Overview of the Fundamentals of Deep Learning Models

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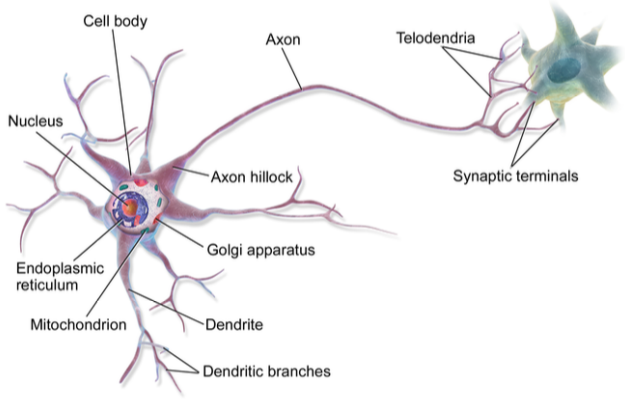
LMAP, University of Pau et Pays de L'Adour (France)

13-15 February 2026

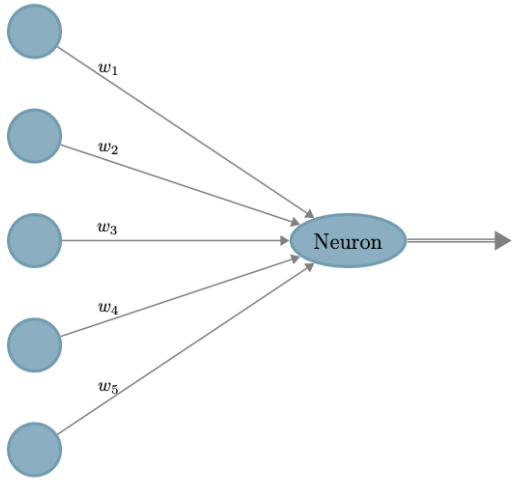
Session: Nuts & Bolts of AI in Neurosurgery (Theoretical and Practical Session)

Ghasem Azemi and Eric Suero Molina

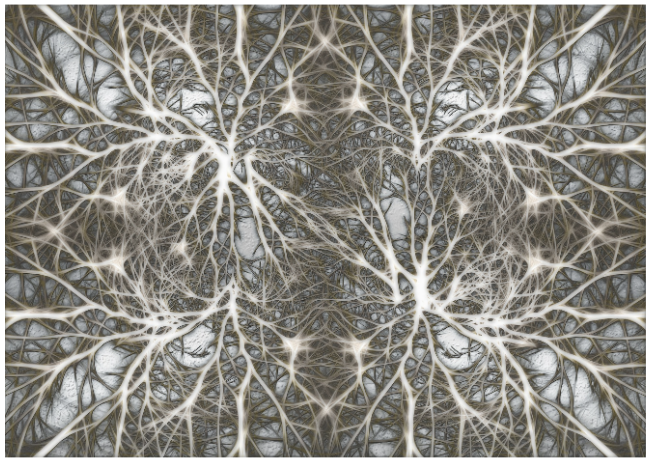
Biological vs. Artificial Neurons: The Building Blocks of Neural Networks



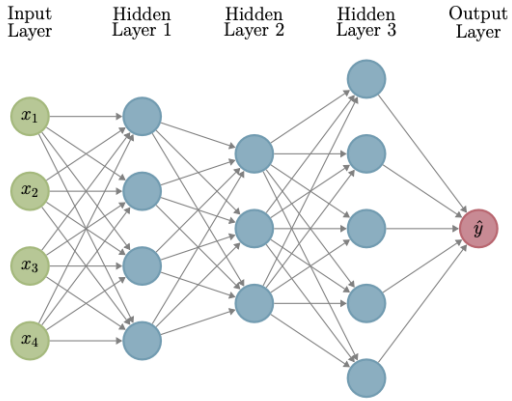
(a)



(b)

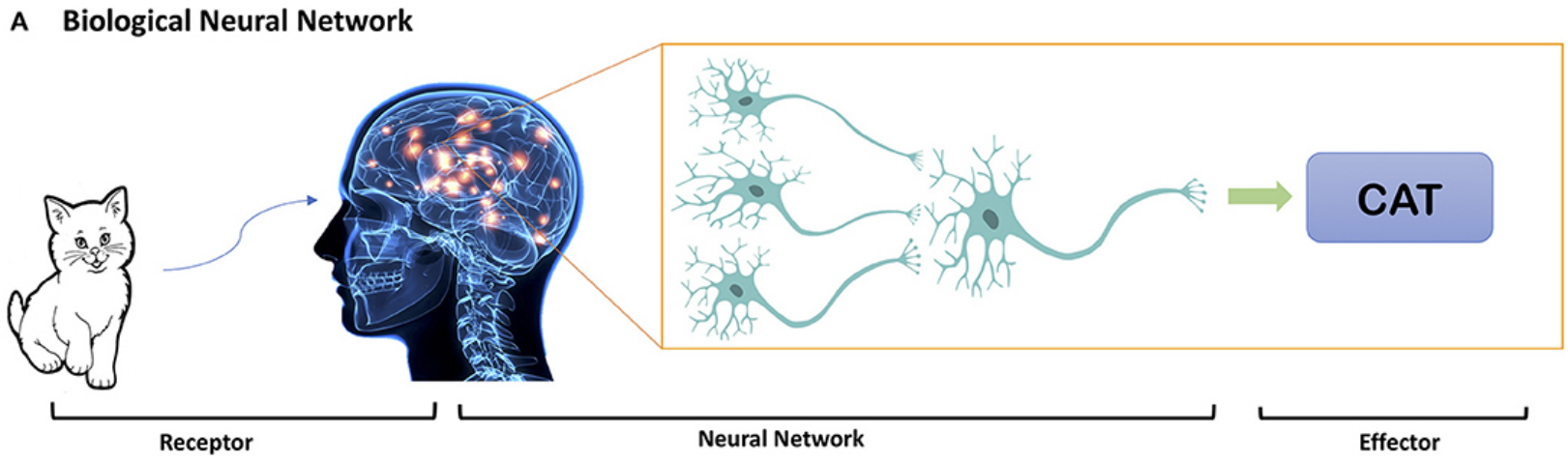


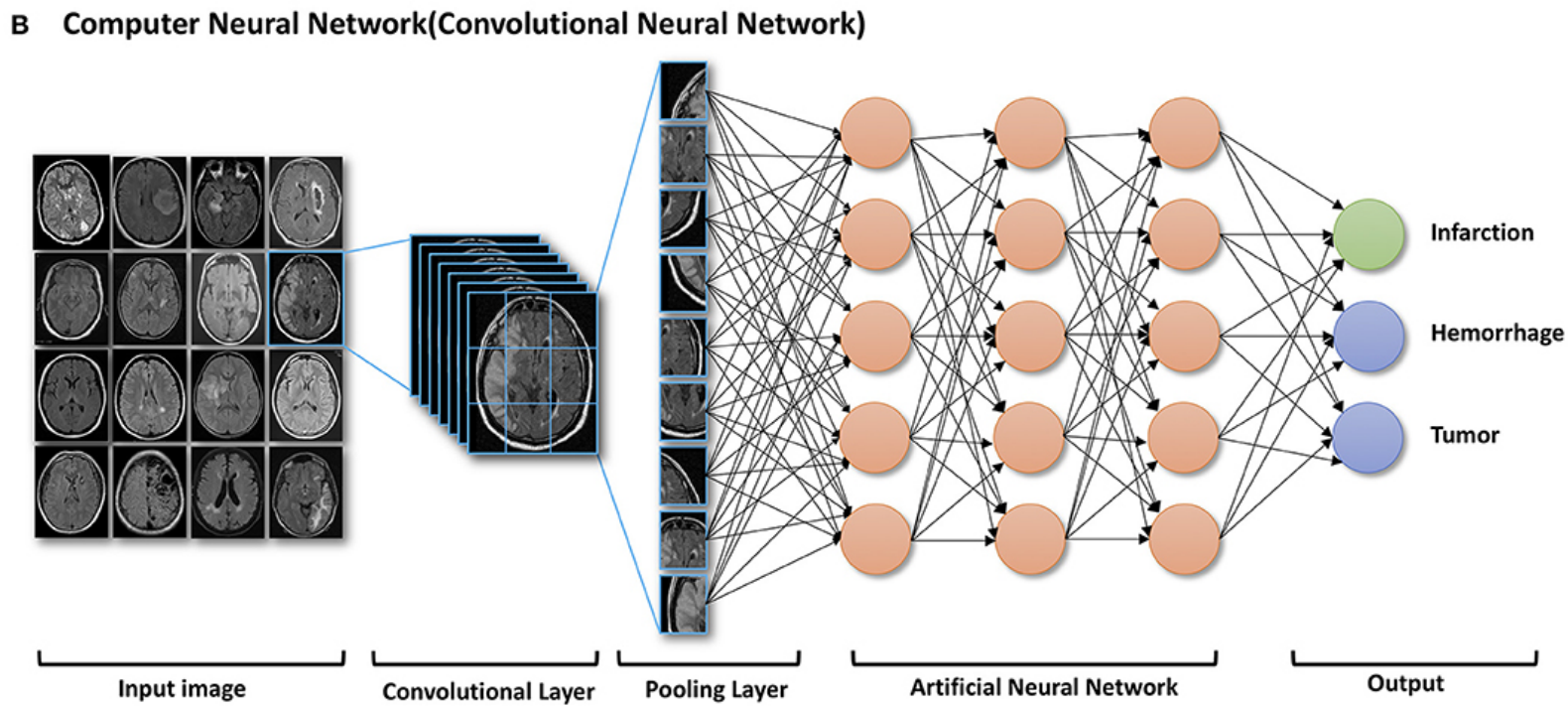
(c)



(d)

Brain Inspiration: CNNs for Medical Imaging

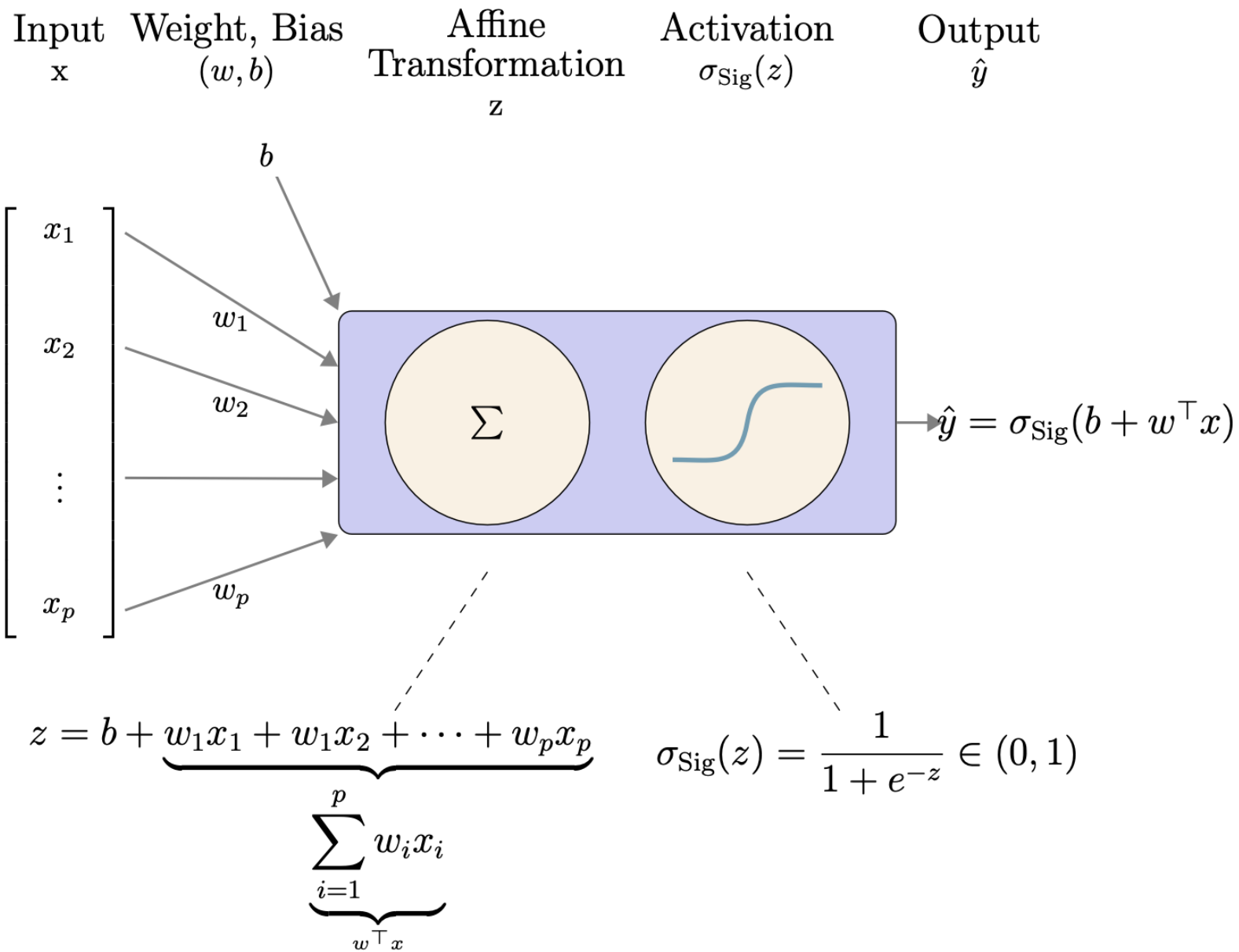




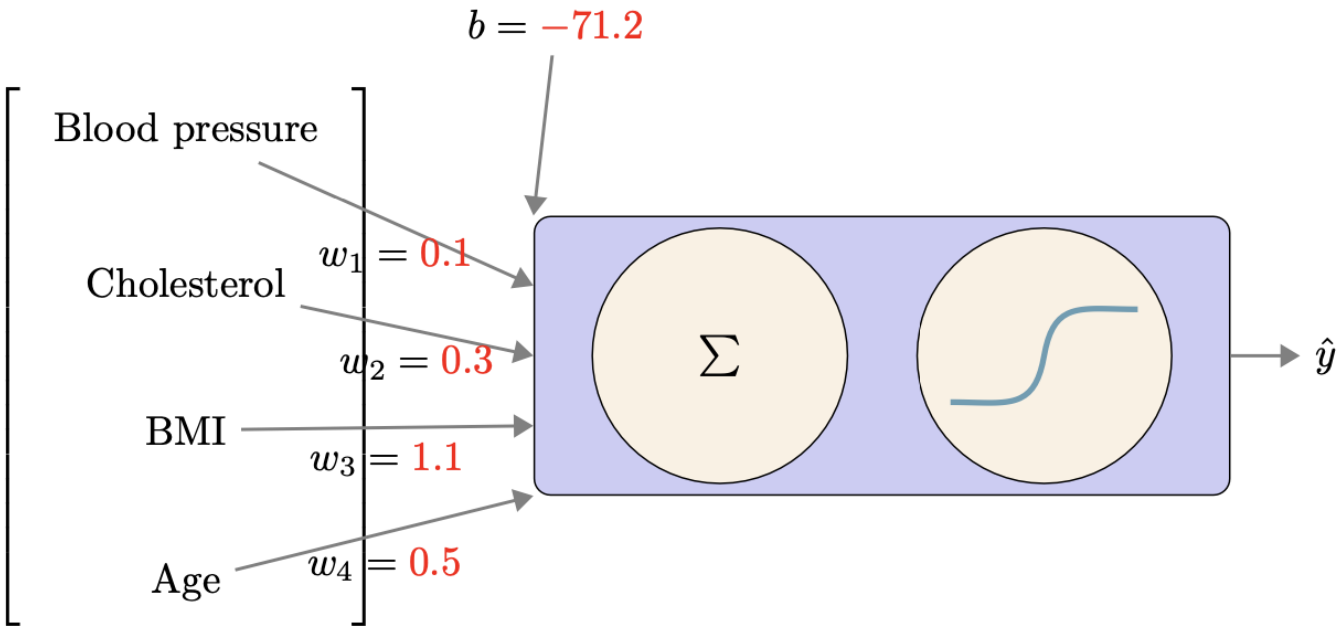
Question 1

How confident are you with the math or algebra behind deep learning models?

Artificial neuron: Sigmoid model (logistic model)



Toy example: predict stroke



Patient 1

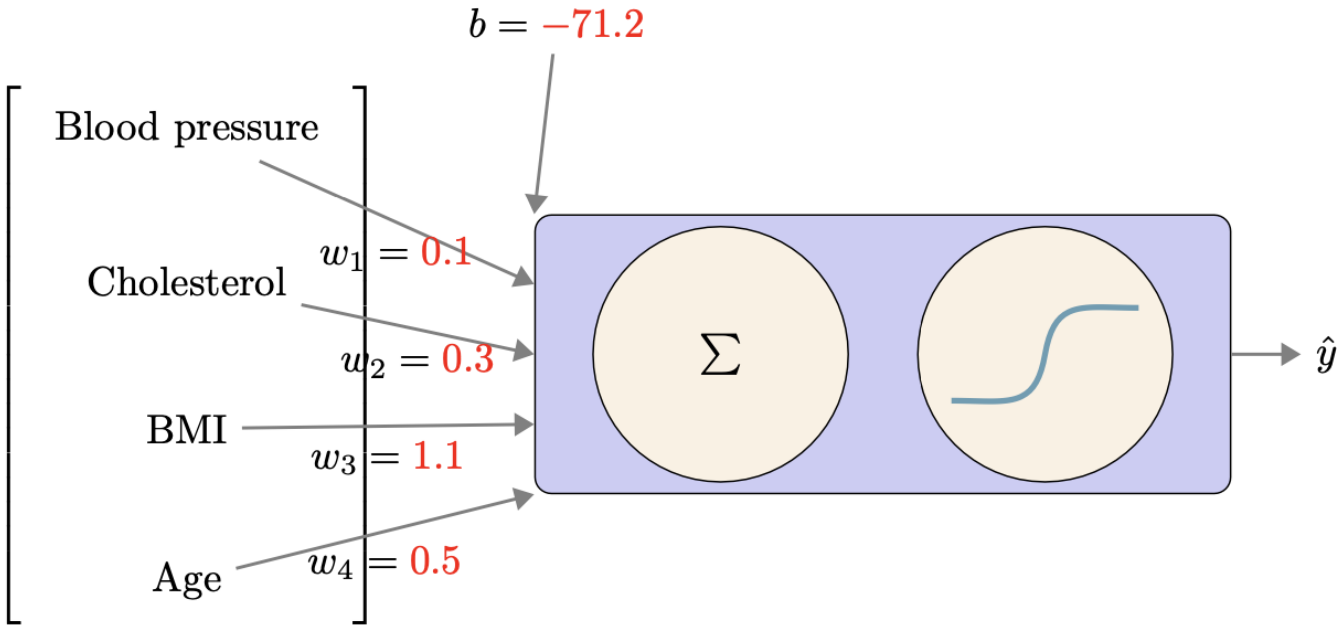
$$x = \begin{bmatrix} 130 \\ 6 \\ 28 \\ 55 \end{bmatrix}$$

$$z = \underbrace{-71.2}_{b} + \underbrace{0.1}_{w_1} \cdot \overbrace{130}^{x_1} + \underbrace{0.3}_{w_2} \cdot \overbrace{6}^{x_2} + \underbrace{1.1}_{w_3} \cdot \overbrace{28}^{x_3} + \underbrace{0.5}_{w_4} \cdot \overbrace{55}^{x_4}$$

$$= 1.9$$

$$\Rightarrow \quad \widehat{y} = \sigma_{\text{Sig}}(z) = \frac{1}{1 - e^{-1.9}} = \underbrace{0.71}_{\textbf{Probability of getting a stroke}}$$

Toy example: predict stroke



Patient 2

$$x = \begin{bmatrix} 130 \\ 5.5 \\ 23 \\ 56 \end{bmatrix}$$

$$z = \underbrace{-71.2}_{b} + \underbrace{0.1}_{w_1} \cdot \overbrace{130}^{x_1} + \underbrace{0.3}_{w_2} \cdot \overbrace{5.5}^{x_2} + \underbrace{1.1}_{w_3} \cdot \overbrace{23}^{x_3} + \underbrace{0.5}_{w_4} \cdot \overbrace{56}^{x_4}$$

$$= -3.25$$

$$\Rightarrow \quad \widehat{y} = \sigma_{\text{Sig}}(z) = \frac{1}{1 - e^{-(-3.25)}} = \underbrace{0.04}_{\textbf{Probability of getting a stroke}}$$

Sigmoid model as a binary classifier

A classifier can be constructed via a *decision rule* based on a threshold τ:

$$\widehat{\mathscr{Y}} = \begin{cases} 0 \text{ (negative)}, & \text{if} \quad \widehat{y} \leq \tau \\ 1 \text{ (positive)}, & \text{if} \quad \widehat{y} > \tau. \end{cases}$$

In many cases one selects the threshold at $\tau = 0.5$.

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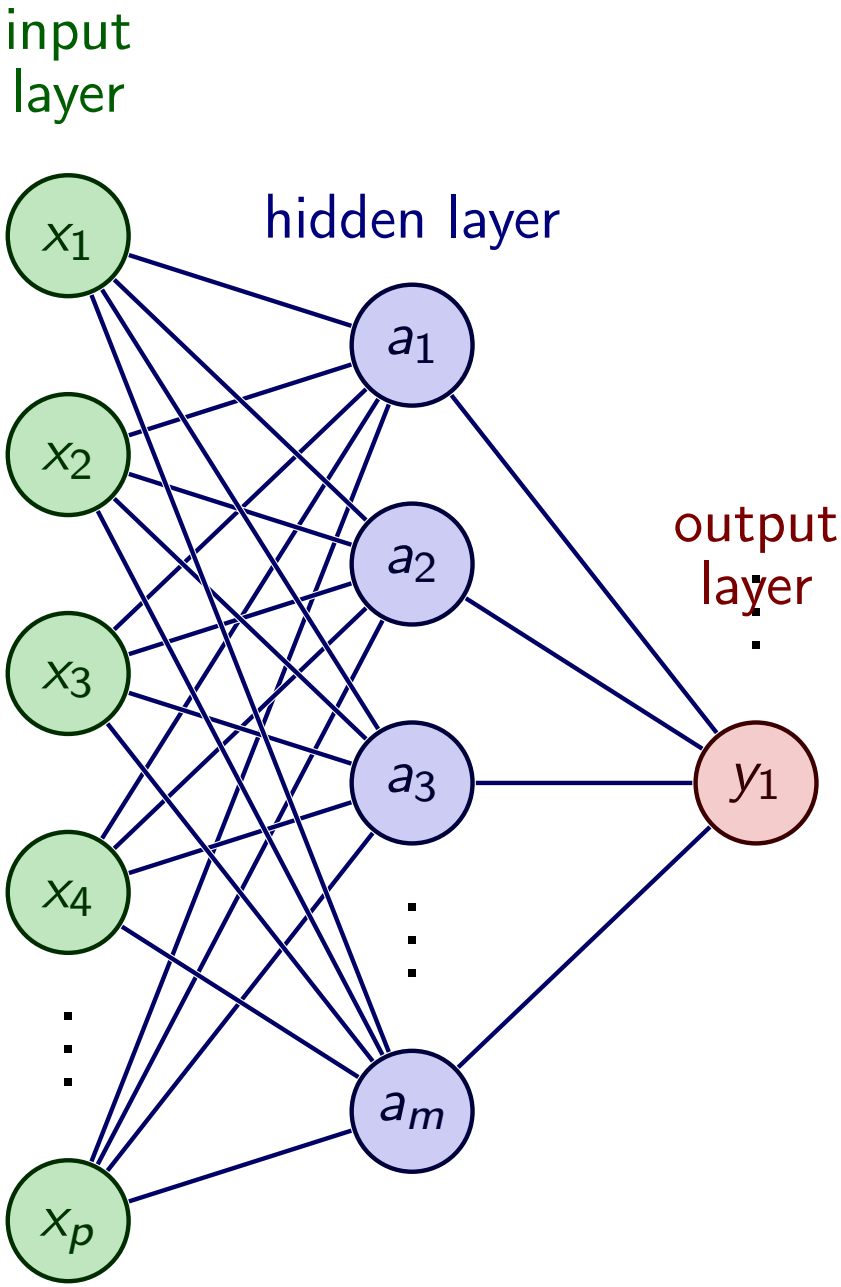
In many cases one selects the threshold at $\tau = 0.5$.

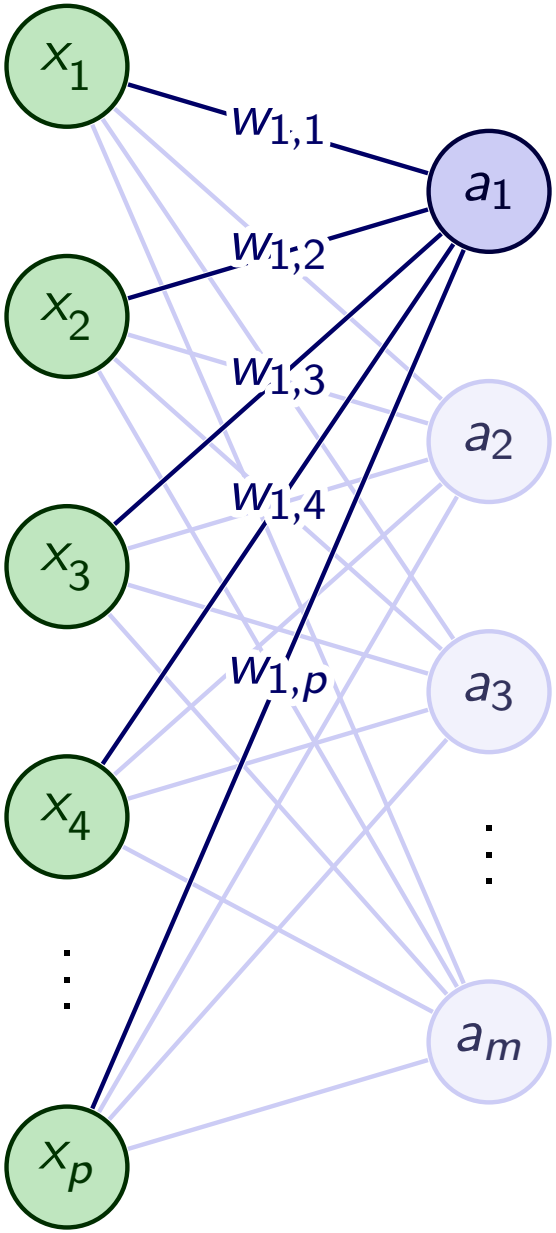
- Patient 1: $\widehat{y} = 0.71 > 0.5 \quad \Rightarrow \quad \widehat{\mathscr{Y}} = 1$ **(Positive)**
- Patient 2: $\widehat{y} = 0.04 \leq 0.5 \quad \Rightarrow \quad \widehat{\mathscr{Y}} = 0$ **(Negative)**

Question 2

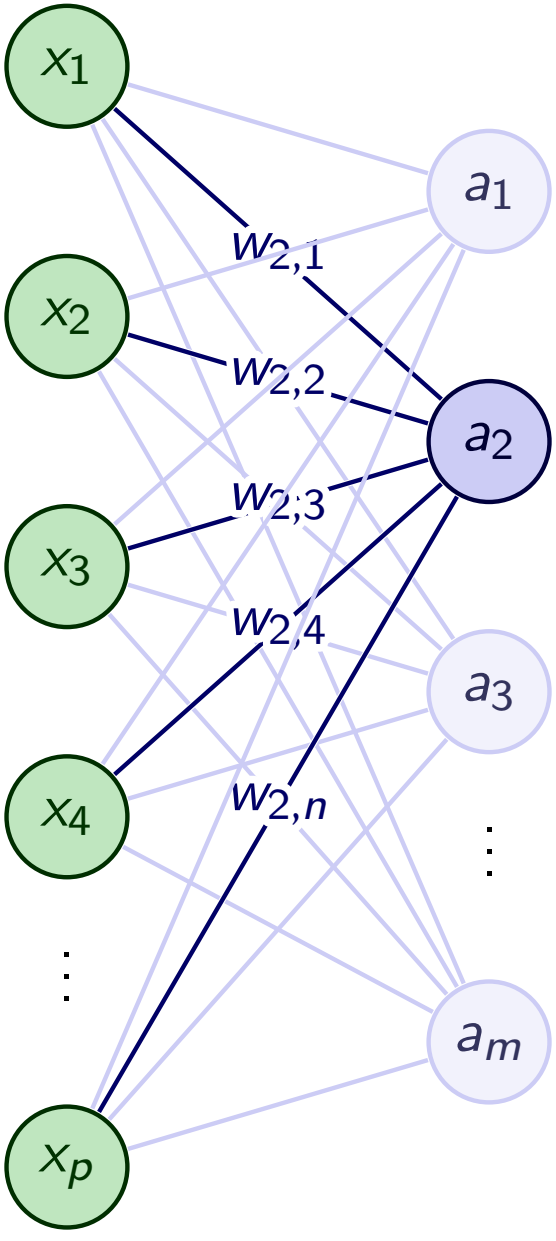
Will changing the threshold after applying the sigmoid function affect the model's performance?

Single hidden layer

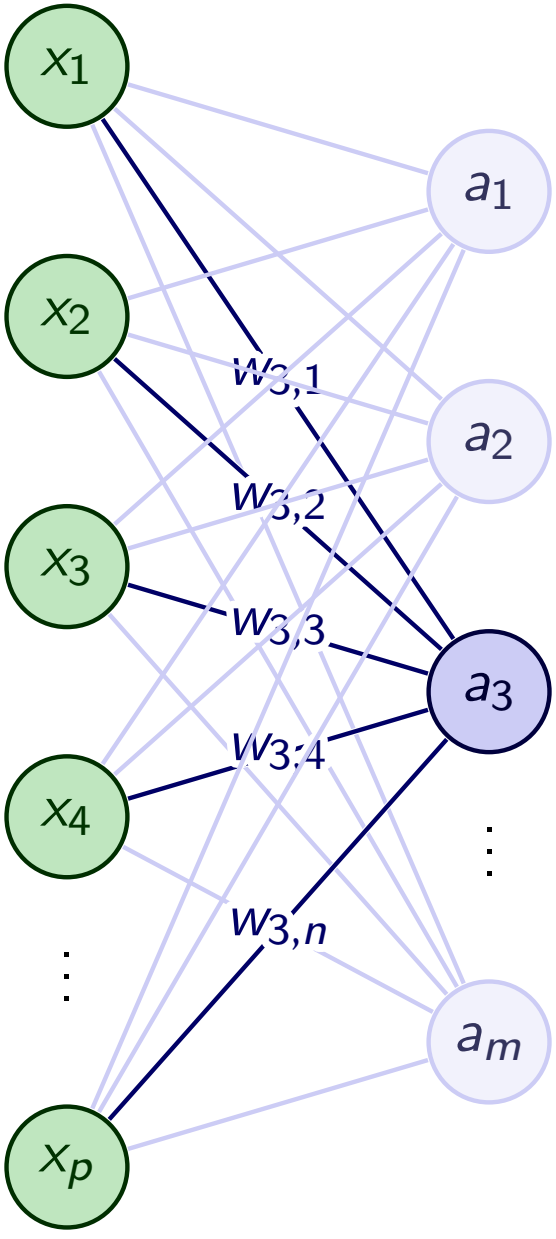




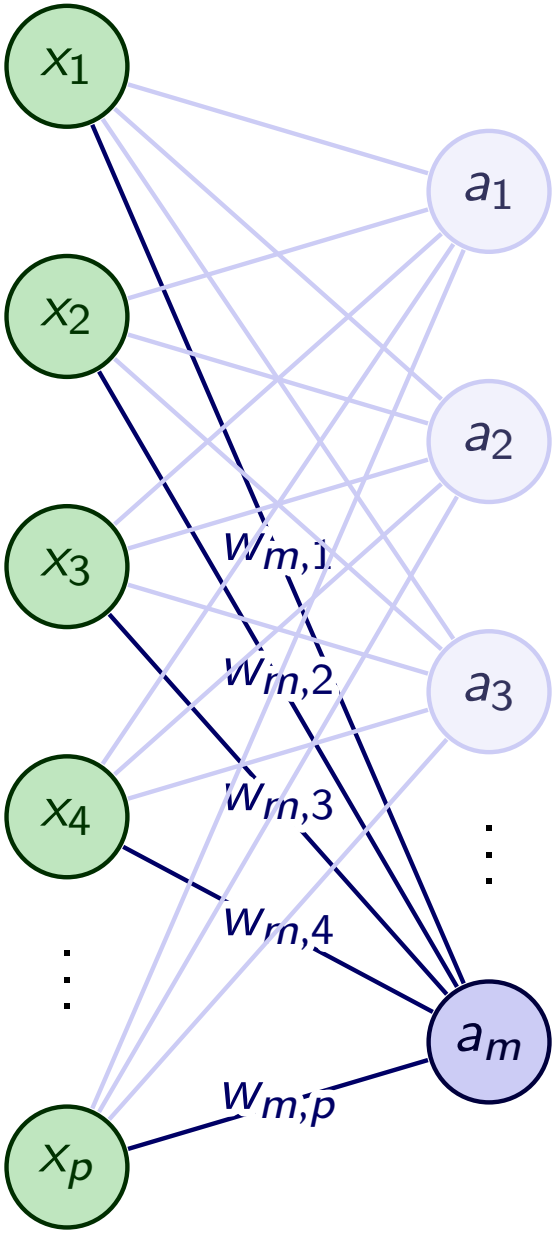
$$= \sigma\left(\underbrace{w_{1,1}x_1 + w_{1,2}x_2 + \ldots + w_{1,p}x_n + b_1}_{z_1}\right) = \sigma(z_1)$$



$$a_2 = \sigma\left(\underbrace{w_{2,1}x_1 + w_{2,2}x_2 + \ldots + w_{2,p}x_n + b_2}_{z_2}\right)$$
$$= \sigma(z_2)$$

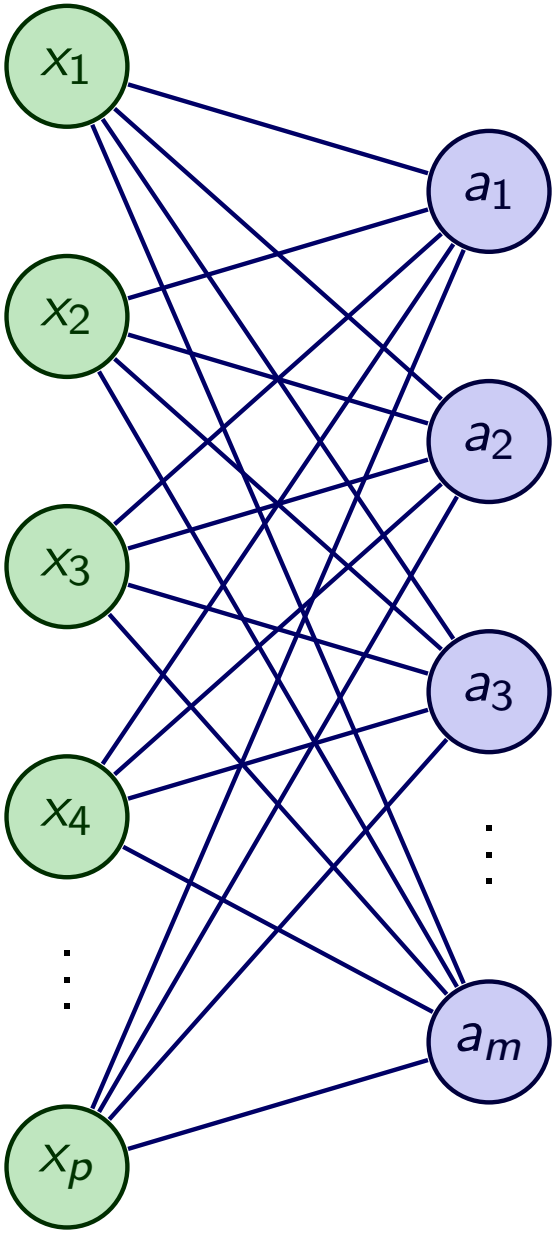


$$a_3 = \sigma\left(\underbrace{w_{3,1}x_1 + w_{3,2}x_2 + \ldots + w_{3,p}x_n + b_3}_{z_3}\right)$$
$$= \sigma(z_3)$$

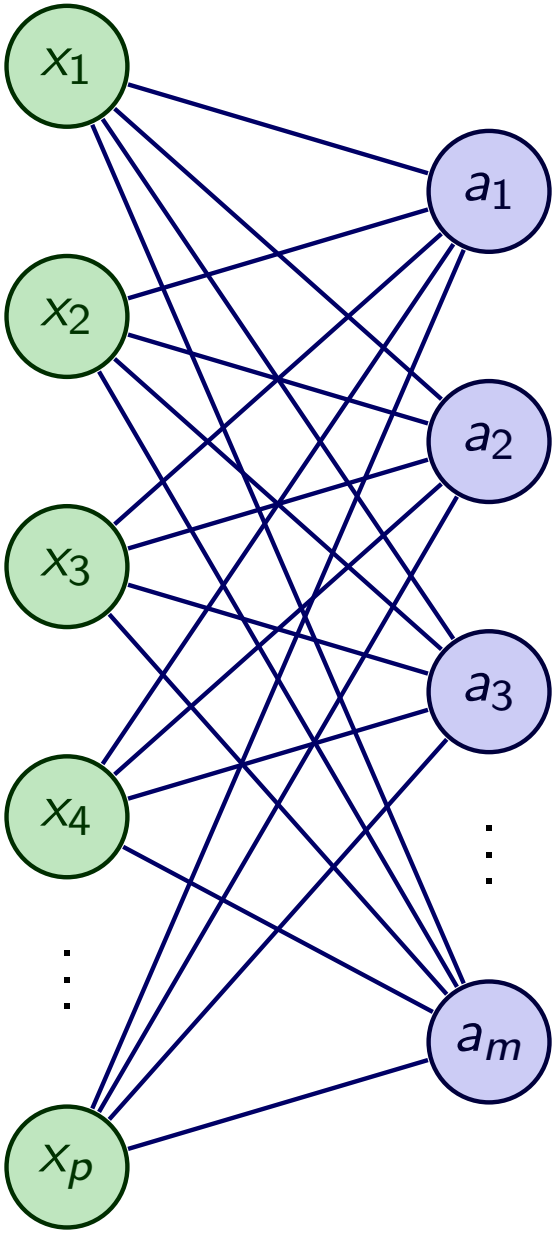


$$a_m = \sigma\left(\underbrace{w_{m,1}x_1 + w_{m,2}x_2 + \ldots + w_{m,p}x_n + b_m}_{z_m}\right)$$

$$= \sigma(z_m)$$



$$
\begin{cases}
a_1 = \sigma\left(w_{1,1}x_1 + w_{1,2}x_2 + \cdots + w_{1,p}x_p + b_1\right) \\
a_2 = \sigma\left(w_{2,1}x_1 + w_{2,2}x_2 + \cdots + w_{2,p}x_p + b_2\right) \\
a_3 = \sigma\left(w_{3,1}x_1 + w_{3,2}x_2 + \cdots + w_{3,p}x_p + b_3\right) \\
\quad \vdots = \qquad\qquad\qquad\qquad \vdots \\
a_m = \sigma\left(w_{m,1}x_1 + w_{m,2}x_2 + \cdots + w_{m,p}x_p + b_m\right)
\end{cases}
$$



$$
\begin{cases}
a_1 = \sigma(w_{1,1}x_1 + \cdots + w_{1,p}x_p + b_1) \\
a_2 = \sigma(w_{2,1}x_1 + \cdots + w_{2,p}x_p + b_2) \\
a_3 = \sigma(w_{3,1}x_1 + \cdots + w_{3,p}x_p + b_3) \\
\vdots = \qquad\qquad\qquad \vdots \\
a_m = \sigma(w_{m,1}x_1 + \cdots + w_{m,p}x_p + b_m)
\end{cases}
$$

$$
a = \sigma(\underbrace{Wx + b}_{z})
$$

Matrix notation

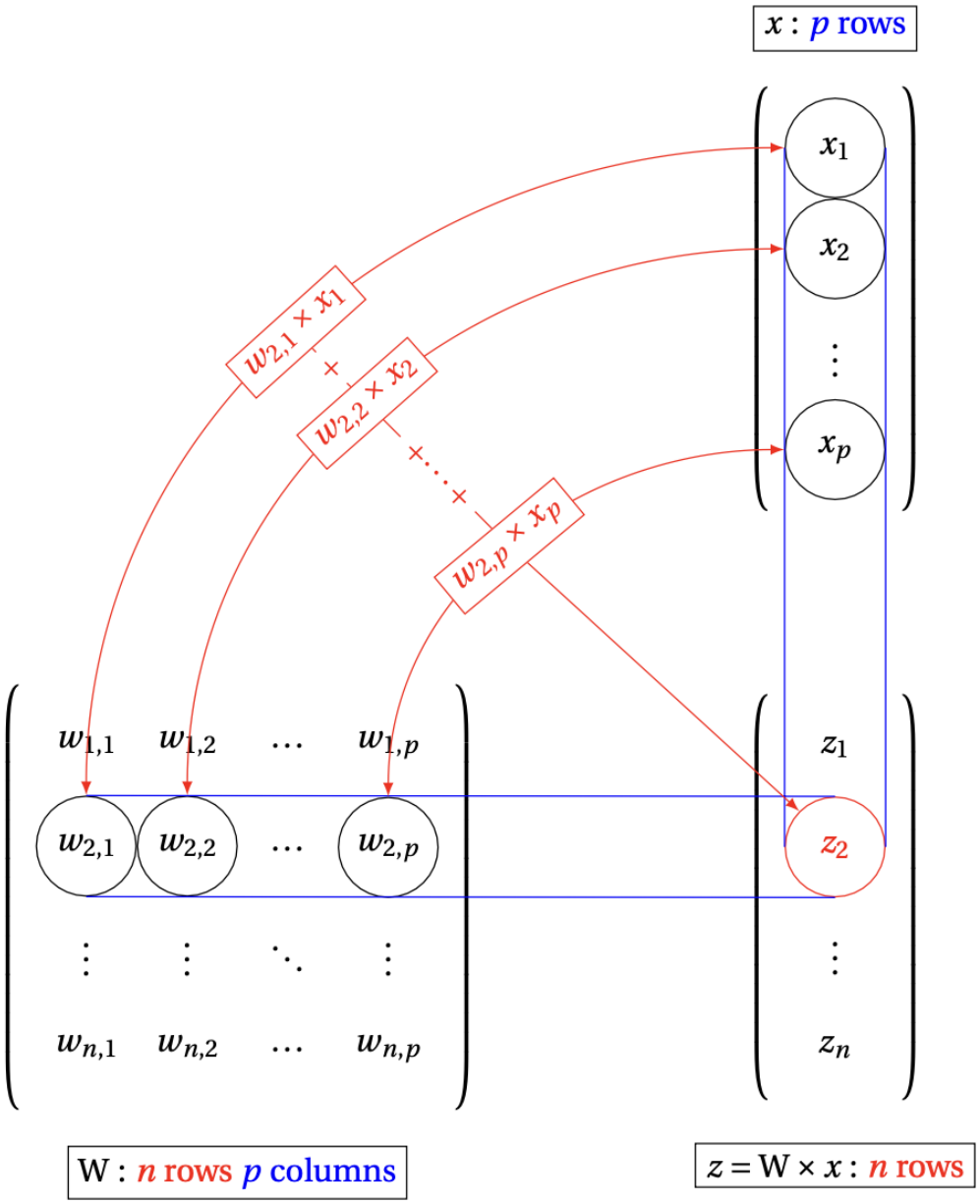
Vectorized form of neuron activations:

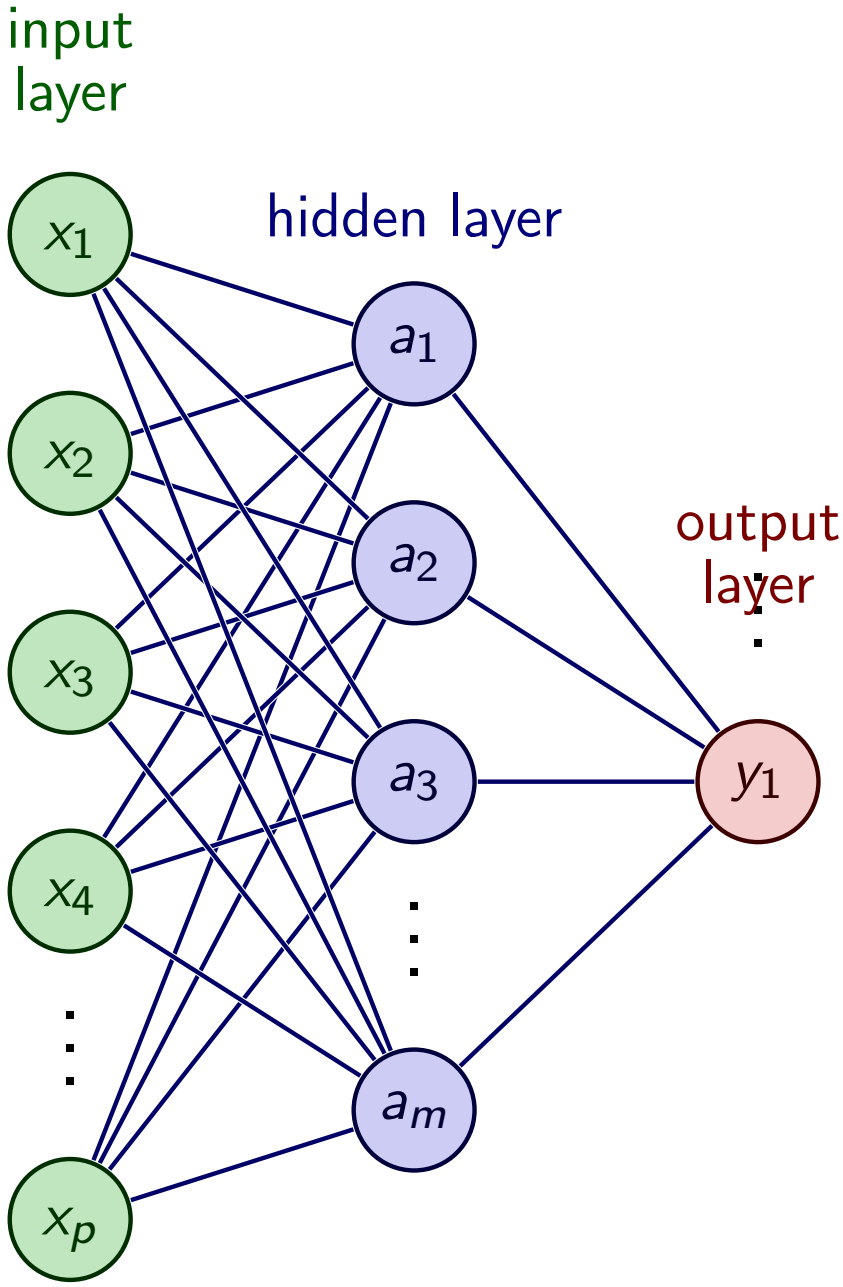
$$a = \sigma(Wx + b)$$

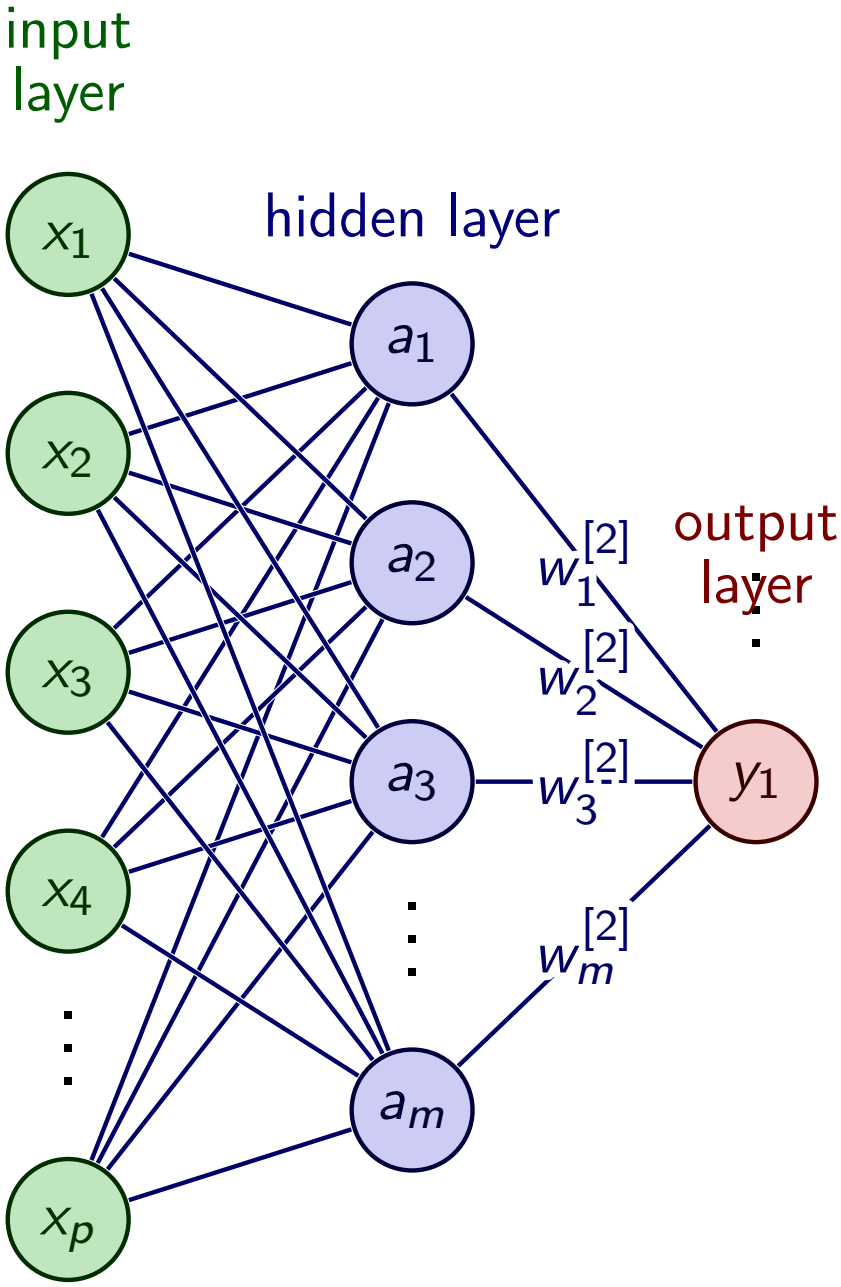
$$\overbrace{\begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix}}^{\text{a}} = \sigma\Bigg(\underbrace{\overbrace{\begin{pmatrix} w_{1,1} & \cdots & w_{1,p} \\ \vdots & \ddots & \vdots \\ w_{m,1} & \cdots & w_{m,p} \end{pmatrix}}^{W} \overbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_p \end{pmatrix}}^{\text{Input } x} + \overbrace{\begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}}^{b}}_{z}\Bigg)$$

$$= \sigma \begin{pmatrix} z_1 \\ \vdots \\ z_p \end{pmatrix} = \begin{pmatrix} \sigma(z_1) \\ \vdots \\ \sigma(z_p) \end{pmatrix}$$

Matrix-vector multiplication: $W \times x = z$ with $b = 0$




input
layer
hidden layer
output
layer
x_1
x_2
x_3
x_4
x_p
a_1
a_2
a_3
a_m
y_1


input
layer
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x_1
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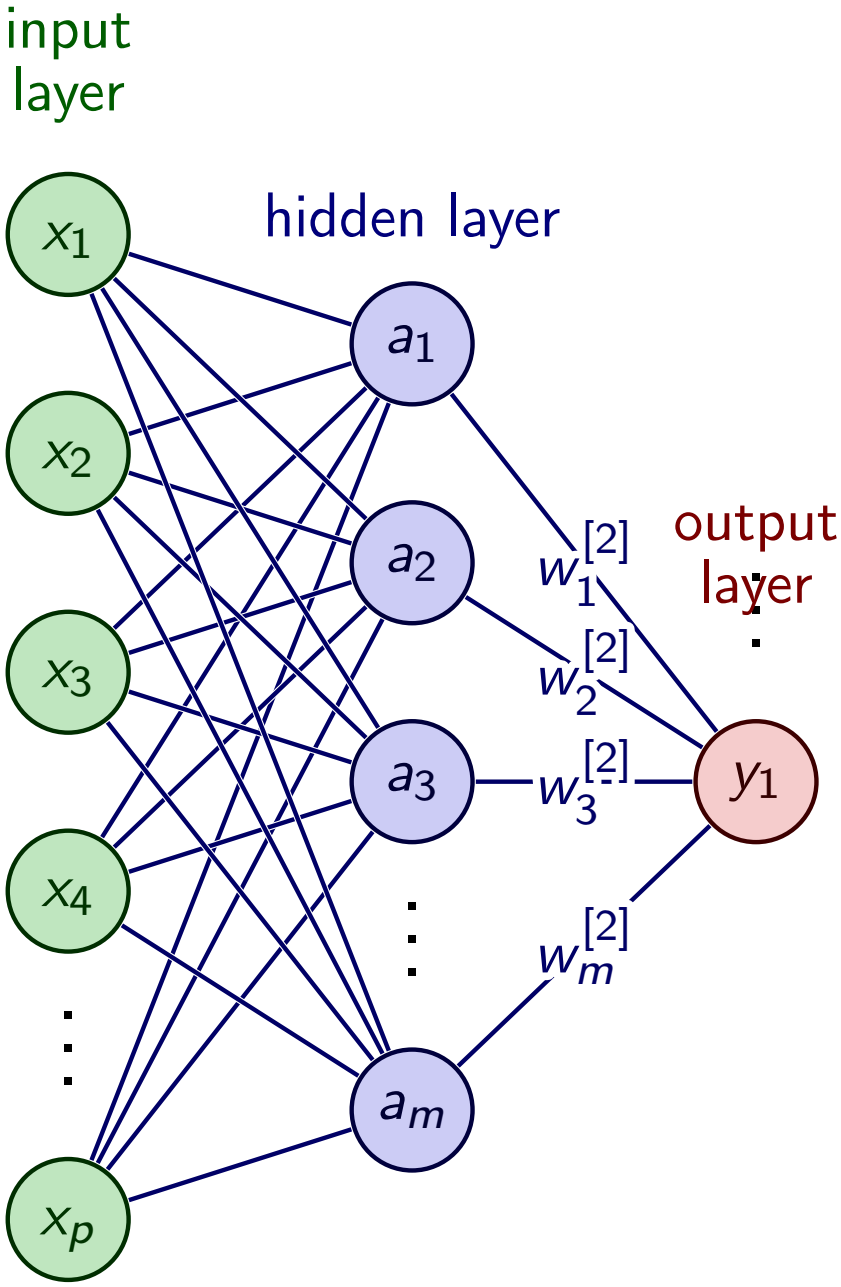
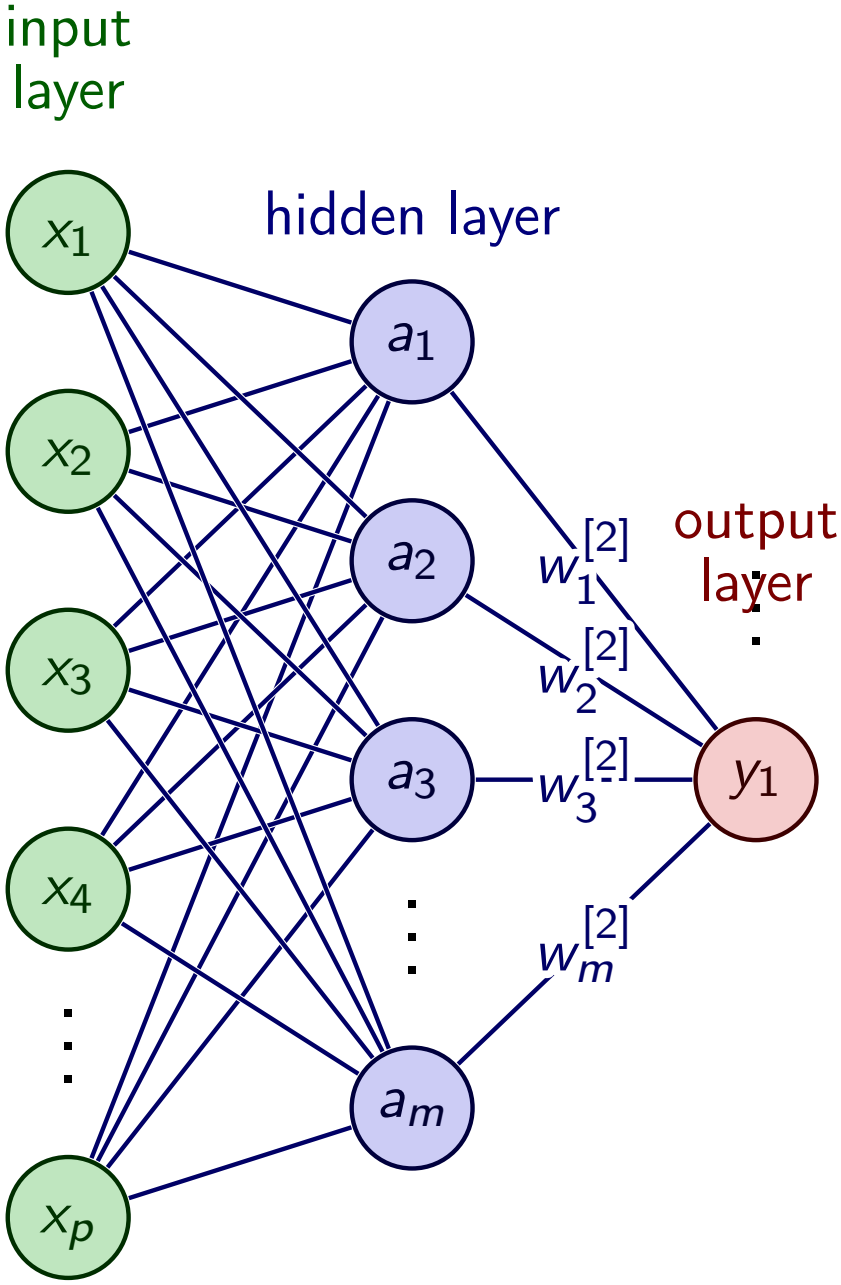
a_3
a_m
y_1
$w_1^{[2]}$
$w_2^{[2]}$
$w_3^{[2]}$
$w_m^{[2]}$



$$y = \sigma\left(w_1^{[2]} a_1 + w_2^{[2]} a_2 + \cdots + w_m^{[2]} a_m + b^{[2]}\right)$$
$$= \sigma\left(W^{[2]} a + b^{[2]}\right)$$

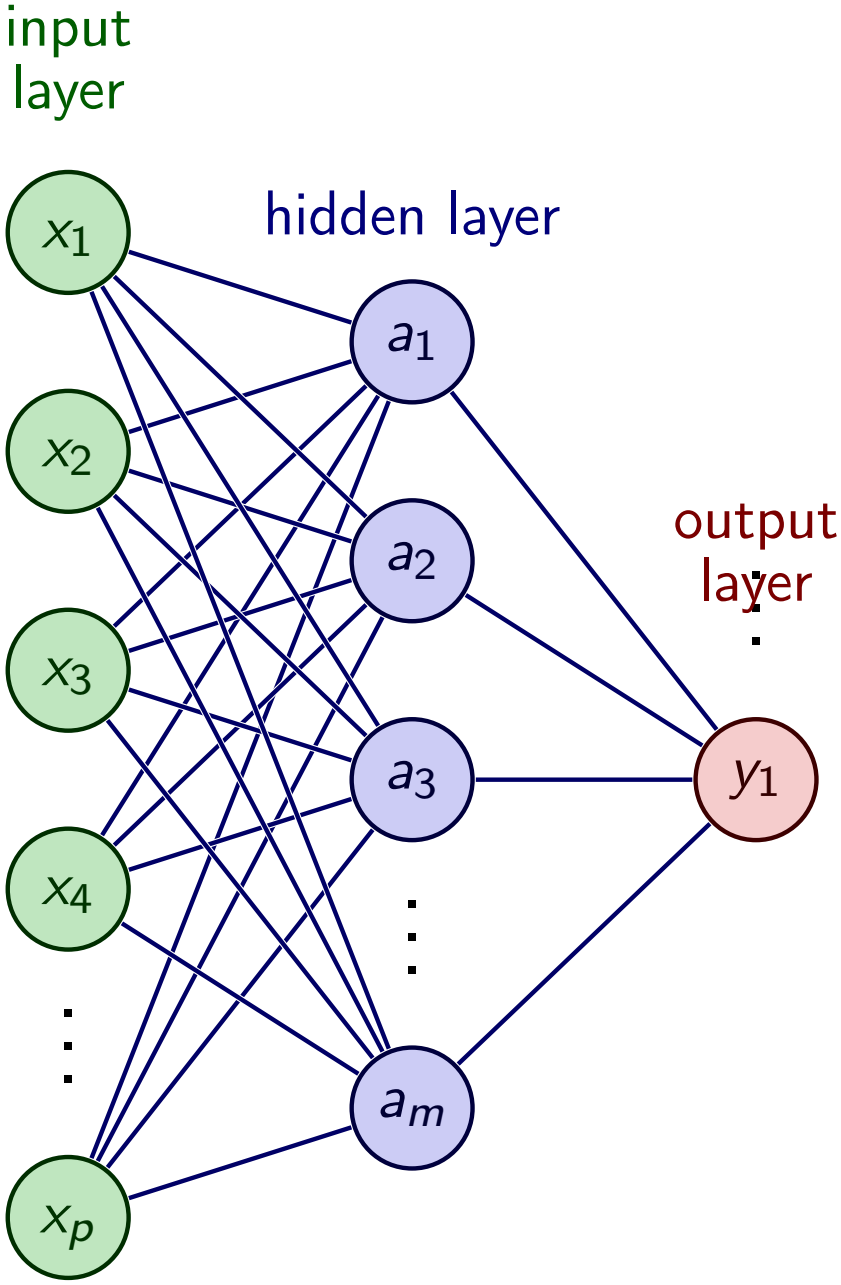


$$y = \sigma\left(\overbrace{w_1^{[2]}a_1 + w_2^{[2]}a_2 + \cdots + w_m^{[2]}a_m}^{W^{[2]}a} + b^{(2)}\right)$$

$$= \sigma\left(W^{[2]}a + b^{(2)}\right)$$

$$W^{[2]}a = \overbrace{\begin{pmatrix} w_1^{[2]} & w_2^{[2]} & \cdots & w_m^{[2]} \end{pmatrix}}^{W^{[2]}} \overbrace{\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}}^{a}$$

$$= \left(w_1^{[2]}a_1 + w_2^{[2]}a_2 + \cdots + w_m^{[2]}a_m\right)$$

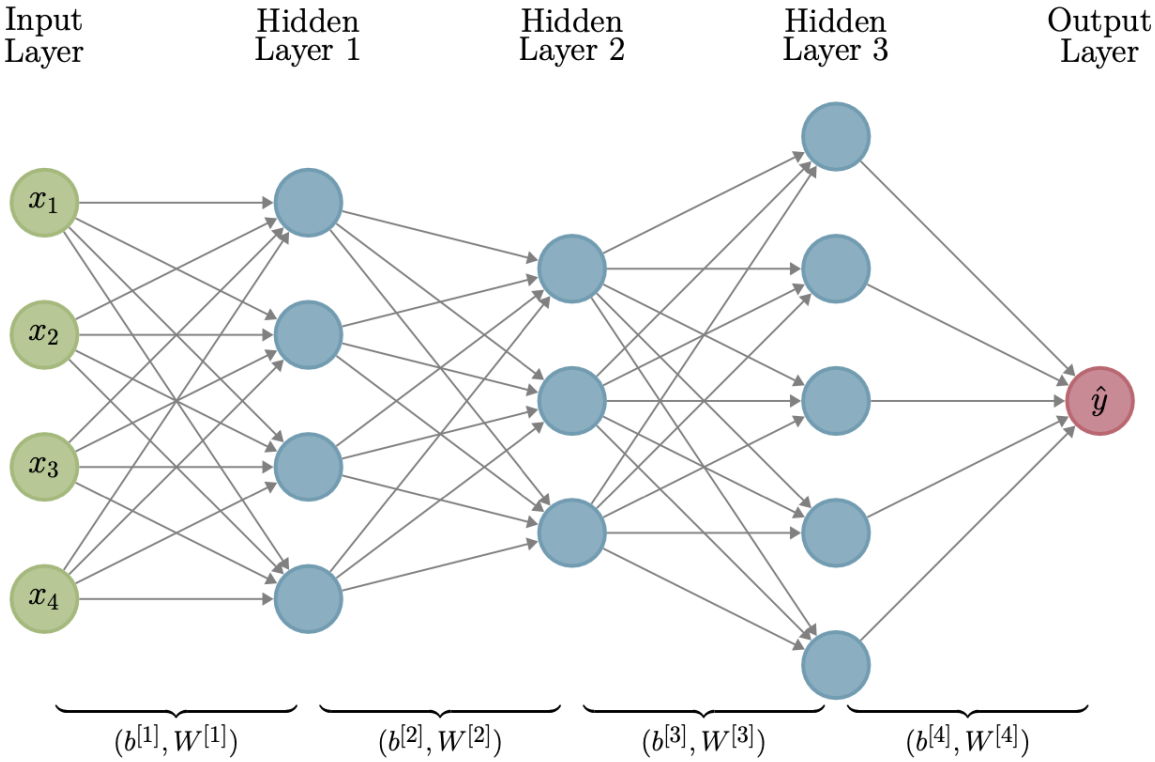


$$y = \sigma\left(W^{[2]} \underbrace{(\sigma(W^{[1]}x + b^{[1]})}_{a^{[1]}} + b^{[2]} \right)$$

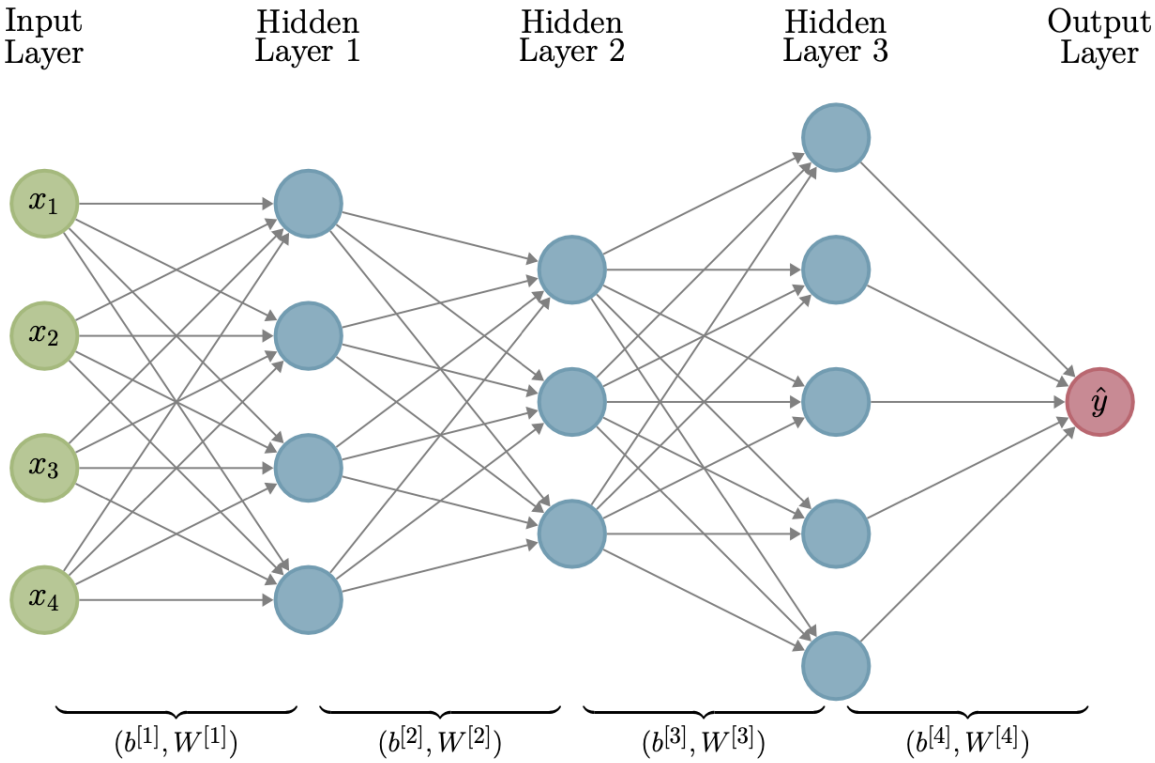
$$= \underbrace{f^{[2]}(f^{[1]}(x))}_{f_\theta(x)}$$

where $\theta = (W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]})$

Fully Connected Neural Network or feedforward networks

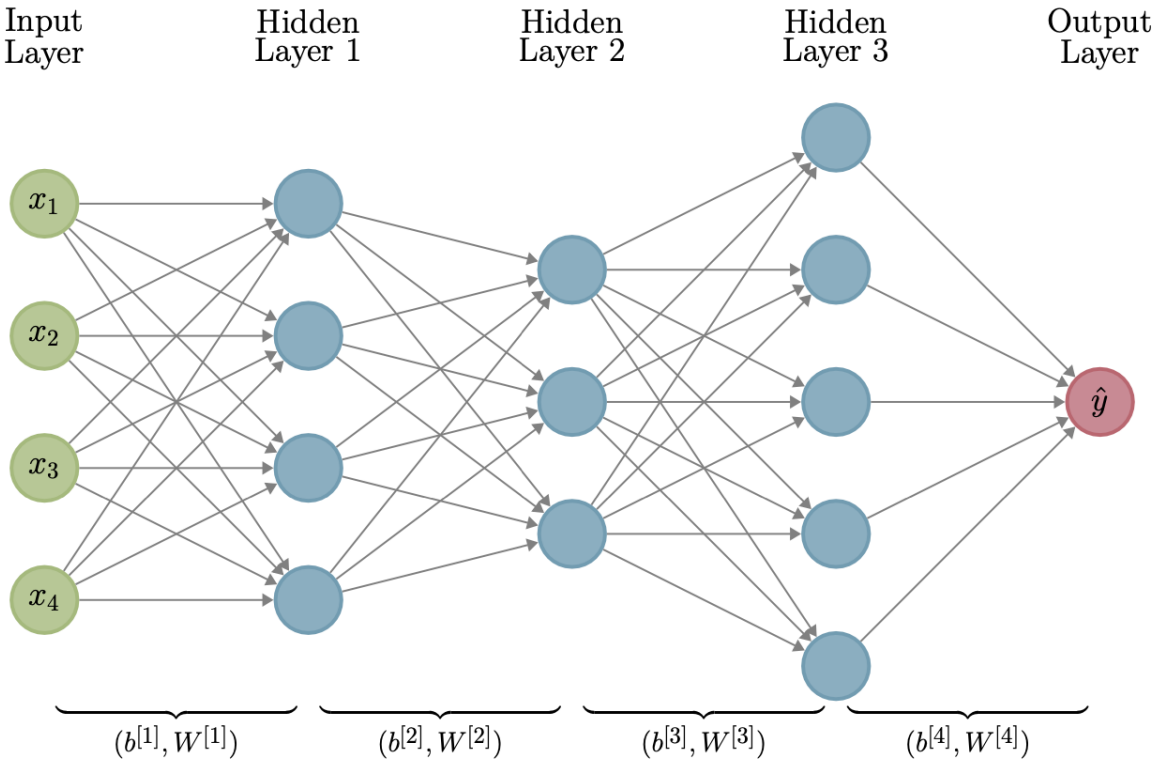


Fully Connected Neural Network or feedforward networks



Composition of function : $\hat{y} = f^{[L]}(f^{[L-1]}(f^{[L-2]}(\ldots(f^{[1]}(x))\ldots)))$

Fully Connected Neural Network or feedforward networks



Composition of function : $\hat{y} = f^{[L]}(f^{[L-1]}(f^{[L-2]}(\ldots(f^{[1]}(x))\ldots)))$

$$\hat{y} = \overbrace{\sigma^{[4]}(W^{[4]}\overbrace{\sigma^{[3]}(W^{[3]}\overbrace{\sigma^{[2]}(W^{[2]}\underbrace{\sigma^{[1]}(W^{[1]}x + b^{[1]})}_{f^{[1]}} + b^{[2]})}^{f^{[2]}} + b^{[3]})}^{f^{[3]}} + b^{[4]})}^{f^{[4]}}$$

Question 3

Does a fully connected neural network with one hidden layer allow interactions between input variables?

Why Single Hidden Layer is Useful?

Universal Approximation Theorem

Consider a continuous function $f^* : \mathcal{K} \to \mathbb{R}^q$ where $\mathcal{K} \subset \mathbb{R}^p$ is a compact set. Then for any any $\sigma(\cdot)$ non-polynomial activation function and any $\varepsilon > 0$ there exists an N_1 and parameters $W^{[1]} \in \mathbb{R}^{N_1 \times p}$, $b^{[1]} \in \mathbb{R}^{N_1}$, and $W^{[2]} \in \mathbb{R}^{q \times N_1}$, such that the function

$$f_\theta(x) = W^{[2]}\sigma(W^{[1]}x + b^{[1]}),$$

satisfies $||f_\theta(x) - f^*(x)|| < \varepsilon$ for all $x \in \mathcal{K}$.

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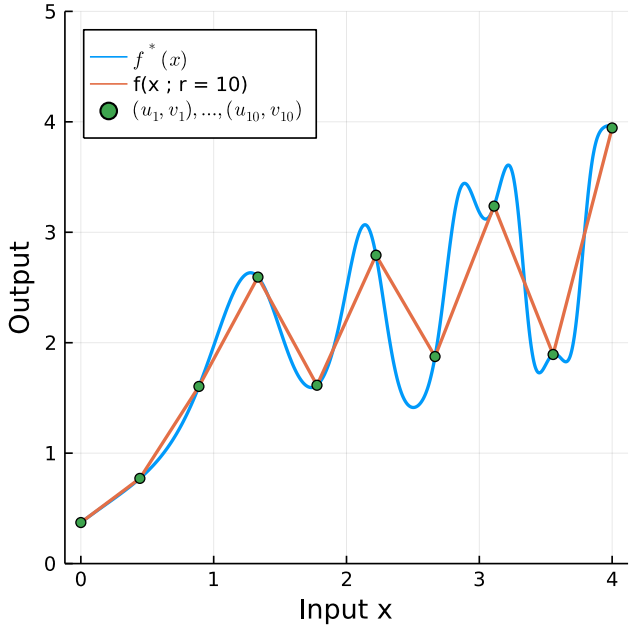
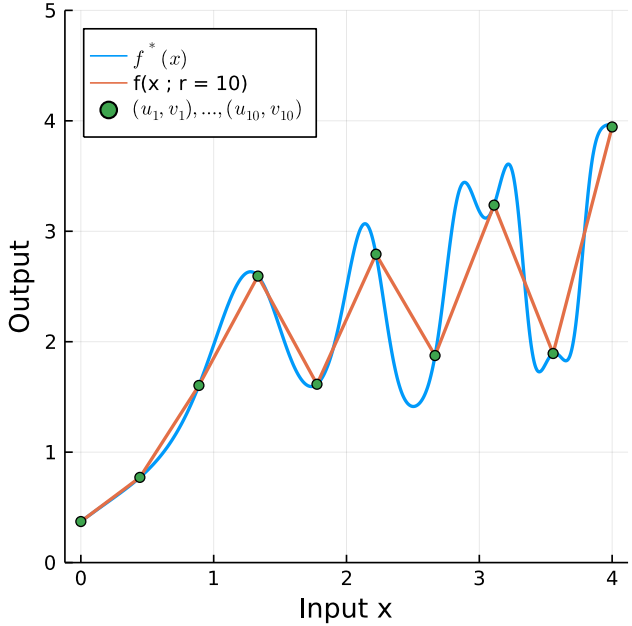
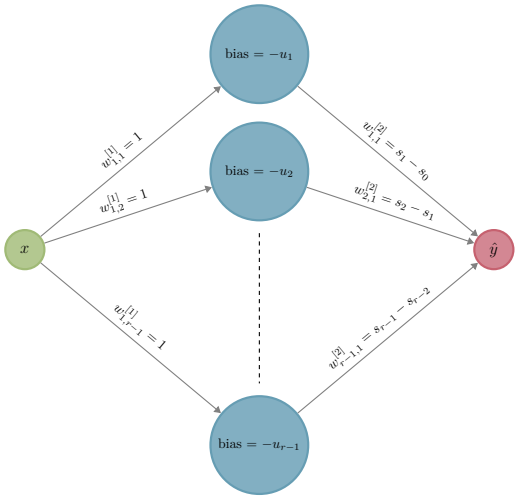
In simple words: *A neural network with just one hidden layer can, in principle, learn to approximate any smooth function, if it has enough hidden neurons.*

Simple Function Approximation

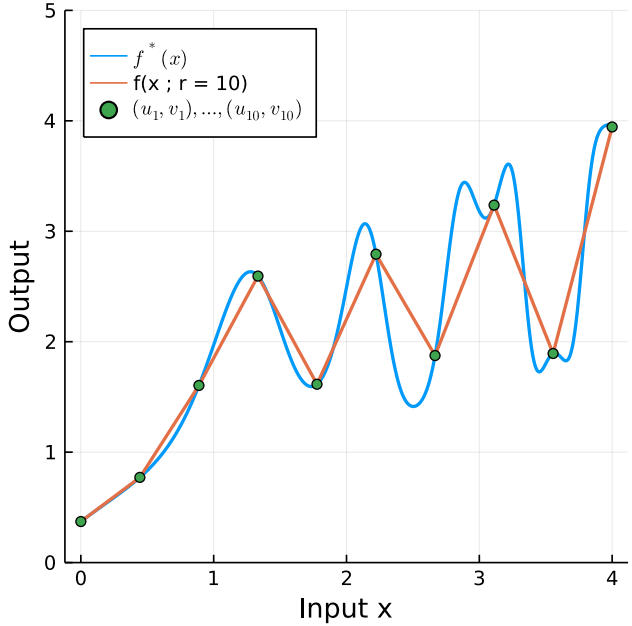


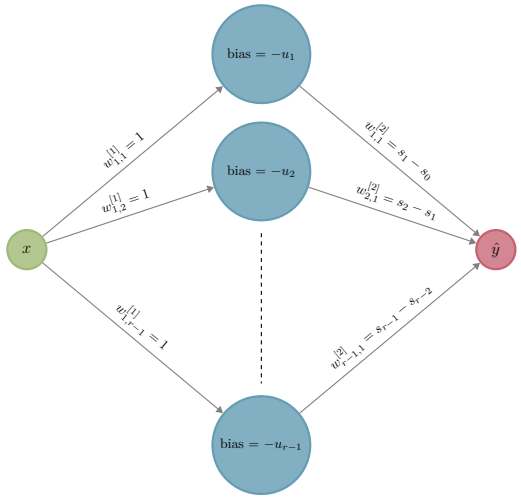
Simple Function Approximation

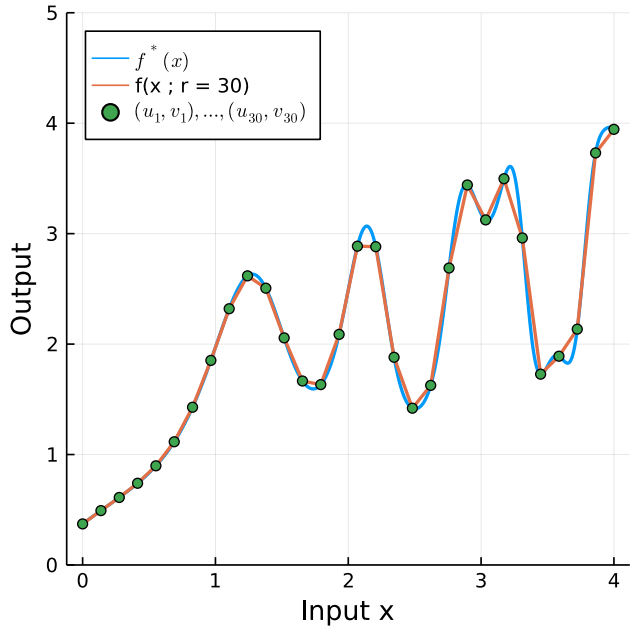




Simple Function Approximation

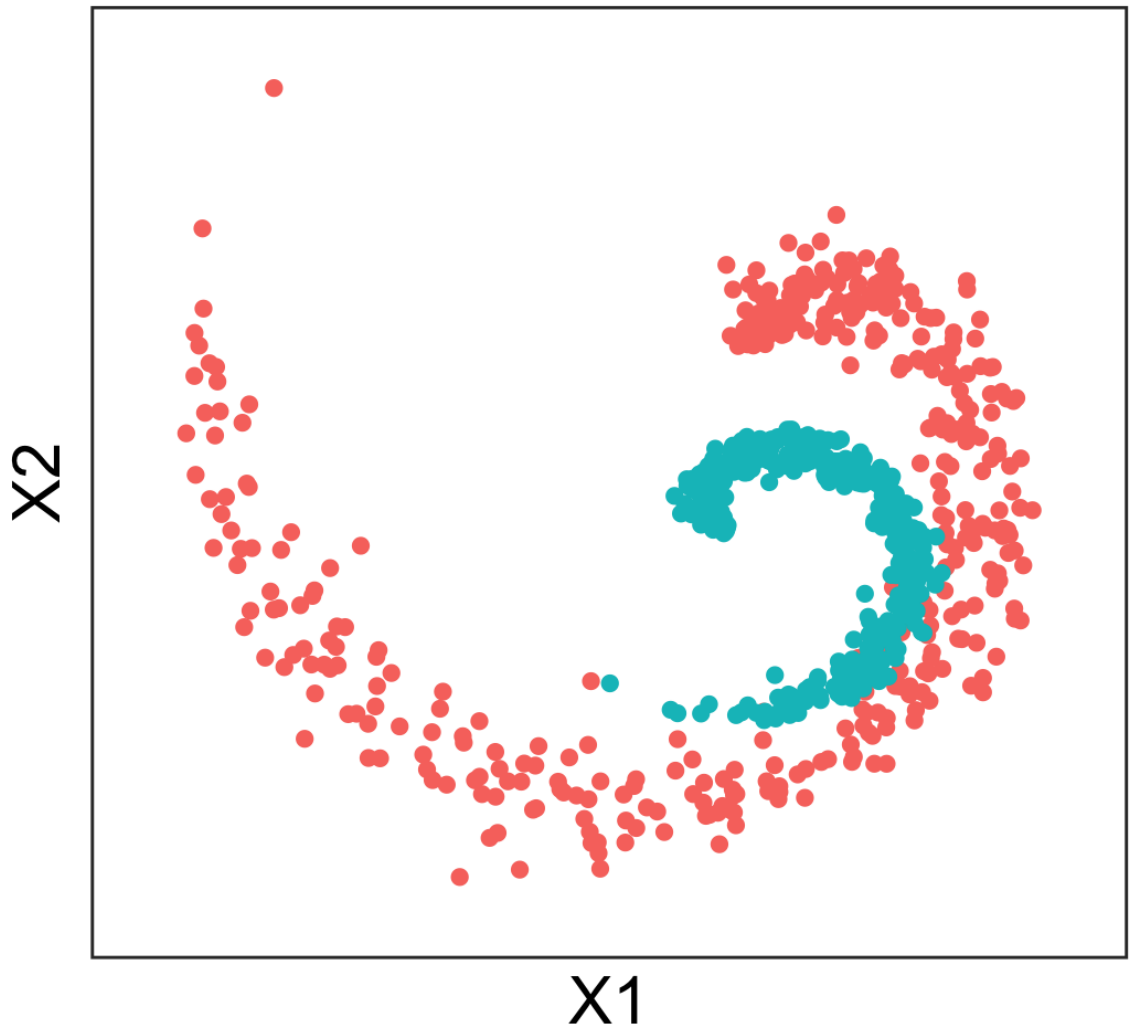






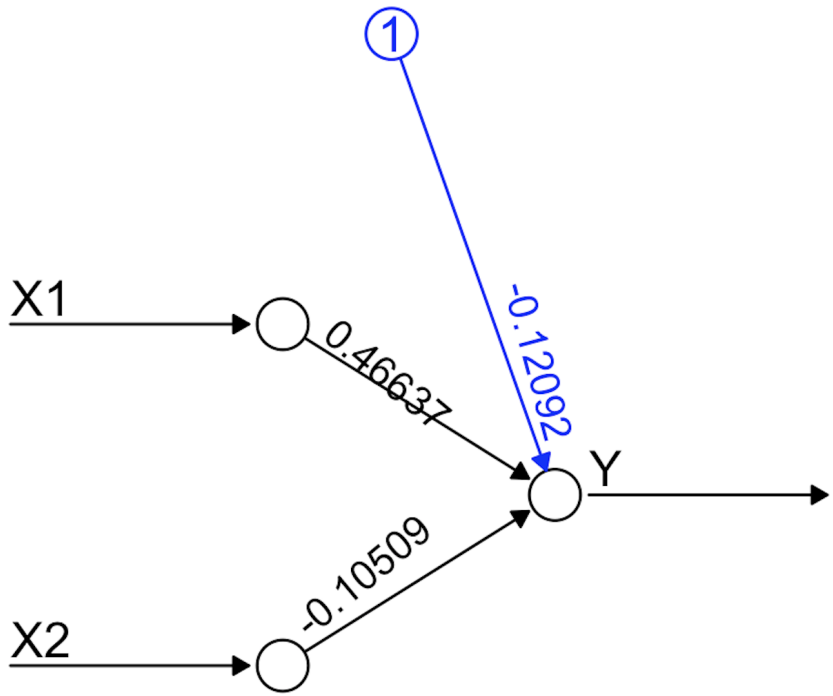
Expressivity:

Binary classification task with two features: x_1 and x_2



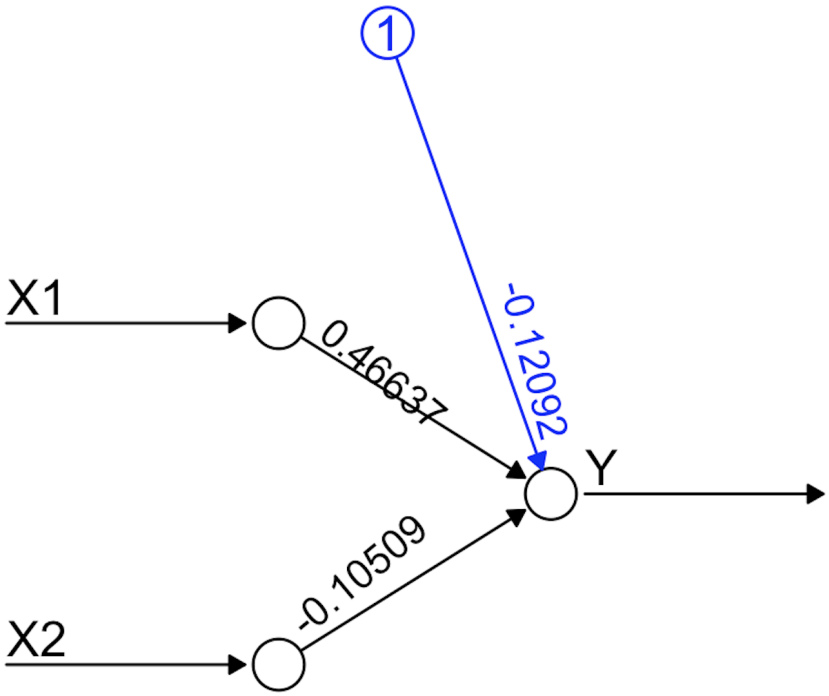
Sigmoid model: no hidden layer

Total parameter: 2 (weights) + 1 (biais) = 3



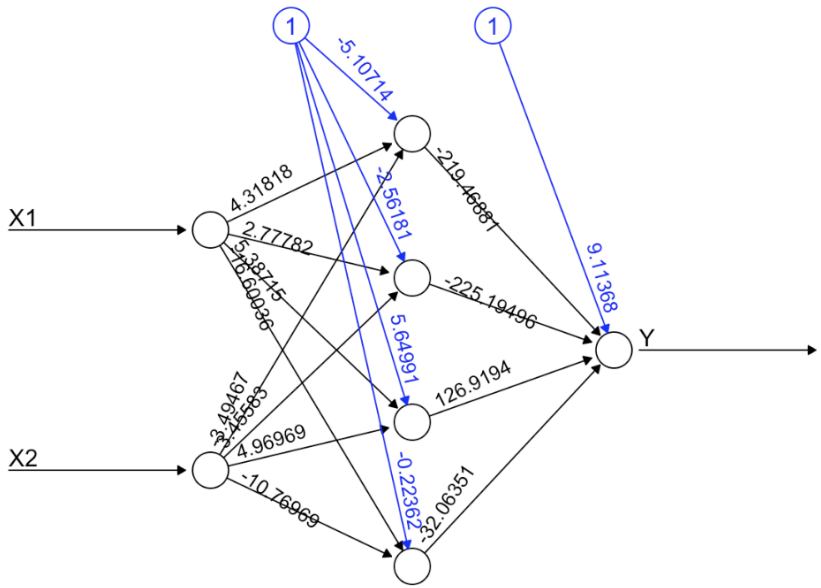
Sigmoid model: no hidden layer

Total parameter: 2 (weights) + 1 (biais) = 3



Expressivity: one hidden layer with 4 neurons

Total parameter: $\overbrace{4 \times 2 + 4}^{\text{hidden Layer 1}} + \overbrace{4 + 1}^{\text{output layer}} = 17$



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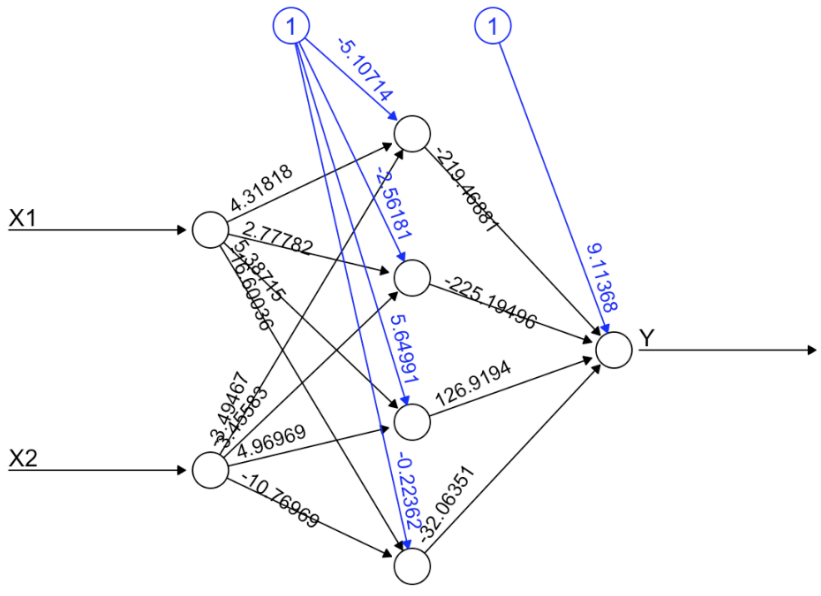
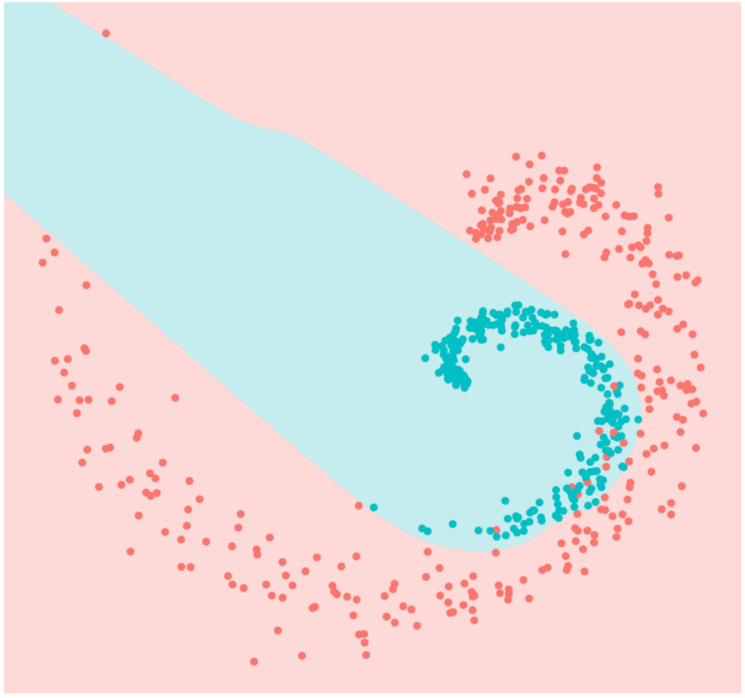
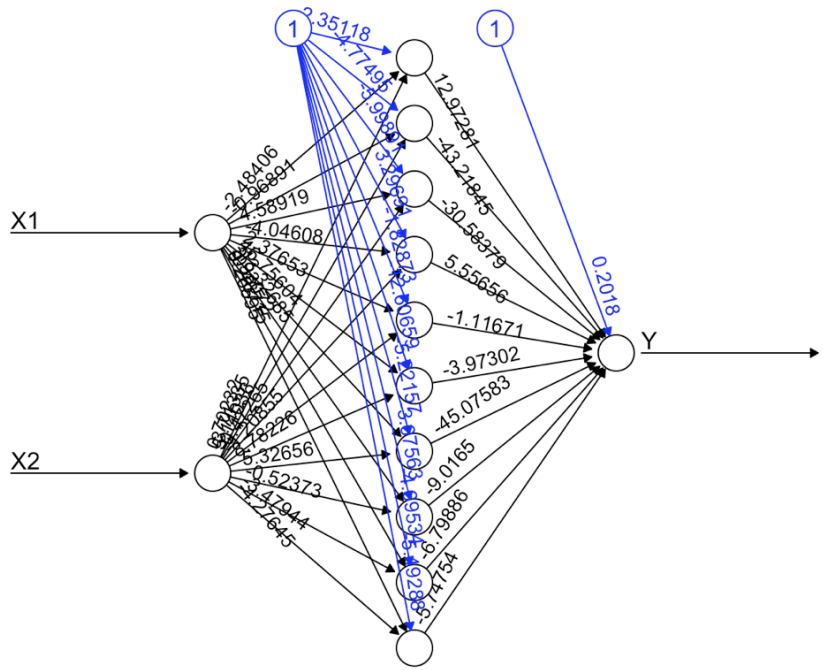




Expressivity: one hidden layer with 10 neurons

Total parameter: $\overbrace{10 \times 2 + 10}^{\text{hidden Layer 1}} + \overbrace{10 + 1}^{\text{output layer}} = 41$



Expressivity: one hidden layer with 10 neurons

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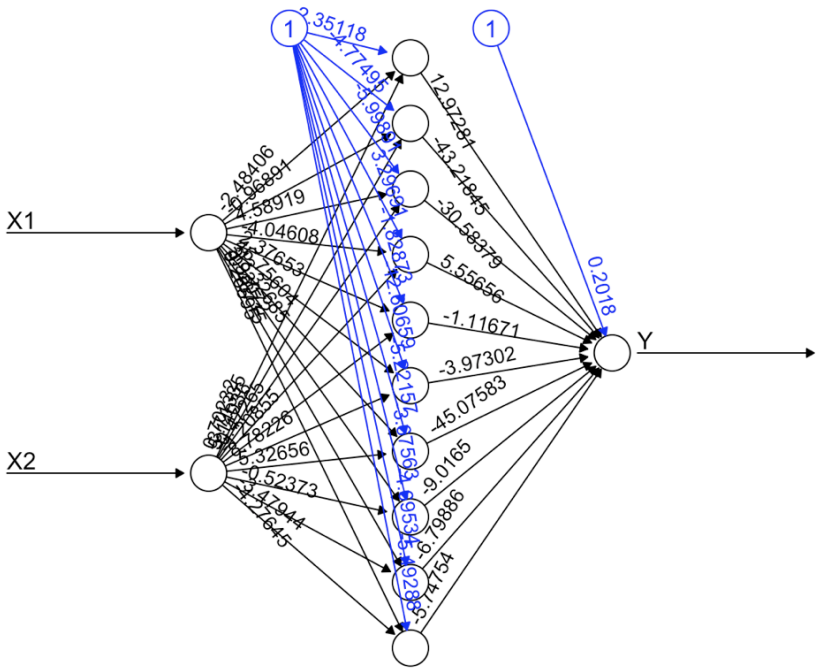
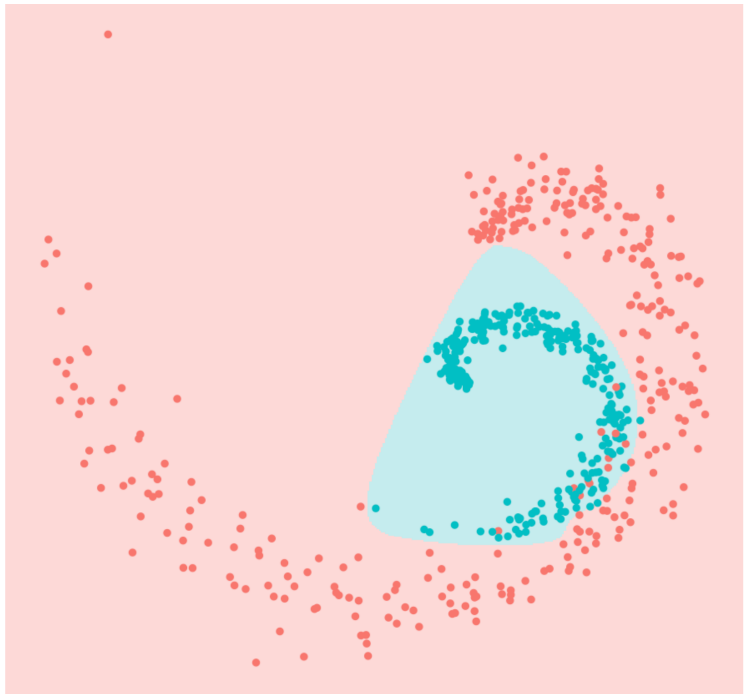
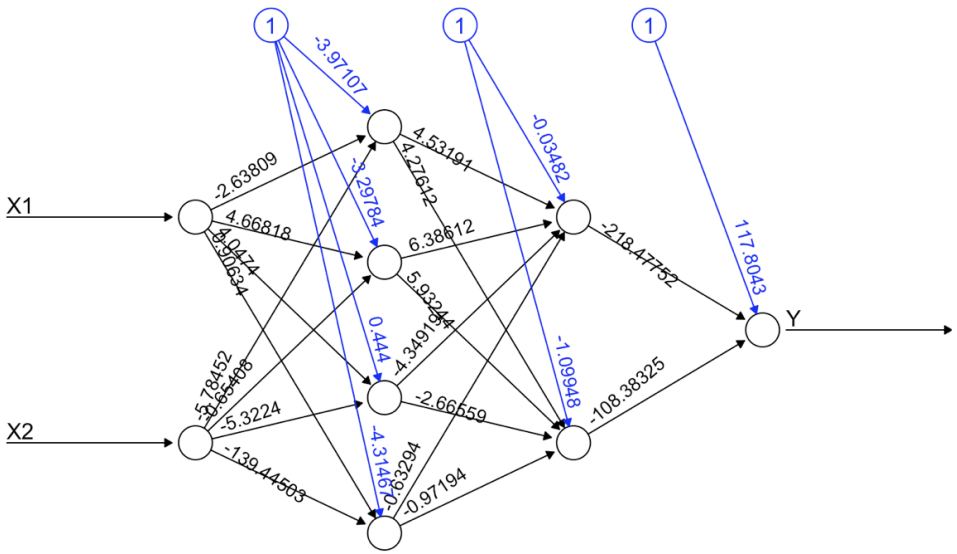




Why not going deep: 2 hidden layers

$$\text{Total parameter: } \overbrace{4\times 2+4}^{\text{hidden Layer 1}} + \overbrace{2\times 4+2}^{\text{hidden Layer 2}} + \overbrace{2+1}^{\text{output layer}} = 25$$



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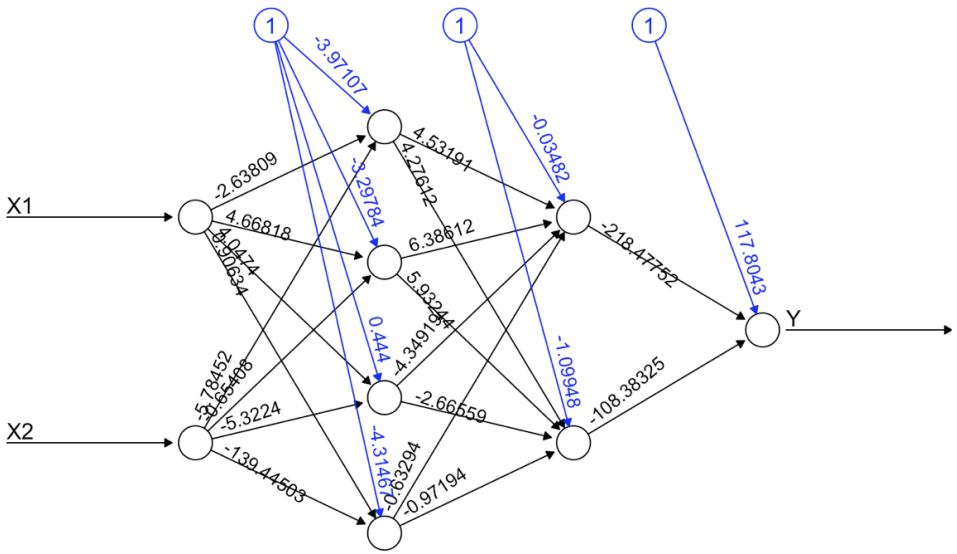
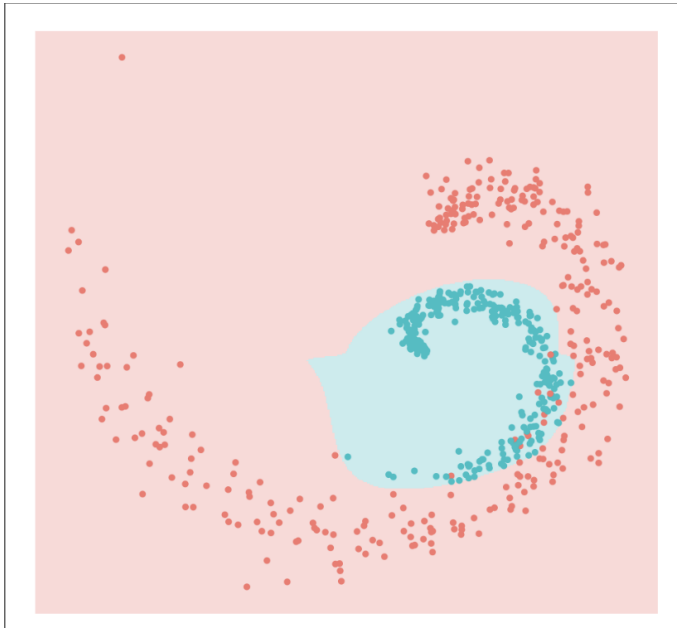
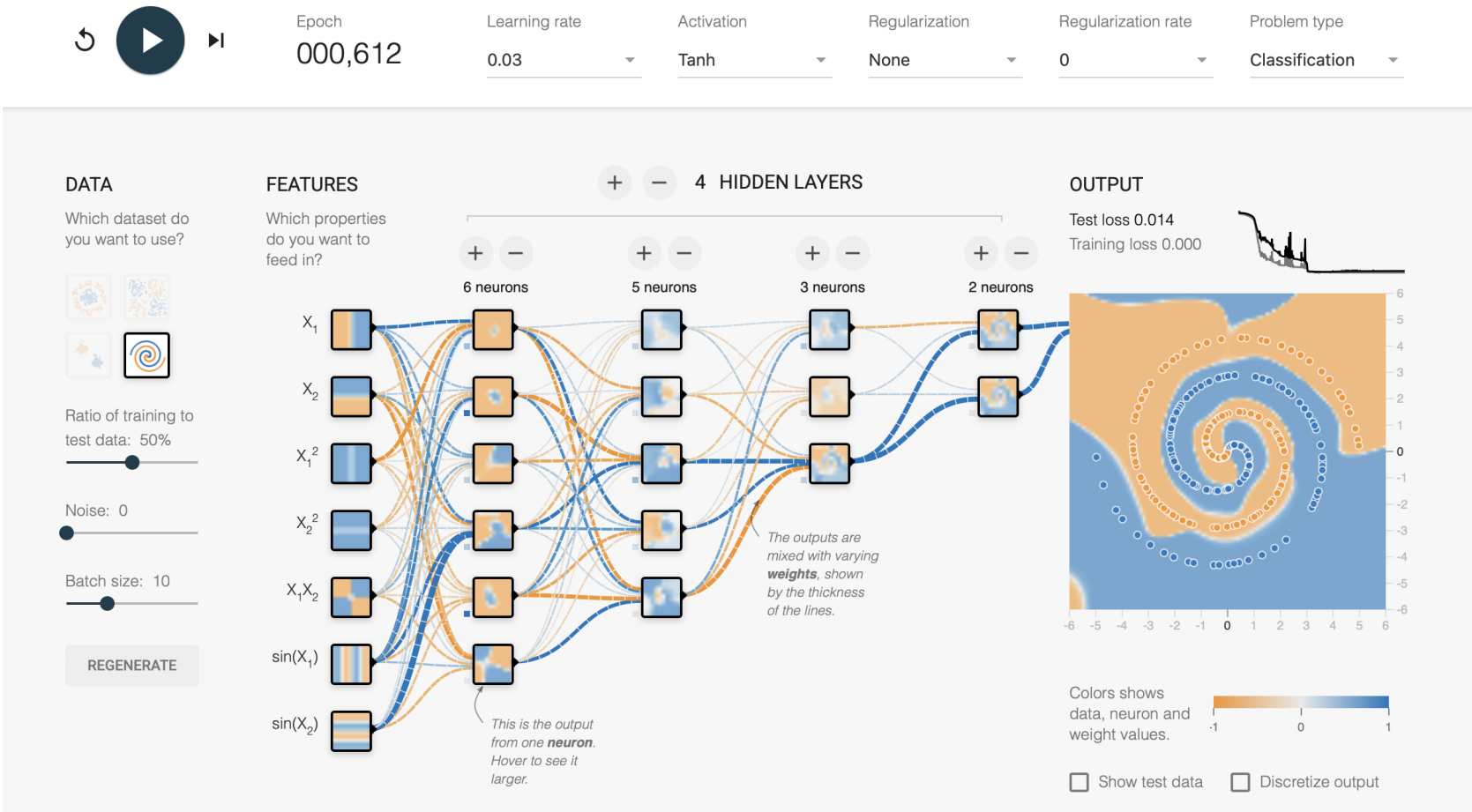




Short demo

https://playground.tensorflow.org



Why Go Deep?

- A single hidden layer can approximate any function
- But it may require a huge number of neurons

Why Go Deep?

- A single hidden layer can approximate any function
- But it may require a huge number of neurons
- Deep networks build functions **hierarchically**

$$\widehat{y} = f^{[L]}(f^{[L-1]}(f^{[L-2]}(\ldots(f^{[1]}(x))\ldots)))$$

- Each layer learns more abstract features

Shallow vs Deep Representations

Shallow network

- Many neurons
- Complex decision boundary learned at once

Deep network

- Fewer neurons per layer
- Decision boundary built step by step

Depth allows feature reuse and composition.

Expressivity of Deep Networks

Key Result

There exist functions that:

- require exponentially many neurons with one hidden layer
- can be represented with polynomial size deep networks

- Telgarsky (2016): *Benefits of depth in neural networks*
- Eldan & Shamir (2016): *The power of depth for feedforward networks*

Question 4

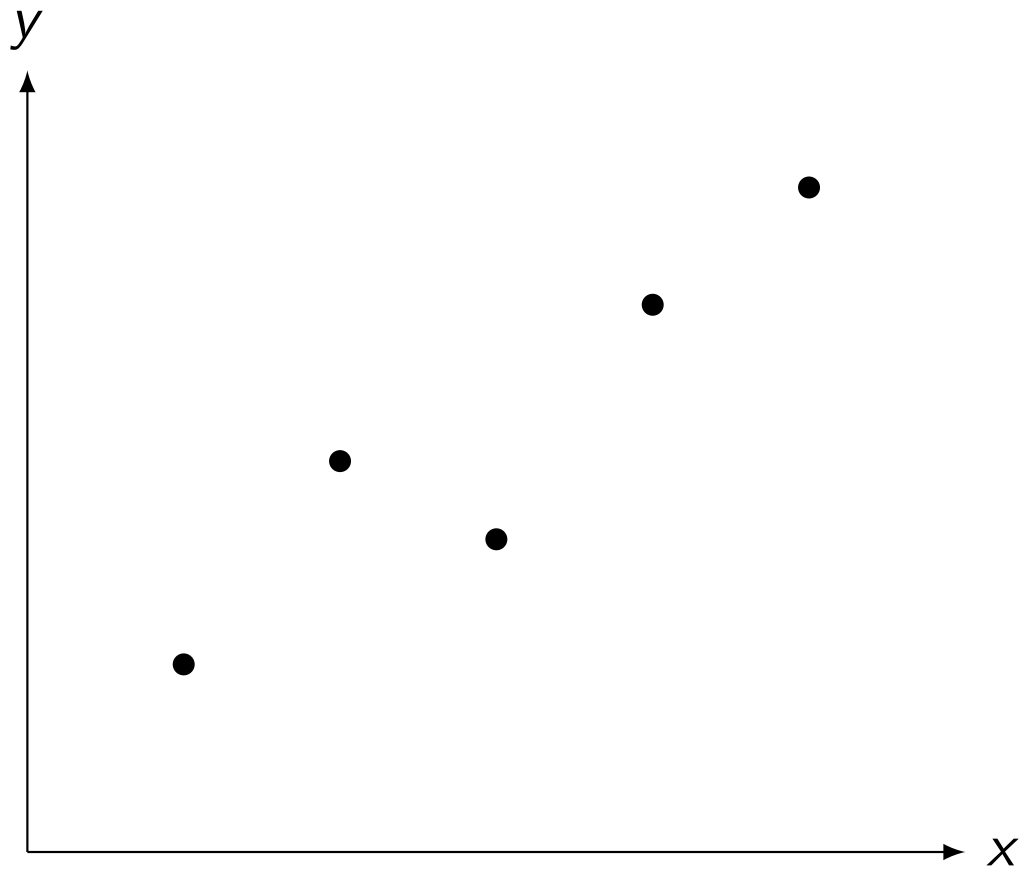
Does the fact that a neural network can approximate very complex functions guarantee that we can easily train it?

From Expressivity to Learning

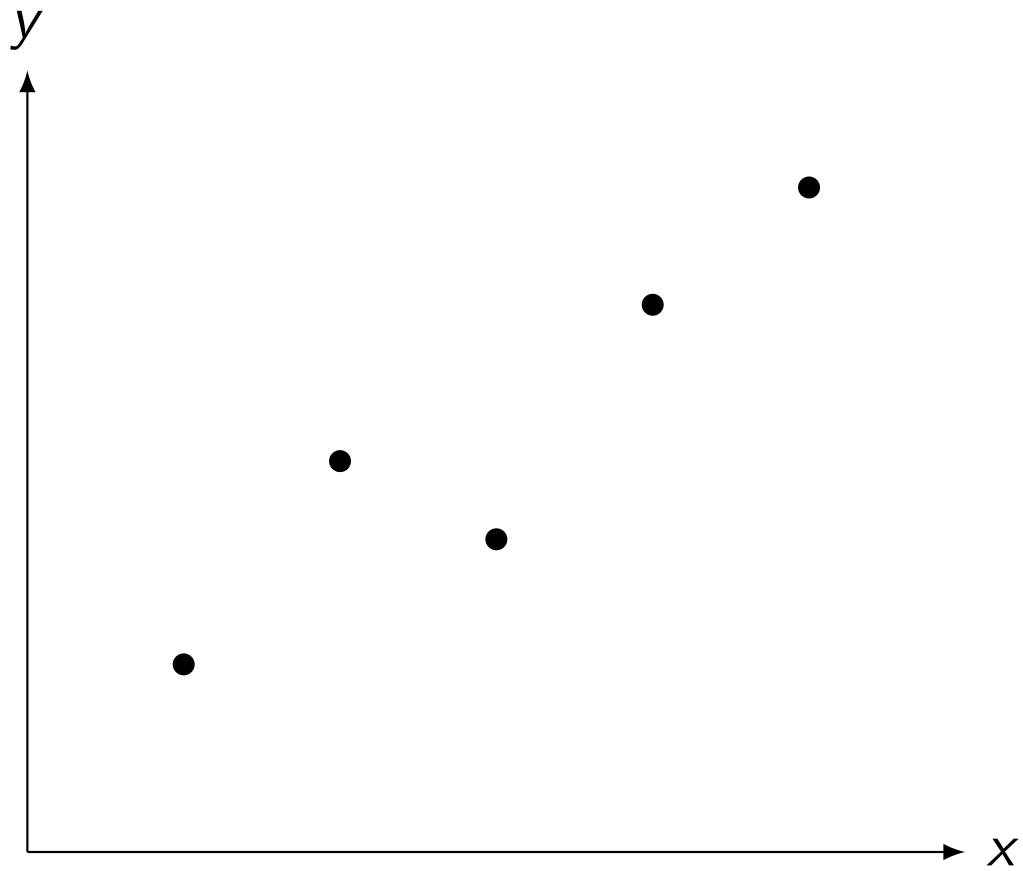
- Deep networks can represent very complex functions
- But representation alone is not enough
- We must **learn** the parameters from data

Learning = minimizing a loss (i.e. error) between predictions ($\widehat{y}$ and observed outcome y)

One-parameter model and no bias ($b = 0$)



One-parameter model and no bias ($b = 0$)

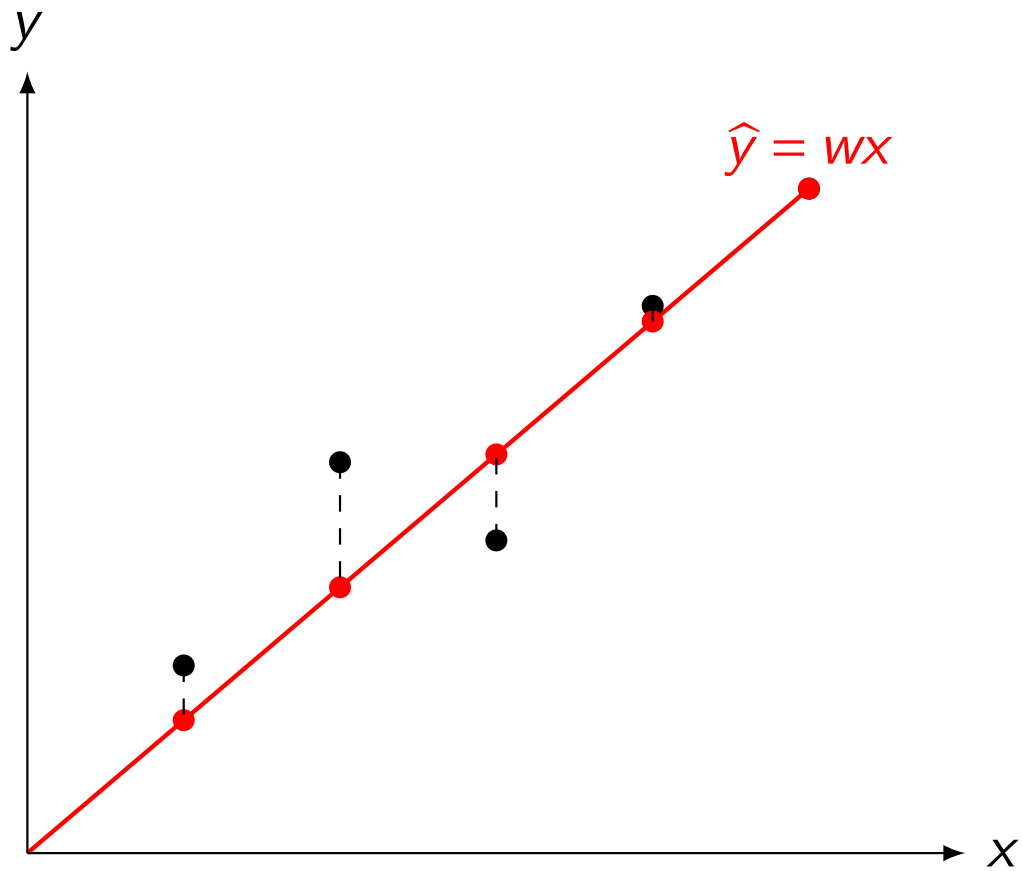


Simple linear model viewed as artificial neuron with a linear activation function ($\sigma(u) = u$).

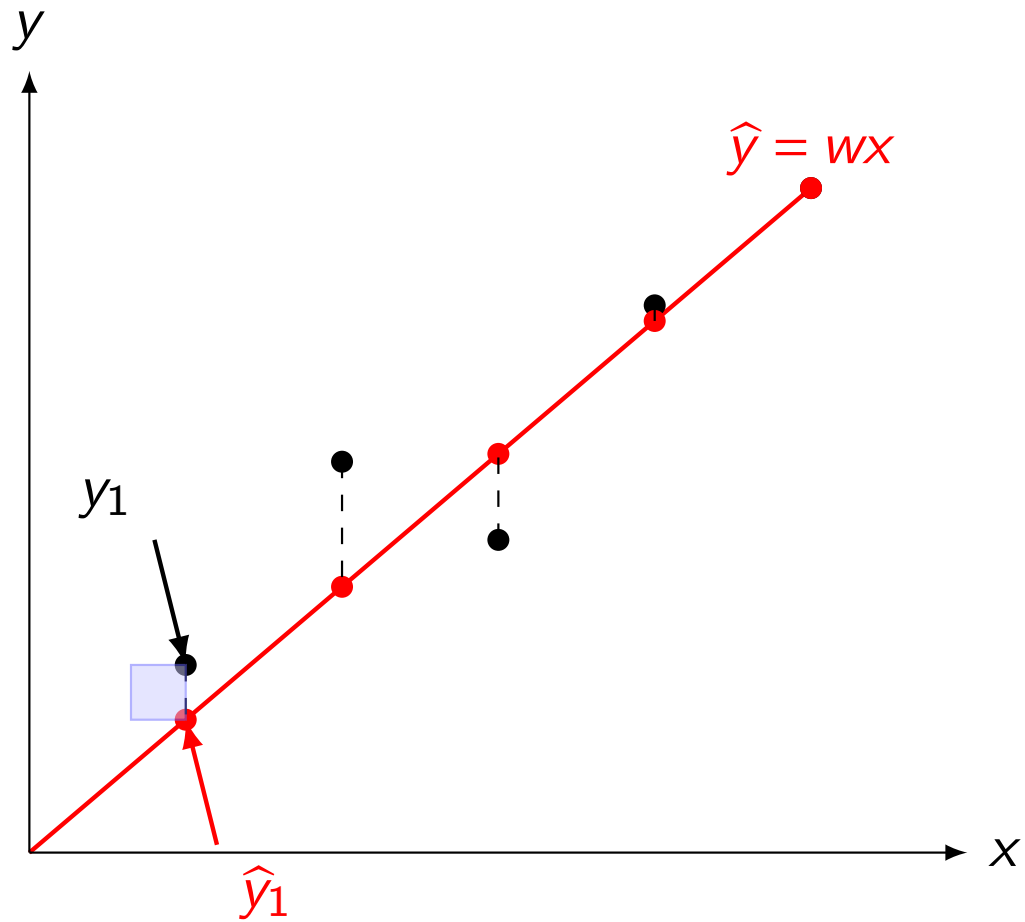
$$y = w \times x, \qquad \text{which best value for } w?$$

.

Find the best lines, i.e choose a suitable value for w

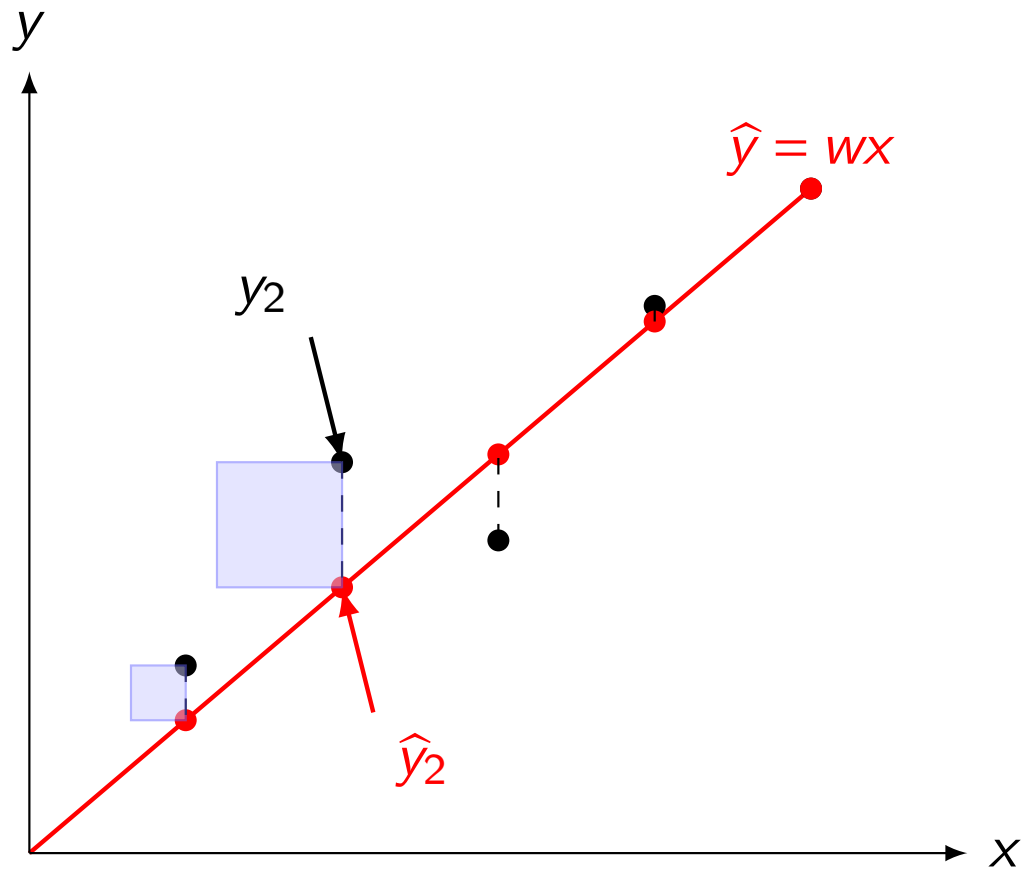


Find the best lines, i.e choose a suitable value for w



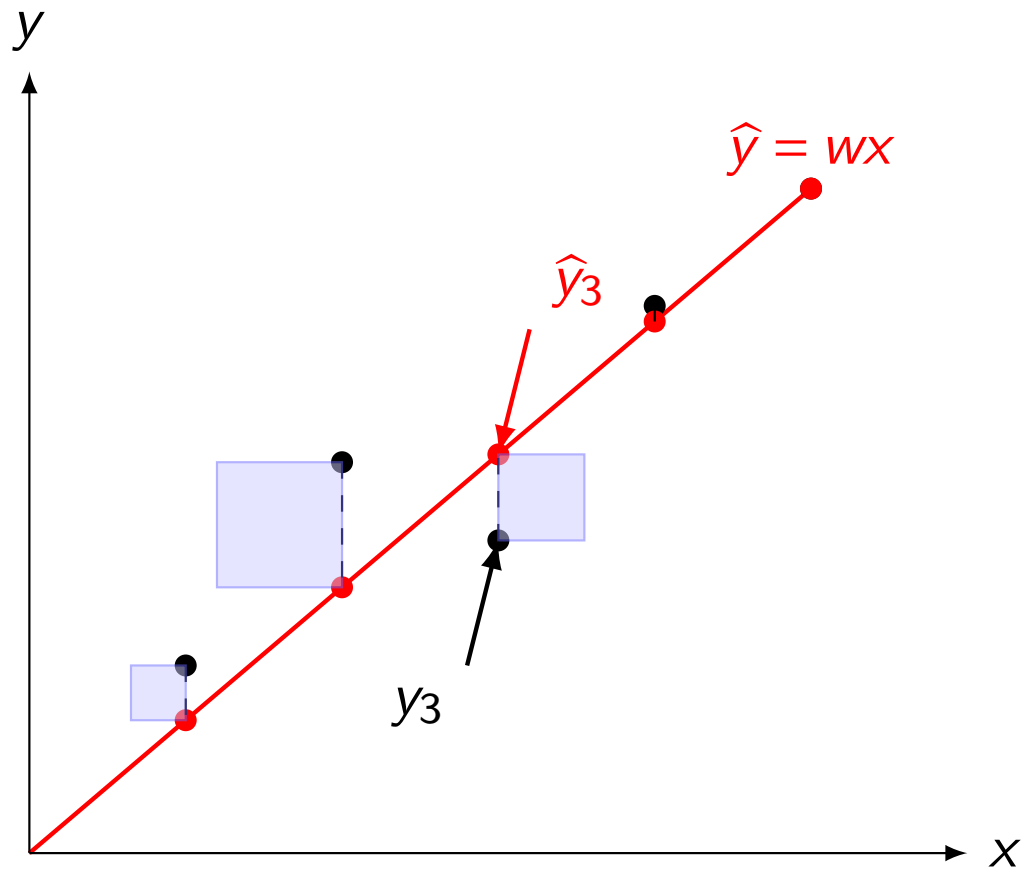
$$C_1(w) = (y_1 - \widehat{y}_1)^2$$

Find the best lines, i.e choose a suitable value for w



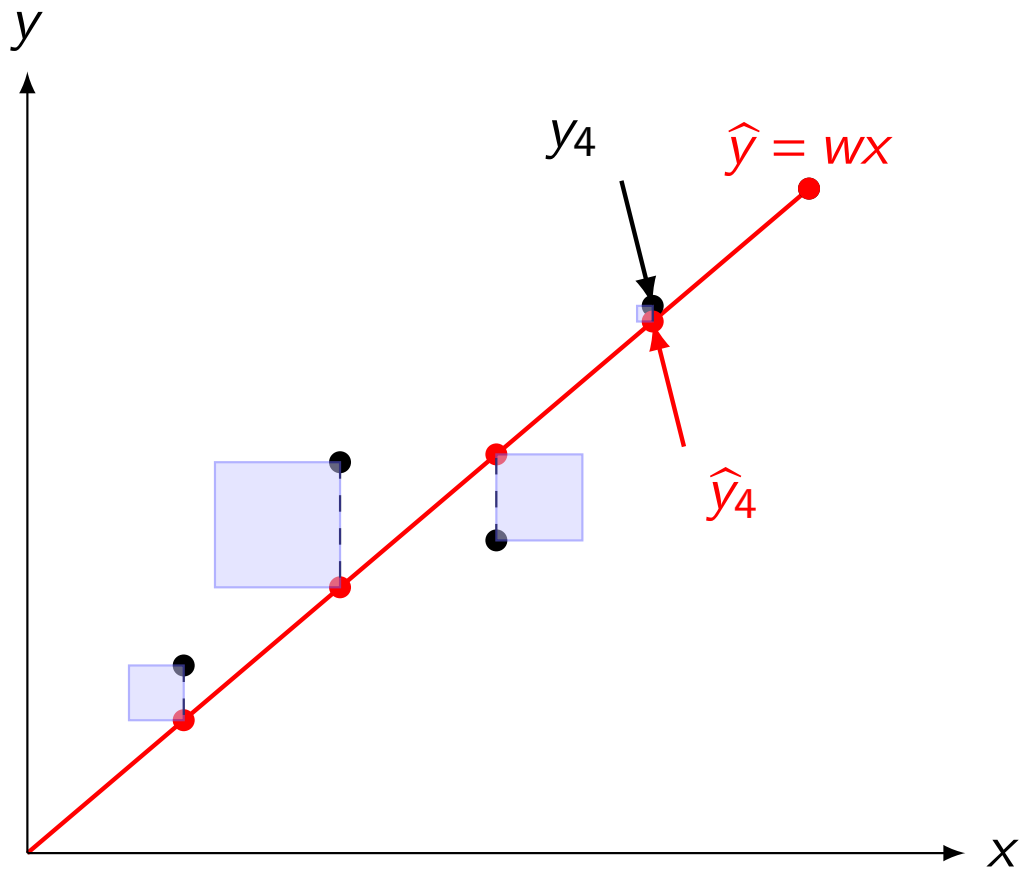
$$C_2(w) = (y_2 - \widehat{y}_2)^2$$

Find the best lines, i.e choose a suitable value for w



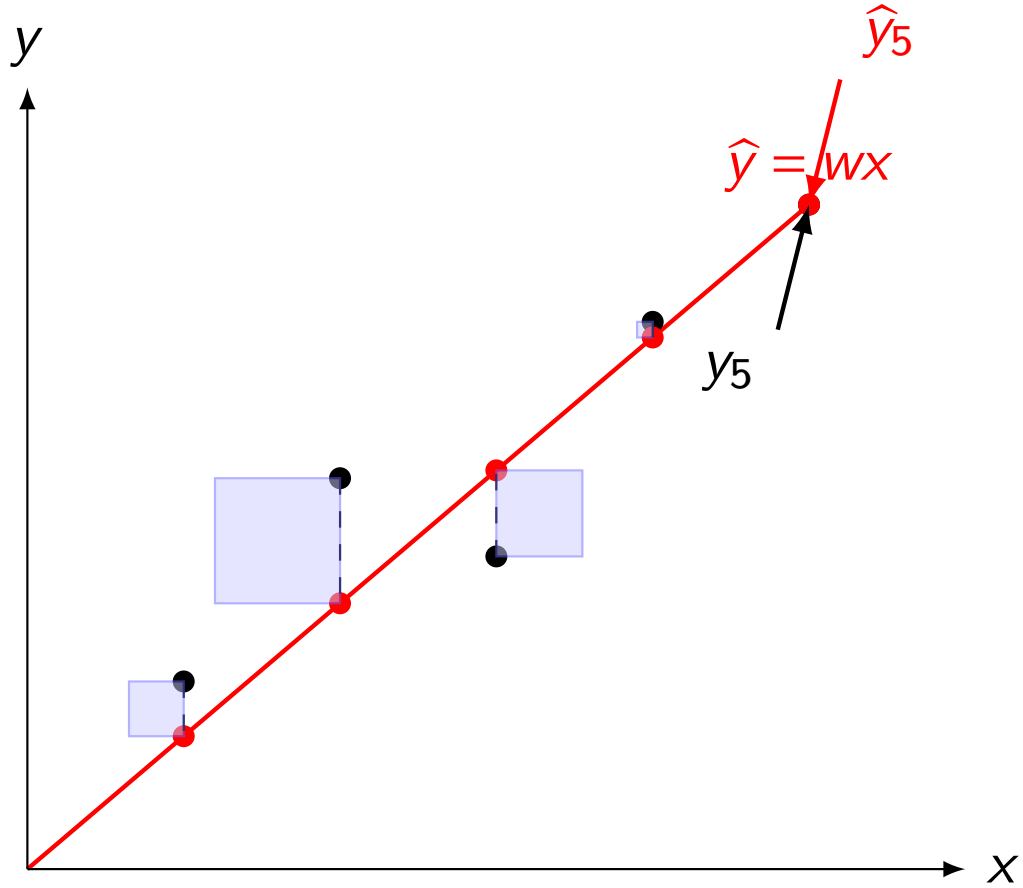
$$C_3(w) = (y_3 - \widehat{y}_3)^2$$

Find the best lines, i.e choose a suitable value for w



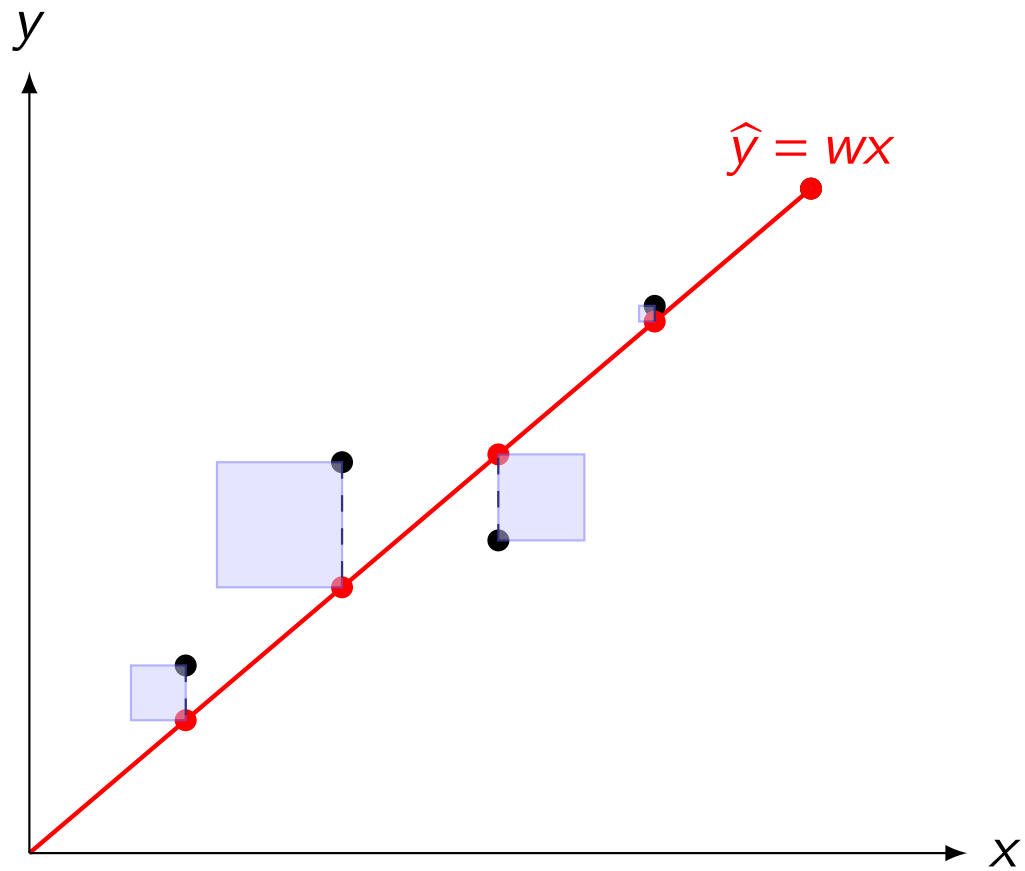
$$C_4(w) = (y_4 - \widehat{y}_4)^2$$

Find the best lines, i.e choose a suitable value for w



$$C_5(w) = (y_5 - \widehat{y}_5)^2$$

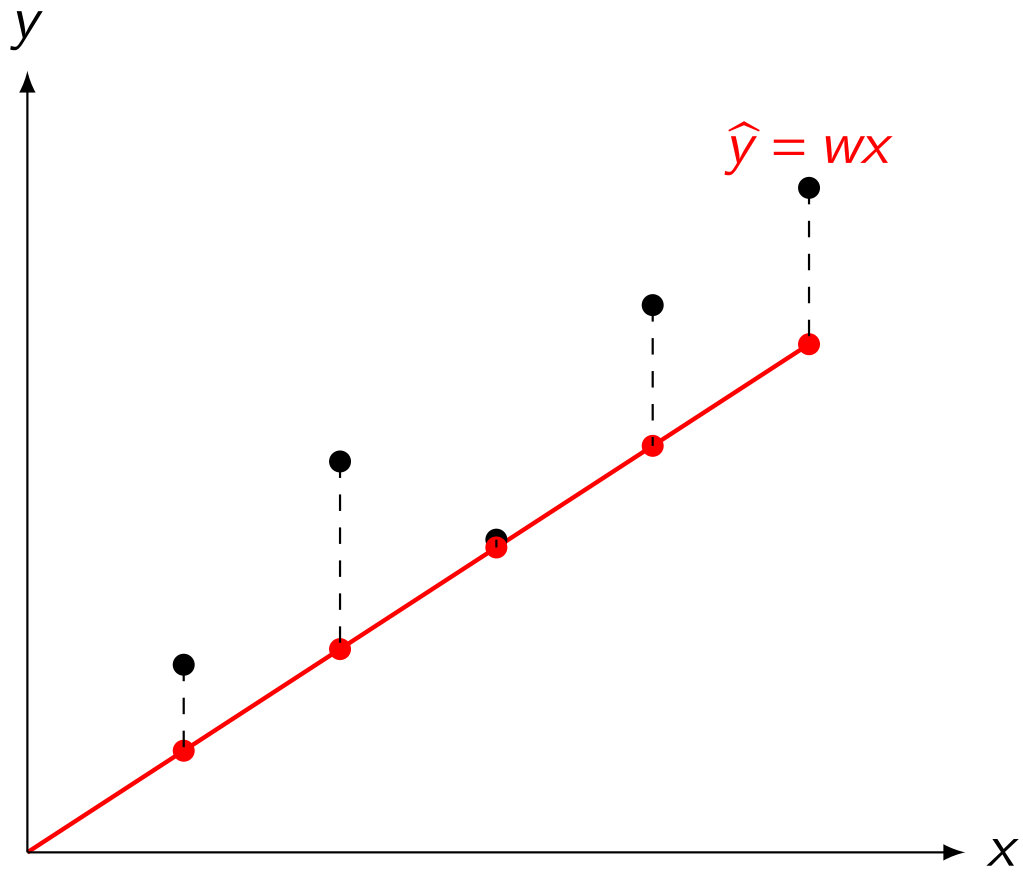
Find the best lines, i.e choose a suitable value for w



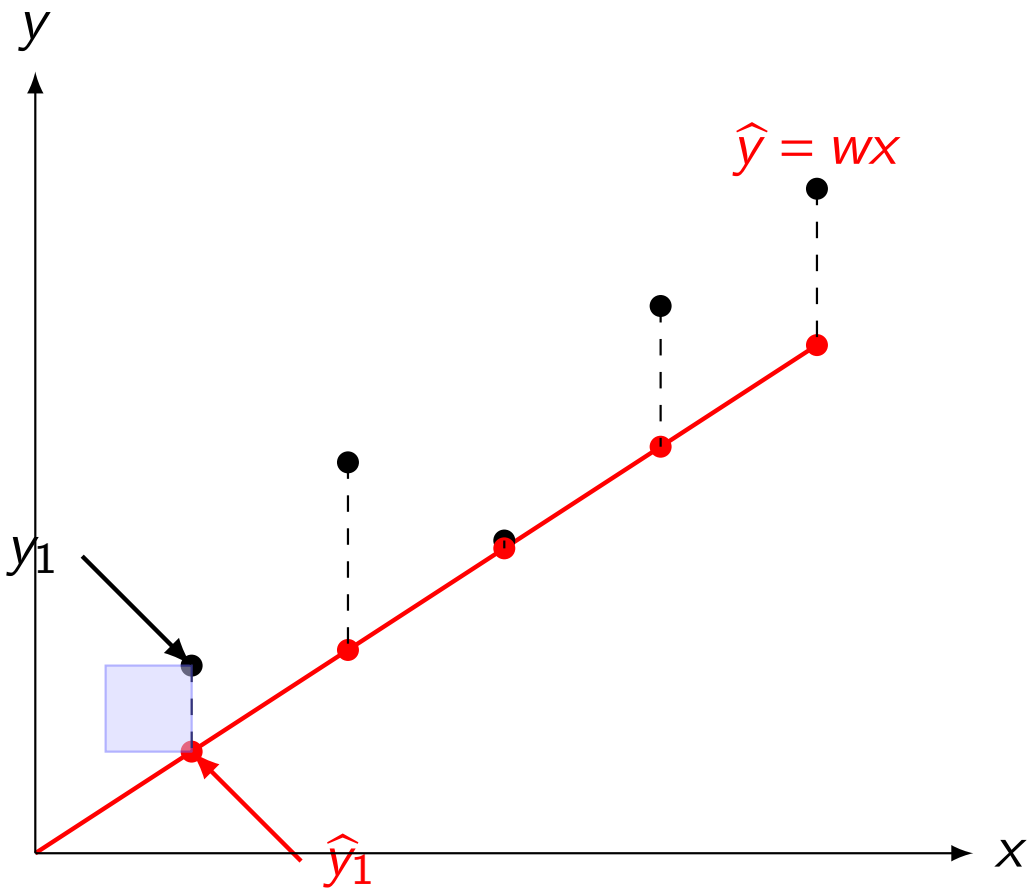
Here $w = 0.85$ and total loss is:

$$C(w) = \sum_{i=1}^{5} C_i(w) = 5.4$$

Another try with $w = 0.65$

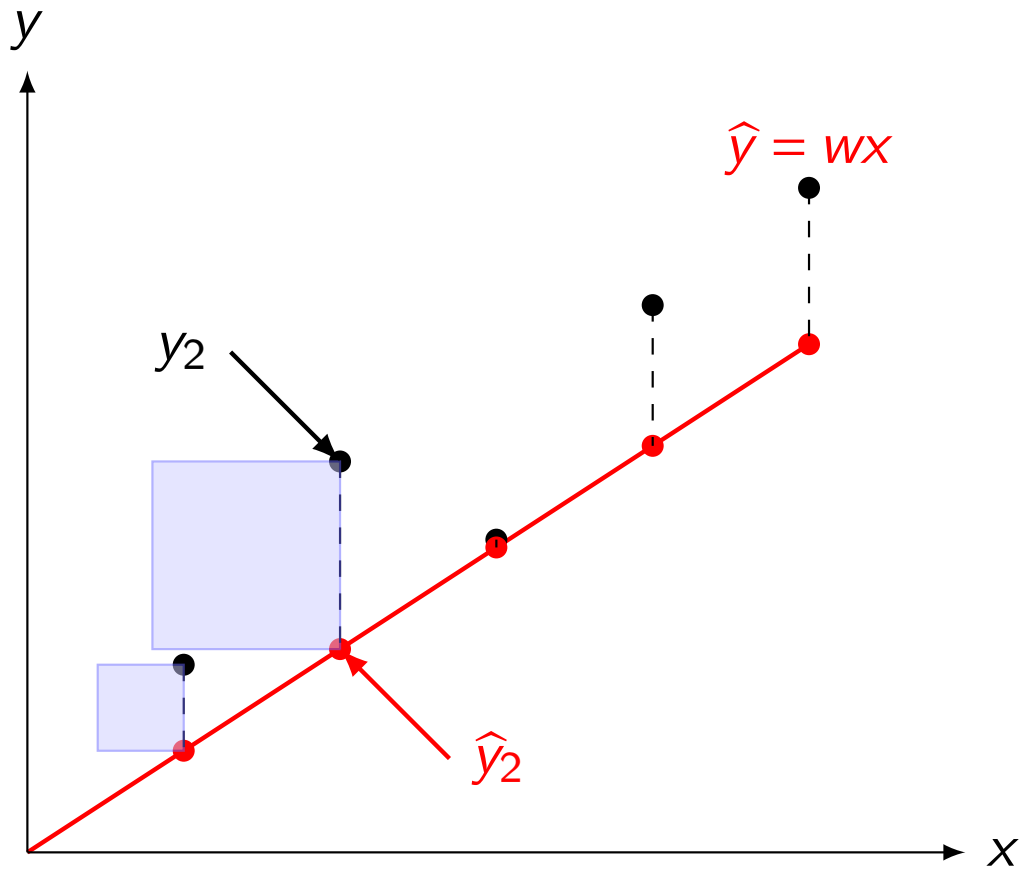


Another try with $w = 0.65$



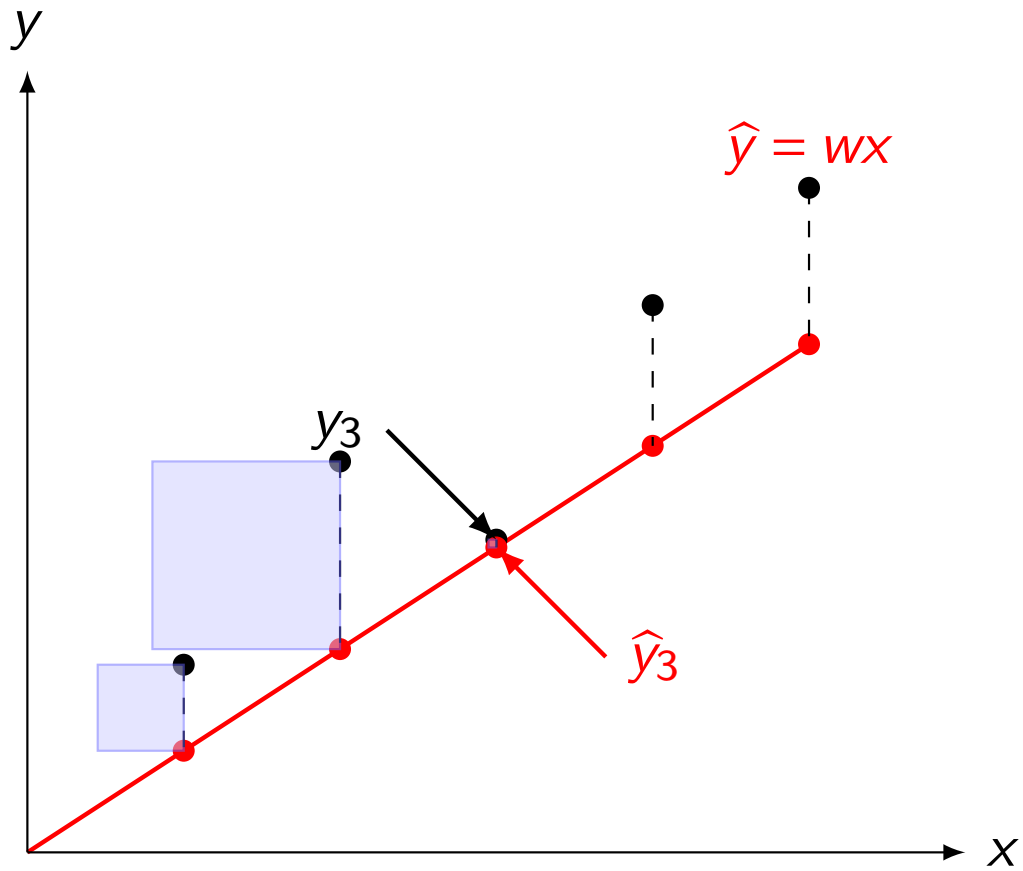
$$C_1(w) = (y_1 - \widehat{y}_1)^2$$

Another try with $w = 0.65$



$$C_2(w) = (y_2 - \widehat{y}_2)^2$$

Another try with $w = 0.65$



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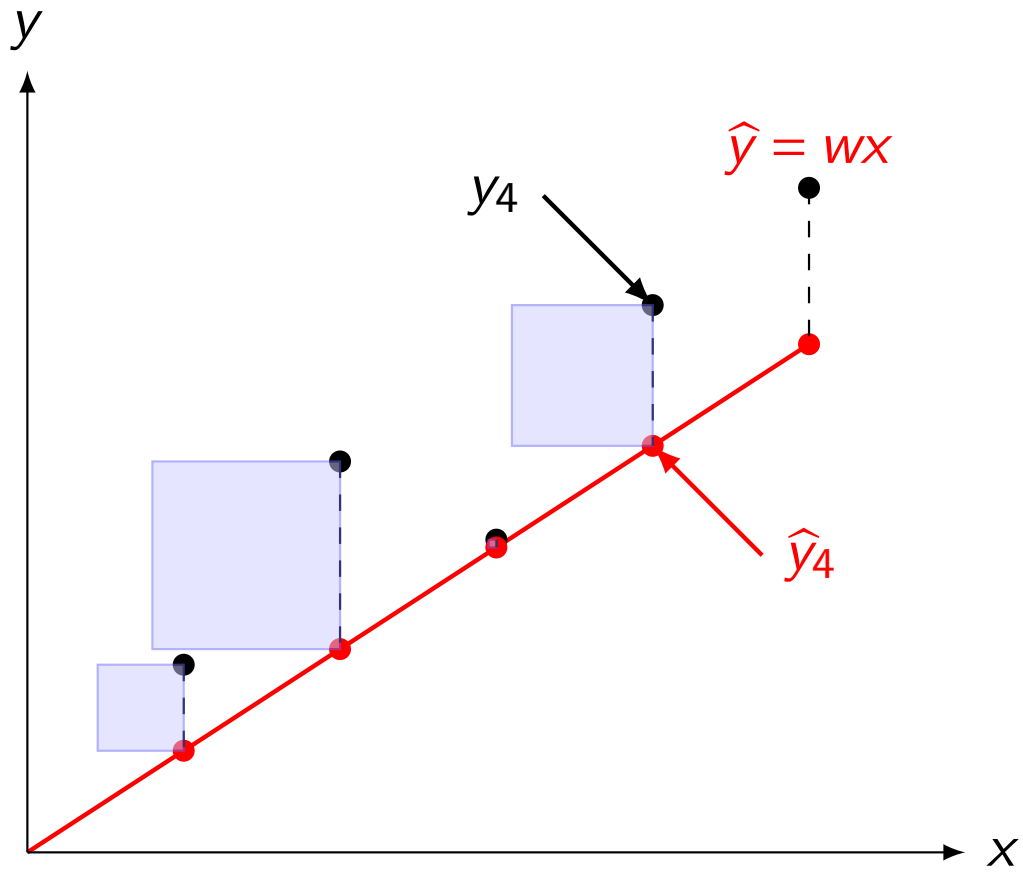
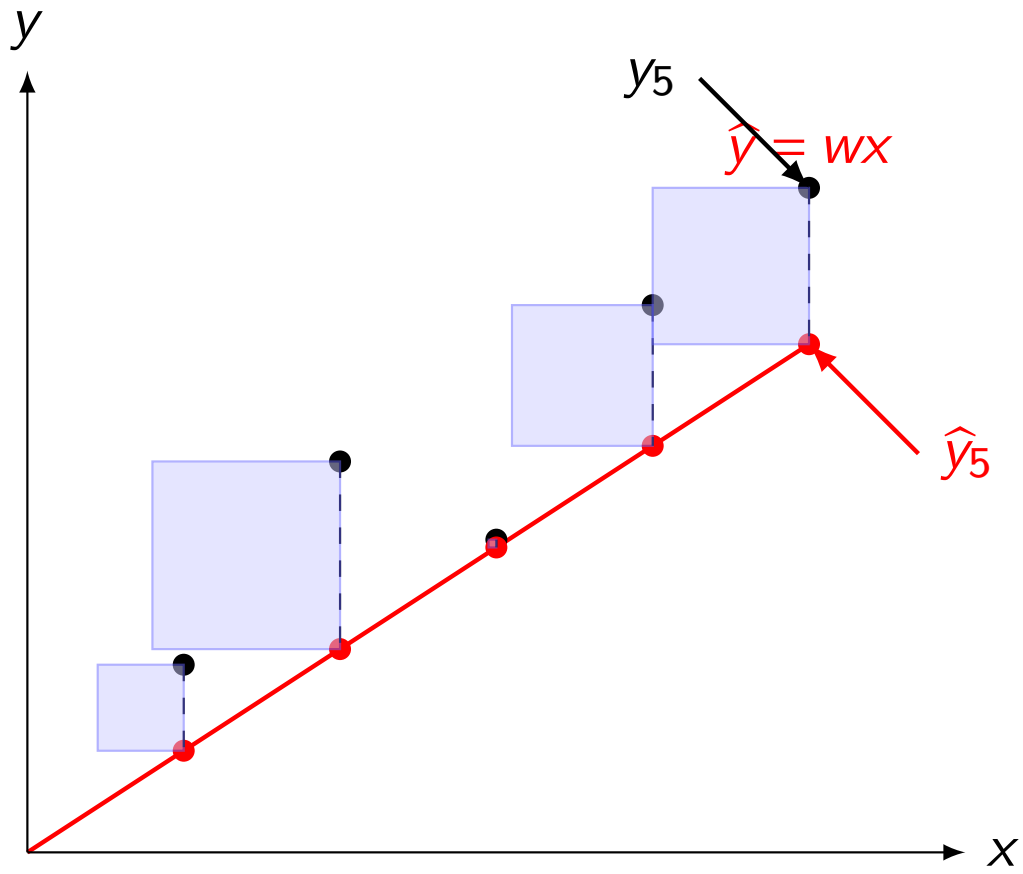


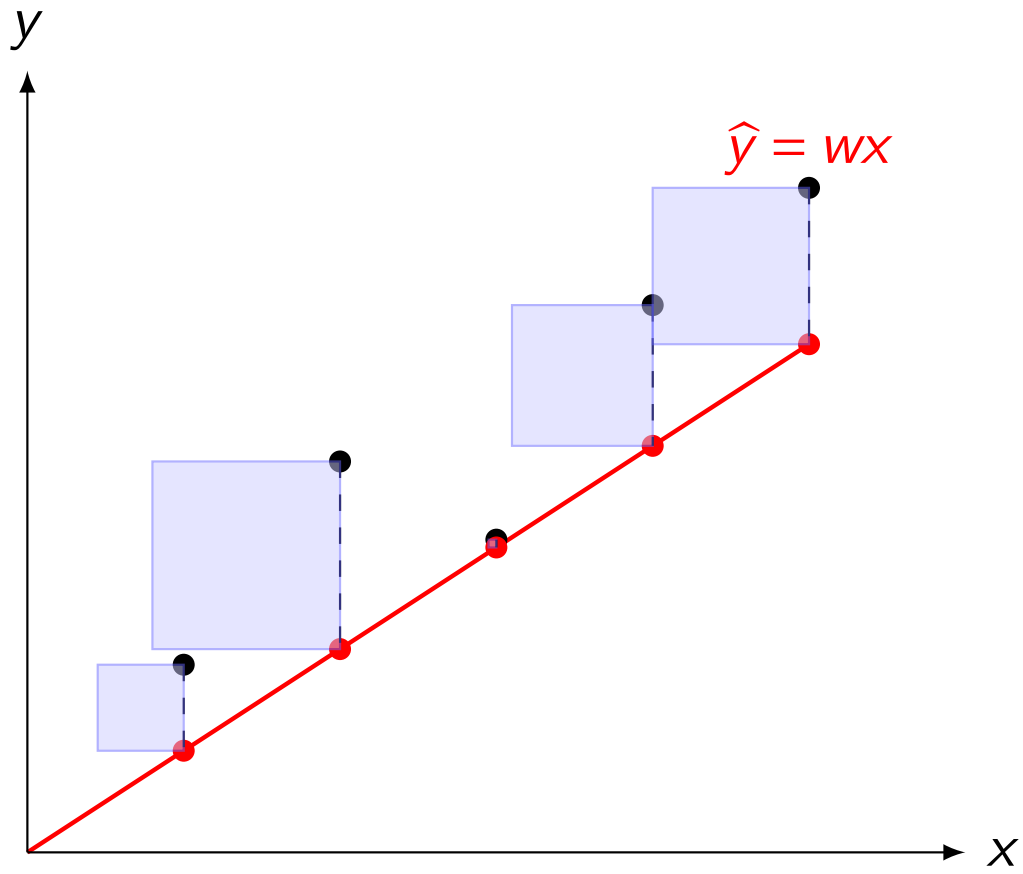
$$C_4(w) = (y_4 - \widehat{y}_4)^2$$

Another try with $w = 0.65$



$$C_5(w) = (y_5 - \widehat{y}_5)^2$$

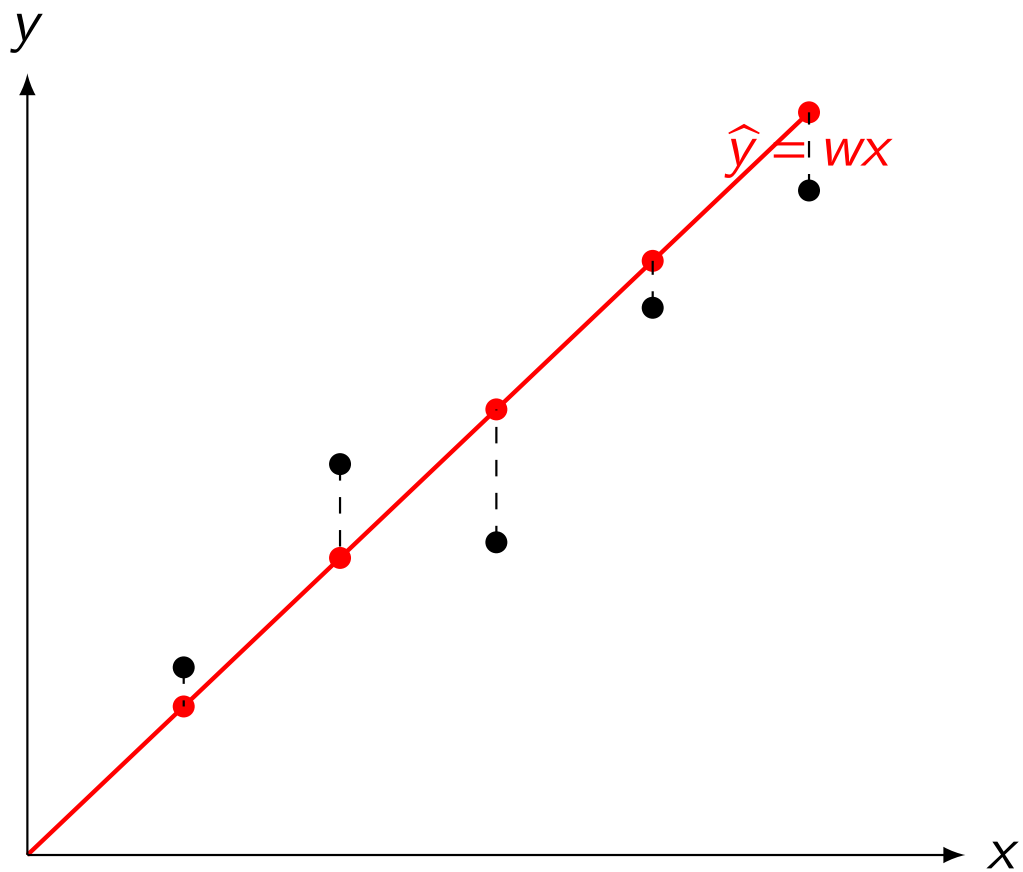
Another try with $w = 0.65$



$$C(w) = \sum_{i=1}^{5} C_i(w) = 12.3$$

Another try with $w = 0.95$

Here a linear activation function $(\sigma(u) = u)$



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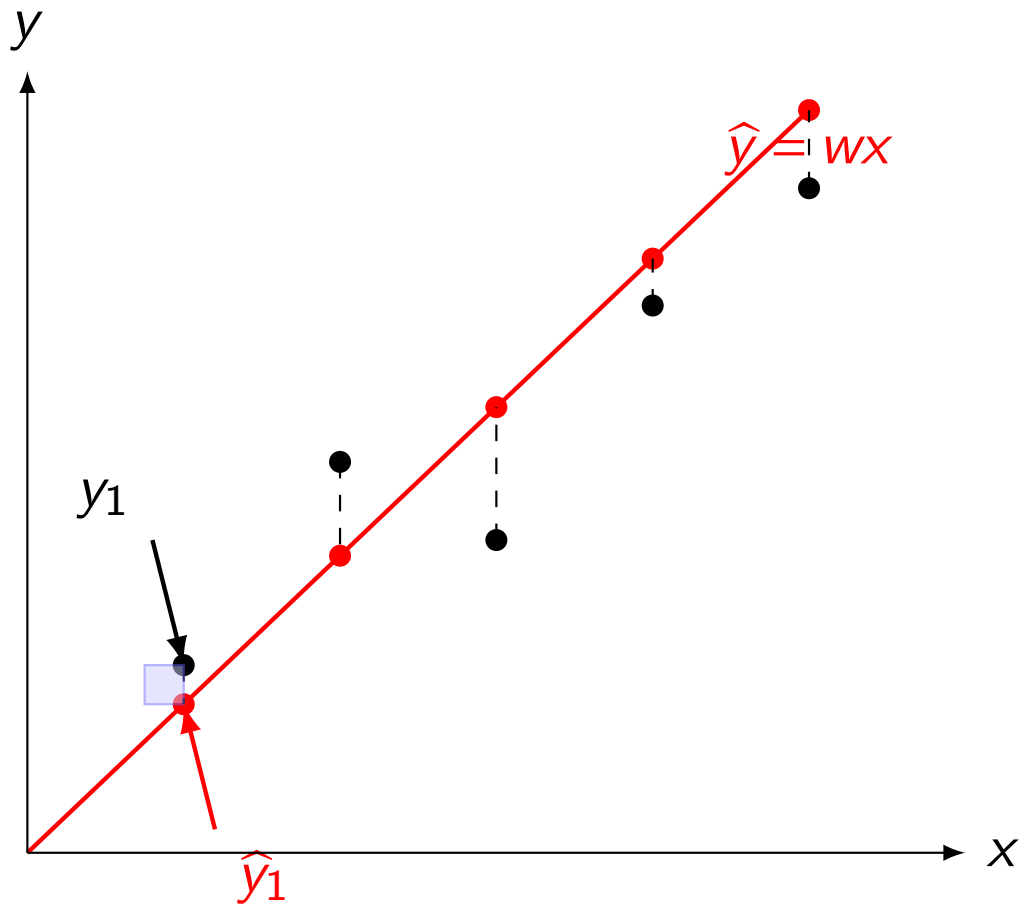

$$C_1(w) = (y_1 - \widehat{y}_1)^2$$

Another try with $w = 0.95$

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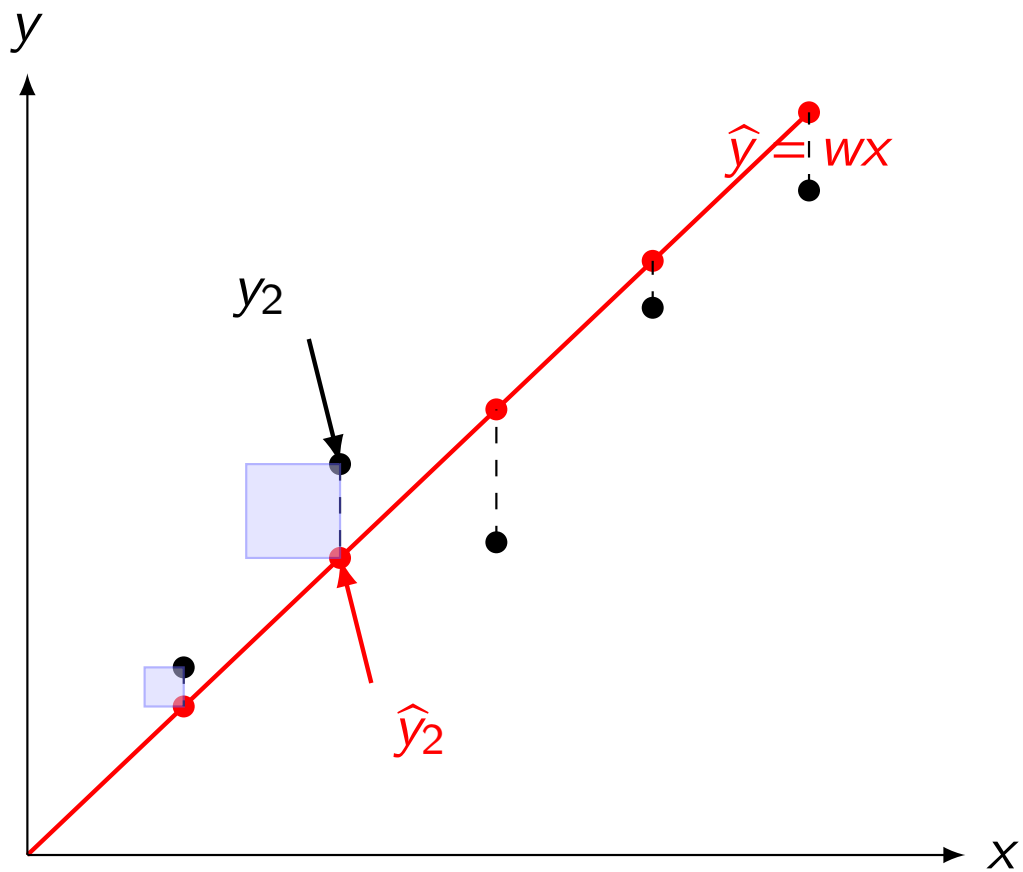

$$C_2(w) = (y_2 - \widehat{y}_2)^2$$

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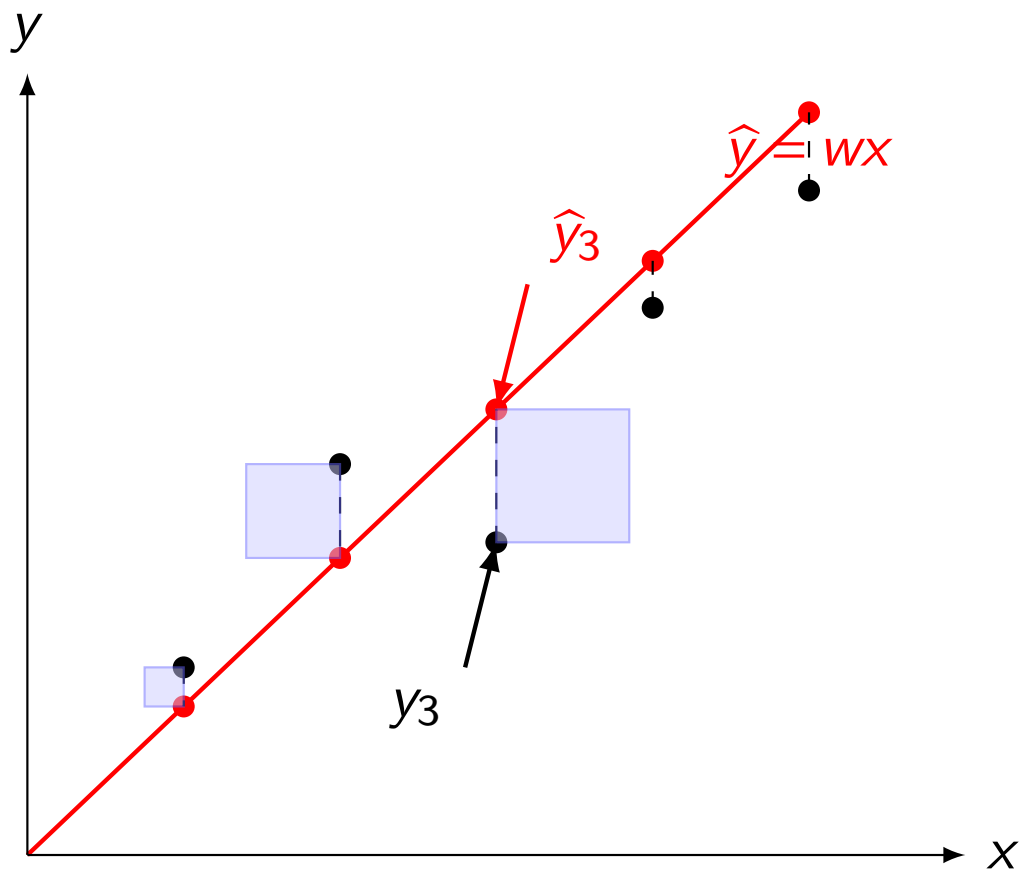

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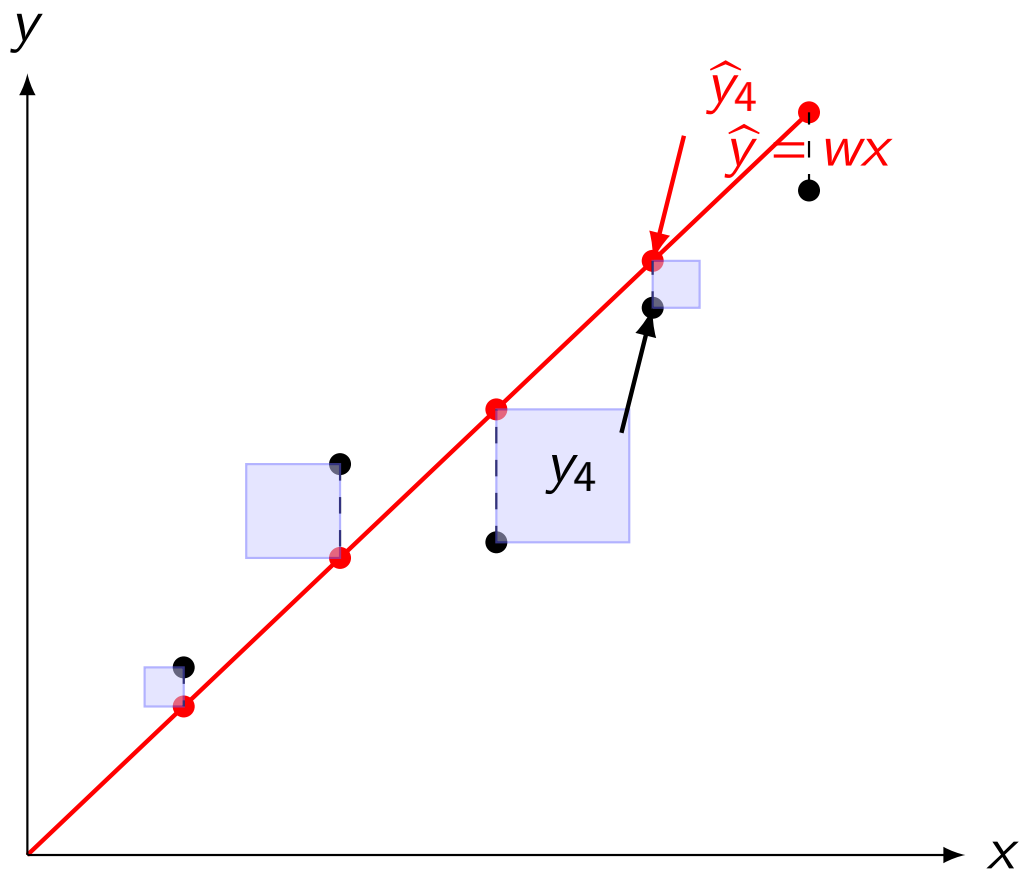

$$C_4(w) = (y_4 - \widehat{y}_4)^2$$

Another try with $w = 0.95$

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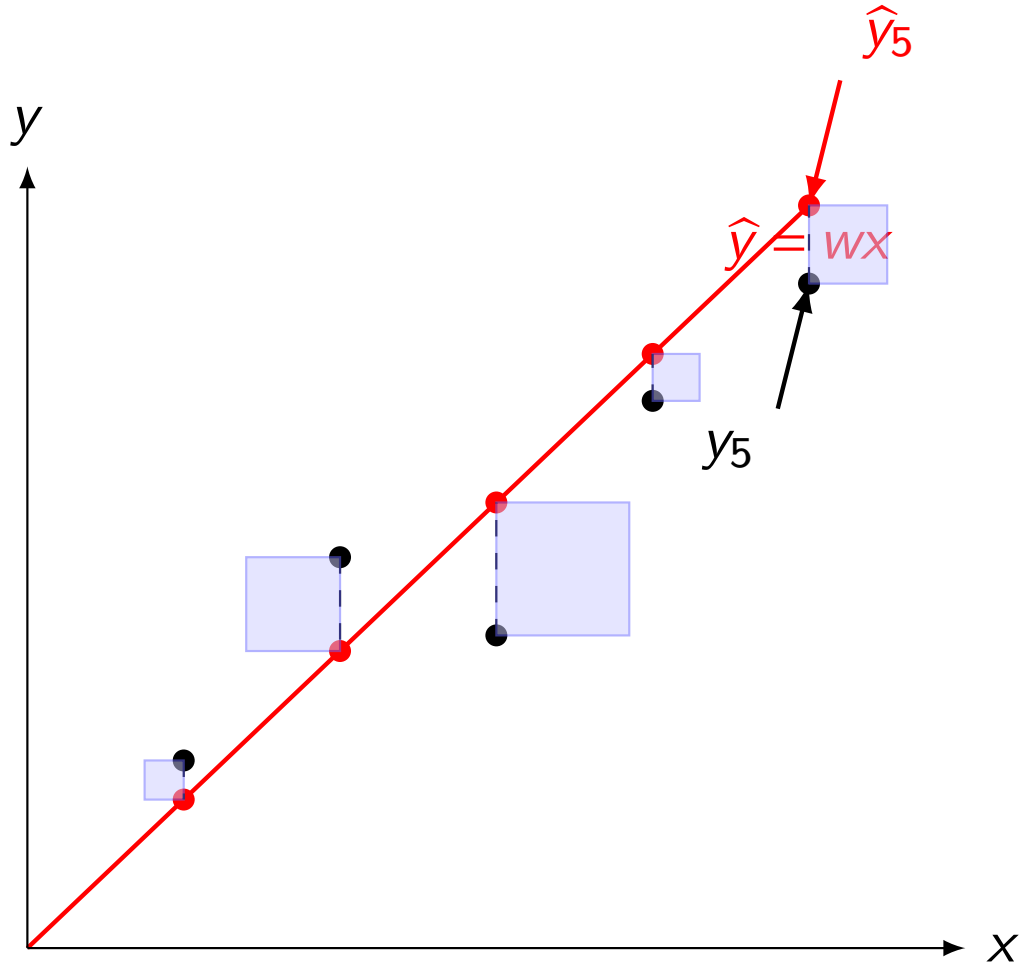
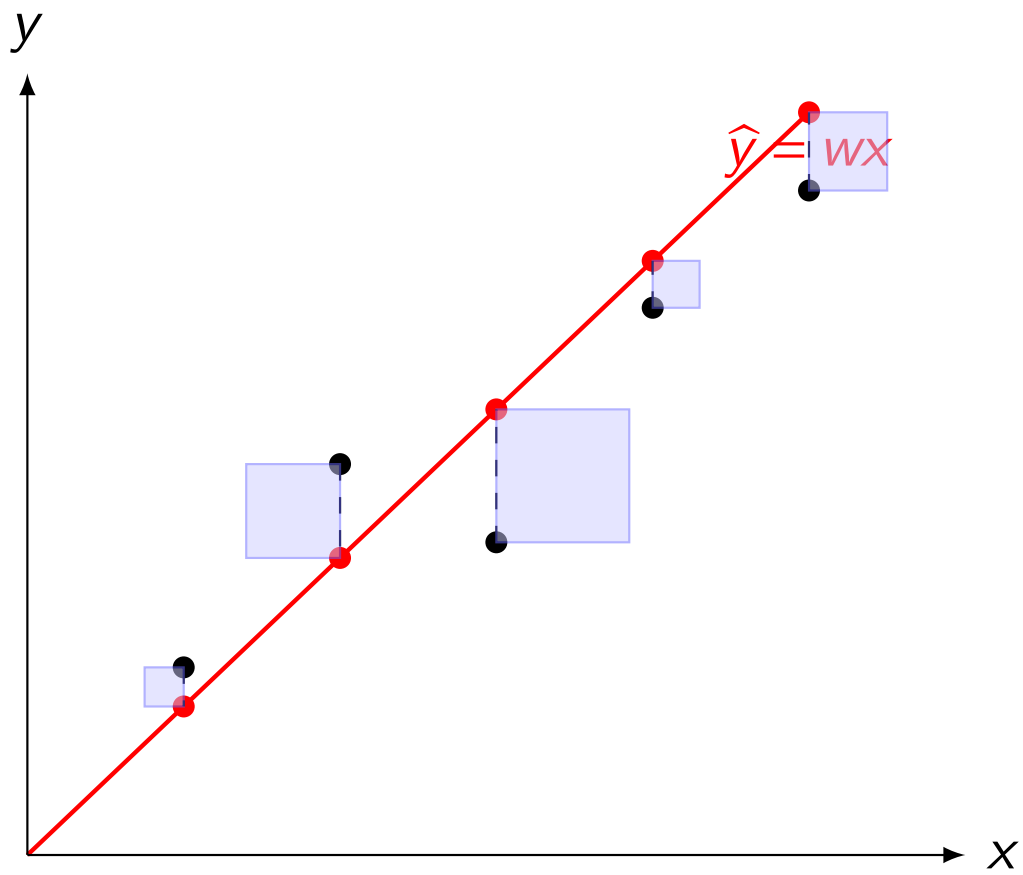


$$C_5(w) = (y_5 - \widehat{y}_5)^2$$

Another try with $w = 0.95$

Here a linear activation function $(\sigma(u) = u)$



$$C(w) = \sum_{i=1}^{5} C_i(w) = 4.9$$

We want to minize the loss function

- DL exploits iterative optimisation method to train (find) suitable parameter.
- Gradient Descent (GD) algorithm and variations are the key tools.
- Gradient means derivative (slope of the function) for one parameter model.
- An estimate of the slope of the function (rise divided by run) at a specific point θ:

$$\frac{\text{Rise}}{\text{Run}} = \frac{C(\theta+\Delta) - C(\theta)}{(\theta+\Delta) - \Delta}$$
$$\approx C'(\theta) \quad \text{when } \Delta \text{ positive small number}$$

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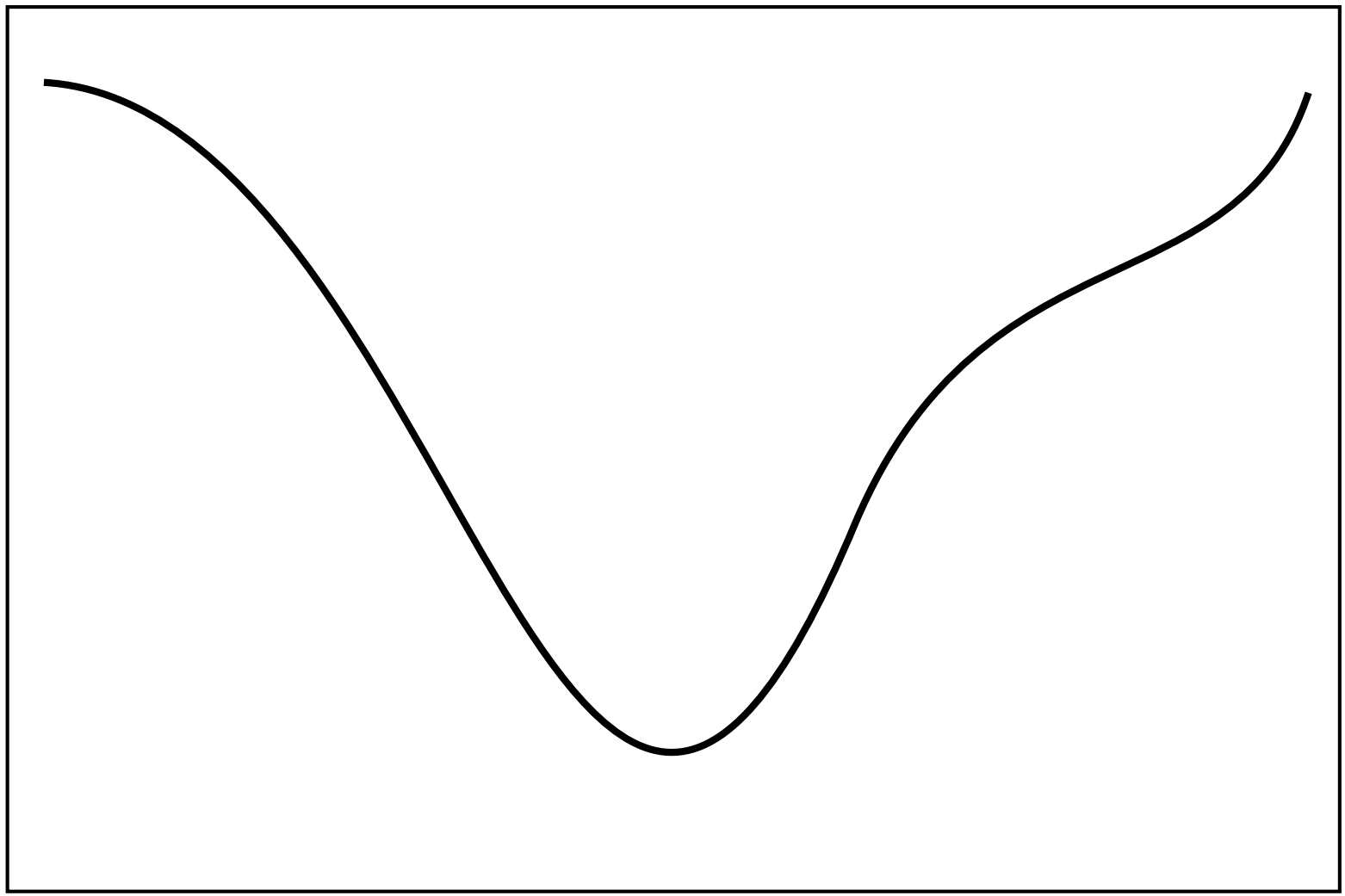
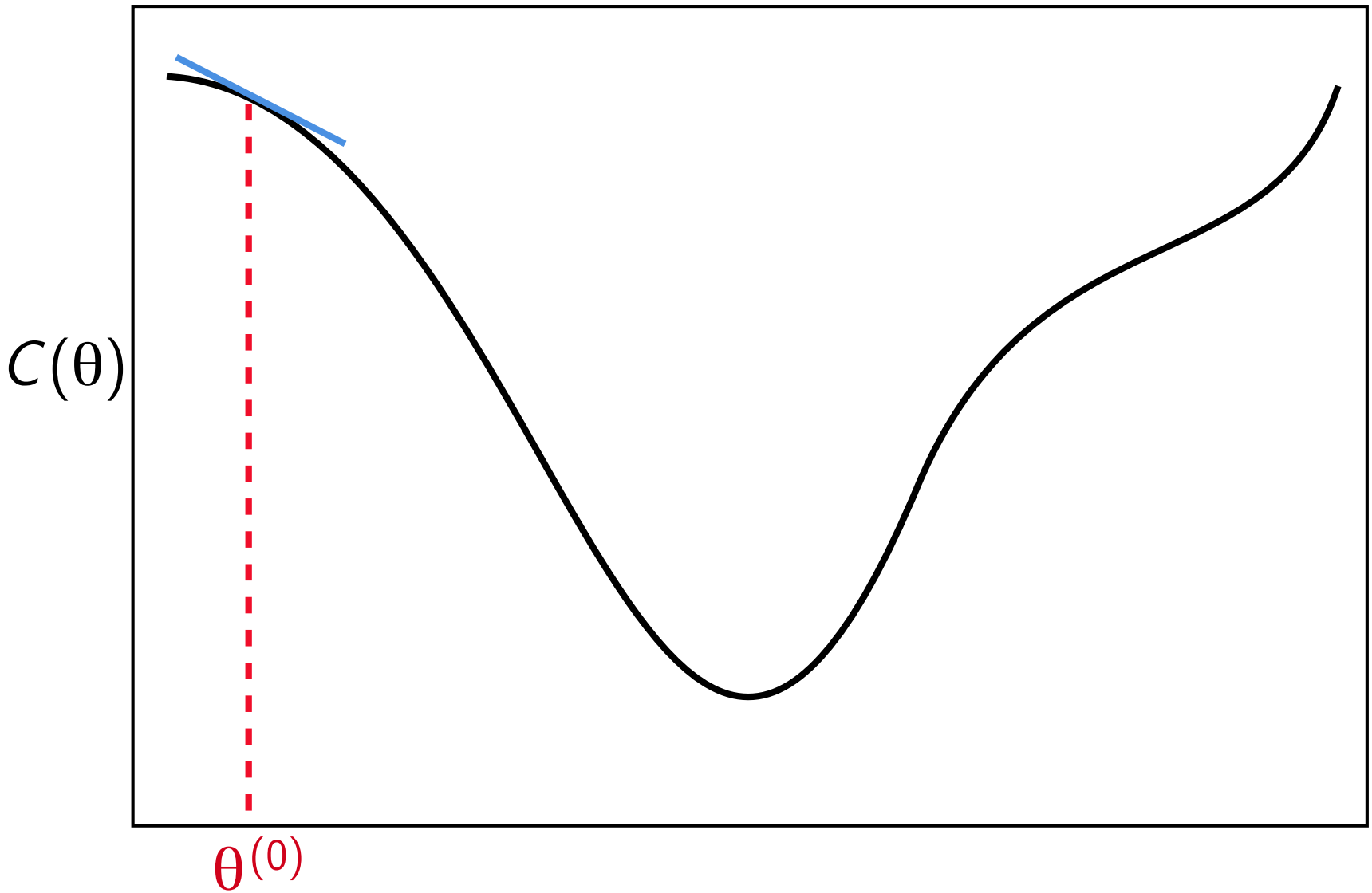
$$\approx C'(\theta) \quad \text{when } \Delta \text{ positive small number}$$

- Gradient is related to vector parameter: noted $\nabla C(\theta)$ $(\theta \in \mathbb{R}^p)$

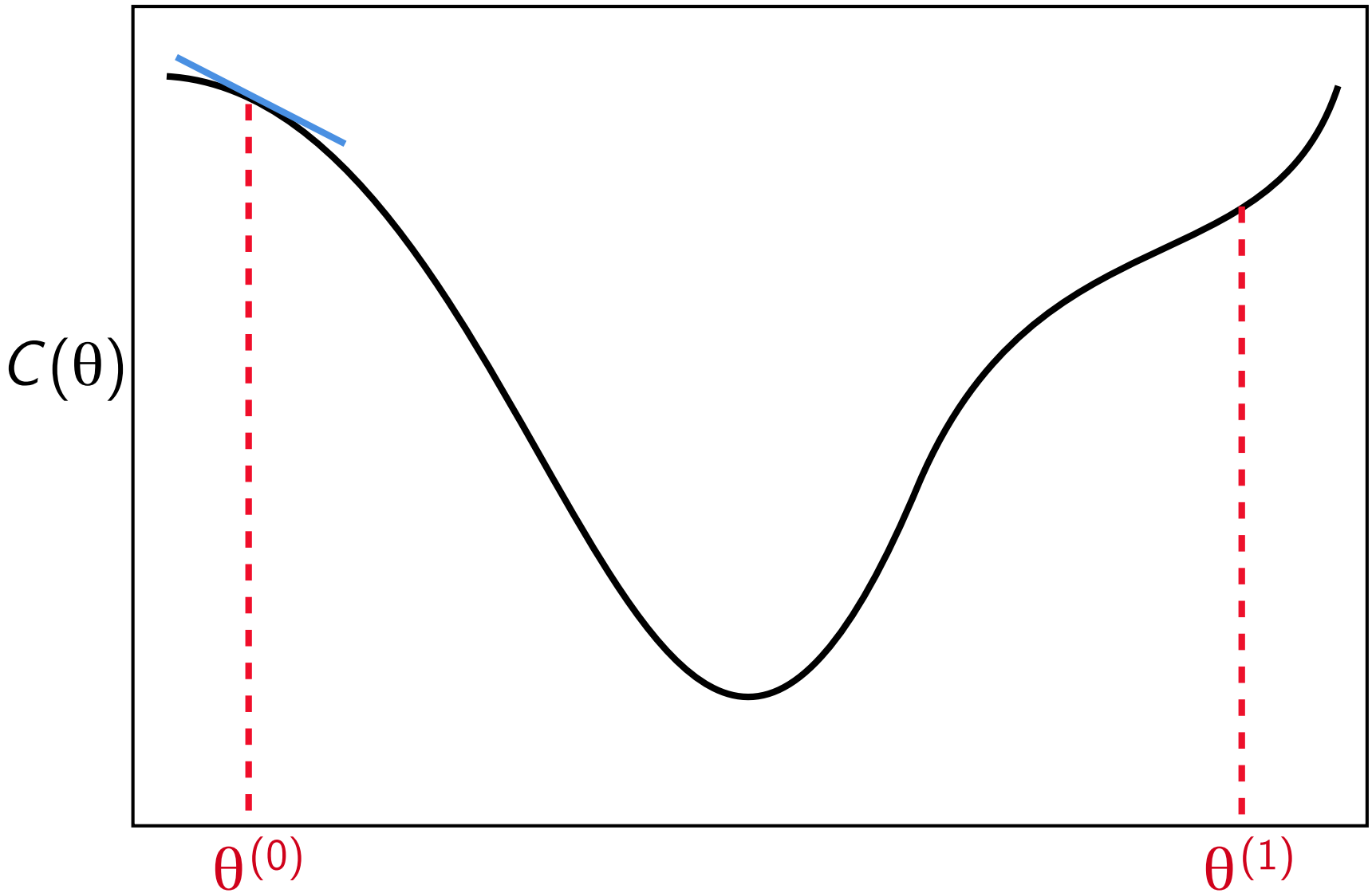
Gradient Descent in action: $\theta^{(t+1)} = \theta^{(t)} - \alpha \nabla C(\theta^{(t)})$



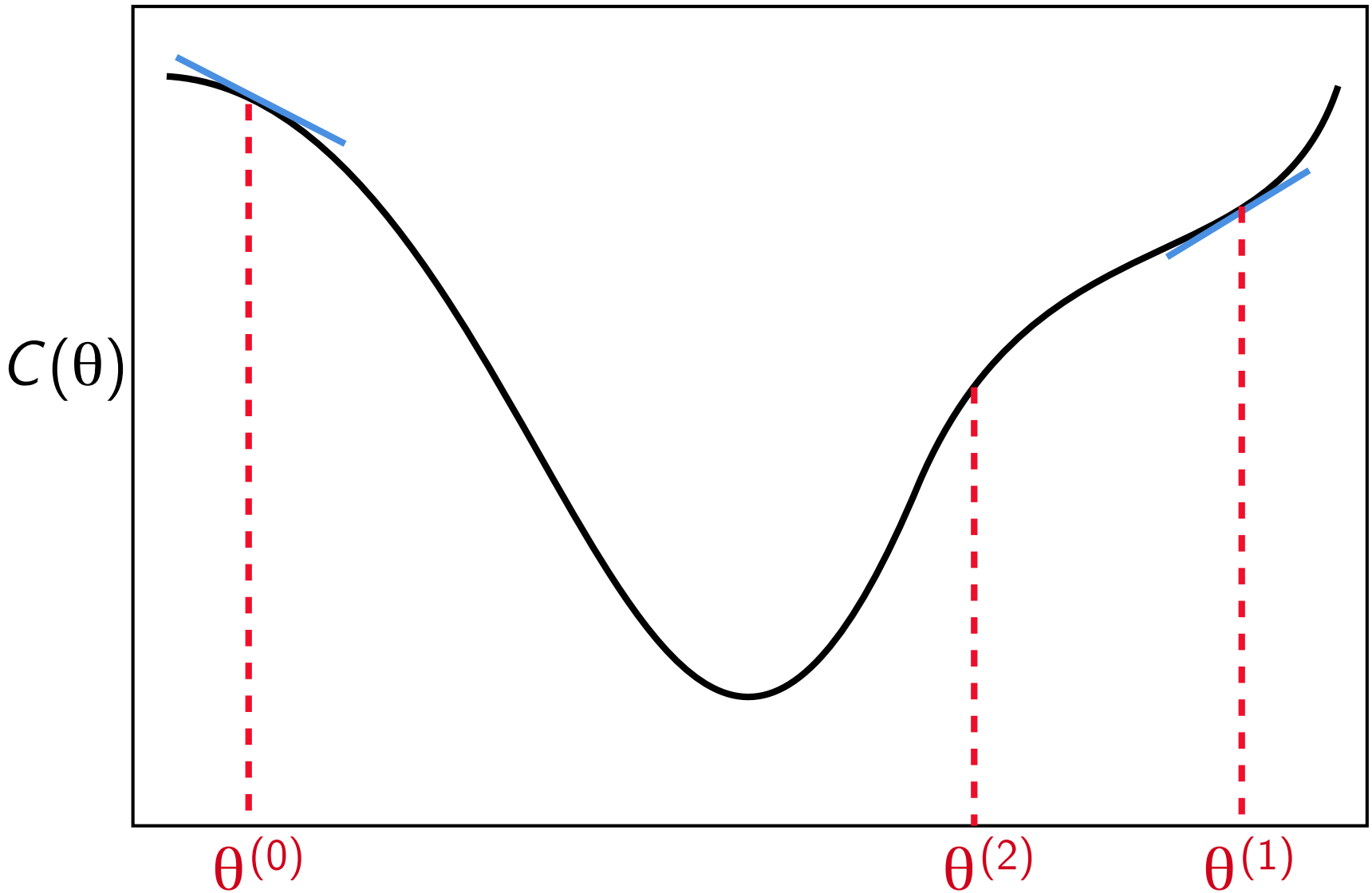
Gradient Descent: Initial guess $\theta^{(0)}$ and compute $\nabla C(\theta^{(0)})$



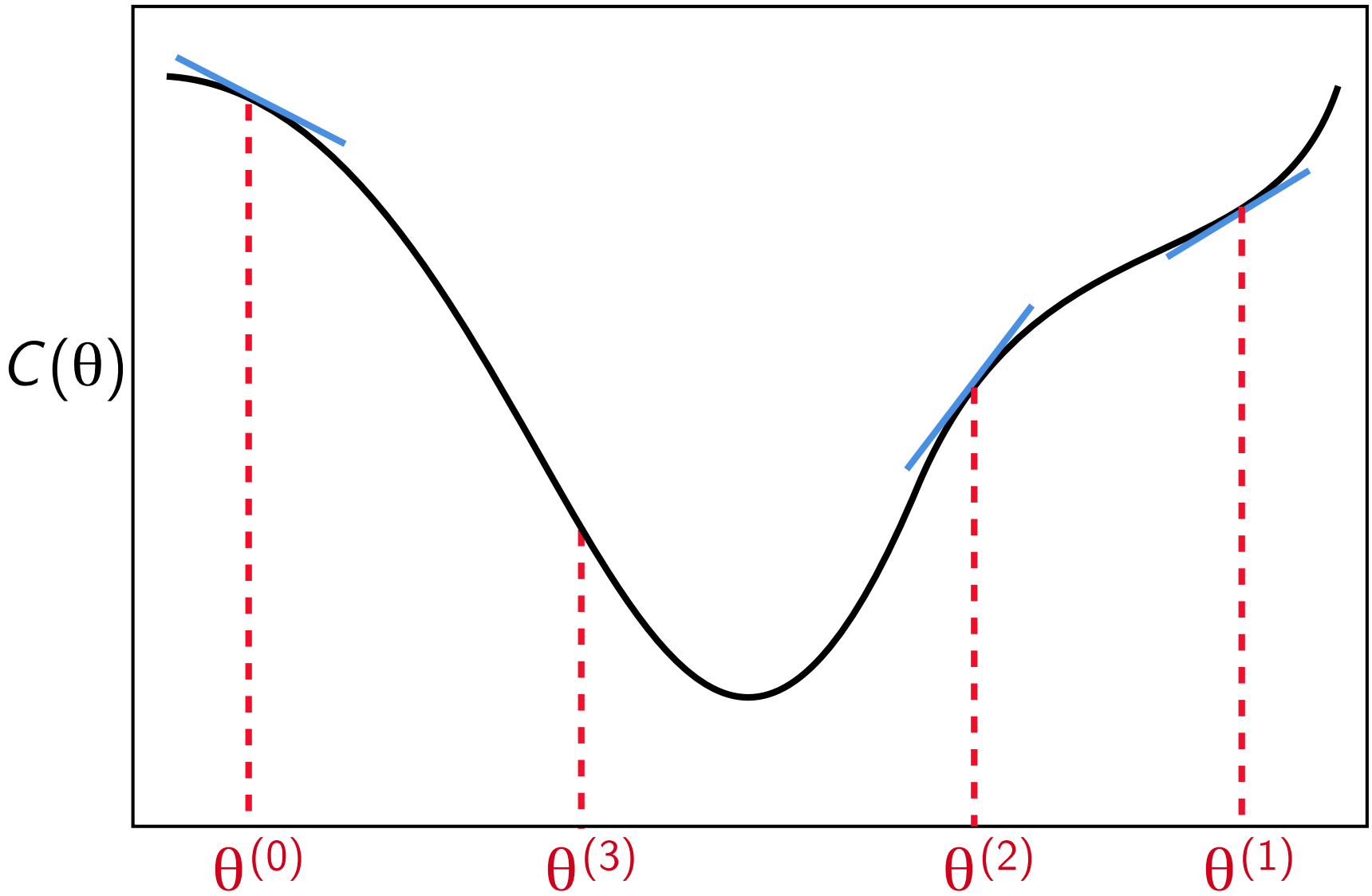
Gradient Descent moves: $\theta^{(1)} = \theta^{(0)} - \alpha \nabla C(\theta^{(0)})$



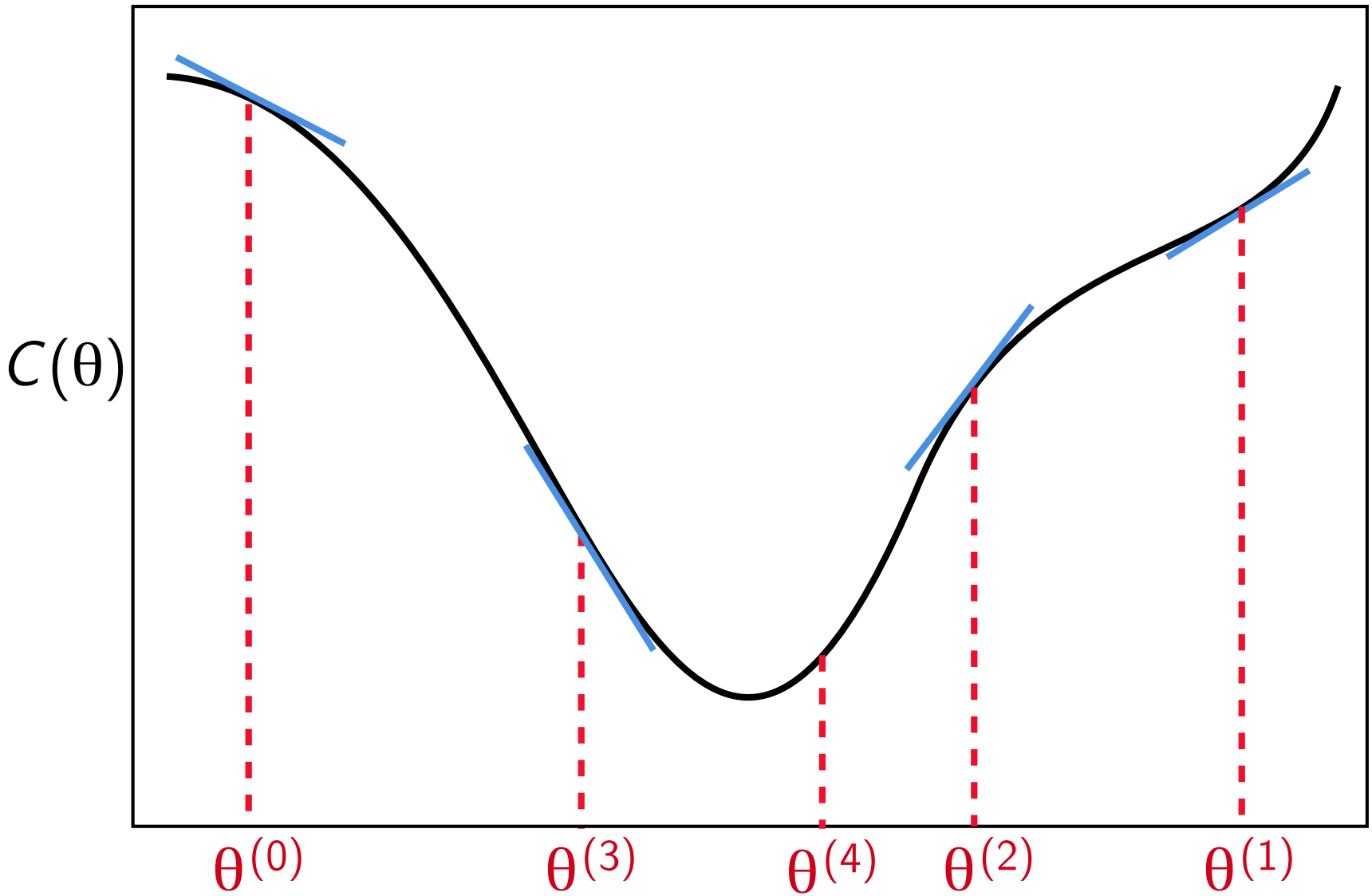
Gradient Descent moves: $\theta^{(2)} = \theta^{(1)} - \alpha \nabla C(\theta^{(1)})$



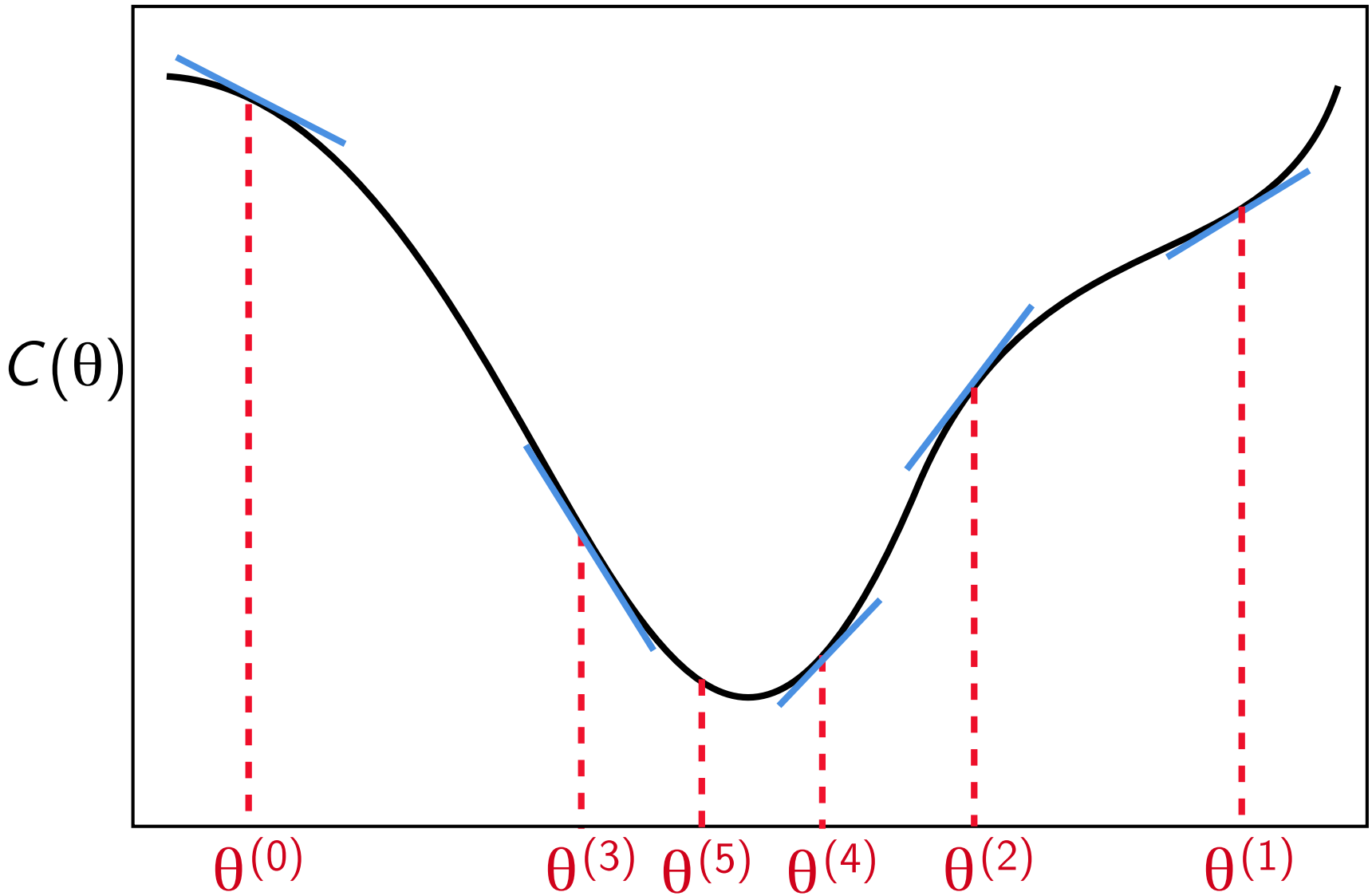
Gradient Descent moves: $\theta^{(3)} = \theta^{(2)} - \alpha \nabla C(\theta^{(2)})$



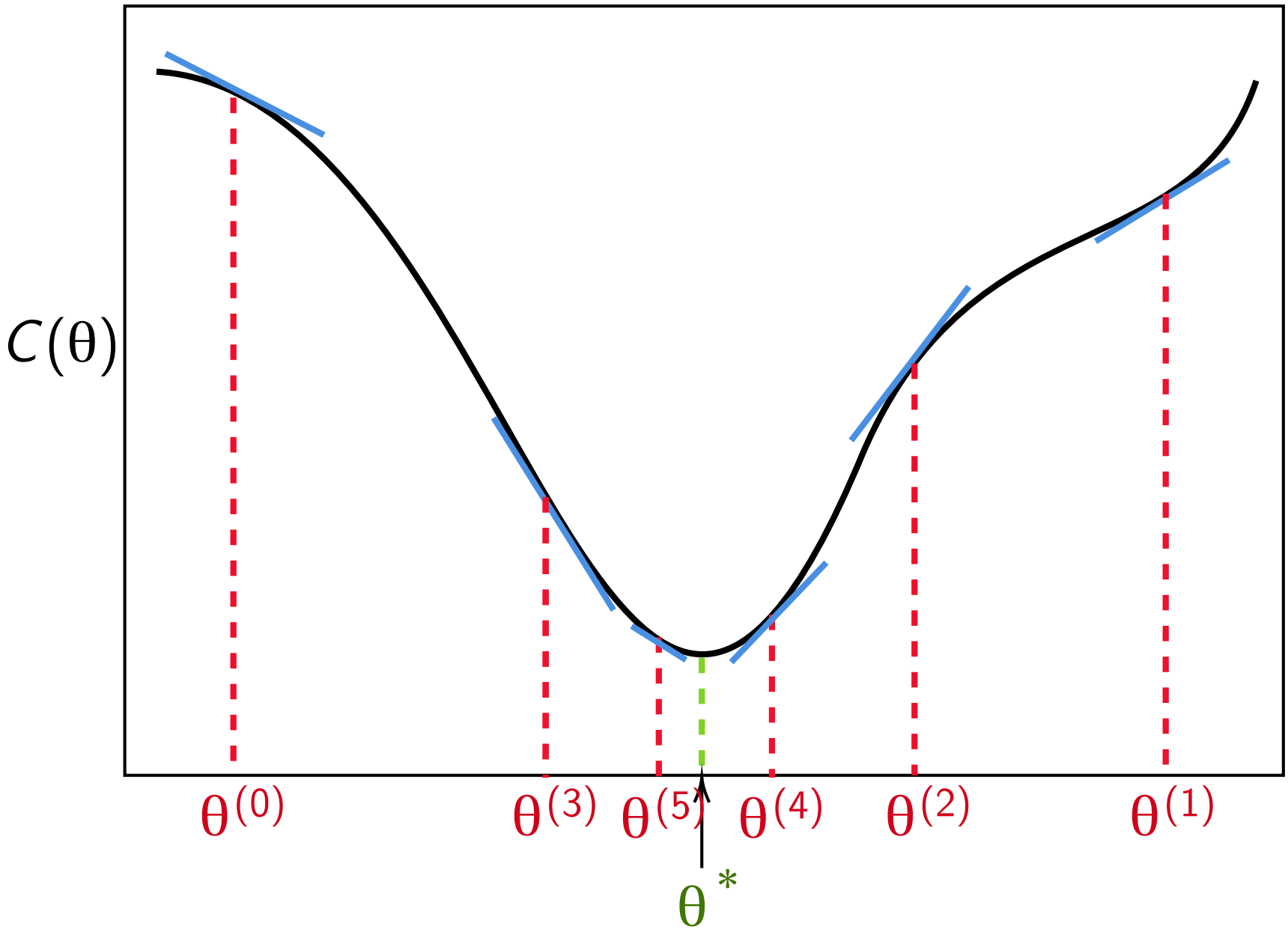
Gradient Descent moves: $\theta^{(4)} = \theta^{(3)} - \alpha \nabla C(\theta^{(3)})$



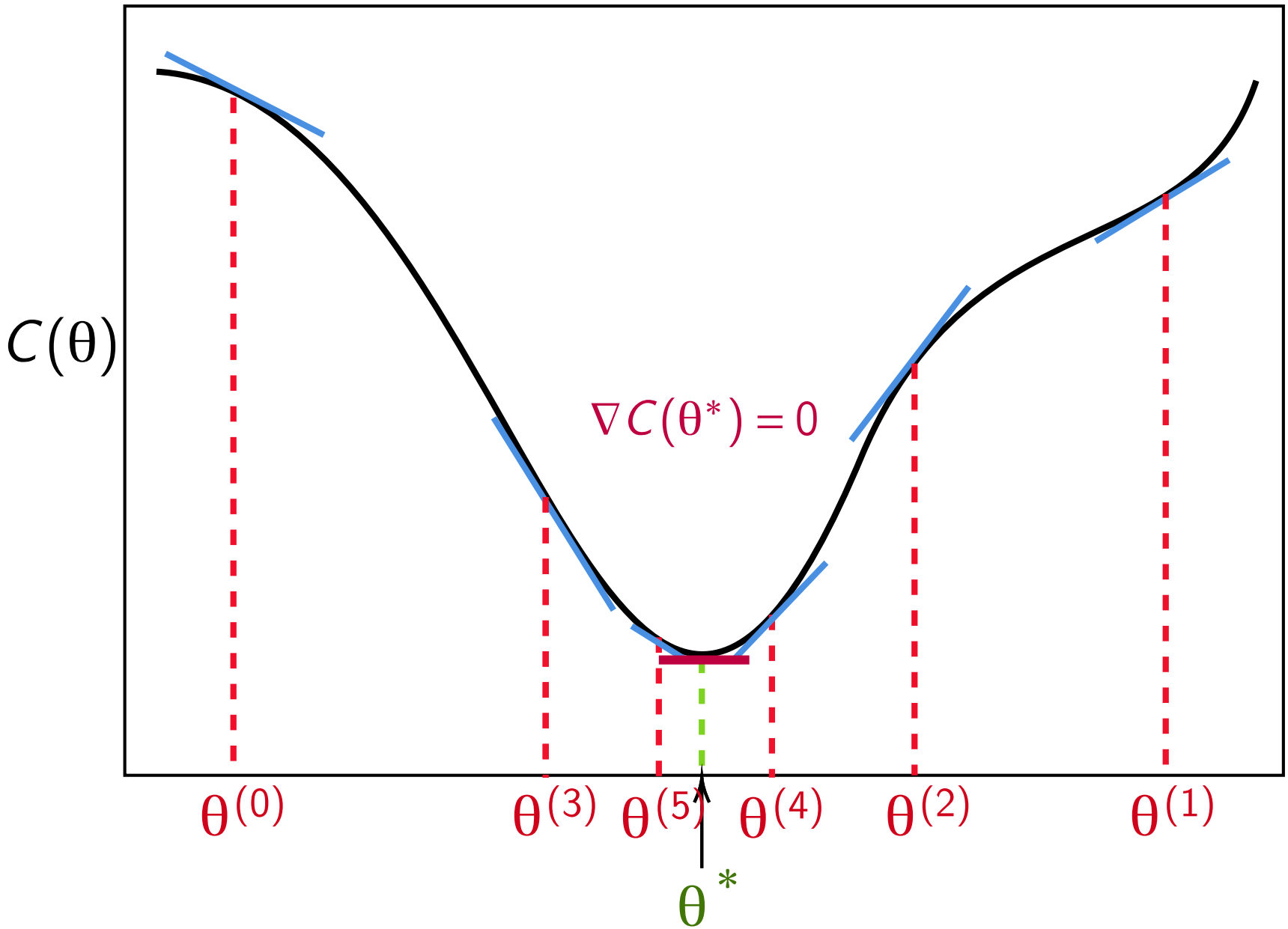
Gradient Descent moves: $\theta^{(5)} = \theta^{(4)} - \alpha \nabla C(\theta^{(4)})$



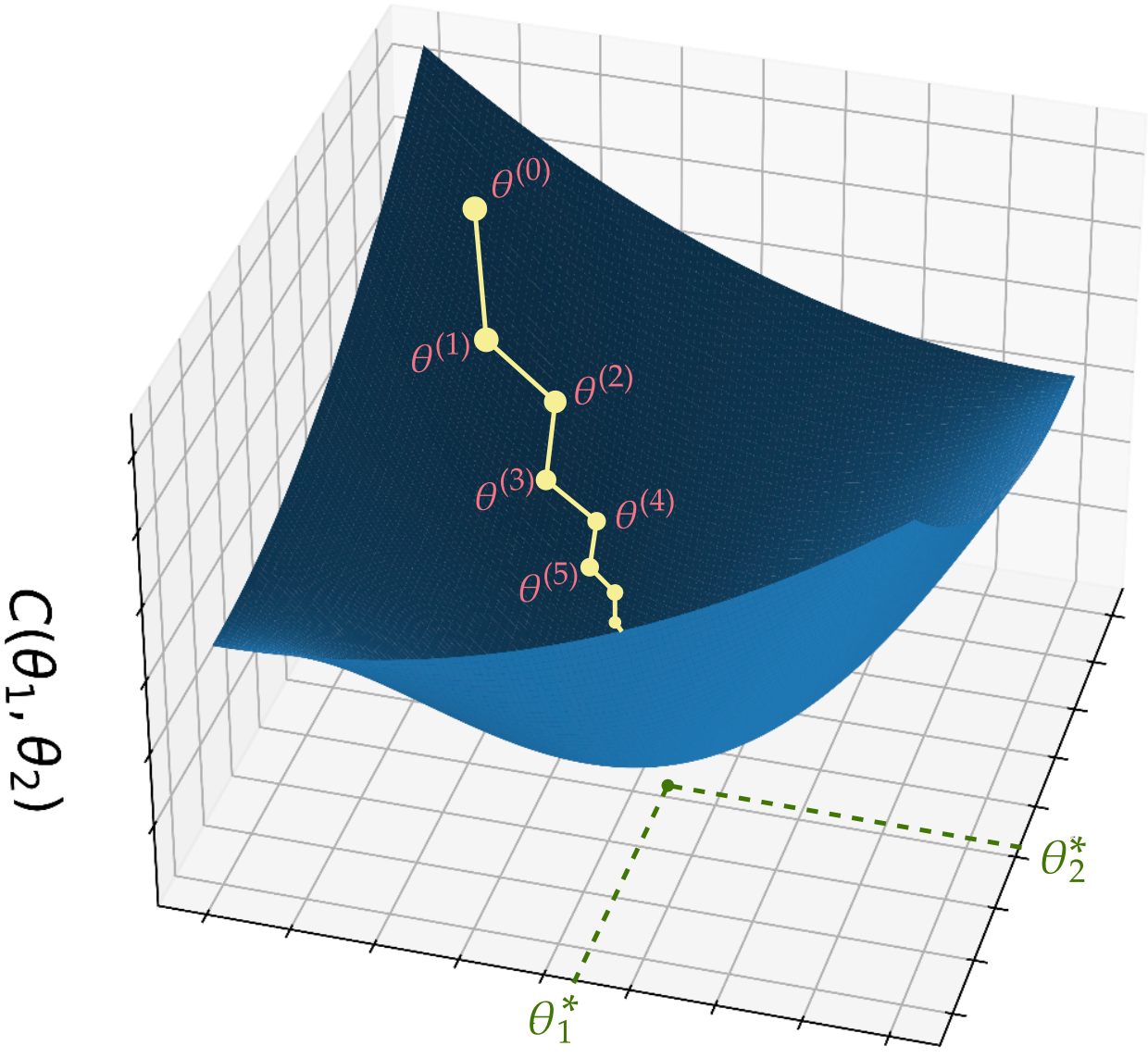
Gradient Descent moves: $\theta^* = \theta^{(5)} - \alpha \nabla C(\theta^{(5)})$



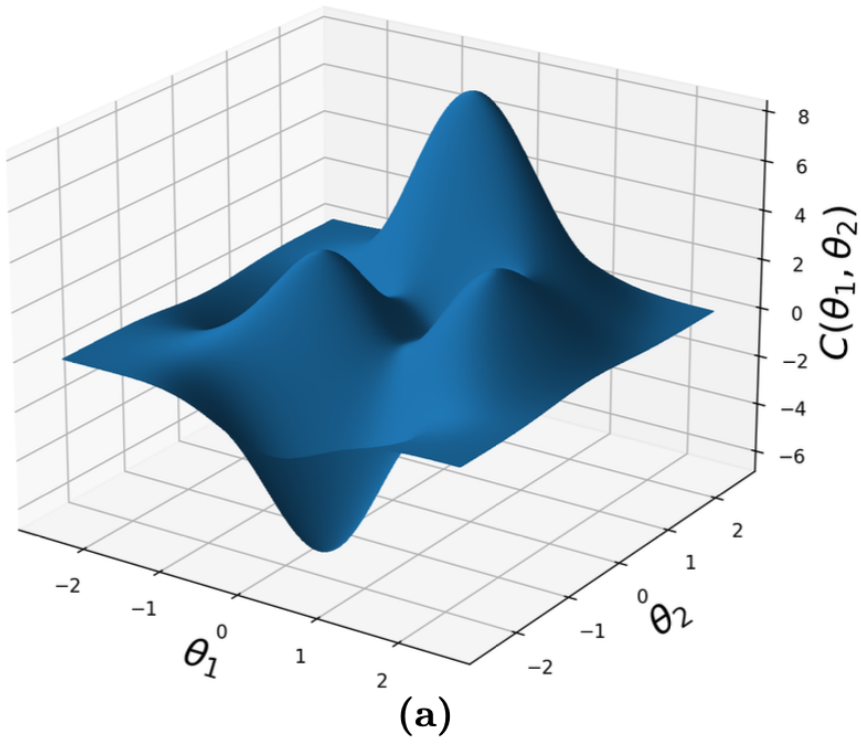
Gradient Descent terminates: $\nabla C(\theta^*) \approx 0$

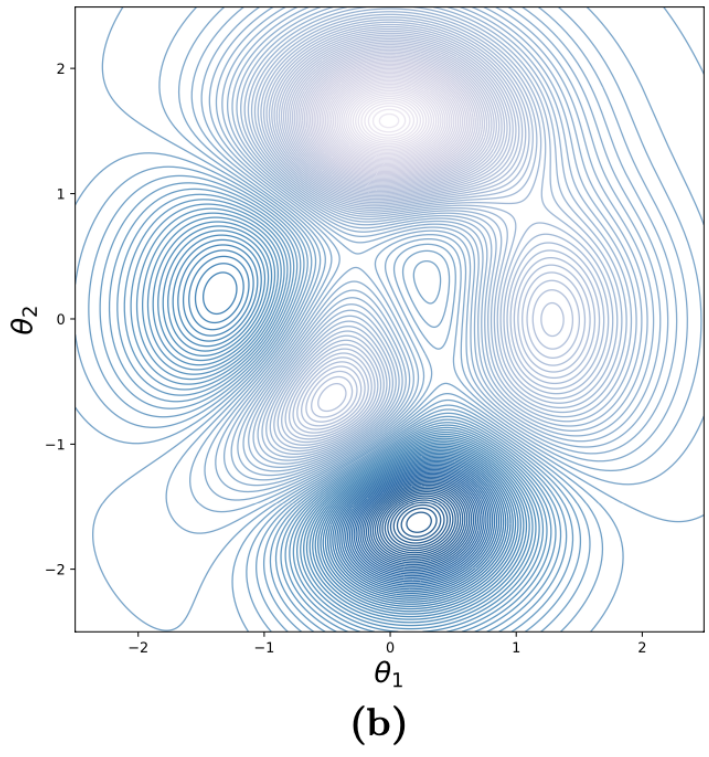


Loss landscape: two parameters



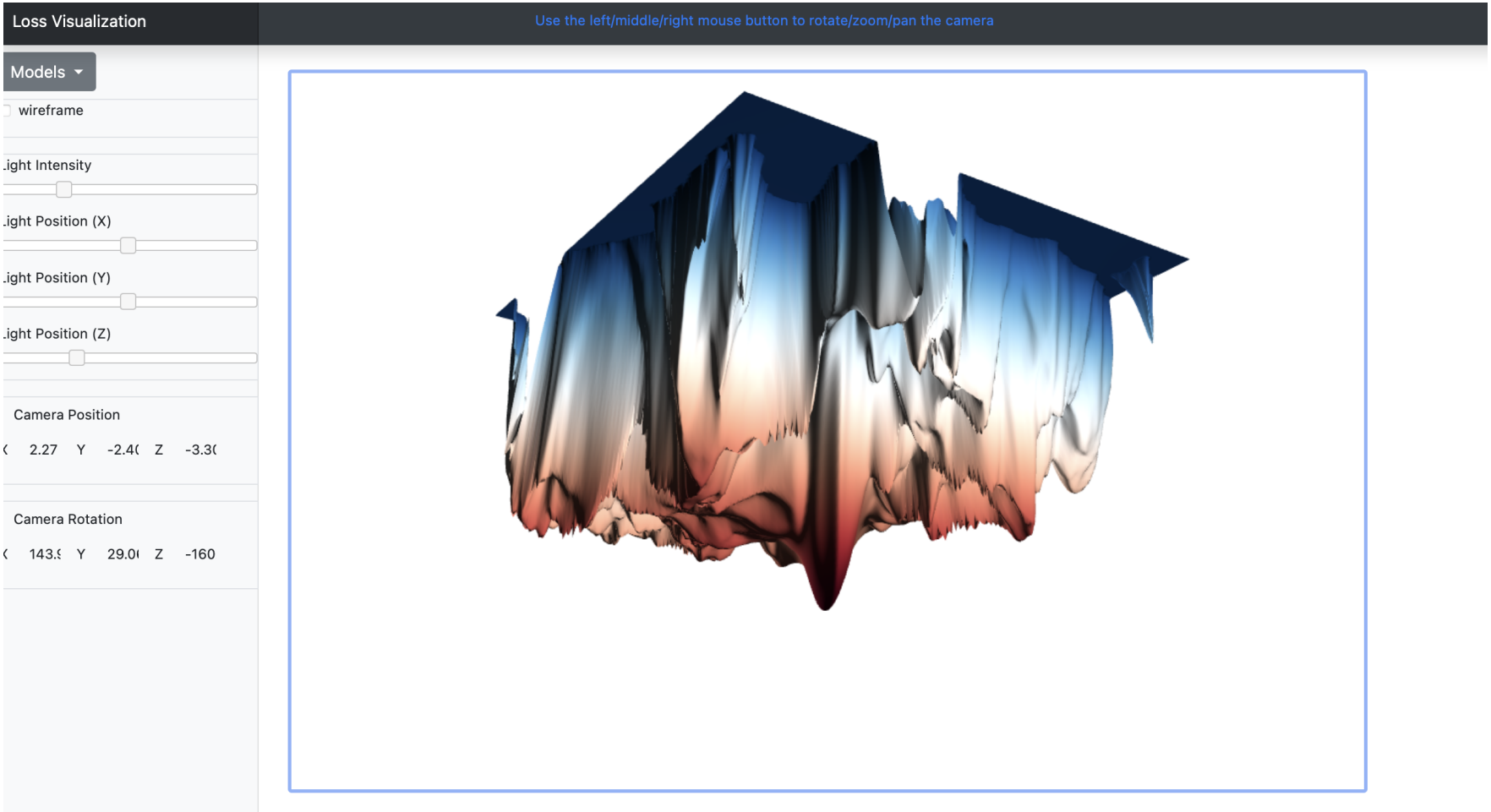
Loss landscape: 3D surface plot and topographical map





Loss landscape is quite complex

Interactive loss landscape visualization



Why Gradient Descent is expensive?

$$C(\theta; \text{Data}) = \frac{1}{n} \sum_{i=1}^{n} C_i(\theta), \qquad i.e, \qquad C_i(\theta) = (y_i - \widehat{y}_i)^2$$

Why Gradient Descent is expensive?

$$C(\theta; \text{Data}) = \frac{1}{n} \sum_{i=1}^{n} C_i(\theta), \qquad i.e, \qquad C_i(\theta) = (y_i - \hat{y}_i)^2$$

- To compute the gradient, GD must look at **all training data** at every step:

$$\nabla C(\theta) = \frac{1}{n} \sum_{i=1}^{n} \nabla C_i(\theta)$$

- For modern datasets: millions of data points and millions of parameters.
- Each update becomes **slow** and **computationally expensive**.

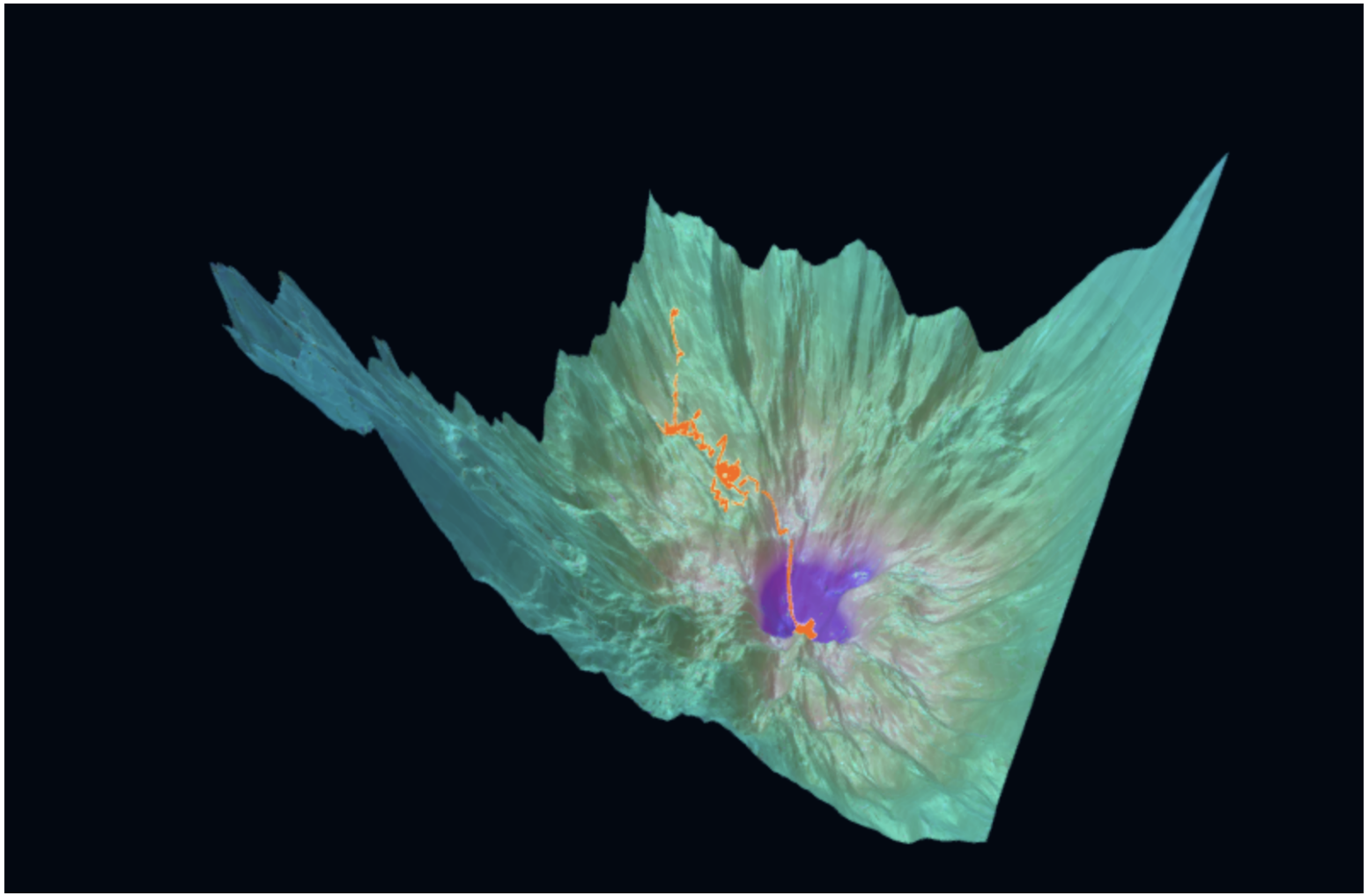
Key idea: *Deep learning replaces full Gradient Descent with faster approximations, such as Stochastic Gradient Descent (SGD) and mini-batch GD, which use only a small subset of the data at each step.*

Question 5

Does gradient descent guarantee that we find the global minimum of the loss function?

Visualisation: `https://losslandscape.com/`

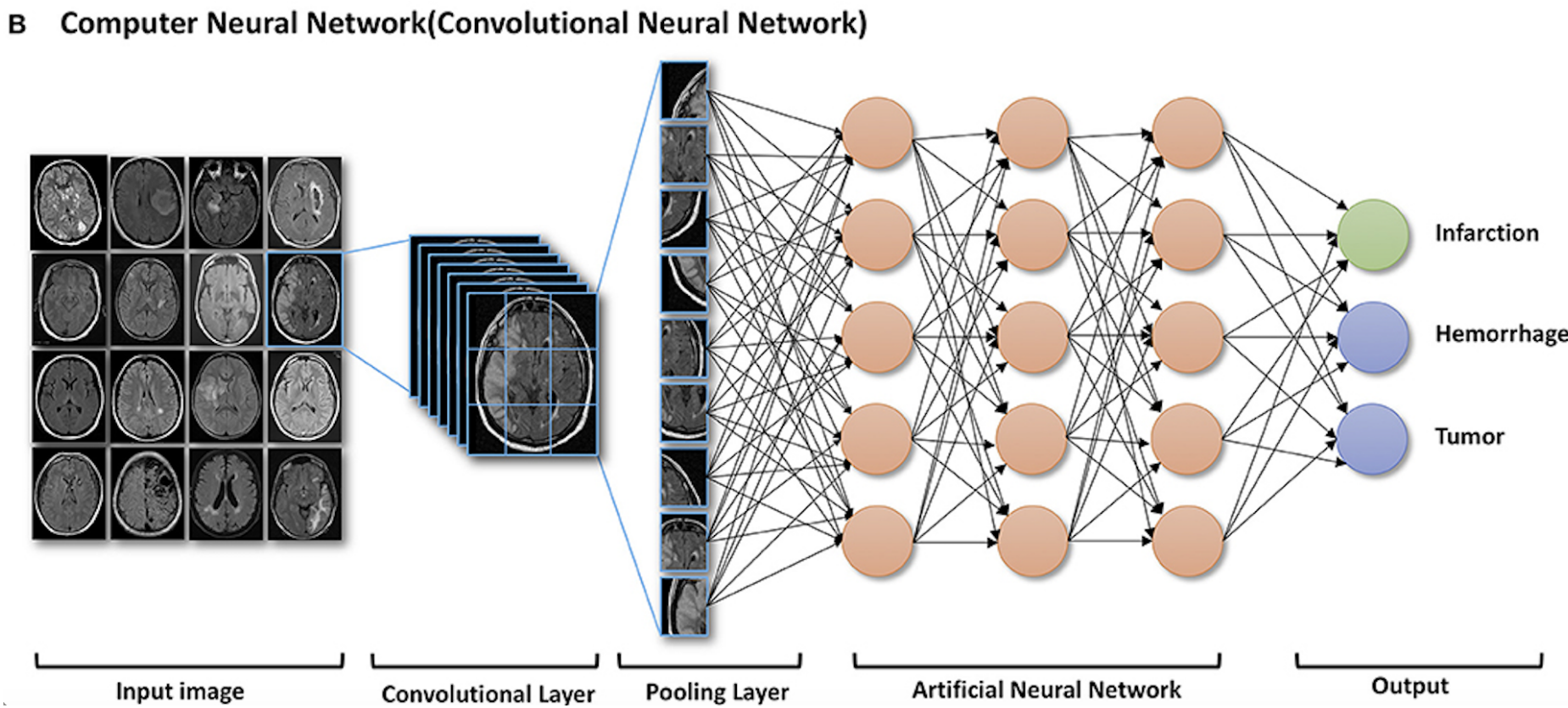
Explorer



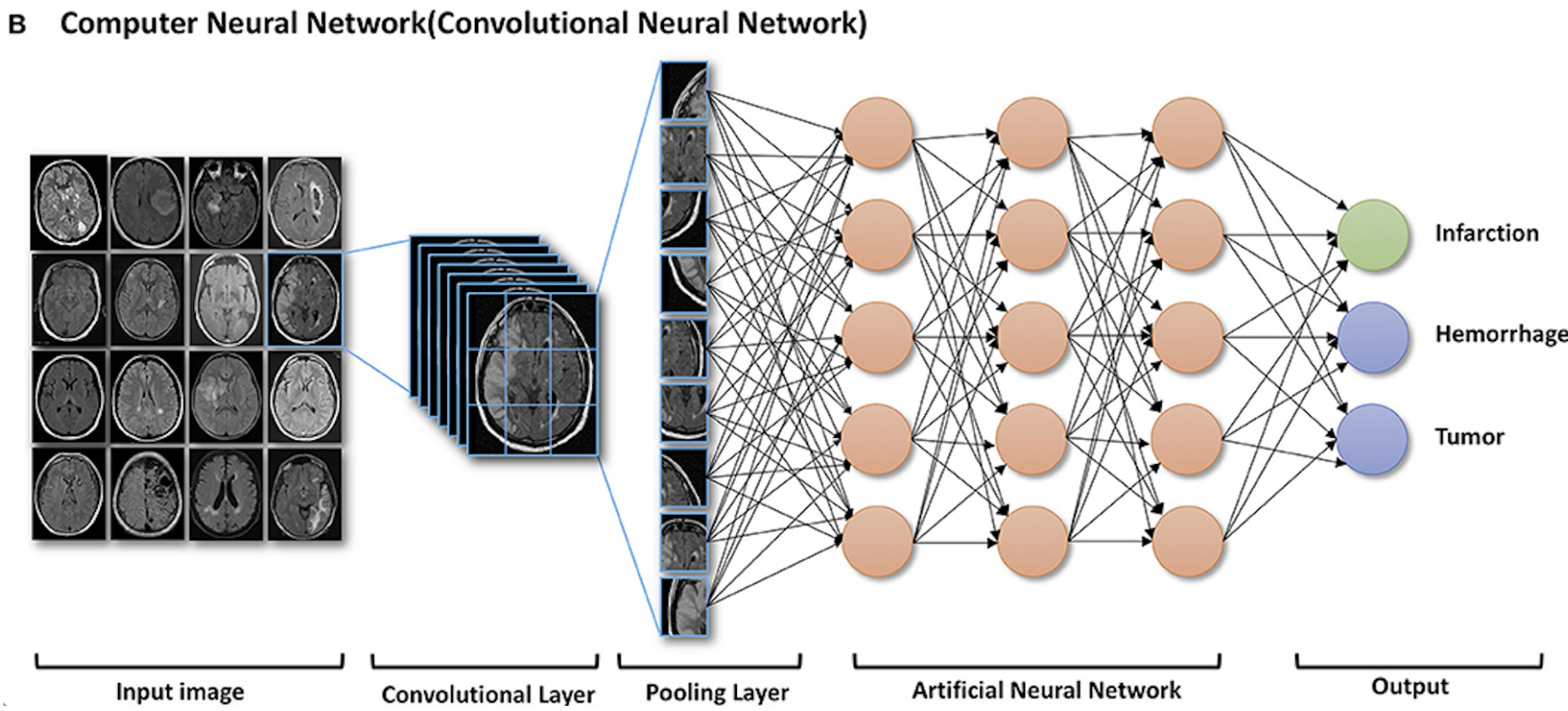
Visualisation: video

https://losslandscape.com/videos/

CNN: the math behind ?



CNN: the math behind ?



The key is the Convolution operation

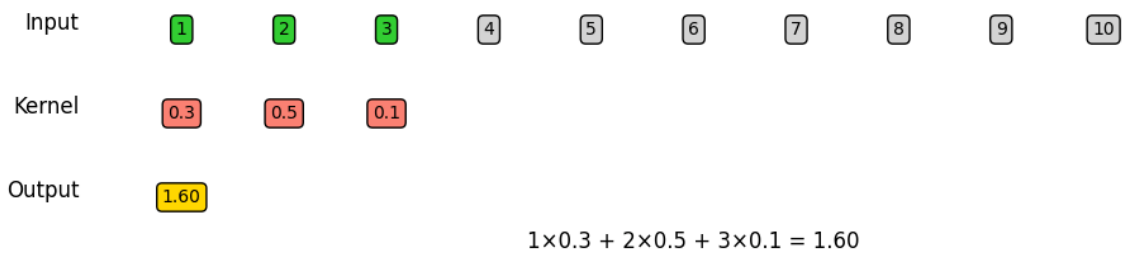
Convolution 1D

Given an input vector $x = [x_1, x_2, \ldots, x_{10}]$ and a kernel (filter) $w = [w_1, w_2, w_3]$, the output z of the 1D convolution operation is :

$$z = w \star x$$

where $\star$ denotes the convolution operation. Each element is computed as:

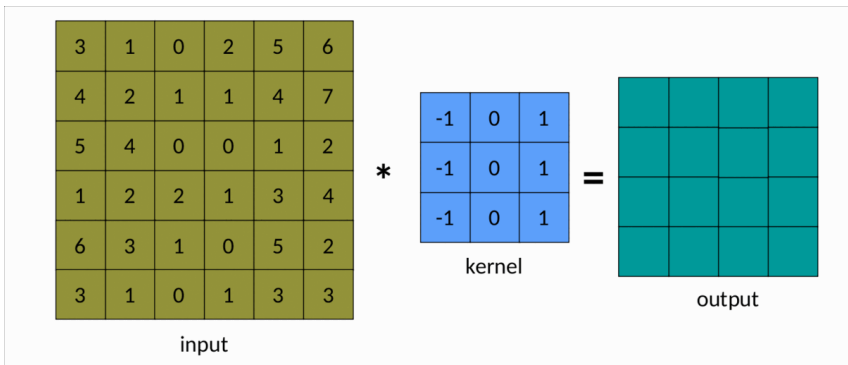
$$z_i = w_1 \cdot x_i + w_2 \cdot x_{i+1} + w_3 \cdot x_{i+2}$$



Convolution 2D: $z = W \star x$

A 3×3 Kernel W applied on an 6×6 input matrix

$$\underbrace{\begin{bmatrix} -11 & -4 & 9 & 12 \\ -7 & -6 & 5 & 11 \\ -9 & -8 & 6 & 7 \\ -7 & -4 & 8 & 7 \end{bmatrix}}_{z} = \underbrace{\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}}_{W} \star \underbrace{\begin{bmatrix} 3 & 1 & 0 & 2 & 5 & 6 \\ 4 & 2 & 1 & 1 & 4 & 7 \\ 5 & 4 & 0 & 0 & 1 & 2 \\ 1 & 2 & 2 & 1 & 3 & 4 \\ 6 & 3 & 1 & 0 & 5 & 2 \\ 3 & 1 & 0 & 1 & 3 & 3 \end{bmatrix}}_{x}$$



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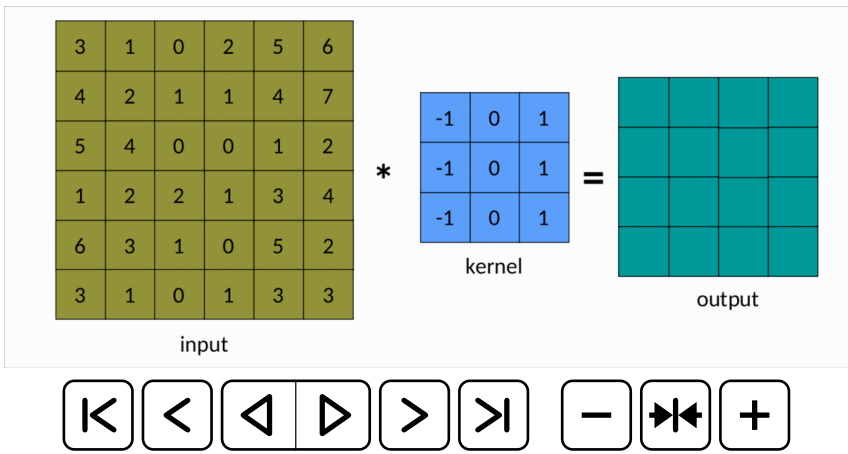
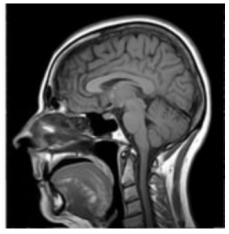


$$z_{i,j} = \sum_{m=0}^{2} \sum_{n=0}^{2} W_{m,n} \cdot x_{i+m-1,j+n-1}$$

Example of 2d convolution filter

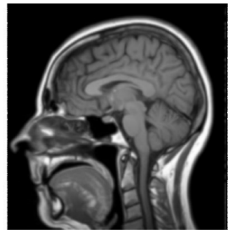
0	0	0
0	1	0
0	0	0



Identity Kernel: Just the starting image itself.

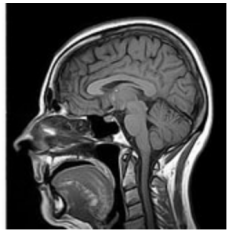
$$\frac{1}{9}$$

1	1	1
1	1	1
1	1	1



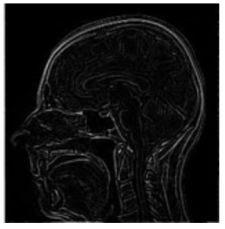
Box Blur Kernel: Each pixel is averaged equally with its 8 neighbors.

0	-1	0
-1	5	-1
0	-1	0



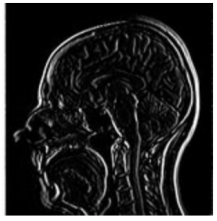
Sharpen Kernel:: Differences emphasized with its adjacent pixel values.

-1	-1	-1
-1	8	-1
-1	-1	-1



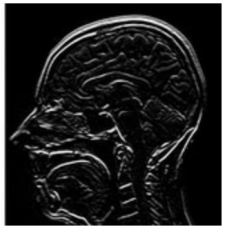
Outline ("Edge") Kernel: Highlights neighboring pixels with large differences of intensity.

1	0	-1
2	0	-2
1	0	-1



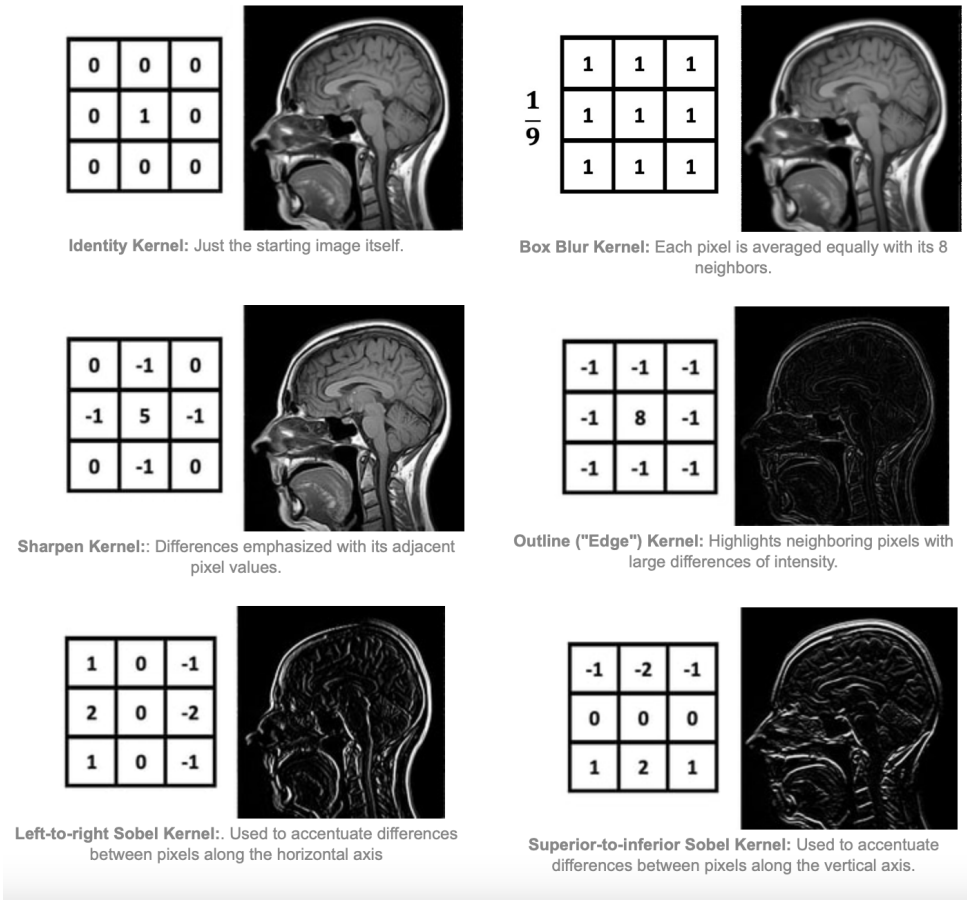
Left-to-right Sobel Kernel:. Used to accentuate differences between pixels along the horizontal axis

-1	-2	-1
0	0	0
1	2	1



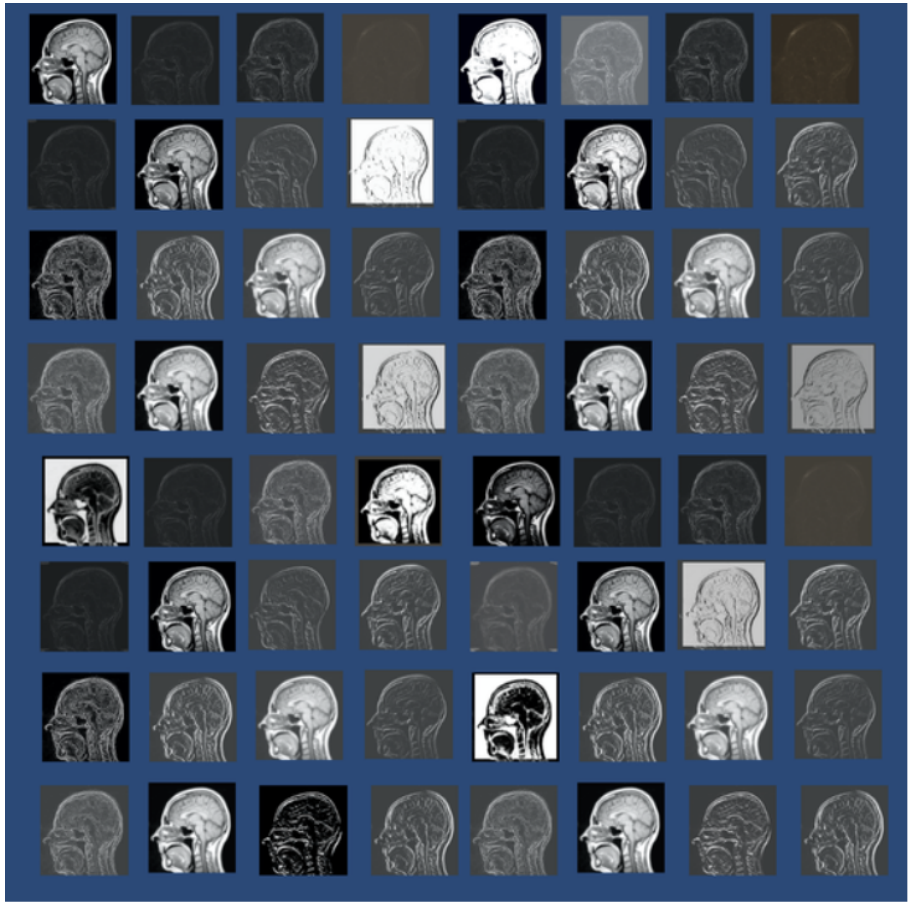
Superior-to-inferior Sobel Kernel: Used to accentuate differences between pixels along the vertical axis.

Example of 2d convolution filter



The key is the Convolution operation

CNN: Kernels are learned



Why Convolutional Kernels?

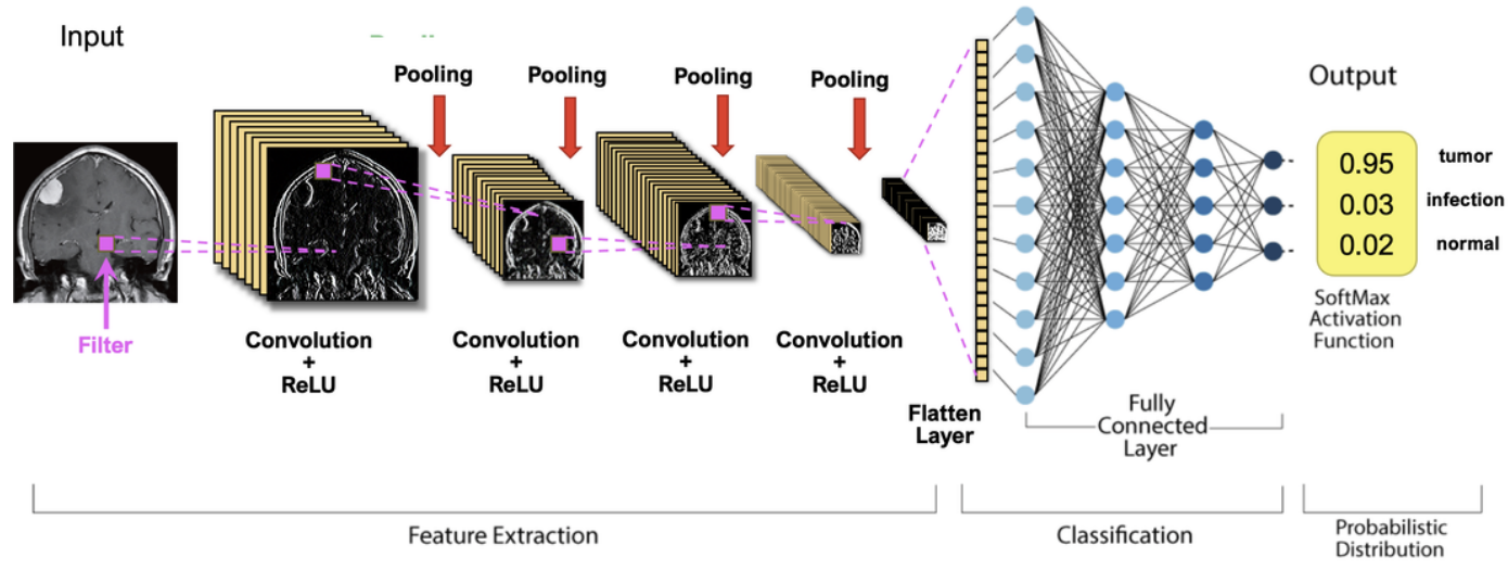
Key idea: Convolutional kernels allow neural networks to efficiently detect local patterns that can appear anywhere in the input.

- **Locality**
  - Nearby pixels (or signal values) are strongly related
  - Clinically relevant structures (edges, textures, lesions) are local
  - Convolution focuses on small neighborhoods
- **Translation invariance**
  - A relevant pattern matters regardless of its position
  - The same kernel is applied across the entire image
  - Enables detection of the same feature anywhere
- **Parameter efficiency**
  - Fully connected layers require many parameters
  - Convolution uses shared weights
  - Fewer parameters $\Rightarrow$ better generalization

Take-home message: *What matters locally matters everywhere.*

CNN as a Hierarchical Feature Extractor

A CNN is a sequence of convolutional layers that progressively extract more abstract features before making a prediction.



Input → **Conv + ReLU** → **Conv + ReLU** → **Conv + ReLU**

Image / Signal | Edges, textures | Shapes, regions | High-level patterns

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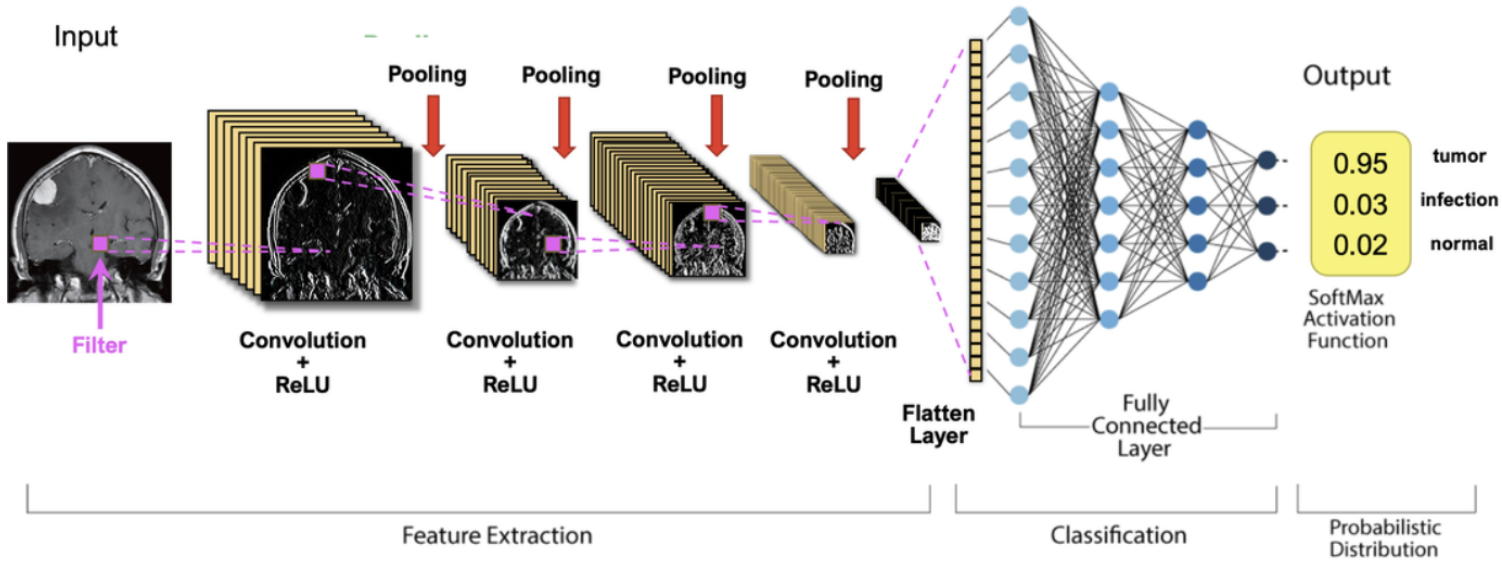

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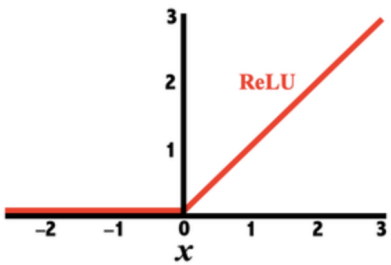

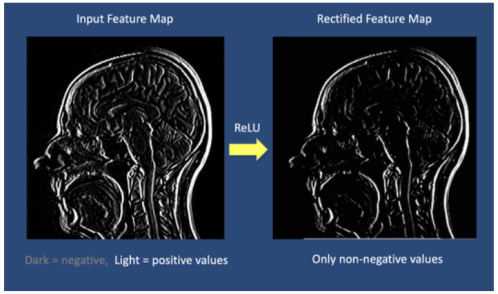

Appearance of one of the above 64 feature maps before and after application of ReLU activation function

CNN as a Hierarchical Feature Extractor

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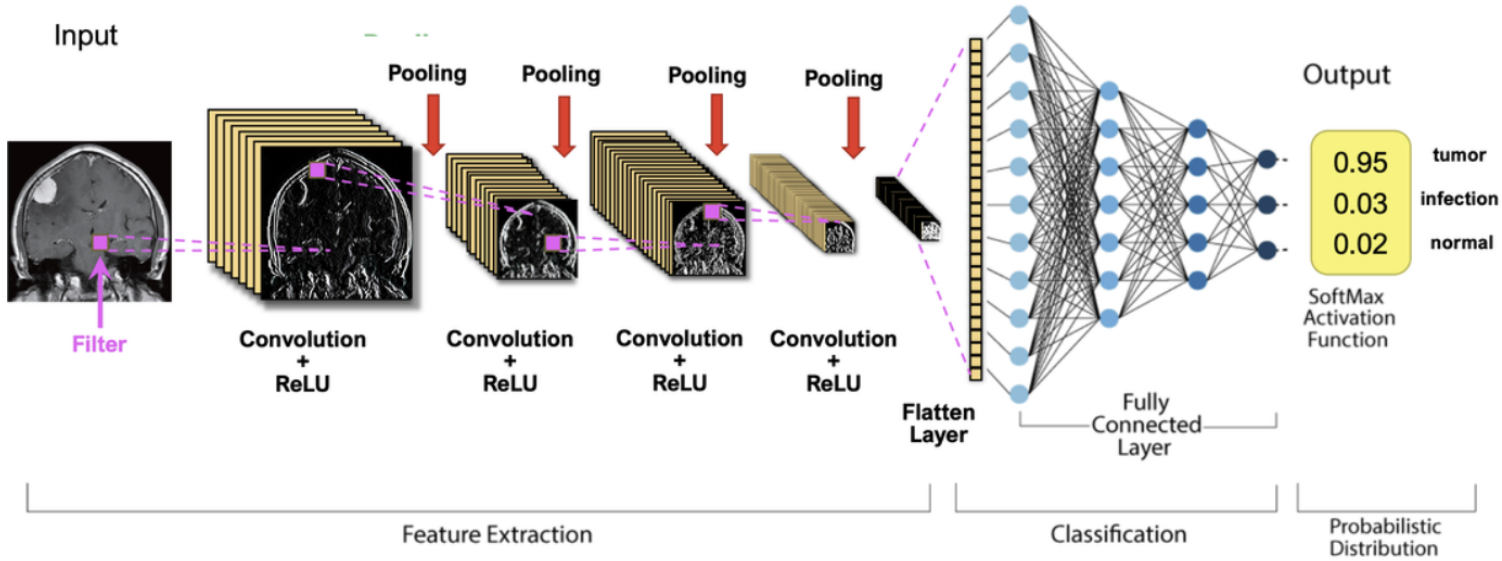


Input →	**Conv + ReLU** →	**Conv + ReLU** →	**Conv + ReLU**
Image / Signal	Edges, textures	Shapes, regions	High-level patterns

- Convolution layers perform **feature extraction**
- Fully connected layers perform **decision making**

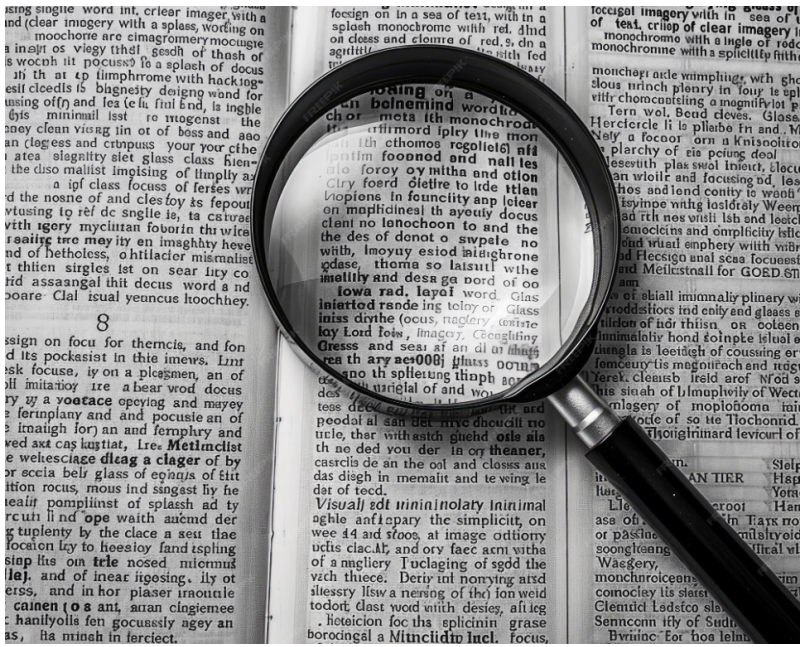
Analogy: from edges → structures → diagnosis

Question 6?

Do convolutional neural networks reduce the number of parameters by sharing the same filter across different parts of the image?

Do I still have your attention?

Now, let's see how machines focus on the important information, just like clinicians do.



"Attention is all you need" – Vaswani et al., 2017

Attention: Selecting Relevant Information

Human Attention

- When analysing imaging, clinicians focus on suspicious regions
- When reading clinical notes, doctors focus on key symptoms

Key Idea

Attention = focusing on relevant information while ignoring distractions

Attention: Selecting Relevant Information

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Key Idea

Attention = focusing on relevant information while ignoring distractions

Example: Clinical Note

> The patient reports a **headache** and took ibuprofen for arthritis.

Clinical Question:
What is the patient's main symptom?

Model Needs To Focus On

Headache

Attention: Selecting Relevant Information

Human Attention

- When analysing imaging, clinicians focus on suspicious regions
- When reading clinical notes, doctors focus on key symptoms

Key Idea

Attention = focusing on relevant information while ignoring distractions

Example: Clinical Note

> The patient reports a **headache** and took ibuprofen for arthritis.

Clinical Question:
What is the patient's main symptom?

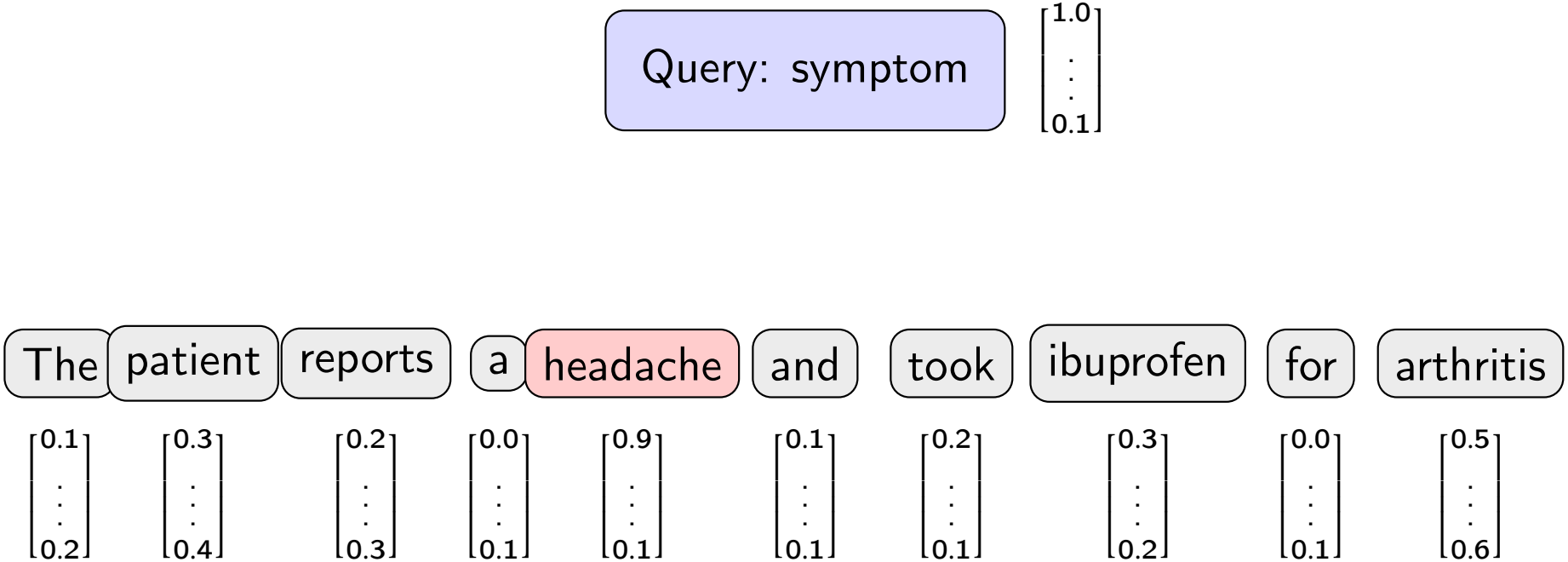
Model Needs To Focus On

Headache

Attention helps models focus on the right words, just like clinicians do.

How Does a Model Use Attention?

Clinical Question: What is the patient's main symptom?



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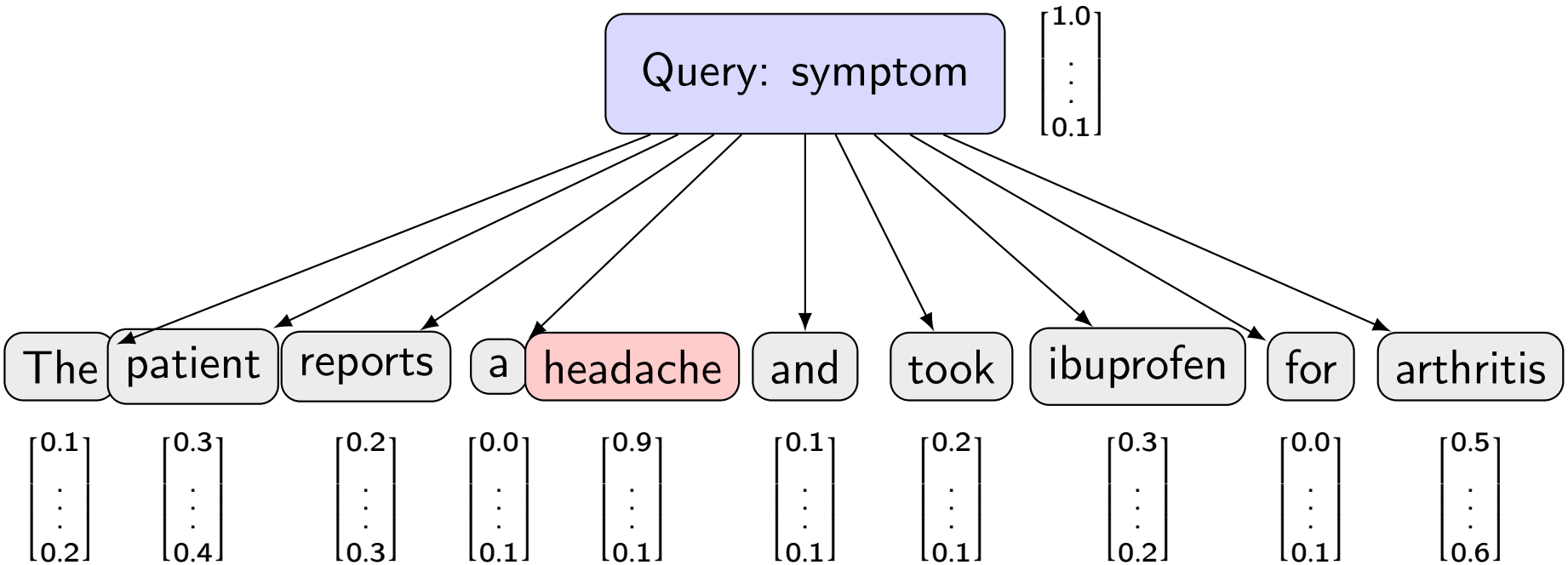
Clinical Question: What is the patient's main symptom?



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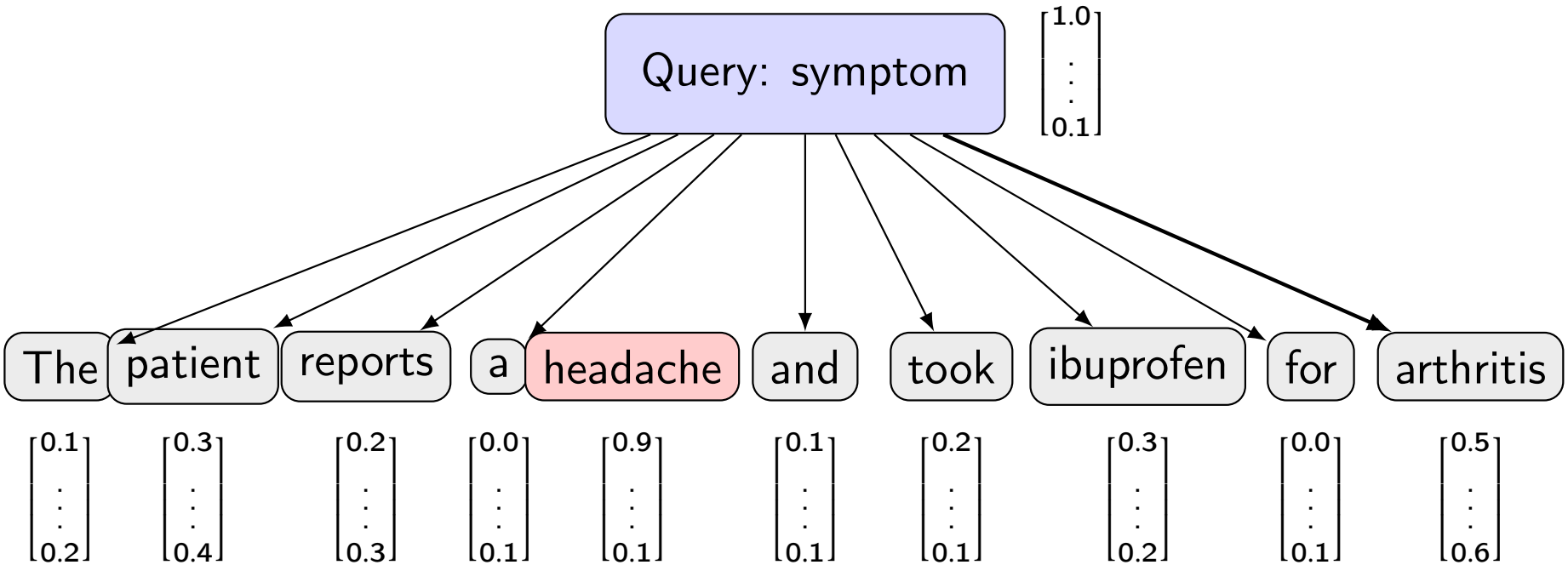

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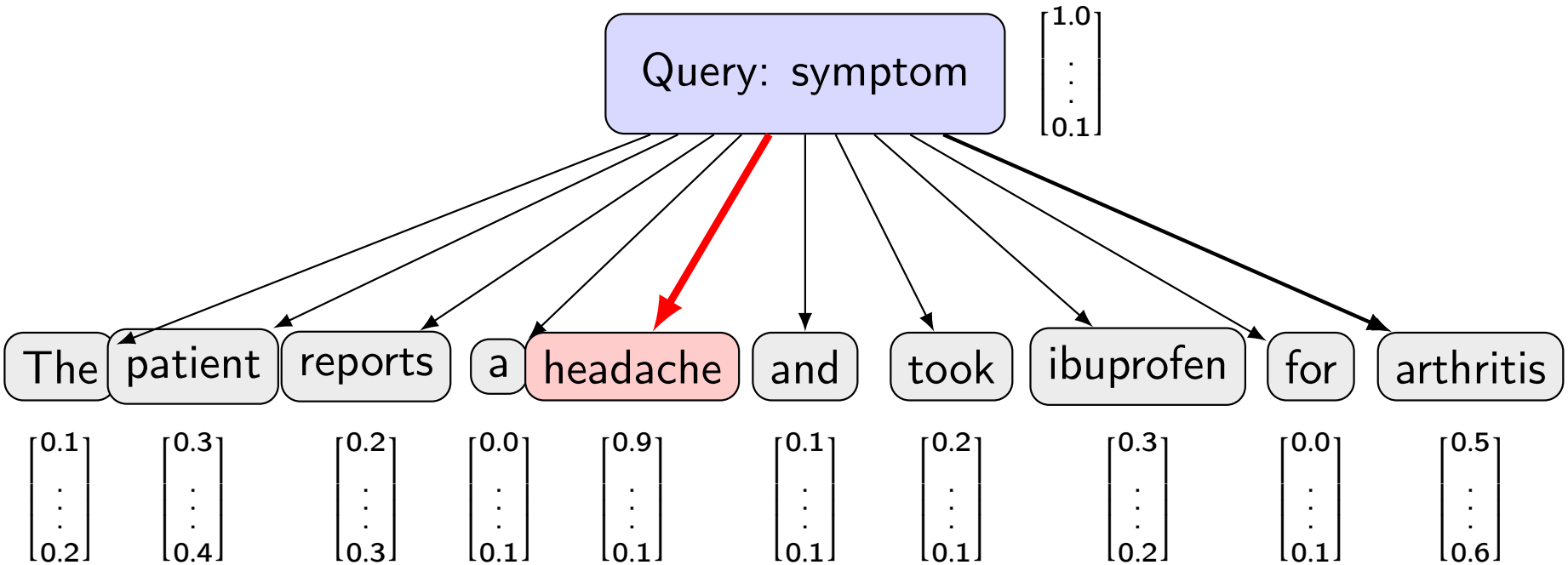

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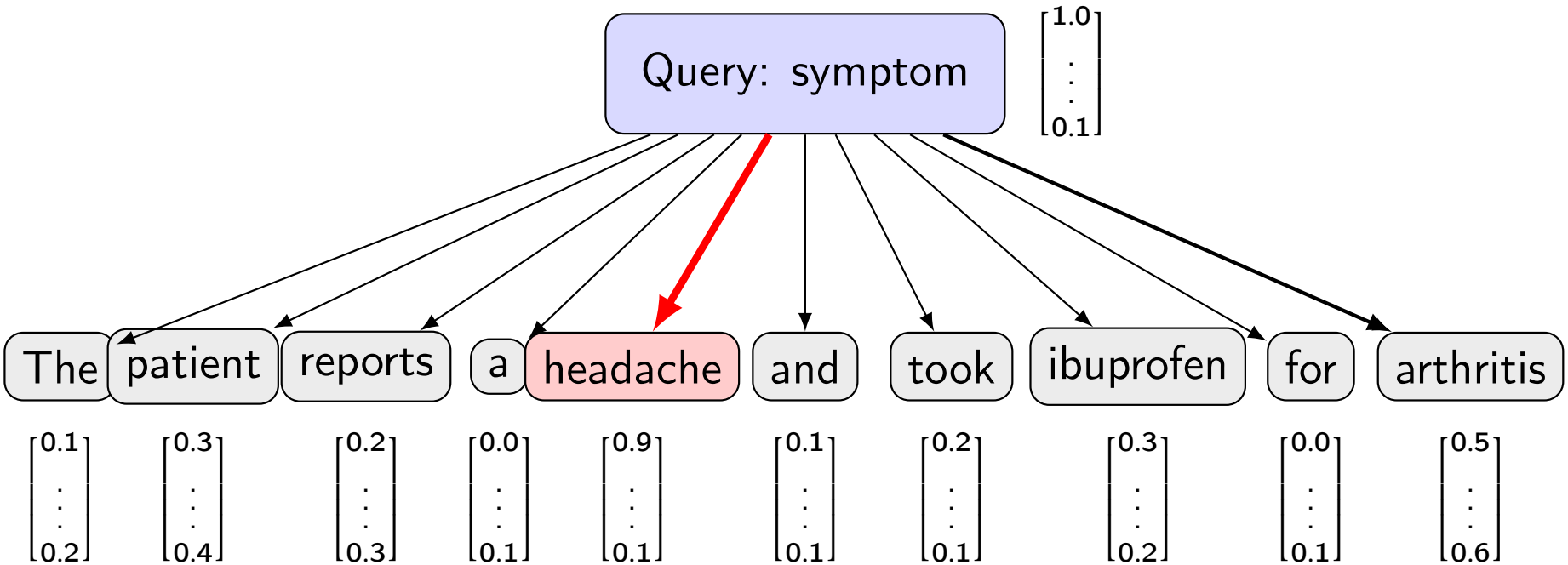
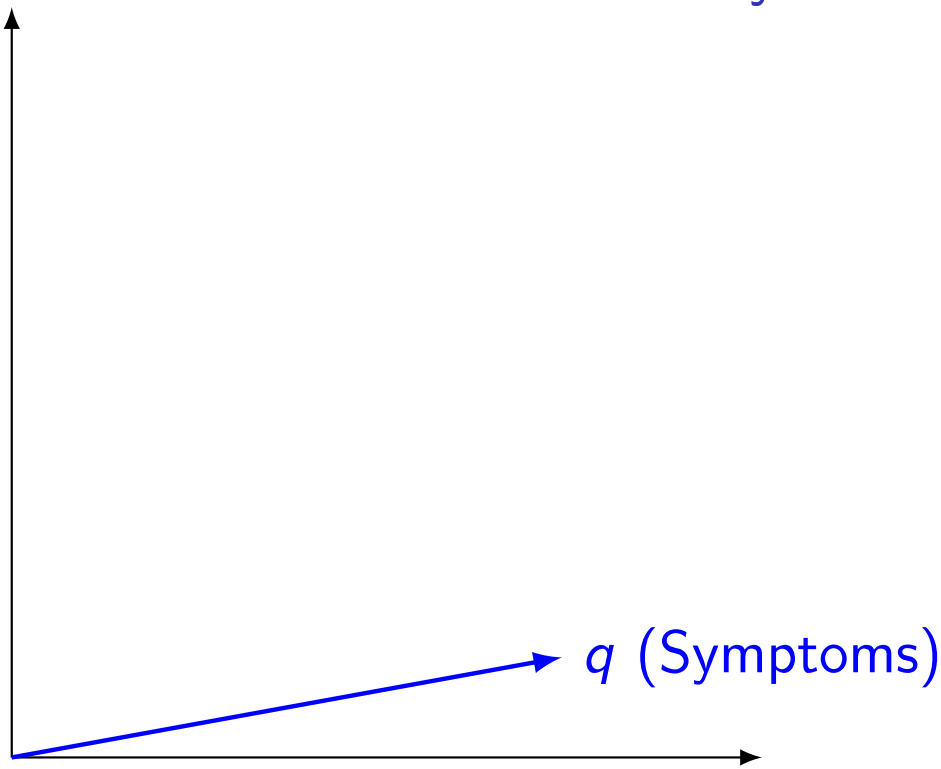


Key Idea

The model compares the question with each word and assigns importance using Dot Product.

Why Does Dot Product Measure Similarity?



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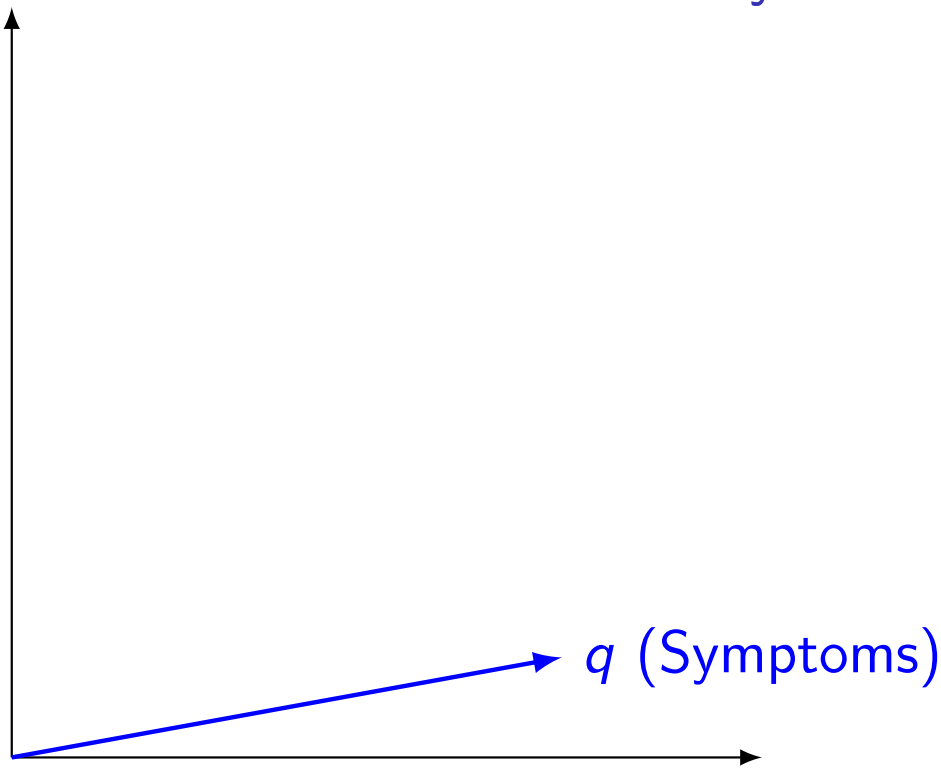
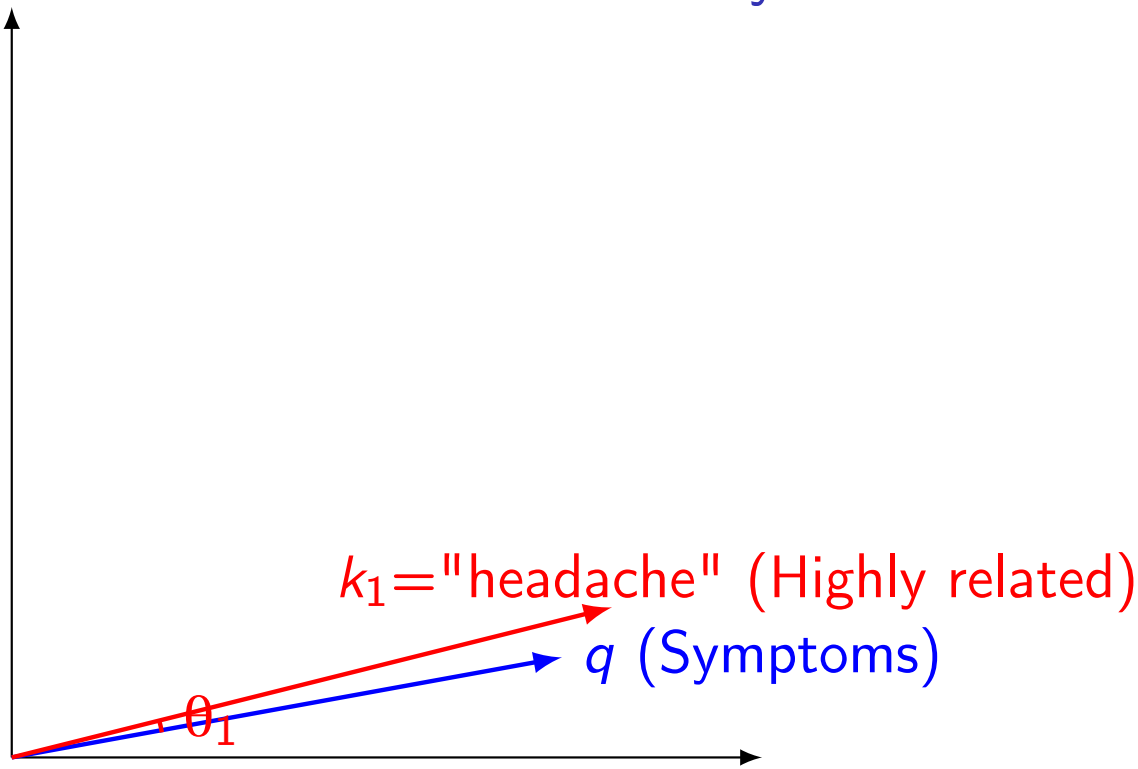


$$q \cdot k = \overbrace{||q||}^{\text{Norm of } q} \times \underbrace{||k||}_{\text{Norm of } k} \times \cos(\theta)$$

- Small angle $\rightarrow$ High similarity
- 90° angle $\rightarrow$ No similarity

Why Does Dot Product Measure Similarity?



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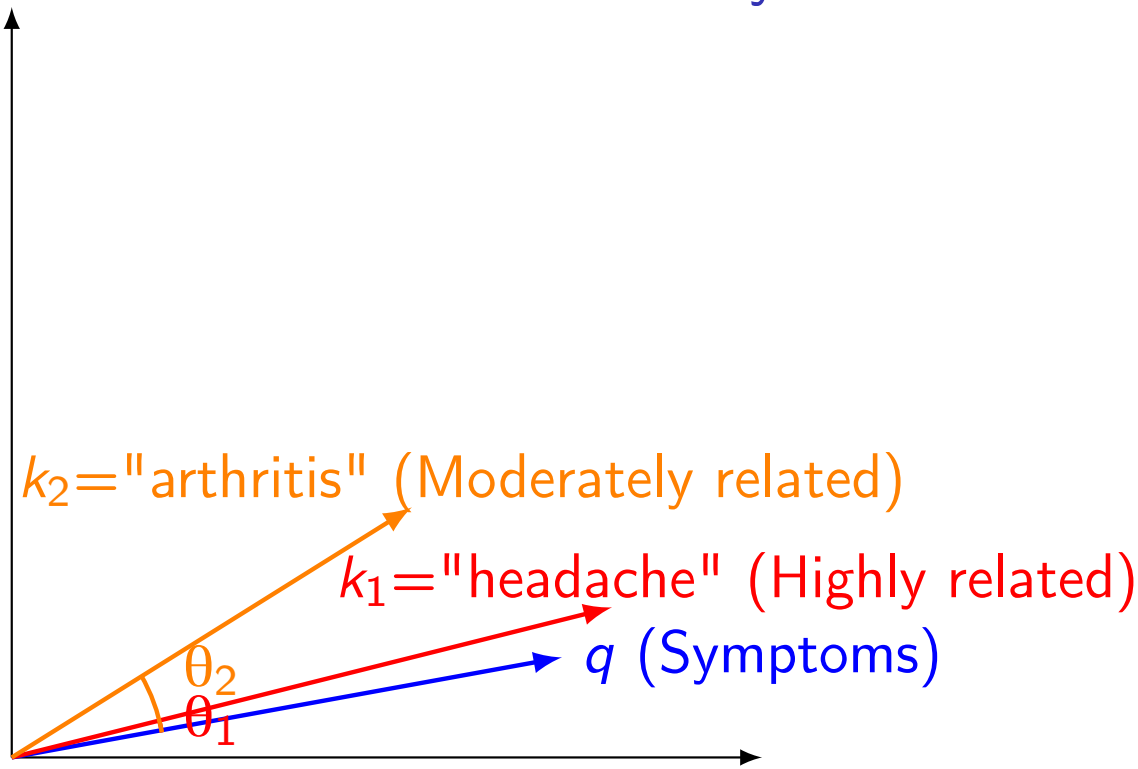

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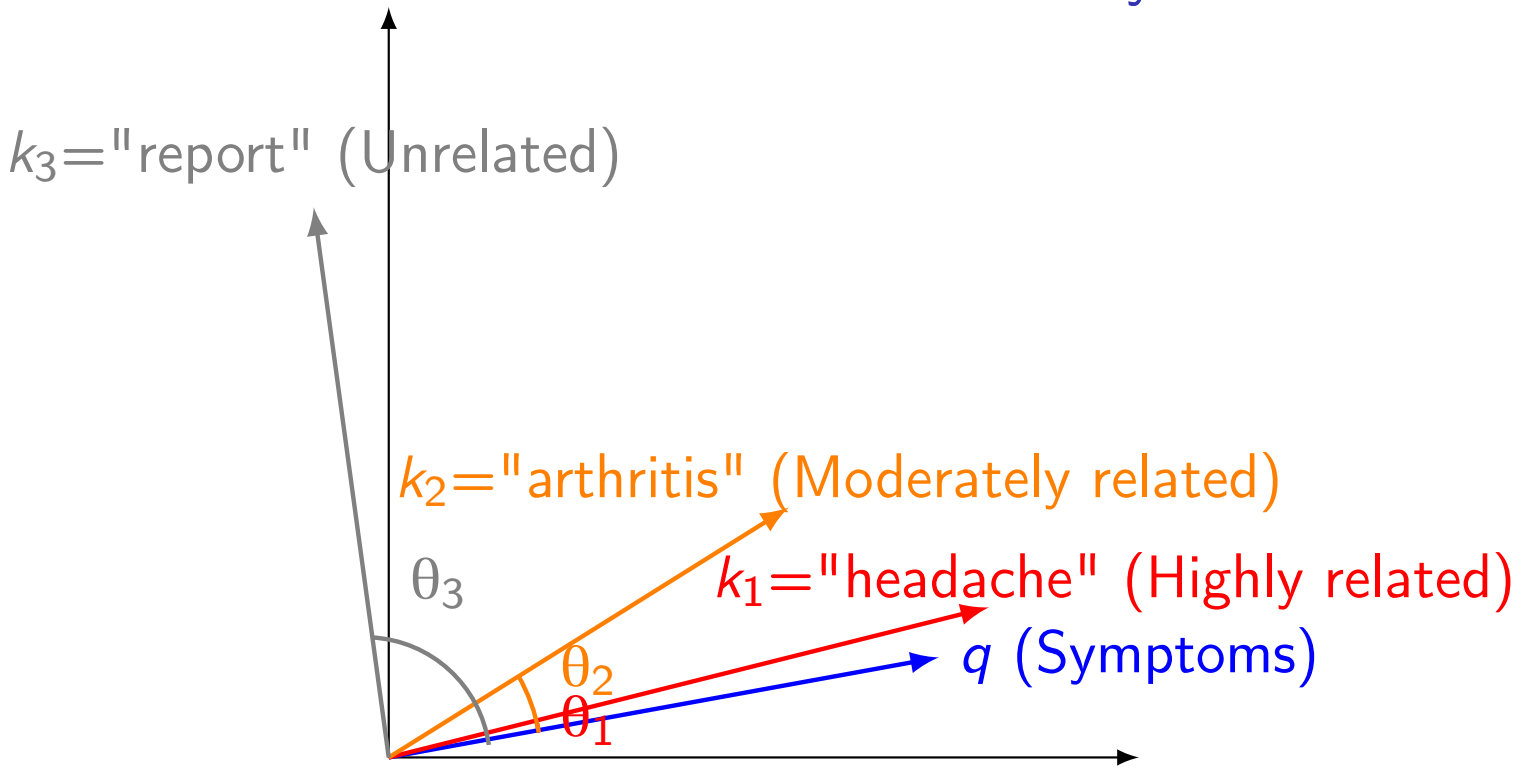
- 90° angle $\rightarrow$ No similarity

Why Does Dot Product Measure Similarity?



$$q \cdot k = \overbrace{||q||}^{\text{Norm of } q} \times \underbrace{||k||}_{\text{Norm of } k} \times \cos(\theta)$$

- Small angle $\rightarrow$ High similarity
- 90° angle $\rightarrow$ No similarity

From Similarity to Answer: Attention Mechanism

Step 1: Compute Attention Weights

- Softmax converts these similarity scores into weights α_i between 0 and 1
- Words more relevant to the question get higher weights

Word (k_i)	Weight α_i
The	0.02
patient	0.05
reports	0.03
a	0.01
headache	**0.75**
and	0.02
took	0.03
ibuprofen	0.05
for	0.02
arthritis	0.02

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Step 2: Build Context Vector

- Multiply each word vector by its attention weight α_i
- Sum weighted vectors to get the context vector:

$$c = \sum_i \alpha_i v_i$$

Context Vector $c \rightarrow$ mostly influenced by headache

Step 3: Model Output

- Context vector c is fed into the model's prediction layer
- Produces the answer to the clinical question

Main Symptom = Headache

Question 7?

Do attention mechanisms allow a model to give different importance to various parts of the input??

Thank you for your attention!

Key takeaway: The math behind these AI models is nothing new, just familiar concepts applied in clever ways.

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Question 8: *After this lecture, do neural networks feel less like a black box?*

Any questions?

Thanks to my co-authors: **Sarat Moka** and **Yoni Nazarathy**

1. Liquet, B., Moka, S., & Nazarathy, Y. (2024). *Mathematical Engineering of Deep Learning, 1st Edition*. Available online: https://deeplearningmath.org

2. Liquet, B., Moka, S., Nazarathy, Y. (2024). Book chapter. *Navigating Mathematical Basics: A Primer for Deep Learning in Science* Available online: https://deeplearningmath.org

Question 9: *Could you provide your email for the practical? We'll send you important information there.*