport]) Model-Easiest one



Radio propagation over a flat reflecting surface

p. 103

- The received signal power Ω_p, is (minus in dB terms=>division in non-dB terms)

$$\Omega_p = k\frac{\Omega_t}{L_p}$$

where Ω_t is the transmit power and L_p is the path loss.

- For the flat earth model, the path loss at distance *d* is

$$L_p = \left[\left(\frac{\lambda_c}{4\pi d}\right)^2 4\sin^2\left(\frac{2\pi h_b h_m}{\lambda_c d}\right)\right]^{-1} \quad (2.227)$$

where λ_c = carrier wavelength, h_b = base station antenna height, and h_m = mobile station antenna height.

- Under the condition that $d >> h_b h_m$

$$L_p = \left(\frac{h_b h_m}{d^2}\right)^{-2}$$

- Propagation over a flat reflecting surface, under the condition $d >> h_b h_m$, differs from free space propagation in two respects:

***** it is not frequency dependent

***** path loss increases with 4th power of the distance

Path Loss Exponent

- In reality, the earth's surface is curved and rough, and the signal strength typically decays with the inverse β power of the distance:

$$\Omega_p = k\frac{\Omega_t}{d^\beta}$$

where *k* is a constant of proportionality. Expressed in units of dB, the received power is

$$\Omega_{p(\mathrm{dB})} = 10\log_{10}(k) + \Omega_{t(\mathrm{dB})} - 10\beta\log_{10}(d)$$

- β is called the **path loss exponent**. Typical values of β have been determined by empirical measurements for a variety of measurements- <u>by linear regression method in statistics</u>
- Flat earth model with β=4 still offer a good approximation (double distance, additional -12 dB)

Terrain	β
Free space	2
Open area	4.35
North American suburban	3.84
North American Urban (Philadelphia)	3.68
North American Urban (Newark)	4.31
Japanese Urban (Tokyo)	3.05

Larger Path Loss Exponent is Good for Cellular Systems!!!

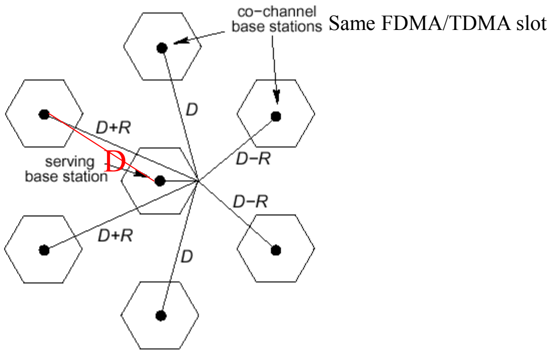
- for larger β, Ω_p is smaller
- although signal is weak, CCI is weaker due to longer distance

BS1——MS————————————BS2

- C/I increases, quality improved!
- We can reduce reuse factor to decrease the quality to minimum requirement

=> Higher capacity for cellular systems!

Co-channel Interference



Worst case co-channel interference on the forward channel

C/I is smallest when MS is at cell edge.

Worst Case Co-channel Interference

- There are **six** co-channel base stations: two each at distances *D* - *R*, *D*, and *D* + *R*. (approximation)
- The worst case carrier-to-interference ratio is

$$\begin{aligned}\frac{C}{I} &= \frac{1}{2}\frac{R^{-\beta}}{(D-R)^{-\beta}+D^{-\beta}+(D+R)^{-\beta}} \\ &= \frac{1}{2}\frac{1}{\left(\frac{D}{R}-1\right)^{-\beta}+\left(\frac{D}{R}\right)^{-\beta}+\left(\frac{D}{R}+1\right)^{-\beta}} \\ &= \frac{1}{2}\frac{1}{\left(\sqrt{3N}-1\right)^{-\beta}+\left(\sqrt{3N}\right)^{-\beta}+\left(\sqrt{3N}+1\right)^{-\beta}}\end{aligned}$$

p.17 eq. (1.2)

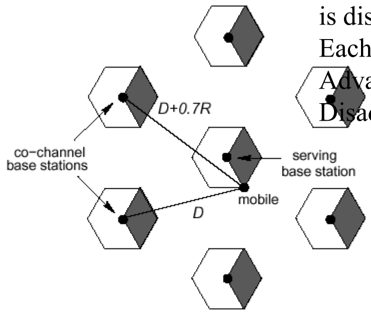
- For $\beta = 3.5$,

$$\frac{C}{I}_{\text{(dB)}} = \begin{cases} 14.3\text{dB} & \text{for } N = 7 \\ 9.2\text{dB} & \text{for } N = 4 \\ 6.3\text{dB} & \text{for } N = 3 \end{cases}$$

- .

Cell sectoring by using directional antennas

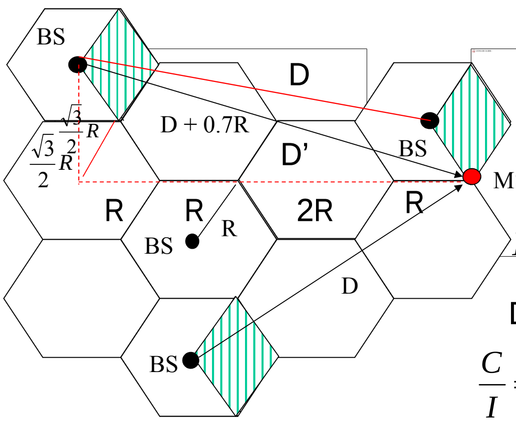
The channels assigned to one cell is distributed among its three sectors.
Each sector uses a disjoint set of channel.
Advantage: CCI decreases
Disadvantage: more handoff



Worst case co-channel interference on the forward channel with 120° cell sectoring

2,3,6 sectors: 3,2,1 co-channel BS, respectively
Work for reuse factor=1 too

Worst Case for Forward Channel Interference in Three-sectors



$$D = \sqrt{\left(4R + \frac{1}{2}R\right)^2 + \left(\frac{\sqrt{3}}{2}R\right)^2}$$

$$D = \sqrt{21}\,R \quad \approx 4.58R$$

$$D' = \sqrt{(5R)^2 + \left(\sqrt{3}R\right)^2} = \sqrt{28}R$$

D'≈5.29R= D+0.7R

$$\frac{C}{I} = \frac{1}{\frac{D^{-\beta}}{R} + \left(\frac{D+0.7R}{R}\right)^{-\beta}}$$

Trigono'metric in high school

Worst Case Co-channel Interference: Sectoring

- 120º cell sectoring reduces the number of co-channel base stations from 6 to 2: at distances D and D + 0.7R(approximation).
- The carrier-to-interference ratio becomes

$$\frac{C}{I} = \frac{R^{-\beta}}{D^{-\beta} + (D + 0.7R)^{-\beta}}$$

$$= \frac{1}{\left(\frac{D}{R}\right)^{-\beta} + \left(\frac{D}{R} + 0.7\right)^{-\beta}}$$

$$= \frac{1}{\left(\sqrt{3N}\right)^{-\beta} + \left(\sqrt{3N} + 0.7\right)^{-\beta}}$$

- Hence

$$\left.\frac{C}{I}\right|_{(\mathrm{dB})} = \begin{cases} 21.1\mathrm{dB} & \text{for } N = 7 \quad (14.3 \text{ for no sectoring}) \\ 17.1\mathrm{dB} & \text{for } N = 4 \quad (9.2 \text{ for no sectoring}) \\ 15.0\mathrm{dB} & \text{for } N = 3 \quad (6.3 \text{ for no sectoring}) \end{cases}$$

- For N = 7, 120° sectoring yields 6.8 dB C/I improvement.
- if the radio receiver can operate at C/I = 15.0 dB, then N = 3 can be used.

Receiver Sensitivity (sec. 1.5, p.23)

- Receiver sensitivity (often -110dBm) refers to the ability of the receiver to detect radio signals. Radio receivers must detect radio waves in noise.

***** external noise sources include atmospheric noise (lightning strikes), ga'lactic (太陽風)noise, man made noise (automobile ignition, wideband).

***** internal noise sources include thermal noise (proportional to degree K).

- The carrier-to-noise ratio Γ is a function of communication link parameters including Ω_t, L_p, receiver antenna gain, and effective input-noise temperature of the receiving system
- The formula that relates the link parameters to Γ is called the **link budget(預算)**

Link Budget (pp.23-28 in Stuber)

- the maximum allowable path loss $L_{p,max}$(dB) $=\Omega_t$(dB)+G_T(dB)+G_R(dB)-S_{Rx}(dB)-L_I(dB)-M_{shad}(dB)+ G_{HO}(dB) (1.27)
- G_T and G_R are the gains of the transmit and receive antennas and S_{Rx} is the receiver sensitivity(RX receive power lower limit).
- To account for interference loading degradation, reduce $L_{p,max}$ by the interference margin, L_I(dB) (cell shrinks).
- To account for shadowing effects, reduce $L_{p,max}$ by the shadow margin, M_{shad}(dB) (details later).
- The soft handoff used in CDMA enhances performance (macrodiversity), thus a handoff gain, G_{HO}(dB) can be added to $L_{p,max}$ (details later).

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- ***The maximum allowable path loss is proportional to β power of distance and thus it decides the cell radius and the coverage.*** $\Omega_p = k\frac{\Omega_t}{d^\beta}$
- LNA is used in BS to compensates lower Ω_t(dB) for MS=> Balancing U/L and D/L cell radius

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Interference Margin L_I

- Must account for the increased traffic load by including interference degradation margin in the link budget. Otherwise, there will be very poor coverage near the planned cell boundaries.
- Thus, the maximum allowable path loss in the link budget must be reduced by an amount equal to the **interference margin** to maintain acceptable quality due to receiver sensitivity.
- CDMA typically requires higher interference margin than TDMA because all signals occupy the same bandwidth (MAI).

Path loss+shadowing

- often modeled as being **log-normally distributed**, meaning the *pdf* of Ω_p(dB) is
$$\Omega_{p(\mathrm{dB})}(d)(x) \sim N(\mu_{\Omega(\mathrm{dB})}(d), \sigma_\Omega^2)$$
- Where $\mu_{\Omega(\mathrm{dB})}(d) = \mu_\Omega(d_0) - 10\beta \log_{10}(d/d_0)$ dB ,

  d is the distance between MS and BS, and d_0 is a reference distance (one point + slope 10β=> a line).
- Ω_p(dB) is called the **local mean** (average over a-few-wavelengths distance to average out multipath fading)
- μ_Ω(dB) is the **area mean (average out shadowing)**.
- σ_Ω typically ranges from 5 to 12 dB and is nearly independent of the path length (for macrocells): field test results from [225]
- σ_Ω=0 => no shadowing effect

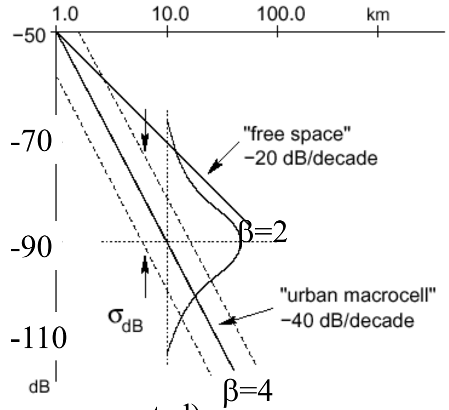
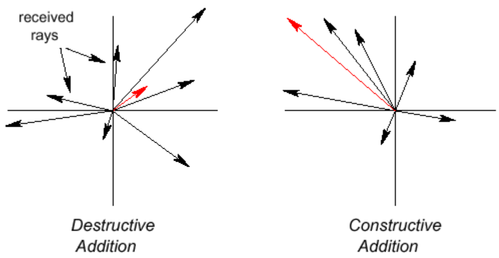


Fig. 1.11 (errors corrected)

Path loss and shadowing in a typical macrocellular environment

Why Multipath => fading?

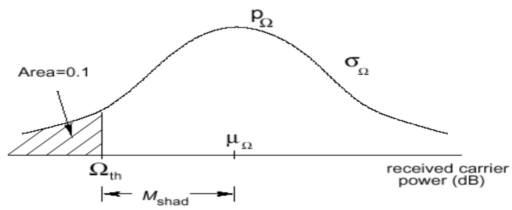


Vector addition of (complex-valued) multiple reflections of a transmitted signal

- Different paths have different phases
- Red arrows indicate the sum of the received components
- Reception of time-varying components results in **multipath fading**.

Outage(停用停電)
=> Shadow Margin (p.25)

- The quality of the radio link is acceptable only when the received signal power Ω_p(dB) is greater than a threshold value Ω_{th}(dB) .
- An **outage** occurs whenever Ω_p(dB)< Ω_{th}(dB) . (discontuined conversation may occurs)
- The **edge outage probability**, *O(R)*, is defined as the probability that Ω_p(dB)< Ω_{th}(dB) at the cell edge.
- To maintain an acceptable outage probability in the presence of shadowing, we must introduce a **shadow margin** to the link budget (cell radius decreases).



Determining the required shadow margin to give $O(R) = 0.1$

$$\text{edge outage probabilit y} = O(R) = \Pr\left(\Omega_{p(dB)} < \Omega_{\text{th(dB)}}\right)$$

$$= \Phi(\frac{-M_{shad}}{\sigma_\Omega}) = Q(\frac{M_{shad}}{\sigma_\Omega})$$

$$\text{where } M_{shad} = \mu_\Omega - \Omega_{\text{th}} > 0 \text{ (all in dB)}$$

Ex. 1.1 Shadow Margin (edge outage prob) (sec. 1.5, p.26)

- Choose M_{shad} so that the shaded area under the Gaussian *pdf* is equal to 0.1. Hence, we solve

$$O(R) = 0.1 = Q\left(\frac{M_{shad}}{\sigma_\Omega}\right) \quad \text{where} \quad Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

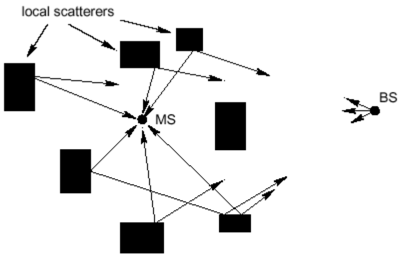
- We have

$$\frac{M_{shad}}{\sigma_\Omega} = Q^{-1}(0.1) = 1.28$$

- For σ_Ω = 8 dB, M_{shad}=1.28*8=10.24dB

Coverage (sec. 1.6, p. 26)

- **Coverage** refers to the number of BSs or cell sites that are required to provide service to an area with an acceptable grade of service.
- Number of cells depends on max. allowable path loss and path loss exponent. Fewer cell sites ➔ smaller infrastructure cost and lower BS fee



A typical macrocellular mobile radio environment

- Macrocell: BS high above local scatterers, MS surrounded (iso'tropic 等方向性)
- Microcell: BS & MS surrounded by local scatterers
- LOS may or may not exist between MS & BS

Ch. 2 Narrowband/Wideband Propagation Modeling

P.S. NOT applied to ultrawideband (UWB)-no longer Rayleigh fading etc.

Bandpass Signal Models

• Bandpass signals have bandwidths that are small (20MHz) compared to the carrier frequency (2GHz)

• Bandpass signals have three representations :

(I) Envelope - phase representation

$$s(t) = A(t) \cdot \cos(2\pi f_c t + \theta(t))$$

(II) Complex envelope representation

$$s(t) = \mathrm{Re}\{\tilde{s}(t) e^{j2\pi f_c t}\}$$

like phasors in electrical circuit/electromagnetic theory course

$$= \mathrm{Re}\{(s_I(t) + js_Q(t))(\cos(2\pi f_c t) + j\sin(2\pi f_c t))\}$$

(III) Quadrature representation **(vector space)**

$$s(t) = s_I(t) \cdot \cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

where $s_I(t) = A(t)\cos\theta(t)$, $s_Q(t) = A(t)\sin\theta(t)$

H, |H|, |H|^2/N
Commonly confused!

OFDM channel coefficients

Complex envelope representation

$$H = H_I + jH_Q$$

$$H_I = |H|\cos\theta$$

$$H_Q = |H|\sin\theta$$

$$SNR = \frac{E|H|^2}{N}$$

Doppler Shift

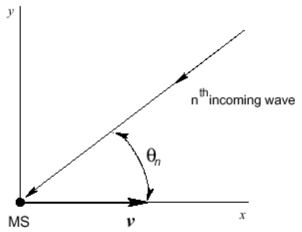


Fig. 2.2 *A typical wave component incident on a MS*

- Doppler shift is frequency shift relative the to carrier frequency of the incoming wave (narrowband only),
- Defined as $f_{D,n}(t) = f_m \cos\theta_n(t)$ Hz (2.1)

where $f_m = v/\lambda = v/c * f$ is the maximum Doppler for velocity v, and λ_c is the wavelength

- Motion toward wave : positive (think!) Doppler shift
- Motion away from wave : negative Doppler shift

Delay/Doppler cause phase changes of paths

- At the receive antenna, the n^{th} plane wave arrives at angle θ_n having Doppler shift $f_{D,n}(t) = f_m \cos\theta_n$ and propagation delay τ_n
- Note that $2\pi(f_c + f_{D,n})(t - \tau_n) = 2\pi(f_c t + f_{D,n} t - f_c\tau_n - f_{D,n}\tau_n)$

and transmitted signal $s(t) = \mathrm{Re}\{\tilde{s}(t)e^{j2\pi f_c t}\}$

The noiseless received bandpass signal is

$$r(t) = \mathrm{Re}\{\tilde{r}(t)e^{j2\pi f_c t}\} = \mathrm{Re}\left\{\sum_n C_n(t)e^{-j\phi_n(t)}\tilde{s}(t - \tau_n)e^{j2\pi f_c t}\right\}$$

where $\phi_n(t) = 2\pi\{(f_c + f_{D,n})\tau_n - f_{D,n} t\}$

- Small changes in τ_n will cause large changes in $\phi_n(t)$ since f_c is large. $f_{D,n}$ also make different phases. This causes multipath fading.
- Baseband equivalent time - varying impulse response of channel at time t due to an impulse applied at time t - τ is

$$g(t,\tau) = \sum_n C_n(t)e^{-j\phi_n(t)}\delta(\tau - \tau_n)$$ **(linear, not time - invariant)**

Paths with different
arrival angle have different Doppler

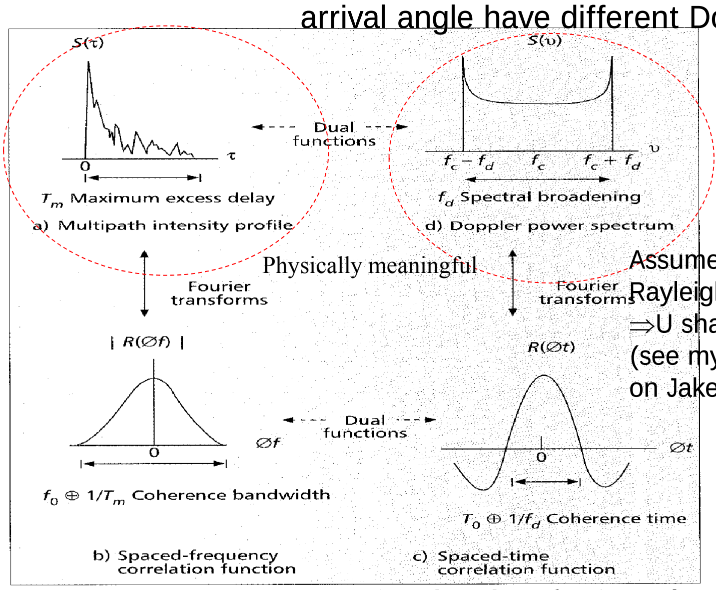


Figure 7. *Relationships among the channel correlation functions and power density functions.*

Assume flat and
Rayleigh fading
⇒U shape
(see my notes
on Jakes model)

- Pilot symbol spacing in time<coherent time
- Pilot symbol spacing in spectrum<coherent bandwidth

Flat fading (max excess delay<symbol interval)

Sec. 2.2, p. 41

If all $\tau_i \approx \hat{\tau}$, we can write

$$g(t,\tau)=\sum_n C_n e^{-j\phi_n(t)}\delta(\tau-\tau_n)=g(t)\delta(\tau-\hat{\tau})$$

* modeled as single - tap channel :

* **coherent BW** > **signal BW** :

channel has constant gain & linear phase response over a BW **(1/max excess delay)** greater than the BW of the transmitted signal **(1/symbol interval)**

* multipath fading affects entire BW of signal in the same way

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Frequency-selective fading (max excess delay>symbol interval)

sec. 2.2, p.70

* modeled as a multi - tap channel :

$$g(t,\tau)=\sum C_n e^{-j\phi_n(t)}\delta(\tau-\tau_n) \quad (2.7)$$

* **coherent BW < signal BW :**

channel possesses constant gain and linear phase response over a BW smaller than BW of signal

* sections of signal BW affected in different ways
* max excess delay > symbol interval :

introduces **intersymbol interference (ISI)**

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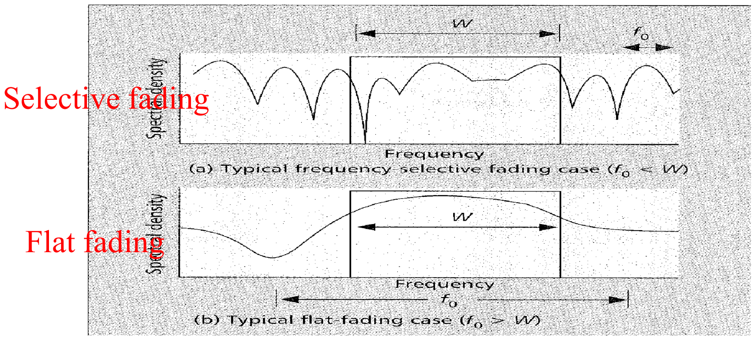
Ref. B. Sklar, IEEE Commun. Magazine, July 1997.



Central Limit Theorem

- A flat fading channel can be characterized by transmission of an unmodulcated carrier: $\tilde{s}(t)=1$.
- The received bandpass signal has the quadrature representation $r(t)=g_I(t)\cdot\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$ (2.10)

where
$$g_I(t)=\sum_n C_n \cos\phi_n(t)$$
$$g_Q(t)=\sum_n C_n \sin\phi_n(t)$$

- By invoking the **central limit theorem**, $g_I(t)$ and $g_Q(t)$ are indep zero-mean Gaussian random processes, i.e., at any time, $g_I(t)$ and $g_Q(t)$ are indep zero-mean Gaussian random variables.=> Rayleigh fading

Isotropic Scattering (Jakes model assumption)

- With 2-D isotropic scattering, the plane waves are conned to the x - y plane and arrive uniformly distributed in angle incidence, i.e., $\boxed{p(\theta)=\frac{1}{2\pi}}$ $\quad -\pi \le \theta \le \pi$
- autocorrelation function of r(t) is (derivation skipped)

$$\phi_{rr}(\tau)=\phi_{g_I g_I}(\tau)\cos 2\pi f_c \tau = \boxed{\frac{\Omega_p}{2} J_0(2\pi f_m \tau)\cos 2\pi f_c \tau}$$

- where

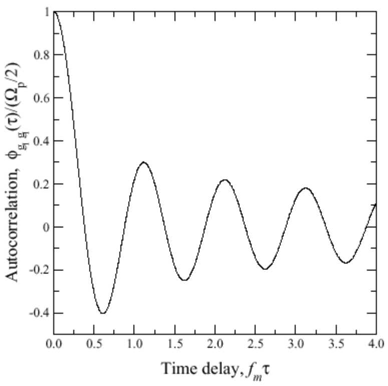
$$J_0(x)=\frac{1}{2\pi}\int_0^{2\pi}\cos(x\cos\theta)d\theta$$

is the zero-order Bessel function of the first kind.

- The psd of the bandpass signal r(t) is (isotropic case)

$$S_{rr}(f)=\frac{1}{2}\left[S_{g_I g_I}(f-f_c)+S_{g_I g_I}(-f-f_c)\right]=\frac{\Omega_p}{4\pi f_m}\frac{1}{\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}, \quad |f-f_c|\le f_m \quad (2.29)$$

Autocorrelation Plot



Fading is correlated

Normalized autocorrelation of the in phase *components. This is typical for a MS.*

U-shaped PSD Matched to experimental results

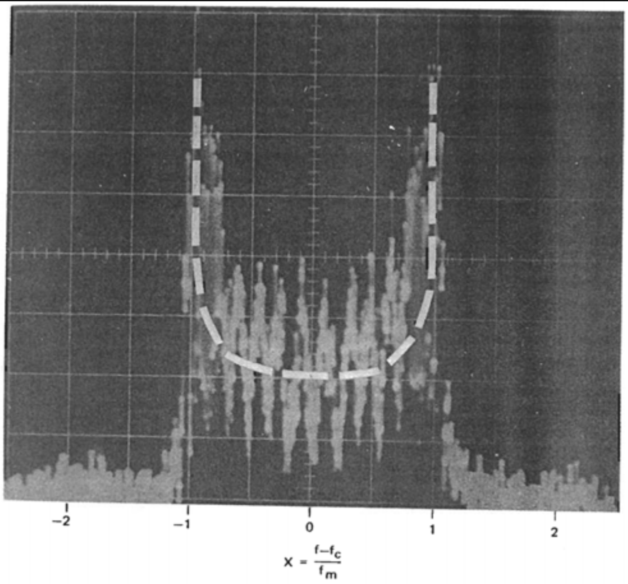


Figure 1.7-5 RF Spectrum of simulated fading carrier. Dashed line is the theoretical spectrum, $(1 - X^2)^{-1/4}$.

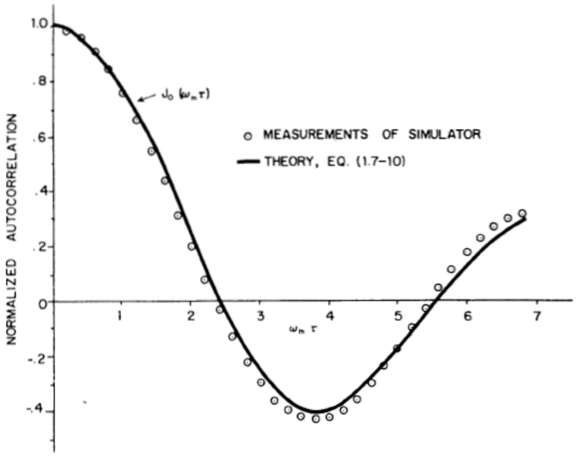


Figure 1.7-4 Comparison of theoretical autocorrelation function of the fading signal with data from a laboratory simulator.

Rayleigh Fading (non line-of-sight) 2.1.2.1, p. 50

- **Rayleigh fading** occurs when there is no LOS or strong specular component in the received signal, i.e., there is no dominant C_n.
- Thus, $g_I(t), g_Q(t) \sim N(0, b_0)$ (p.54) where N denotes the normal, or Gaussian, distribution (by CLT), and $g(t)=g_I(t)+jg_Q(t)$ is a complex Gaussian random process.
- Envelope of received signal, $\alpha(t)=|g(t)|=\sqrt{g_I^2(t)+g_Q^2(t)}$, is **Rayleigh distributed:**

$$p_\alpha(x)=\frac{x}{b_0}\exp\left\{-\frac{x^2}{2b_0}\right\}=\frac{2x}{\Omega_p}\exp\left\{-\frac{x^2}{\Omega_p}\right\} \quad x\geq 0$$

Important

where $\Omega_p = \mathrm{E}\left[\alpha^2(t)\right]=2b_0 = \mathbf{1}$ **(same average power as AWGN)** is **the average envelope power**.

Squared envelope $\alpha^2(t)=|g(t)|^2$ is **exponentially distributed**

$$p_{\alpha^2}(x)=\frac{1}{\Omega_p}\exp\left\{-\frac{x}{\Omega_p}\right\}$$

and related to the instantaneous received power.

Crossing Rates & Fade Durations

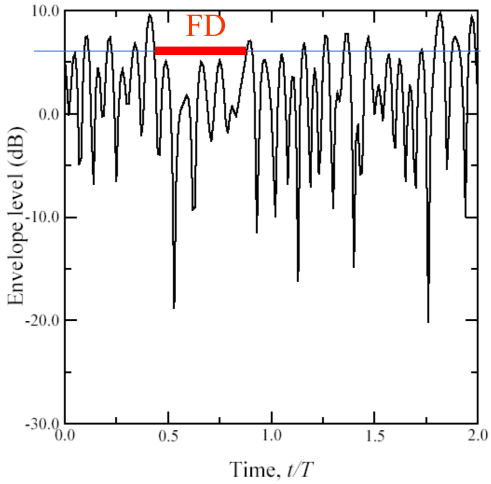
- The **level crossing rate** (LCR) is the rate at which the signal envelope crosses a specified level in the positive (or negative) going direction.
- can be used to estimate velocity
- The **average fade duration** (AFD) is the average length of time that the **envelope** remains below the specified level.
- AFD impacts the outage probability and quality of service

=> **Related to burst errors length**

=> **Affect the choices of interleavers and error correction codes (4G: turbo/convolutional code, 5G: LDPC/Polar code NICT next semester)**

- Both the LCR and AFD depend on the MS velocity.

Envelope Fading



Rayleigh faded envelope with 2-D isotropic scattering

Modified Jakes Simulator

References:

●P. Dent, G. E. Bottomley, and T. Croft, "Jake fading model revisited," Electronics Letters, vol. 29, pp. 1162-1163, June 1993.
●W. C. Jakes, Microwave Mobile Communications, IEEE Press, 1994(reprint), Sec. 1.7

Background

1) Simulation of ***frequency-flat, one-tap, time-correlated Rayleigh fading channels***.
2) Assume N equal-strength rays arrive at a moving receiver with uniformly distributed arrival angles $\alpha_n=(n-\boxed{0.5})2\pi/N$. (isotropic: non-directional)
3) The ray n experiences a Doppler shift $\omega_n=\omega_m\cos\alpha_n$, where $\omega_m=2\pi fv/c$ is the maximum Doppler shift, v is the relative velocity between the transmitter and receiver, c is the speed of light, and f is the carrier frequency.

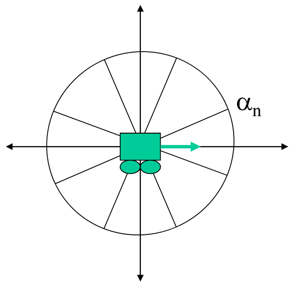


Fig. 1. N=8 case. Note that quadrantal symmetry

Derivation

The (electrical and magnetic) field is

$E(t) = \mathrm{Re}[T(t)\exp(j2\pi f_c t)]$, where

$$T(t) = \sqrt{\frac{2}{N_0}} \sum_{n=1}^{N_0} \left[\cos(\beta_n) + I \sin(\beta_n)\right] \cos(\omega_n t + \theta_n)$$

is the sum of N_0 oscillators

1) $I = \sqrt{-1}$

2) $N_0 = N/4$ because of quadrantal symmetry

3) $\sqrt{\frac{2}{N_0}}$ is normalization factor such that $< T(t)\, T^*(t) >= 1$

(explained later)

4) $\exp(I\beta_n)=\cos(\beta_n)+I\sin(\beta_n)$ is the phase of ray n (equal power in assumption).
5) $\beta_n = \pi n/N_0$ such that real and imaginary part of T(t) uncorrelated and large enough N_0 such that real and imaginary part of T(t) are jointly Gaussian (Central limit theorem) =>T(t) has Rayleigh magnitude and uniformly phase over $[0,2\pi]$

Explain Rayleigh distribution

X,Y~$N(0,\sigma^2)$ and independent
Let $z=\mathrm{sqrt}(x^2+y^2)$ and $w=\tan^{-1}(y/x)$.
Then z is Rayleigh distributed and
w is uniformly distributed on $[0,2\pi]$.

Explain : β_n

$$\mathrm{Re} = \mathrm{Re}\{T(t)\} = \sqrt{\frac{2}{N_0}} \sum_{n=1}^{N_0} \cos(\beta_n)\cos(\omega_n t + \theta_n)$$

where r.v. θ_n is uniform $[\mathbf{0}, 2\boldsymbol{\pi}]$

$$\mathrm{Im} = \mathrm{Im}\{T(t)\} = \sqrt{\frac{2}{N_0}} \sum_{n=1}^{N_0} \sin(\beta_n)\cos(\omega_n t + \theta_n)$$

Because they are uncorrelated,

$$< \mathrm{Re}, \mathrm{Im} >= \frac{2}{N_0} \sum_{n=1}^{N_0} \frac{1}{2}\sin(2\beta_n)\frac{1}{2} = 0 \text{倍角公式}$$

where <> denotes time average, so < sinusoid of time t terms >= 0.

$2\beta_n$ divide 2π equally, so $\beta_n = \pi n / N_0$

($2\boldsymbol{\beta}_n : 0 \sim 2\boldsymbol{\pi} \Rightarrow \boldsymbol{\beta}_n : 0 \sim \boldsymbol{\pi}$)

How large N is enough?

- Autocorrelation $R(\tau)=<E(t)E(t+\tau)>= b_0 J_0(\omega_m \tau)$, where J_0 is zero-order Bessel function of the first kind
- Eq. (1.3-7) of [Jakes] derived for a continum of arrival angles
- It has 8 significant digits for N>=32. Thus N_0>=8.

Extension to generate K uncorrelated Rayleigh fading processes

- Waveform correlation is determined by the sum of the products of oscillator coefficients (*vector inner product*)
- Thus Hadamard matrices (vector inner product of any two rows are zero) can be used

coordinates

$$T(k,t)=\sqrt{\frac{2}{N_0}}\sum_{n=1}^{N_0} H(k,n)\left[\cos(\beta_n)+I\sin(\beta_n)\right]\cos(\omega_n t+\theta_n)$$

Orthogonal basis

k=1,2,…,K user index n: ray index

Hadamard matrix

$$\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$$

$$H_1=[1]$$

$$H_2=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4=\begin{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & -\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

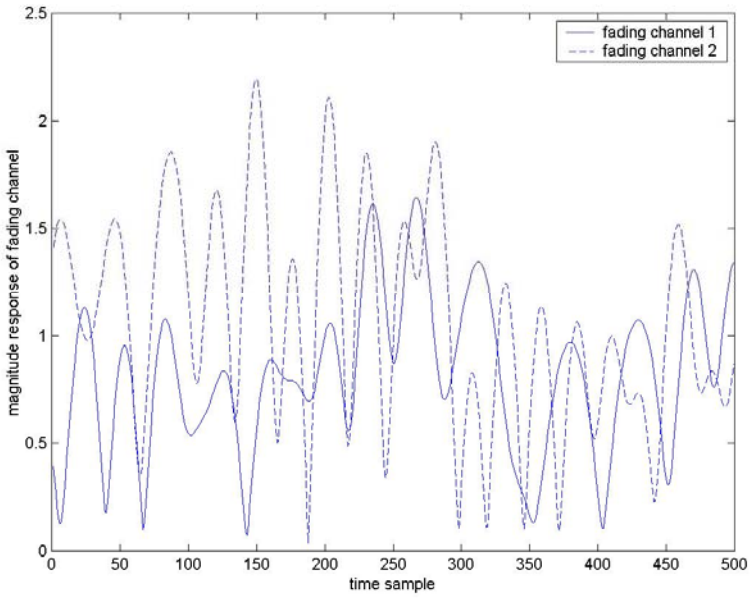
Matlab program

```
function g=rayleigh2(branches,length,dop_norm)
% branches: no. of indep fading processes to be generated
% length: no of symbols in a block
% dop_norm: normalized Doppler rate=Doppler frequency
    shift*symbol duration
temp=zeros(branches, length);
N0=16;
N=64;
gc=zeros(1,N0); gs=zeros(1,N0); w=zeros(1,N0);
H=hadamard(N0);
```



```
%Frequency components: ωn=ωmcosαn
w(1:N0)=2*pi*dop_norm*cos(2*pi*((1:N0)-0.5*ones(1,N0))/N);
%The gain coefficients cos(βn),sin(βn)
  gc(1:N0)=cos(pi*(1:N0)/N0);
  gs(1:N0)=sin(pi*(1:N0)/N0);
%normalize
  gc = sqrt(2/N0)*gc;
  gs = sqrt(2/N0)*gs;
%The phase for different user and oscillator \theta_n
phs=2*pi*rand(1,N0);

for k=1:branches
  for n=1:N0
     temp(k,1:length)=temp(k,1:length)+H(k,n)*(gc(n)+j*gs(n))*cos(w(n)*(
     1:length)+phs(n));
  end
end
 g=temp;
```



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Normalized Doppler rate example

- In our simulation, we assume the carrier frequency is 2 GHz, a mobile user is moving at 100km/hr, and thus the maximum Doppler shift f_m=200 Hz.
- For a symbol rate at 10000=1/T symbols/s (GSM 9.6Kbps), The normalized Doppler rate is thus f_dT=0.02. => **Coherence time=50 symbols**

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Multiple correlated fading channels example

We consider three-path correlated Rayleigh fading channels. The three independent fading processes

$\tilde{\mathbf{x}}_{l,k} = \left[\tilde{x}_{l,k}^{(1)} \quad \tilde{x}_{l,k}^{(2)} \quad \tilde{x}_{l,k}^{(3)}\right]^T$ for $l \geq 1$

are generated by a modified Jakes simulator [Dent]. Let the correlation matrix of vector $\mathbf{x}_{l,k}$ at the zero time shift be $\mathbf{R}_{\mathbf{x}}$ and the correlated fading processes are generated by

$$\mathbf{x}_{l,k} = (\mathbf{R}_x)^{1/2} \tilde{\mathbf{x}}_{l,k}$$

where we set

$$\mathbf{R}_x = \begin{bmatrix} 1 & 0.9 & 0.81 \\ 0.9 & 1 & 0.9 \\ 0.81 & 0.9 & 1 \end{bmatrix}$$

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Fixed total energy

- For single user, the total transmitted energy is fixed, say 1.
- If the energy is equally divided into 3 paths, each Jakes fading resolvable paths (fingers) must be multiplied 1/sqrt(3).
- Otherwise M-path case will have as M times total energy as single path case.

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Ch. 6. Diversity

Diversity Techniques

- Diversity exploits the random nature of radio propagatio n by finding uncorrelat ed signal paths for communicat ion.
  - ∗ provide multiple, independen tly faded replicas of the same signal
  - ∗ if one radio path undergoes a deep fade, another independen t path may have a strong signal
- Microscopi c diversity – exploit th e rapidly va rying signal to fight fading.
- Macroscopi c diversity – exploit large separation of two BSs (receivers) to fight shadowing and fading.
- Several mechanisms to generate uncorrelat ed faded signals

Diversity Methods

- Space diversity - spatial diversity between multiple antennas is chosen so that branches experience uncorrelated fading.
- Frequency diversity - use multiple channels that are separated by at least the coherence bandwidth of the channel.

(e.g. MC DS CDMA)

- Multipath diversity - resolve multipath components at different delays in DS-SS (RAKE receiver).
- Time diversity - transmit same signal at multiple time periods that are separated by at least the coherence time of the channel.

(e.g. redundency bits + interleaver)

Popular combing methods

- Selection combining (SC)
- Maximal-ratio combining (MRC)
- Equal-gain combining (EGC)

Diversity Combining

- L is the number of diversity branches.
- M is the size of the signal constellation.
- The noise processes $\tilde{n}_k(t)$ are usually uncorrelated from branch to branch.
- $g_k = \alpha_k e^{j\phi_k}$ is the complex fading gain associated with the k - th branch.
- Corresponding complex lowpass received signal vector (complex envelope)

$$\tilde{r}_k = g_k \tilde{s}_k + \tilde{n}_k, \ k = 1,...,L$$

- The fading gains typically have some correlation. the degree of which depends on type of diversity and propagation channel.
- Branch correlation reduces the achievable diversity gain.

- greater gain for Rayleigh than Ricean fading

SC

- Branch giving the highest SNR is selected at any instant.
- When selective combining is used, the symbol energy - to - noise ratio at the combiner output is $\mathbf{r}_b^s = \max\{r_1, r_2, ..., r_L\}$,

where $\mathbf{r}_k$ is the received **vector** at k - th branch

- γ_k is the received symbol energy - to - noise ratio on the k - th diversity branch, and has the exponential distribution(Rayleigh^2) :

$$p_{\gamma_k}(x) = \frac{1}{\bar{\gamma}_c} e^{-x/\bar{\gamma}_c}$$

- For independently faded branches, the order statistics gives the

cdf $F_{\gamma_b^s}(x) = \Pr[\gamma_1 \le x \cap \gamma_2 \le x \cap ... \cap \gamma_L \le x] = \left[1 - e^{-x/\bar{\gamma}_c}\right]^L$

(max <= x, so each <= x)

pdf $P_{\gamma_b^s}(x) = \frac{L}{\bar{\gamma}_c}\left[1 - e^{-x/\bar{\gamma}_c}\right]^{L-1} e^{-x/\bar{\gamma}_c}$

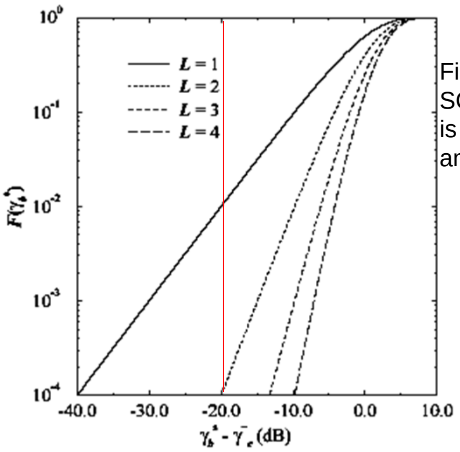


Fig. 6.2. the probability that
SC result <=average-20dB (deep fade)
is 10^-2 for (L=1)
and 10^-4 for L=2

MRC
sec. 6.3, p. 180

- With MRC, the diversity branches must be weighted by their respective complex fading gains, and combined.
- MRC results in a ML receiver and gives the best performance among the diversity techniques.
- The vector $\tilde{\mathbf{r}} = [\tilde{r}_1, ..., \tilde{r}_L]$ has the multivariate Gaussian distributed likelihood function

(assume indep branches and Gaussian **noise psd** = $\mathbf{N_0}$ (2Dim))

$$\mathrm{p}(\tilde{\mathrm{r}}|\mathrm{g}, \tilde{\mathrm{s}}_m) = \frac{1}{(2\pi N_0)^L} \exp\left\{-\frac{1}{2N_0}\sum_{k=1}^{L}\left\|\tilde{\mathbf{r}}_k - g_k\tilde{\mathbf{s}}_m\right\|^2\right\}$$

*w*here $\mathrm{g_k} = \alpha_\mathrm{k} \mathrm{e}^{\mathrm{j}\phi_\mathrm{k}}$ is the fading gain of the channel.

MRC is ML

$$\mu(\tilde{\mathbf{s}}_m) = -\sum_{k=1}^{L} \left\| \tilde{\mathbf{r}}_k - g_k \tilde{\mathbf{s}}_m \right\|^2 = -\sum_{k=1}^{L} \left\{ \left\| \tilde{\mathbf{r}}_k \right\|^2 - 2\,\mathrm{Re}(g_k^{*} \tilde{\mathbf{s}}_m^{*}, \tilde{\mathbf{r}}_k) + |g_k|^2 \left\| \tilde{\mathbf{s}}_m \right\|^2 \right\}$$

$(,)$ denotes vector inner product

$$=> \sum_{k=1}^{L} \mathrm{Re}(g_k{}^{*} \tilde{\mathbf{s}}_m{}^{*}, \tilde{\mathbf{r}}_k) - \frac{1}{2} \sum_{k=1}^{L} |g_k|^2 \left\| \tilde{\mathbf{s}}_m \right\|^2$$

1st term is ignored

$$= \sum_{k=1}^{L} \mathrm{Re}\left\{ \int_0^T \left(g_k^{*} \tilde{s}_m{}^{*}(t) \right) \tilde{r}_k(t) \right\} dt - \frac{1}{2} \sum_{k=1}^{L} |g_k|^2 \left\| \tilde{s}_m(t) \right\|^2$$

vector inner product => continuous - time function inner product(correlation)

$$= \int_0^T \mathrm{Re}\left\{ \left(\boxed{\sum_{k=1}^{L} g_k^{*} \tilde{r}_k(t)} \right) \tilde{s}_m{}^{*}(t) \right\} dt - \frac{1}{2} \sum_{k=1}^{L} |g_k|^2 \left\| \tilde{s}_m(t) \right\|^2 \quad (6.18)$$

the last termcan be ignored if the signal energy is equal (PSK for example)

MRC also used in HARQ

- Hybrid automatic repeat request (hybrid ARQ or HARQ) is a combination of high-rate forward error-correcting coding and ARQ error-control. (pure ARQ no FEC)
- Chase combining: every re-transmission contains the same information (data and parity bits). The receiver uses maximum-ratio combining to combine the received bits with the same bits from previous transmissions.

Hybrid ARQ with Chase combining

We consider an example of which the receiver is endowed with N antennas. In this case, the received vector y is

$$y = hs + \rho n \quad (1)$$

where n is noise vector $n - CN(0, I_{N\times N})$. Following the ML detection criterion the detection procedure may be written as

$$\tilde{s} = argmin_{s\in QPSK}|\hat{s} - s|^2, \quad (2)$$

where $\hat{s}$ is the least square solution to the above model.

$$\hat{s} = (h^* h)^{-1} h^* y. \quad (3)$$

The least square solution in this case is also known as maximal-ratio-combining (MRC). In the case of N antennas the LS can be written as

$$\hat{s} = \boxed{\frac{h_0^* y_0 + h_1^* y_1 + \ldots + h_{N-1}^* y_{N-1}}{|h_0|^2 + |h_1|^2 + \ldots + |h_{N-1}|^2}}, \quad (4)$$

New input to demodulator in RX
(h is scalar) |signal|=1

which means that the signal from each antenna is rotated and weighted according to the phase and strength of the channel, such that the signals from all antennas are combined to yield the maximal ratio between signal and noise terms.

MRC

- Assuming the k branches are independent and identically distributed, the symbol energy - to - noise ratio (E/N_0) γ_s^{mr} has a Chi - square distribution with 2L degrees of freedom

$$P\gamma_s^{mr}(x) = \frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c} \quad (6.23)$$

where $\bar{\gamma}_c = E[\gamma_k]$
(L Rayleigh/Rician terms sum up,
each Rayleigh/Rician is composed of two indep Gaussian r.v.)

- The cdf of γ_s^{mr} is

$$F_{\gamma_s^{mr}}(x) = 1 - e^{-x/\bar{\gamma}_c} \sum_{k=1}^{L-1} \frac{1}{k!}\left(\frac{x}{\bar{\gamma}_c}\right)^k \quad (6.25)$$

- The average bit energy - to - noise ratio with MRC is $\gamma_s^{mr} = L\bar{\gamma}_c$

BER of BPSK/MRC

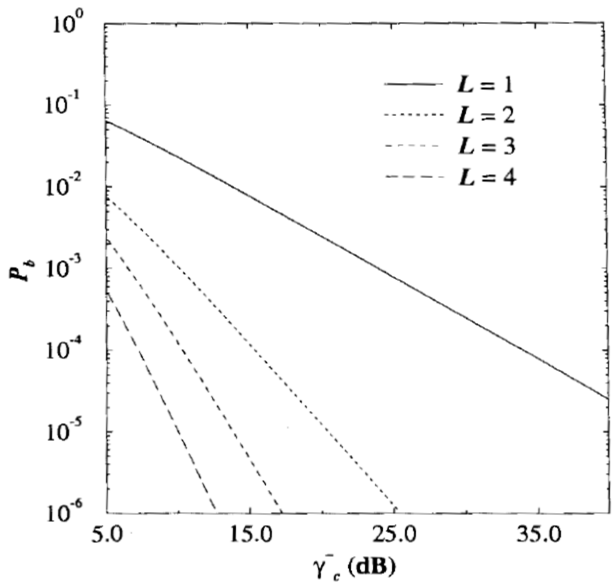
- The bit error probability of BPSK is

$$P_b = \int_0^\infty P_b(x) P_{\gamma_b^{mr}}(x)\,dx = \int_0^\infty Q\left(\sqrt{2x}\right)\frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c}\,dx$$

$$\begin{pmatrix} \text{substituting (6.23)} \\ \text{where x is instaneous } E/N_0, \\ Q\left(\sqrt{2x}\right) = Q\left(\sqrt{E/(N_0/2)}\right) = Q\left(\sqrt{SNR}\right) \text{for 1 - dim modulation} \end{pmatrix}$$

$$= \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+K}{k} \left(\frac{1+\mu}{2}\right)^k, \quad \text{where} \quad \mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}$$

BER for BPSK with MRC (Fig. 6.6)



Equal Gain Combining (EGC)

- EGC is different from MRC in that the diversity branches are not weighted (by α_{k}).
- Useful for equal energy symbols (otherwise, complete $\{\mathrm{g}_{\mathrm{k}}\}_{\mathrm{k}=1}^{\mathrm{L}}$ is needed anyway and MRC should be used).
- The coherent EGC receiver maximizes

$$\mu(\tilde{\mathrm{s}}_{\mathrm{m}}) = \sum_{k=1}^{L} \mathrm{Re}\left\{e^{-j\phi_k} \int_0^T \tilde{\gamma}_k(t)\tilde{s}_m^*(t)dt\right\} = \int_0^T \mathrm{Re}\left\{\sum_{k=1}^{L} e^{-j\phi_k} \tilde{\gamma}_k(t)\right\}\tilde{s}_m^*(t)dt$$

- The cdf and pdf for $\gamma_{\mathrm{s}}^{\mathrm{eg}}$ do not exist in closed form for L > 2.
- The mean value of the bit energy - to - noise ratio

$$\bar{\gamma}_{\mathrm{s}}^{\mathrm{eg}} = \bar{\gamma}_c\left(1 + (L-1)\frac{\pi}{4}\right)$$

Monte Carlo simulation

- 一平方公尺的方形區域中，有一個圓圈。以亂數產生許多在方形區域的點，看多少比例落在圓圈內，即為圓圈佔方形區域的比例。
- 一波蘭數學家Ulam以大量亂數估計贏排機率，故以Monaco最有名的Casino-Monte Carlo Casino來命名。