

# Data Detection/Interference Cancellation for Any Multiple Access Schemes (3 hours)

Same matrix form for general interference  
cancellation: CDMA multiple access interference  
OFDM inter-carrier interference, 4G MIMO/MU-  
MIMO inter-antenna interference (SIC/PIC). 4G  
other cell interference (SIC), 5G NOMA  
interference etc.

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## Outline

- Introduction
- Single-user detection
- Multi-user detection
- Comparison

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# Introduction

Disadvantages of CDMA conventional receiver

- MAI (Multiple access interference)  $\leq$  users are not disjoint in time or spectrum
- near-far problem  $\leq$  interfering user is nearer than intended user.

Strategies

- I.C./advanced data detection
- Power control (reduce near-far)
- Decrease number of interferences (smart antennas)
- Good design of radio link (channel coding)

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# Complexity

Single-user detection

- conventional detector  $O(K)$
- enhanced single user detector  $O(K)$

Multiuser detection

- ML  $O(2^K)$
- Decorrelator (zero-forcing) / MMSE  $O(K^3)$
- SIC/PIC (*most practical*)  $O(K)$

*P.S. matrix inverse (Gaussian elimination) requires  $O(K^3)$  computational complexity*

*Strang, Linear Algebra, 1988, p.15*

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- K terminals transmit asynchronously to a single base station
- Each user has L paths, spreading code  $S_k(t)$ , data symbol  $d_k$ .
- The received signal can be expressed as

$$r(t) = \sum_{k=1}^K \sum_{l=1}^L \sqrt{2P_k} \cdot h_{k,l} \cdot d_k(t - \tau_{k,l}) \cdot \quad (1)$$

$$S_k(t - \tau_{k,l}) \cdot \cos(\omega_c t + \varphi_{k,l}) + w(t)$$

where  $h_{k,l}$  is the channel coefficient,  $\tau_{k,l}$  is the channel delay,  $\varphi_{k,l}$  is the phase.

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## Notation

- Unless otherwise mentioned, we assume users are synchronous and single path ( $L=1$ ) for simplicity.

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## Single-user detection

- The matched filter output of the  $k$ th user's  $l$ th path for the  $i$ -th bit at the time instant giving as

$$t = iT_b + \tau_{k,l}$$

$$y_{k,l}^{(i)} = \int_{(i-1)T_b + \tau_{k,l}}^{iT_b + \tau_{k,l}} r(t) \cdot S_k(t - \tau_{k,l}) \cdot \cos(w_c t + \varphi_{k,l}) dt \quad (2)$$

$$= \text{Signal} + \text{MAI} + \text{noise}$$

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## Linear algebra- column space

- By **matrix** algebra expression

$\mathbf{r} = \mathbf{S}\mathbf{H}\mathbf{d} + \mathbf{w}$ , where

$\mathbf{S}$ : user's spreading code and delay

$\mathbf{H}$ : channel impulse response

$\mathbf{d}$ : transmitted data

Assume  $K=3$ ,  $L=1$ , no ISI,  $N$  chip per symbol

$$\mathbf{r} = [\mathbf{s}_1 \mathbf{s}_2 \mathbf{s}_3] \begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} + \mathbf{w} = \mathbf{s}_1 h_1 d_1 + \mathbf{s}_2 h_2 d_2 + \mathbf{s}_3 h_3 d_3 + \mathbf{w} \quad (3)$$

where  $\mathbf{s}_1 \mathbf{s}_2 \mathbf{s}_3$  are columns of  $\mathbf{S}$ ,  $\mathbf{r}$  is of  $N \times 1$

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- The whole bank of matched filter statistics (multiply  $S^T$ )

$$\mathbf{y} = \mathbf{S}^T \mathbf{r} = \mathbf{S}^T \mathbf{S} \mathbf{H} \mathbf{D} + \mathbf{S}^T \mathbf{w} = \mathbf{R} \mathbf{H} \mathbf{D} + \mathbf{z} \quad (4)$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & \rho_{1,2} & \rho_{1,3} \\ \rho_{2,1} & 1 & \rho_{2,3} \\ \rho_{3,1} & \rho_{3,2} & 1 \end{bmatrix} \begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} h_1 d_1 + \rho_{1,2} h_2 d_2 + \rho_{1,3} h_3 d_3 \\ \\ \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (5)$$

$\mathbf{R}$  : correlation matrix

$\rho_{i,j}$  : correlations coefficient between different users' code

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## Drawbacks

- Even if perfect power control and codes with low cross-correlation, the interference level increases as the number of interfering user increases.
- The performance is limited and so is the system capacity (the number of users supported).

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- This category has similar levels of complexity to the conventional filter, yet offer considerable gains in capacity.
- For example, use MMSE filter (adaptive filter NWTT9) to suppress inter-cell interference, intra-cell interference, and noise (all are part of “errors”) and try to minimize these errors.

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## Introduction to Multiuser detection

- *suitable for uplink* because all user data are needed to be detected anyway and BS can afford more complexity than MS.
- *Can only suppress intra-cell interference because BS know only users' information of its own.* (disadvantage relative to MMSE enhanced single-user detectors)

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## Multi-user detection

- Joint Detection
- Interference Cancellation schemes
- Combined schemes

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## Joint detection

- Maximum Likelihood Sequence Estimator (ML)
- Decorrelator Detector
- Minimum Mean Square Error Detector (MMSE)
- Iterative detection

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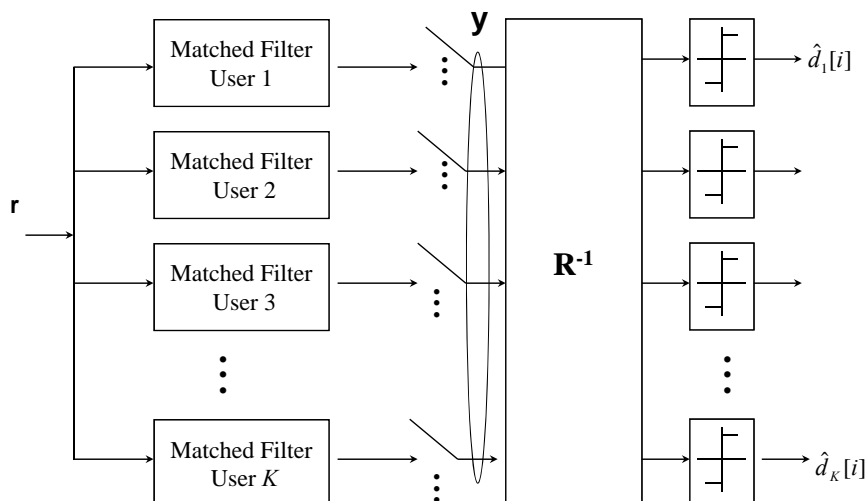
## ML sequence estimator

- Compute all  $2^K$  likelihood functions for BPSK.
- Performance is optimum but computation complexity grows exponentially with the number of users ( $>100$ ).

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## Decorrelator



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- $\mathbf{y} = \mathbf{R}\mathbf{H}\mathbf{d} + \mathbf{n}$  (6) same as (4)

$\mathbf{R}$  : correlation matrix

$\mathbf{H}$  : received amplitude

$\mathbf{d}$  : transmitted data

- $\hat{\mathbf{d}} = \text{sgn}(\mathbf{R}^{-1}\mathbf{y}) = \text{sgn}(\mathbf{H}\mathbf{d} + \mathbf{R}^{-1}\mathbf{n})$  (7)

- Unbiased, near - far resistant

- But enhances the noise, and must consider the existence of  $\mathbf{R}^{-1}$

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## MMSE

- Based on MMSE criterion used

for equalization against ISI, etc.

- By a transformation minimizes the MSE of the bit estimate

$$E[(\hat{\mathbf{d}} - \mathbf{d})^T (\hat{\mathbf{d}} - \mathbf{d})] \quad (9)$$

- The data estimates are

$$\hat{\mathbf{d}} = \text{sgn}\left(\left(\mathbf{R} + \frac{N_0}{2}\mathbf{H}^{-2}\right)^{-1} \cdot \mathbf{y}\right) \quad (10)$$

$$\hat{\mathbf{d}} = \text{sgn}\left(\left(\mathbf{R} + \frac{1}{\text{SNR}}\mathbf{I}\right)^{-1} \cdot \mathbf{y}\right)$$

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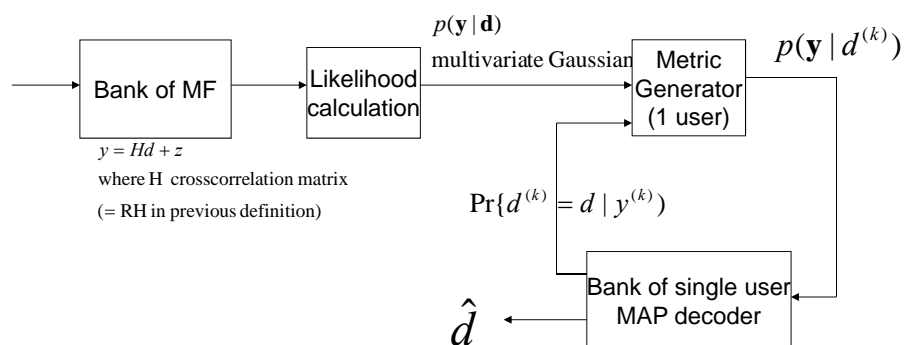
## MMSE is between decorrelator and conventional detector

- When noise  $\rightarrow 0$ , approach to decorrelator; as noise increase, reduce to conventional receiver.
- Imposes the same computational disadvantage as decorrelator and require knowledge regarding the receiver powers of the interfering users.

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## Iterative (MAP) Multiuser Detection



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$\mathbf{d} = [d^{(1)}, d^{(2)}, \dots, d^{(K)}]$  assume  $d^{(i)}$  are i.i.d.

$\mathbf{y} = [y^{(1)}, y^{(2)}, \dots, y^{(K)}]$

$$p(\mathbf{y}, d^{(k)} = +) = \sum_{\mathbf{d}, d^{(k)} = +} p(\mathbf{y} | \mathbf{d}) \Pr\{\mathbf{d}\}$$

$$= \sum_{\mathbf{d}, d^{(k)} = +} p(\mathbf{y} | \mathbf{d}) \prod_{i=1}^K \Pr\{d^{(i)} = d\}$$

metric generator output

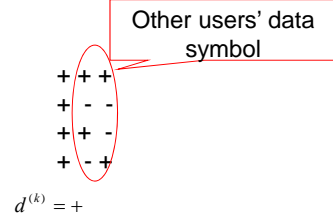
$$p(\mathbf{y} | d^{(k)} = +) = \frac{p(\mathbf{y}, d^{(k)} = +)}{\Pr\{d^{(k)} = +\}}$$

$$= \sum_{\mathbf{d}, d^{(k)} = +} p(\mathbf{y} | \mathbf{d}) \prod_{\substack{i=1 \\ i \neq k}}^K \Pr\{d^{(i)} = d\}$$

Metric generator two inputs

where  $p(\mathbf{y} | \mathbf{d})$  and  $\Pr\{d^{(i)} = +\} = \Pr\{d^{(i)} = + | y^{(i)}\}$

are from the metric generator inputs



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- The iterative MAP detection strategy is based on that used for turbo codes. Furthermore, interleaving and encoding can also be included.
- As the number of iteration increases, the BER performance improves.

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## Multi-user detection

- Joint Detection
- Interference cancellation schemes (IC)
- Combined schemes

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## Interference Cancellation

- Successive IC (SIC)
- Parallel IC (PIC)

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## Successive IC

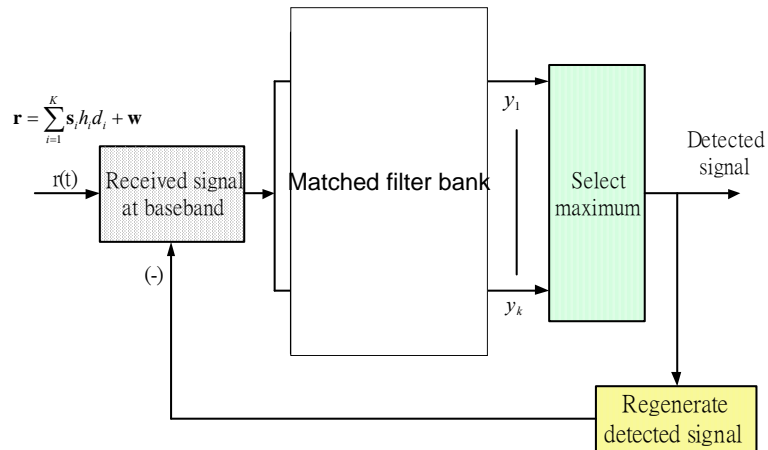


Fig 6. The SIC scheme

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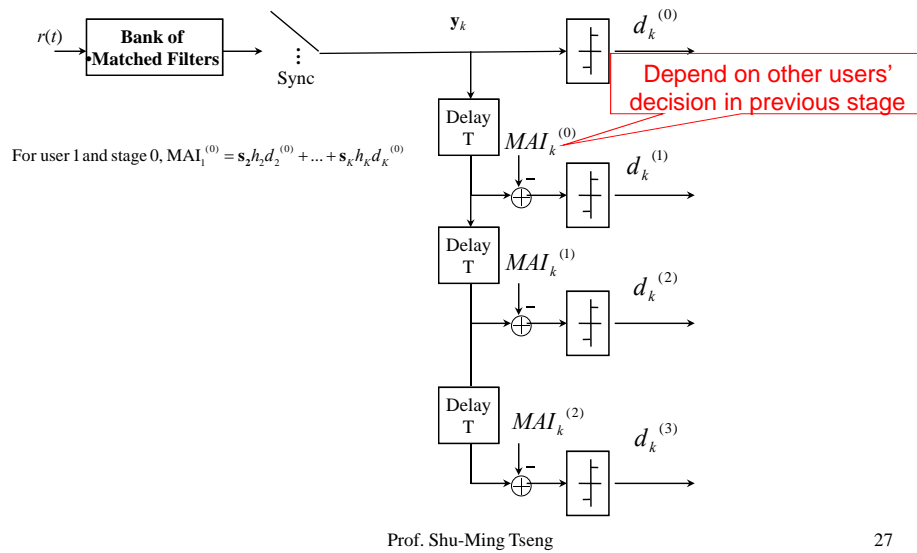
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- The detection/cancellation process occurs successively until all users have been detected.
- But the successive approach leads to a long delay when the number of users is large.

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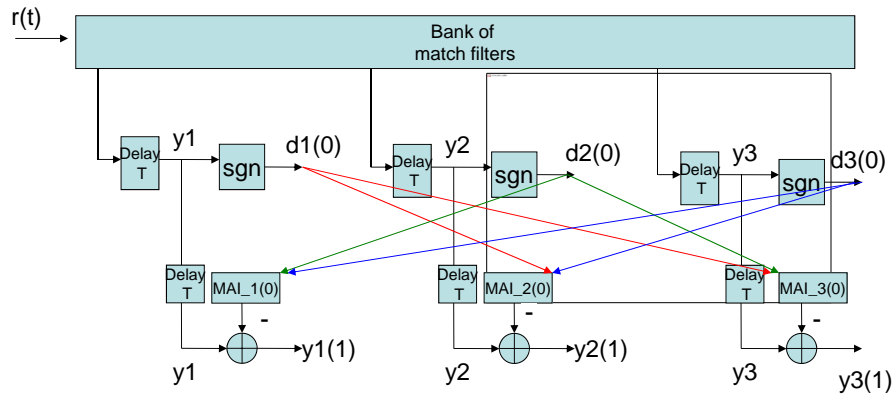
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## PIC: only user k is shown



- Canceling all users simultaneously: an alternative to the SIC.

## Ex: PIC M(# of stages)=2 K=3



$$\begin{aligned}
 y1(1) &= h1d1 + h2\rho_{1,2}(d2 - d2(0)) + h3\rho_{1,3}(d3 - d3(0)) + Z1 \\
 y2(1) &= \rho_{2,1}h1(d1 - d1(0)) + h2d2 + h3\rho_{2,3}(d3 - d3(0)) + Z2 \\
 y3(1) &= \rho_{3,1}h1(d1 - d1(0)) + h2\rho_{3,2}(d2 - d2(0)) + h3d3 + Z3
 \end{aligned}$$

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## Decorrelating Decision Feedback (actually partially decorrelator/SIC)

$$y = RHD + z$$

$$R = F^T F$$

where F is lower triangle matrix (suited for SIC)

$$(F^T)^{-1} y = FHD + (F^T)^{-1} z$$

$$= \begin{bmatrix} F_{1,1} & & \\ F_{2,1} & F_{2,2} & \\ & & F_{K,K} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_K \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_K \end{bmatrix} + (F^T)^{-1} z$$

國中消去法解聯立方程式

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## Partially Decorrelator/SIC

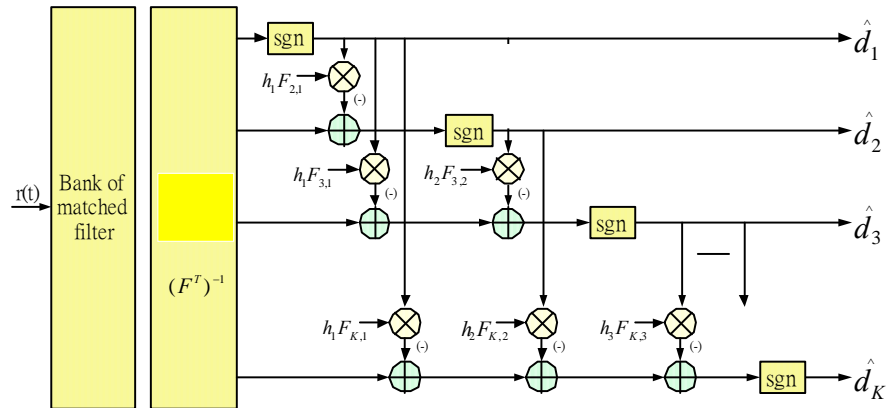


Fig 13. The decorrelating decision feedback detector.

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- This scheme inherits most advantages of SIC. Nevertheless, if data estimation subtracted are correct, all users experience complete MAI reduction.

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## Similar case for triangular matrix decomposition in Multi-user

### MIMO

- [Wan14] Xianan Wang, Xiaoxiang Wang, and Yanyan Qi, “Low-Complexity Transceiver Design for MIMO Broadcast Systems,” IEEE Communications Letters, vol. 18, no. 9, pp.1661-1664, 2014.
- QR decomposition  $H=QR$ , Q: orthonormal basis and R is upper triangular matrix (for SIC)

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## Required Knowledge for Detectors

- A: The signature waveform of the desired user
- B: The signature waveforms of the interfering users
- C: The timing of the desired user
- D: The timings of each of interfering users
- E: The received amplitudes of interfering users

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## Comparison of Detection Schemes

detection scheme	complexity	requirement
Single-user detector	$O(K)$	AC
Optimum multiuser detector	$O(2^K)$	ABCDE
Decorrelating detector	$O(K^3)$	ABCD
MMSE detector	$O(K^3)$	ABCDE
Parallel interference cancellation	$O(K)$	ABCDE
Successive interference cancellation	$O(K)$	ABCDE

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## Pro and Con

Detection scheme	pro	con
Single-user detector	simplicity	Low capacity
Optimum multiuser detector	High capacity	NP complete
Decorrelating detector	Near-far resistance	Noise enhancement Worse BER in low SNR
MMSE detector	Better BER than decorrelator in low SNR	Require amplitude estimate
Parallel interference cancellation	High capacity if good power control	Require perfect amplitude estimate
Successive interference cancellation	High capacity if severe near-far problem Prof. Shu-Ming Tseng	Long delay Require amplitude estimate

