Some statistical approaches for movement ecology

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Plan

Movement Ecology

Definition : The study of the mechanisms responsible for the movement of invividuals.

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Goals : Identifying patterns in trajectories for

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- understanding the use of space (utilization distribution function),

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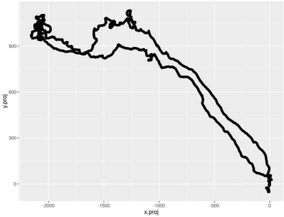

Peruvian booby data courtesy of Sophie Bertrand

Question : Identification of the diving events or ARS (Area-Restricted search) .

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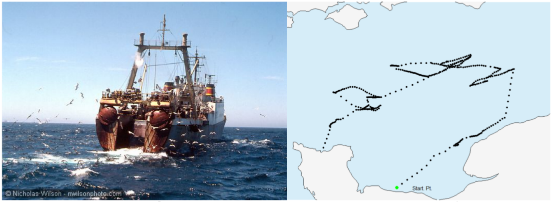

Fishing vessel trip in the Channel- Recopesca data of IFREMER

Question : Identification of the activity : Fishing or not fishing ?

How do they use space ?

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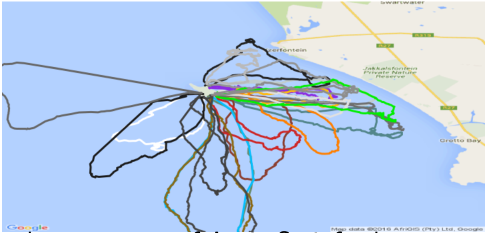


African penguin data courtesy of Antje Steinfurth

Question : Clustering individuals according to their feeding strategy

Recording movement - tagging animals

GPS and accelerometer tag © Sophie Bertrand

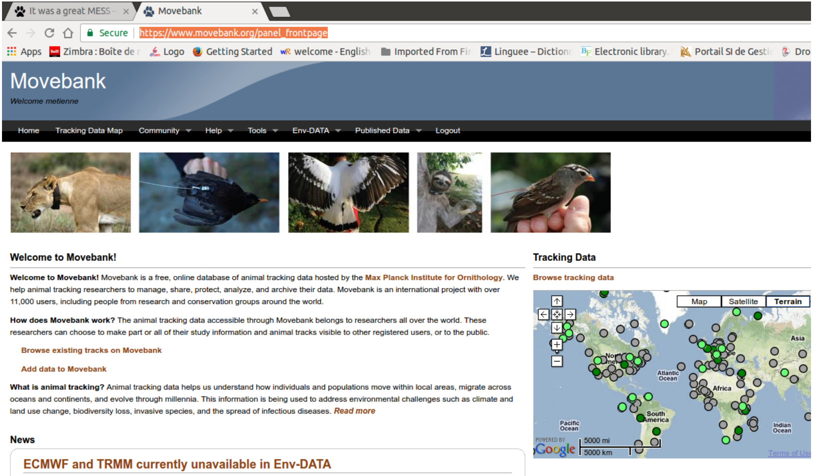
Recording movement - tagging animals

Masked booby with GPS and accelerometer ©Sophie Bertrand

Recording movement - tagging animals

Apollo, a famous hiena © MESS Workshop

Recording movement - tagging animals



Move Bank website https://www.movebank.org/panel_frontpage

Movement ecology data

A trajectory is, at least, a set of recorded location (relocations),

Time	Lat	Long
t_0	lat_1	$Long_1$
...		
t_n	lat_n	$Long_n$

Movement ecology data

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Precision of the relocations

Movement ecology data

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Time	Lat	Long
t_0	lat_1	$Long_1$
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Precision of the relocations

Highly dependant of the technology

Main technologies, different precisions

- Archival Global Location Sensing (GLS) tags
- ARGOS
- GPS

Main technologies, different precisions

- Archival Global Location Sensing (GLS) tags : record and store information on date, time, swim depth, water temperature, body temperature, and light levels (every few minutes over several years).
- ARGOS
- GPS

Main technologies, different precisions

- Archival Global Location Sensing (GLS) tags
- ARGOS : transmit data to satellites, no need to recapture the animal.

Table 1. Argos Doppler location class (LC) accuracy as estimated and documented by CLS (2011), based on the least-squares method of location derivation

Location class	Estimated error radius	Number of transmissions
3	<250 m	≥ 4
2	250–500 m	≥ 4
1	500–1500 m	≥ 4
0	>1500 m	≥ 4
A	No estimation	3
B	No estimation	2
Z	Invalid location	

Extract from [Douglas et al., 2012]

- GPS

Main technologies, different precisions

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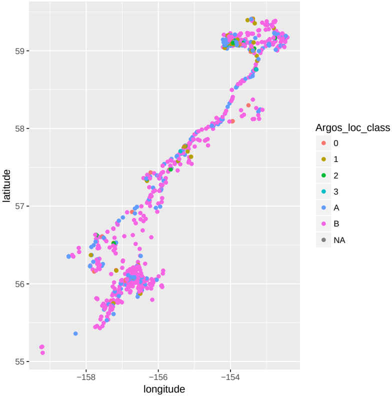


From [Johnson et al., 2008] Harbour seals trajectory

- GPS

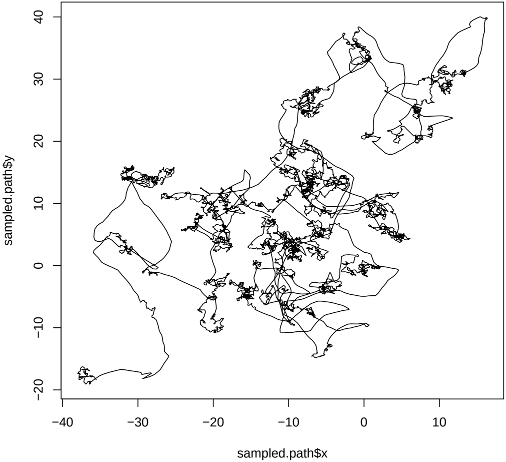
Main technologies, different precisions

- Archival Global Location Sensing (GLS) tags
- ARGOS
- GPS : receive data from satellites, very precise, need to be recovered.

Main technologies, different precisions

- Archival Global Location Sensing (GLS) tags
- ARGOS
- GPS : FOCUS

Effect of sampling step



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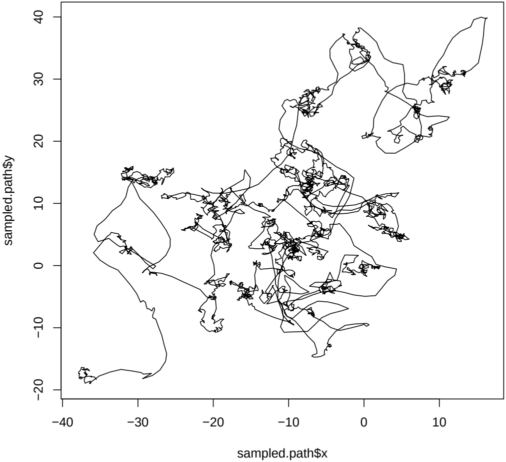

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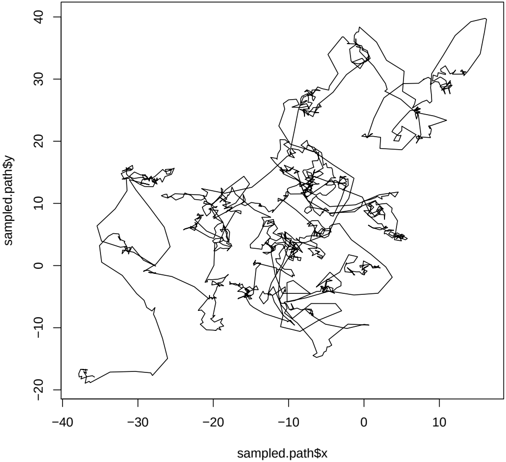

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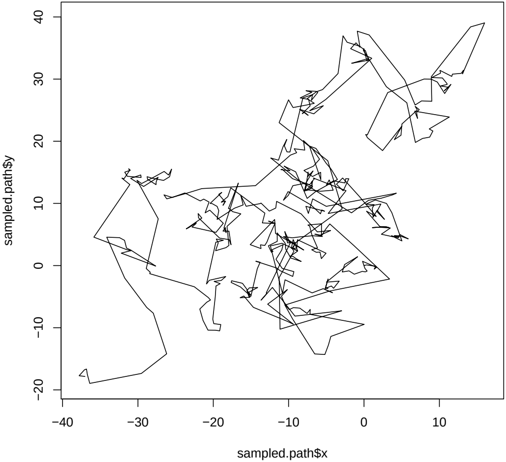

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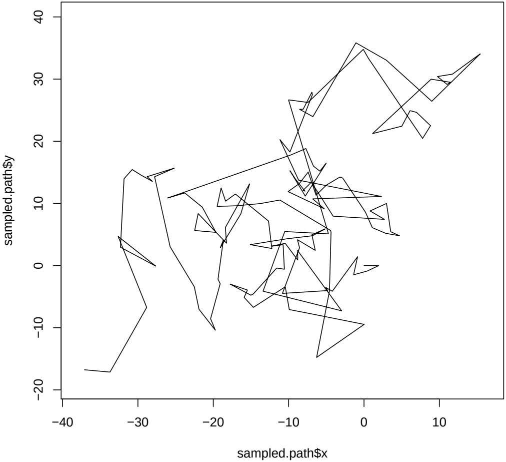

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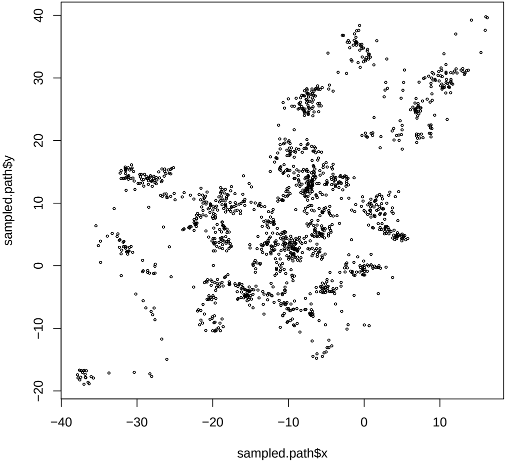


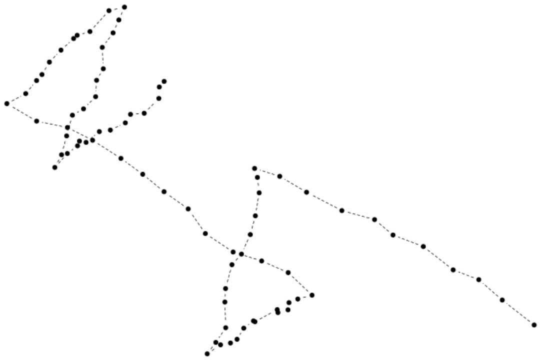
Methodological questions

- Segmentation of the trajectory to identify activities
- Reconstruction of the utilization distribution function
- (Filtering/Smoothing methods to reconstruct the 'true' position)

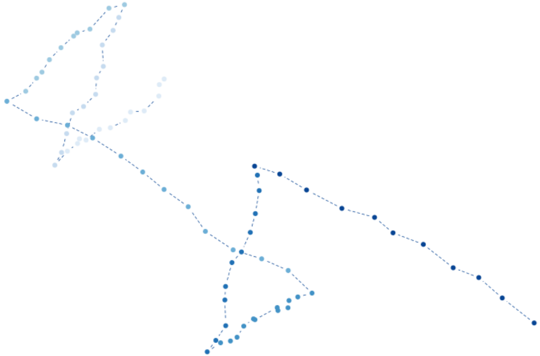
Plan

Segmentation and clustering

Segmentation and clustering



Segmentation and clustering



Segmentation and clustering

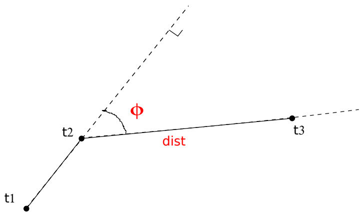
Summarising trajectories

$(t_1, \ldots, t_N)$ denotes the time acquisition and
$((x_1, y_1), \ldots, (x_N, y_N))$ the position at those times.
Trajetories as Turning angle and Speed

$$\mathbf{\Phi} = (\phi_2, \ldots, \phi_N)$$

$$\boldsymbol{S} = (S_2, \ldots, \S_N),$$

with $S_i = dist_i/(t_i - t_{i-1})$



Summarising trajectories

$(t_1, \ldots, t_N)$ denotes the time acquisition and
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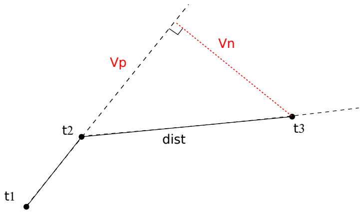
Trajectories as Persistent and Normal Velocity

$$\boldsymbol{V}^P = (V_2^P, \ldots, V_N^P)$$

$$\boldsymbol{V}^N = (V_2^N, \ldots, V_N^N)$$

with

$$V_i^P = S_i\, cos(\phi_i)$$
$$V_i^N = S_i\, sin(\phi_i)$$



Summarising trajectories

Using `adehabitatLT`, [Calenge, 2006].

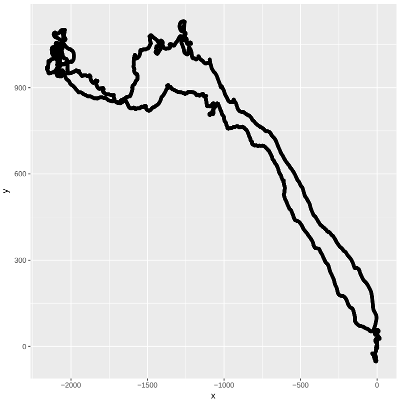
```
suppressMessages(library(adehabitatLT))
head(tab,n=4)
```

```
##              datetime       lon       lat    x.proj    y.proj
## 1 2012-11-15 18:22:33 -77.26383 -11.77127 1.1235933 -8.099363
## 2 2012-11-15 18:22:34 -77.26384 -11.77118 0.9896146 -6.542616
## 3 2012-11-15 18:22:35 -77.26384 -11.77104 1.0000083 -4.199344
## 4 2012-11-15 18:22:36 -77.26384 -11.77085 0.9247431 -1.132206
```

```
booby.ltraj <- as.ltraj(xy = tab[,which(names(tab)%in%c('x.proj', 'y.proj'))],
                        typeII = FALSE, id=rep(1,nrow(tab)))
plot(booby.ltraj)
head(booby.ltraj[[1]], n=4)
```

```
##           x         y date          dx       dy     dist dt       R2n
## 1 1.1235933 -8.099363    1 -0.13397870 1.556746 1.562501  1  0.000000
## 2 0.9896146 -6.542616    2  0.01039374 2.343272 2.343295  1  2.441409
## 3 1.0000083 -4.199344    3 -0.07526522 3.067138 3.068061  1 15.225416
## 4 0.9247431 -1.132206    4  0.26737063 3.414525 3.424977  1 48.580806
##   abs.angle   rel.angle
## 1  1.656648          NA
## 2  1.566361 -0.09028728
## 3  1.595331  0.02896985
## 4  1.492652 -0.10267876
```

From trajectories to (bi)variate signal



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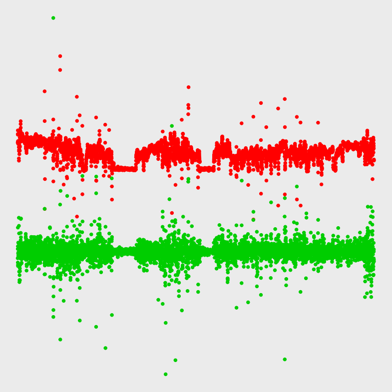
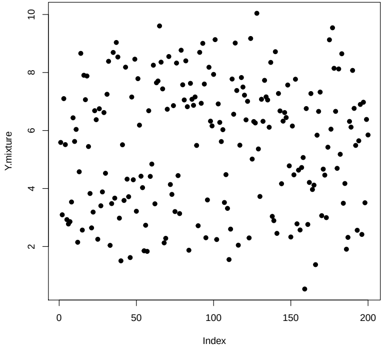


Signal processing methods

Method 1 - Clustering methods



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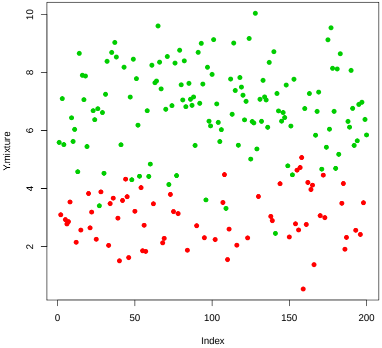


Method 1 - Clustering methods

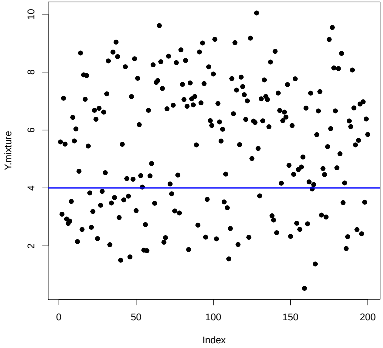
Basic idea :

"Expert" threshold s

$$State_i = 1 \quad \text{if} Y_i < s$$

$$State_i = 2 \quad \text{if} Y_i \geq s$$

`VMSbase`, `vmstools` packages as done in the ICES scientific groups



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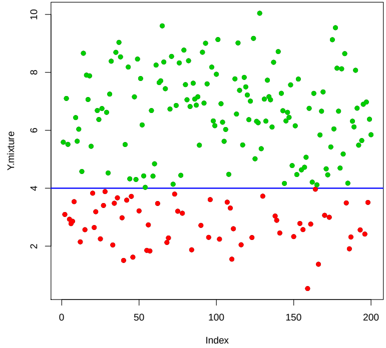
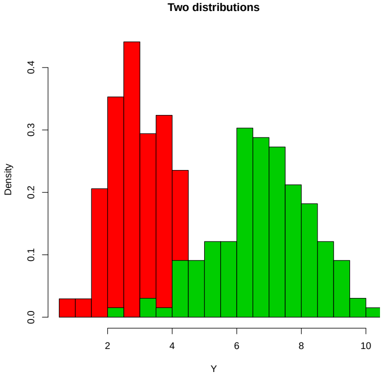
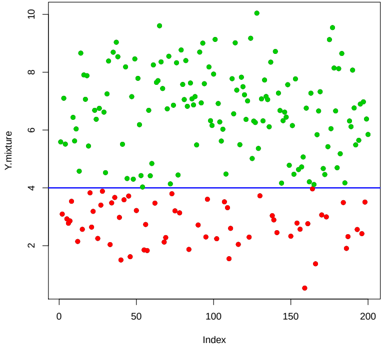


Method 1 - Clustering methods

Improvement :

- K-means, Random Forest, (in [Joo et al., 2013])
- Mixture model `Rmixmod`, `mclust` packages



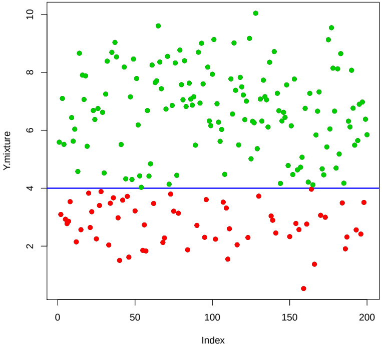


Method 1 - Clustering methods

Clustering methods :

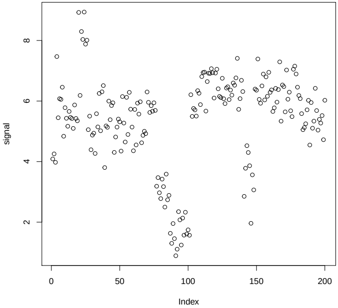
- "Expert" threshold
- K-means,
- Mixture model

No time coherence



Method 2 - Segmentation (Change point detection)

According to [Lavielle, 2005],



Goal : Identifying homogenous regions and abrupt changes in the signal.

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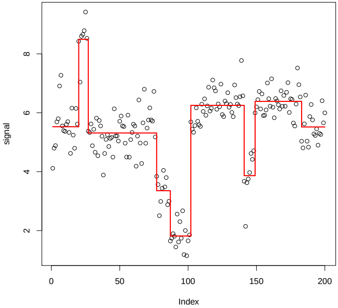


Goal : Identifying homogenous regions and abrupt changes in the signal.

These regions may be interpreted afterwards.

Underlying model

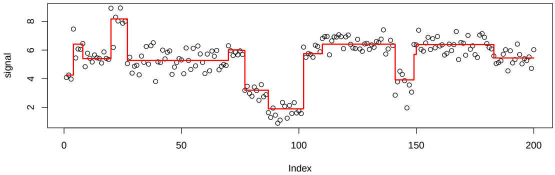
K being given, $t_1 < t_2 < \ldots < t_{K-1}$, the breakpoints :

$$Y_t \overset{i.i.d}{\sim} f(\theta_k) \text{ if } t \text{ in region } I_k = [t_{k-1}+1, t_k]$$

Underlying model

K being given, $t_1 < t_2 < \ldots < t_{K-1}$, the breakpoints :

$$Y_t \overset{ind}{\sim} \mathcal{N}(\mu_k, \sigma^2) \text{ if } t \text{ in portion } I_k, \text{ for } k = 1, \ldots, K.$$



Remark : $K - 1$ change points $\Leftrightarrow$ K regions.

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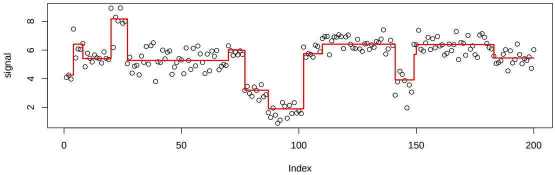

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Based on dynamic programming, available in `adehabitatLT`.

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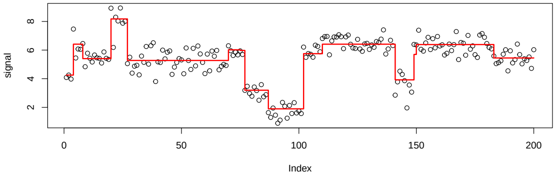

Remark : $K - 1$ change points $\Leftrightarrow$ K regions.

Based on dynamic programming, available in `adehabitatLT`.

Extended to univariate AR signal by [Chakar et al., 2017] in `AR1seg` and to bivariate signal by [Etienne et al., 2017] to be implemented in `adehabitatLT`

Underlying model

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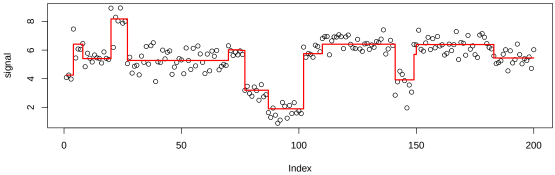


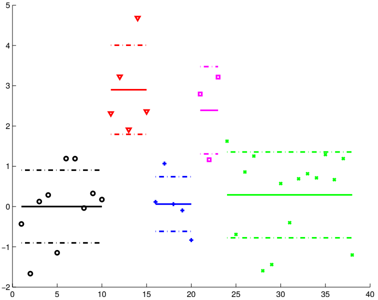
Remark : $K - 1$ change points $\Leftrightarrow$ K regions.

Does not define activities !

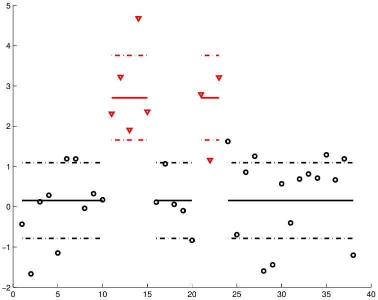
Method 3 - Segmentation/Clustering

When Segmentation is not sufficient - clustering segmentation model

Pure segmentation



Segmentation + classification



Segmentation-Clustering

- The distribution of the signal given the group of the segment is

$$t \in I_k, k \in p \qquad \Rightarrow \qquad Y_t \sim \mathcal{N}(m_p, \sigma^2)$$
$$Y^k | Z_{kp} = 1 \sim \mathcal{N}(m_p, \sigma^2).$$

- It is a model of segmentation / clustering.

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- Model parameters are $\boldsymbol{\theta} = (\mathbf{\Pi}, \gamma)$ and the breakpoint positions $\mathbf{T} = (t_1, ..., t_{K-1})$.

Segmentation-Clustering

- The distribution of the signal given the group of the segment is

$$
\begin{aligned}
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$$

- It is a model of segmentation / clustering.
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Described in [Picard et al., 2011], available in `cghseg` for univariate signal, on going work for extension to bivariate signal (to be implemented in `adehabitatLT`)

Method 4 - State Space Model

Model For a given number of states K,

- Hidden States $\mathbf{Z}$ model : $\mathbf{Z}$ is assumed to follow a Markov Chain model with unknown initial distribution $\boldsymbol{\nu}$ and transition matrix $\mathbf{\Pi}$.
- Observations $\mathbf{Y}$ model : $(Y_i|, Z_i = k) \overset{i.i.d}{\sim} f_{\gamma_k}()$.

Model parameters are $\boldsymbol{\theta} = (\boldsymbol{\nu}, \mathbf{\Pi}, \gamma)$

Estimation used either EM algorithm or Stochastic algorithms.

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Popular movement models in ecology

Discrete time, discrete space models

- Markov model on lattice (Tuna, [Pedersen et al., 2011])

Discrete time, continuous space models

- Random walk with Drift (RWD,[Patterson et al., 2008]), Vector Auto Regressive model (VAR,[Gloaguen et al., 2015]), Continuous time correlated Randow walk (CTCRW) : integrated Ornstein Uhlenbeck process
- Turning angles and speed ([Walker and Bez, 2010]).

Continuous time, continuous space models

- Successive moves (duration, direction and angles), Levy flights
- Stochastic Differential Equations

Plan

Charaterizing habitat

Home Range (HR) :

the territory of an individual, groups or species

Utilization Distribution (UD) :

probability of presence

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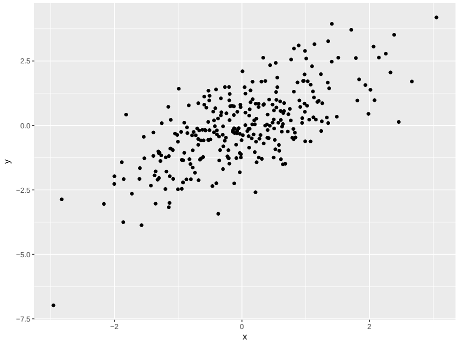

GPS positions

Charaterizing habitat

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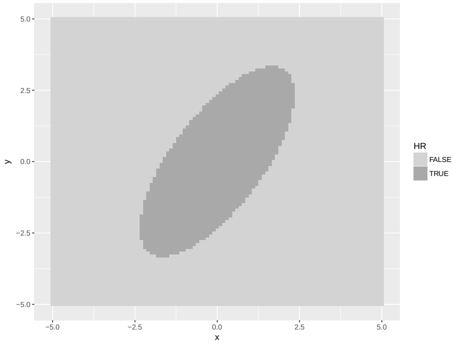

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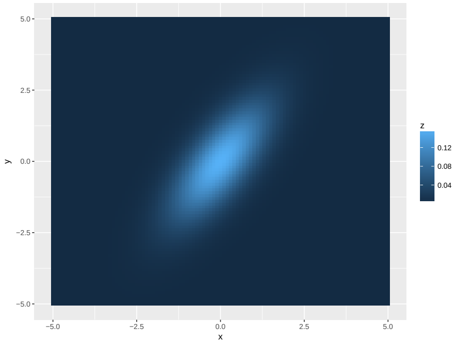

Utilization distribution function

Forgetting time with kernel methods

Denoting by P_t, the position at time t, GPS data consists in $\mathbf{P} = (P_{t_1}, \ldots, P_{t_n})$.
The classical methods for characterizing utilization distribution rely on kernel density estimation.

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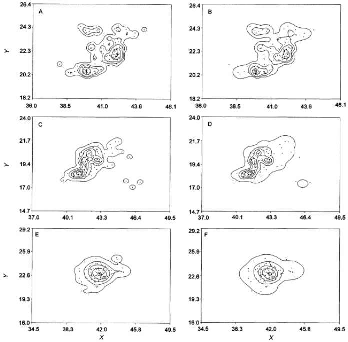

Fig. 5. Telemetry locations with fixed (A, C, E, G, I) and adaptive (B, D, F, H, J) kernel contours for five black bear home ranges: bear 106 (A, B); bear 70 (C, D); bear 163 (E, F); bear 72 (G, H); bear 61 (I, J). Contours and symbols are as in Fig. 3. Axis values are truncated UTM coordinates (km).

UD for 5 black bears in [Seaman and Powell, 1996]

Forgetting time with kernel methods

Denoting by P_t, the position at time t, GPS data consists in $\mathbf{P} = (P_{t_1}, \ldots, P_{t_n})$.
The classical methods for characterizing utilization distribution rely on kernel density estimation.

Corresponding UD definition :

$$U(s) = \frac{1}{T} \int_0^T \mathbf{1}_{(P_t \in ds)} dt$$

Some assumption on ergodicity or stationnarity, mostly independance.

Forgetting time with Brownian Bridge methods

Assuming the movement is a Brownian Motion (Ohhh !), the Brownian Bridge methods compute the probability of presence between 2 successive relocations

Forgetting time with Brownian Bridge methods

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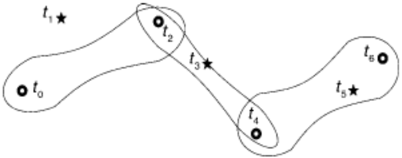

FIG. 2. Example of three Brownian bridges connecting even observations at time intervals $[t_0, t_2]$, $[t_2, t_4]$, and $[t_4, t_6]$. The in-between observations at times t_1, t_3, and t_5 are independent observations from these Brownian bridges and can be used to estimate the Brownian motion variance parameter.

From [Horne et al., 2007]

Forgetting time with Brownian Bridge methods

Assuming the movement is a Brownian Motion (Ohhh !), the Brownian Bridge methods compute the probability of presence between 2 successive relocations

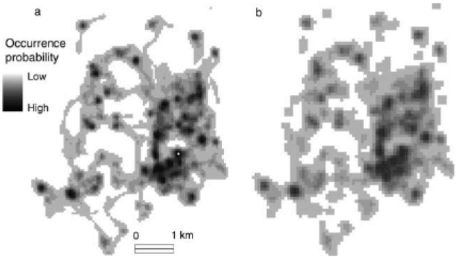

FIG. 3. Estimated home range (i.e., utilization distribution) of male black bear in northern Idaho, USA. The range in (a) is calculated using the Brownian bridge movement model with a variance parameter = 642.44. The range in (b) is calculated using a fixed kernel density estimate with a smoothing parameter = 76.95. In both panels, the outer contour represents the 99% contour.

From [Horne et al., 2007]

Forgetting time with Brownian Bridge methods

Assuming the movement is a Brownian Motion (Ohhh !), the Brownian Bridge methods compute the probability of presence between 2 successive relocations

Corresponding UD function :

$$\tilde{U}(s) = \frac{1}{T} \int_0^T \mathbb{P}(P_t \in ds | \mathbf{P}) dt$$

The differences are properly developped in [Gloaguen, 2015].

Improving movement model

Improving movement model

Discrete time, continuous space models

- Random walk with Drift (RWD), Vector Auto Regressive model (VAR), Continuous time correlated Randow walk (CTCRW) : integrated Ornstein Uhlenbeck process
- Turning angles and speed.

Continuous time, continuous space models

- Successive moves (duration, direction and angles), Levy flights
- Stochastic Differential Equations

Improving movement model

[Brillinger et al., 2002] models the movement as a consequence of some unknown potential function.

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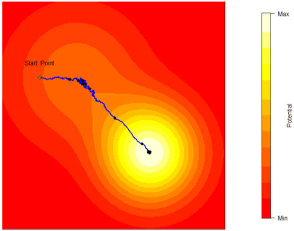


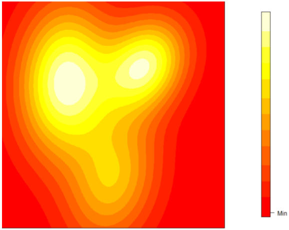
Movement driven by a potential map.

$$dX_t = \nabla P(X_t) + \gamma dW_t$$

- Deterministic part $P(\cdot)$ the potential function.
- Stochastic part γ a diffusion coefficient and $(W_t, t \geq 0)$ a standard Brownian motion.

Improving movement model

[Brillinger et al., 2002] models the movement as a consequence of some unknown potential function.



Some parametric form ?

$$dX_t = \nabla P(X_t) + \gamma dW_t$$

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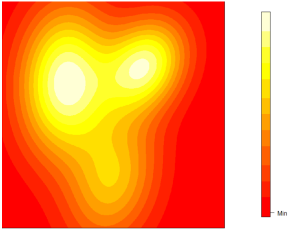

Some parametric form ?

$$dX_t = \nabla P(X_t) + \gamma dW_t$$

- Deterministic part
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Improving movement model

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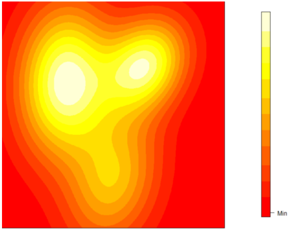


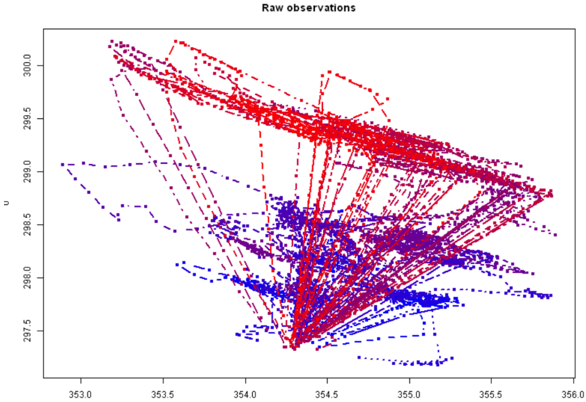
Some parametric form ?

- Deterministic part
  Gaussian shape function with K componants

$$P(X_t) = \sum_{k=1}^{K} \pi_k \varphi_k(X_t)$$

- Stochastic part γ a diffusion coefficient and $(W_t, t \geq 0)$ a standard Brownian motion.

Application on Fishermen trajectories



Application on Fishermen trajectories

Comparing 4 estimation methods

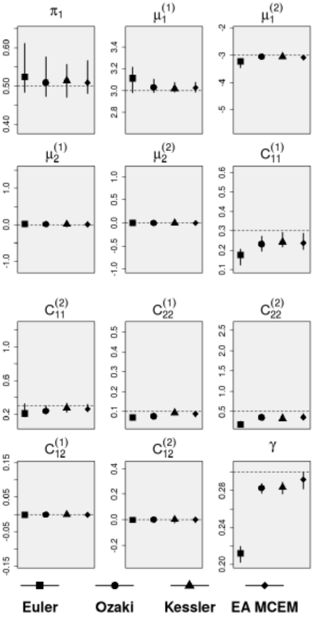
- Euler approximation (normal transition, 1st order)
- Ozaki approximation (normal transition, 2d order, [Shoji and Ozaki, 1998])
- Kessler approximation (normal transition, adptative mean, [Kessler et al., 2012])
- Beskos exact algorithm (MCEM, exact simulation, [Beskos et al., 2009])

Application on Fishermen trajectories

On simulated dataset

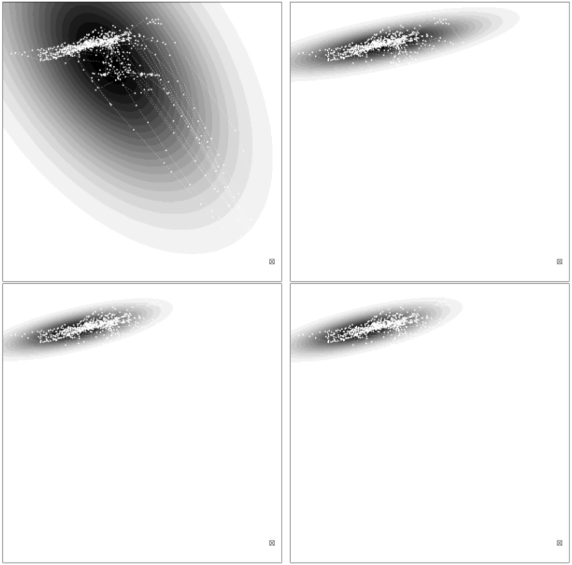
- Euler approximation : poor, biased estimation especially for low frequency data
- Ozaki approximation : good
- Kessler approximation : good, some unstability issue
- Beskos exact algorithm : good, as soon as no bugs left

Application on Fishermen trajectories



Application on Fishermen trajectories

(a) Euler (b) Kessler



(c) Ozaki (d) EA MCEM

Plan

Conclusions

- Identifying activities,
- Potential maps for flexible movement models using SDE,

Perspectives

- Switching SDE,
- Interaction between individuals.

Thank you for your attention

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Popular movement models in ecology

Discrete time, discrete space models

- Markov model on lattice (Tuna, [Pedersen et al., 2011])

Discrete time, continuous space models

- Random walk with Drift (RWD,[Patterson et al., 2008]), Vector Auto Regressive model (VAR,[Gloaguen et al., 2015]), Continuous time correlated Randow walk (CTCRW) : integrated Ornstein Uhlenbeck process
- Turning angles and speed ([Walker and Bez, 2010]).

Continuous time, continuous space models

- Successive moves (duration, direction and angles), Levy flights
- Stochastic Differential Equations

Popular movement models in ecology

Discrete time, continuous space models

- Random walk with Drift (RWD),

$$Y_{i+1} = Y_i + \boldsymbol{\nu} + E, \quad E \sim \mathcal{N}(0, \boldsymbol{\Psi})$$

- Vector Auto Regressive model (VAR),

$$Y_{i+1} = \boldsymbol{R} Y_i + \boldsymbol{\nu} + E, \quad E \sim \mathcal{N}(0, \boldsymbol{\Psi})$$

- Continuous time correlated Randow walk (CTCRW) : integrated Ornstein Uhlenbeck process

$$dX_t = -\tau X_t + \boldsymbol{\nu} + \boldsymbol{\Psi}^{1/2} dW_t$$
$$Y_{i+1} = \int_{t_{i-1}}^{t_i} X_t dt$$

Gaussian explicit transition density functions

Popular movement models in ecology

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Gaussian explicit transition density functions