# RMT Summer School in Japan 2025

*September 8–12, 2025 — Kyoto University*

## Schedule

| Time | 8 (Mon) | 9 (Tue) | 10 (Wed) | 11 (Thu) | 12 (Fri) |
|---|---|---|---|---|---|
| 09:20–10:40 | — | Guionnet I | Guionnet II | Nahmod IV | Guionnet IV |
| 11:10–12:30 | — | Nahmod II | Erdős II | Guionnet III | Erdős IV |
| 12:30–14:00 (Lunch) | | | | | |
| 14:00–15:20 | Erdős I | Nahmod III | — | Erdős III | — |
| 15:50–17:10 | Nahmod I | Poster session | — | Poster session | — |

**Location:** Room 401, Building 6

## Lecturers and Abstracts

### Alice Guionnet (ENS Lyon)

**Lecture:** Large deviations for the largest eigenvalue of large random matrices, and applications

**Abstract:** In this mini-course I will review several recent large deviation results for large random matrices, and their relations with spin glasses and the study of the volume of local minima of random functions. We will discuss in particular their universality and how universality is broken by the localization phenomenon.

### Andrea Nahmod (University of Massachusetts Amherst)

**Lecture:** Random Tensor Theory, Propagation of Randomness, and Nonlinear Dispersive PDE

**Abstract:** The study of randomness in partial differential equations (PDEs) goes back more than seventy years. Nonlinear dispersive PDEs naturally appear as models describing wave phenomena in quantum mechanics, nonlinear optics, plasma physics, water waves, and atmospheric sciences. One way in which randomness enters the field of nonlinear dispersive PDE is via the random data Cauchy initial value problem for (deterministic) equations, such as the nonlinear Schrödinger (NLS) and the nonlinear wave equations (NLW). The interest comes from two fundamental problems: (1) invariance of measures such as Gibbs measures which are physical equilibria for these systems; arising naturally in statistical mechanics and closely related also to QFT models such as the $\Phi^4$, and (2) the study of generic behavior of solutions in the probabilistic sense, and how they are expected to be better than worst case (exceptional) scenarios.

The study of this subject in the context of dispersive PDEs can be traced back to Lebowitz–Rose–Speer (1988, 1989) and Bourgain (1994, 1996) concerning the Gibbs measure for NLS. Since then there have been substantial developments of their ideas by many different researchers, extending them in different directions (geometric, infinite volume, other dispersive relations). In recent years, this field has seen significant progress and many new ideas and methods have been introduced. In this mini-course we will explain the method of random averaging operators and the theory of random tensor (both by Y. Deng, A.N. and H. Yue) as well as new bilinear tensor norms introduced in subsequent work also with B. Bringmann. These new methods have led to the resolution of several important open questions in this field, and are expected to play more important roles in future developments. The aim of this course is to provide the foundations upon which these recent developments have built upon, in particular Bourgain's seminal work in the subject.

### László Erdős (IST Austria)

**Lecture:** Multi-resolvent local laws and their applications

**Abstract:** Classical local laws in random matrix theory assert that the resolvents of large random matrices tend to be deterministic even for spectral parameters very close to the real axis. They are robust and provide essential a priori bounds for eigenvalue and eigenvector distributions that are routinely used in more sophisticated analysis. Products of resolvents also tend to be deterministic, but they are not simply given as a product of single resolvent approximations. In this series of lectures we present a theory of multi-resolvent local laws. The proofs are dynamical, they rely on the so-called zig-zag strategy; a successive alternate application of two different stochastic flows. In the second part of the lectures we focus on applications that include the proof of the Eigenstate Thermalisation Hypothesis and the Law of Fractional Logarithm for general Hermitian random matrices, as well as CLT for linear eigenvalue statistics and the Gumbel law for extremal eigenvalues for non-Hermitian random matrices.

## Poster Sessions

Poster sessions will take place on Tuesday and Thursday from 15:50 to 17:10.

## Poster Titles

## Tuesday, 09-09

- Raunak Shevade: Limiting Spectral Distribution of large Kendall's correlation matrix and its application under moderately high dimensions

  **Abstract:** This work establishes the Limiting Spectral Distribution (LSD) of Kendall's correlation matrix for a moderately high dimensional setup. The existing approaches in the literature establish LSD for Kendall's correlation matrix when the observations are independent and identically distributed in a high dimensional setup. These approaches further as- sume that the observations are from distributions that are continuous with respect to Lebesgue measure. This work establishes LSD of Kendall's correlation matrix in a moderately high dimensional setup when the observations are in- dependent but not necessarily identically distributed. The assumption regarding the continuous nature of observations is also relaxed. This relaxation on the assumptions enhances the scope of distributions for which our result holds. It paves way to include observations from heavy tailed distributions of discrete, continuous and mixed setups. An important application of this result is developed in the form of a graphical test of independence of features. Since the class of observations for which the result holds is large, the test developed is more versatile/robust in nature. The behaviour and performance of the test is also analysed numerically.

- Wonjun Seo: Phase Transitions for Linear Discriminant Analysis under Ultra-high Dimensional Regime

- Samuel Gurrola Viramontes: Bounds on the Energy of Graphs: Applications in Random Graphs

  **Abstract:** Graph energy has been a subject of increasing interest due to its connections to various fields, including chemistry, where it is used to model molecular stability, and network theory, where it provides insights into structural properties. A new

bound on the energy of graphs in terms of their degrees and the number of leaves adjacent to each vertex was established. Additionally, it explored applications in random graph models like Barabási–Albert trees and Erdős–Rényi graphs. For more details see arXiv:2502.01065.

- Oleksii Kolupaiev: Law of fractional logarithm and decorrelation transition in the Wigner minor process
- Patrick Oliveira Santos: Graphon-Theoretic Approach to Central Limit Theorems
- Yeonggwang Jung: Spectral analysis of $q$-deformed unitary ensembles with the Al-Salam & Carlitz weight
- Yoochan Han: Spectral properties and weak detection in stochastic block models

  **Abstract:** We consider the spectral properties of balanced stochastic block models of which the average degree grows slower than the number of nodes (sparse regime) or proportional to it (dense regime). For both regimes, we prove a phase transition of the extreme eigenvalues of SBM at the Kesten-Stigum threshold. We also prove the central limit theorem for the linear spectral statistics for both regimes. We propose a hypothesis test for determining the presence of communities of the graph, based on the central limit theorem for the linear spectral statistics.

- Danilo Jr Dela Cruz: Spectral Properties of the Signature Transform
- Anas Rahman: A multiscale cavity method for sublinear-rank symmetric matrix factorization

  **Abstract:** We consider a statistical model for symmetric matrix factorization with additive Gaussian noise in the high-dimensional regime where the rank $M$ of the signal matrix to infer scales with its size $N$ as $M = o(\sqrt{\ln N})$. Allowing for an $N$-dependent rank offers new challenges and requires new methods. Working in the Bayes-optimal setting, we show that whenever the signal has i.i.d. entries the limiting mutual information between signal and data is given by a variational formula involving a rank-one replica symmetric potential. In other words, from the informationtheoretic perspective, the case of a (slowly) growing rank is the same as when $M = 1$ (namely, the standard spiked Wigner model). The proof is primarily based on a novel multiscale cavity method allowing for growing rank along with some information-theoretic identities on worst noise for the Gaussian vector channel. We believe that the cavity method developed here will play a role in the analysis of a broader class of inference and spin models where the degrees of freedom are large arrays instead of vectors.

- Taegyun Kim: Heavy-Tailed Mixed p-Spin Spherical Model: Breakdown of Ultrametricity and Failure of the Parisi Formula

  **Abstract:** We prove that the two cornerstones of mean-field spin glass theory—the Parisi variational formula and the ultrametric organization of pure states—break down under heavy-tailed disorder. For the mixed spherical $p$-spin model whose couplings have tail exponent $\alpha < 2$, we attach to each $p$ an explicit threshold $H_p^*$. If any coupling exceeds its threshold, a single dominant monomial governs both the limiting free energy and the entire Gibbs measure; the resulting energy landscape is intrinsically probabilistic, with a sharp failure of ultrametricity for $p \geq 4$ and persistence of only a degenerate 1-RSB structure for $p \leq 3$. When all couplings remain below their thresholds, the free energy is $O(n^{-1})$ and the overlap is near zero, resulting in a trivial Gibbs geometry. For $\alpha < 1$ we further obtain exact fluctuations of order $n^{1-p}$.

Our proof introduces *Non-Intersecting Monomial Reduction* (NIMR), an algebraic–combinatorial technique that blends convexity analysis, extremal combinatorics and concentration on the sphere, providing the first rigorous description of both regimes for heavy-tailed spin glasses with $p \geq 3$.

- Inyoung Yeo: Local laws and spectral properties of deformed sparse random matrices

**Abstract:** We consider deformed sparse random matrices of the form $H = W + \lambda V$, where $W$ is a real symmetric sparse random matrix, $V$ is a random or deterministic, real, diagonal matrix whose entries are independent of $W$, and $\lambda = O(1)$ is a coupling constant. Under mild assumptions on the matrix entries of $W$ and $V$, we prove local laws for $H$ that compares the empirical spectral measure of it with a refined version of the deformed semicircle law. By applying the local laws, we also prove several spectral properties of $H$, including the rigidity of the eigenvalues and the asymptotic normality of the extremal eigenvalues. This is based on a joint work with Ji Oon Lee. (arXiv:2507.02298)

- Lu Chenhao: On the Eigenvalue Rigidity of Jacobi Unitary Ensemble

- Fred Rajasekaran: Gaussian Limits of Lattice Yang-Mills-Higgs Models with Complete Symmetry Breaking

**Abstract:** Given any compact connected matrix Lie group $G$ and any lattice dimension $d \geq 2$, we construct a massive Gaussian scaling limit for the $G$-valued lattice Yang-Mills-Higgs theory in the "complete breakdown of symmetry" regime. This limit arises as the lattice spacing tends to zero and the (inverse) gauge coupling constant tends to infinity sufficiently fast, causing the theory to "abelianize" and yield a Gaussian limit. This complements a recent work by Chatterjee, which obtained a similar scaling limit in the special case $G = \mathrm{SU}(2)$. This is joint work with Oren Yakir and Yanxin Zhou.

- Chin-Yen Lee: Free regular measures and global inversion theory

- Raghavendra Tripathi: Khintchine's inequality for $\epsilon$-independent semicircles

- Ziyun Xu: Free probability and moduli space of 2d CFT

- Manasa Nagatsu: Weingarten calculus for centered random permutation matrices

- Mahmoud Khabou: Poisson Autoregression on a Large Network with a Stochastic Block Model Structure

- Adriana Climescu-Haulica: Spectral analysis of some random graphs using Voiculescu's free probability calculus

**Abstract:** Using the free probability calculus initiated by Dan Voiculescu we study the spectral measures of random graphs inspired from communication networks and neuro-physiology. With these results we obtain a large deviation theorem which calibrates the probability of error for a new class of space-times codes.

## Thursday, 09-11

- Yong-Woo Lee: Large deviations for the number of real eigenvalues of the real elliptic Ginibre ensemble

- Jiaqi Fan: Localization-Delocalization Transition for a Random Block Matrix Model at the Edge

- Teodor Bucht: Quantitative Tracy-Widom laws for sparse random matrices

  **Abstract:** We consider the fluctuations of the largest eigenvalue of sparse random matrices, the class of random matrices that includes the normalized adjacency matrices of the Erdős-Rényi graph $G(N,p)$. We show that the fluctuations of the largest eigenvalue converge to the Tracy–Widom law at a rate almost $O(N^{-1/3}+p^{-2}N^{-4/3})$ in the regime $p \gg N^{-2/3}$. Our proof builds upon the Green function comparison method initiated in (Erdős, Yau and Yin 2012). To show a Green function comparison theorem for fine spectral scales, we implement algorithms for symbolic computations involving averaged products of Green function entries.

- Martin Peev: Solving Singular SPDEs in Free Probability and Beyond

- Rubén Jiménez Lumbreras: Fractional signature: a generalisation of the signature inspired by fractional calculus

  **Abstract:** In this poster, we introduce the fractional signature, a generalisation of the classical path signature inspired by fractional calculus, which effectively characterizes solutions to linear Caputo-controlled fractional differential equations (FDEs). We then propose a second, machine learning–friendly generalisation that builds on the same principles while offering improved computational efficiency. Finally, we demonstrate the practical benefits of this new signature in a toy machine learning task, where it significantly outperforms the original signature in terms of accuracy. This is a joint work with José Manuel Corcuera Valverde (arXiv:2407.17446).

- Zikun Ouyang: Stationary measures of inverse-Wishart polymer

- Ziran (Ryan) Liu: Cognition Meets Coulomb: VERA Beliefs Flow Like Log-Gases – A Γ-Convergence Perspective

- Chunxiao Liu: Universality classes for purification in nonunitary quantum processes

  **Abstract:** We consider the universal aspects of two problems: (i) the singular value structure of a product $M = m_t m_{t-1} \cdots m_1$ of many large independent random matrices, and (ii) the slow purification of a large number of qubits by repeated quantum measurements. Both processes are associated with the decay of natural measures of entropy as a function of time or of the number of matrices in the product. We argue that for a broad class of models, each process is described by universal scaling forms for purification and that (i) and (ii) represent distinct "universality classes" with distinct scaling functions. Using the replica trick, these universality classes correspond to effective one-dimensional statistical mechanics models for a gas of "kinks", representing domain walls between elements of the permutation group. This is an instructive low-dimensional limit of the effective statistical mechanics models for random circuits and tensor networks.

- Chizuru Soukejima, Raian Suzuki: Hydrodynamic limit of radial multiple Schramm–Loewner evolution

  **Abstract:** Schramm–Loewner evolution (SLE) generates a random curve in two-dimensions which is conformally invariant and has the domain Markov property. The typical setting for SLE is called chordal, in which an SLE curve evolves in a simply-connected domain from a boundary point to another boundary point. In its variant called radial, however, an SLE curve evolves from a boundary point to a distinguished interior point. Multiple SLE is another variant which deals with multiple random curves. In this presentation, we report multiple SLE in the radial setting, which we call radial multiple SLE. In our ongoing study with Makoto Katori (Chuo Univ.) and Shinji Koshida (Aalto Univ.), we proved that coupling between the radial multiple SLE and the Gaussian free field on a unit disk is established if and only if the former is driven by the circular Dyson Brownian motions. We are studying the hydrodynamic limit of the radial multiple SLE, where the number of SLE curves tends to infinity and a connected domain called the SLE hull is formulated. In this limit, the circular Dyson Brownian motions converge to a measure-valued process described by the inviscid complex Burgers equation on a unit circle. We will show that the radial multiple SLE in the hydrodynamic limit shows a novel transition at time $t = 1$, where topology of the SLE hull is changed and a critical phenomenon is observed in the edge behavior of the SLE-hull boundary.

- Lamia Lamrani: Holdout method error and optimal split for large non-Gaussian covariance matrix estimation using Weingarten calculus
- Ayesha Irfan: The logarithmic derivative of characteristic polynomials
- Ruihan Liu: Estimation of cp rank of the high-dimensional spiked tensor model
- Eui Yoo: Three topological phases of the elliptic Ginibre ensembles with a point charge

  **Abstract:** In the large $N$ limit, random matrix models exhibit limiting spectra in the complex plane whose supports are called the droplets. In our research, we consider the elliptic Ginibre ensembles conditioned to have a deterministic eigenvalue with its multiplicity proportional to the matrix size. We prove that the droplets are simply connected, doubly connected, or composed of two simply connected components. Moreover, we present the explicit description of the droplet and electrostatic energies for the simply and doubly connected case. This is based on a joint work with Sung-Soo Byun (arXiv:2502.02948).

- Yu Kitagawa: Free multiplicative convolution with an arbitrary measure on the real line
- Yanic Cardin: Photon-number moments and cumulants of Gaussian states

  **Abstract:** In this poster, we develop closed-form expressions for the moments and cumulants of Gaussian states when measured in the photon-number basis. We express the photon-number moments of a Gaussian state in terms of the loop Hafnian, a function that when applied to a (0, 1)-matrix representing the adjacency of a graph, counts the number of its perfect matchings. Similarly, we express the photon-number cumulants in terms of the Montrealer, a newly introduced matrix function that when applied to a (0, 1)-matrix counts the number of Hamiltonian cycles of that graph. We analyze the dependence of the cumulants as a function of the type of Gaussian state : squeezed, lossy squeezed, squashed, or thermal. We find that thermal states perform much worse than other classical states, such as squashed states, at mimicking the photon-number cumulants of lossy or lossless squeezed states

- Daniel Munoz George: Second order cumulants: product commutator and anticommutator

  **Abstract:** Since the seminal work of Voiculescu on free independence or simply freeness, many other notions of independence have emerged. Mingo and Speicher introduced second order freeness and their cumulants. In this work we preset by first time the second order cumulants of two second order free random variables of their product, their commutator and their anticommutator. All of our formulas are indexed by combinatorial objects such as graphs or non-crossing permutations.

- Seungjoon Oh: Spectral moments of non-Hermitian random matrices