A step-by-step guide to Black-Litterman model by T. Idzorek, 2004

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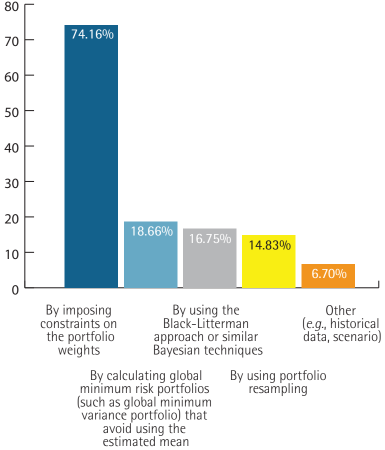
On Black-Litterman model

- Developed by Black and Litterman (1991), He and Litterman (1999), GS investment team
- Suppose you don't want to rely too much on historical returns, and have specific views on the next periods return for certain assets: how to implement these views in a portfolio optimization setup ?
- "Bayesian" portfolio optimization: blending prior, subjective investor views with $E(r_t)$ in a standard Mean-Variance optimization
- Overcome traditional problems with mean-variance optimization: highly concentrated portfolios, large rebalancing, etc...

On Black-Litterman model

Exhibit 5: How do you deal with estimation risk/problems of estimating the expected returns?



Source: EDHEC survey Jan 2009

This paper

- "User-friendly" implementation of Black-Litterman model by a practitioner. Idzorek now head of research at Morningstar.
- Expression of investor's subjective view with percentage of confidence (ie. "I think return of asset A will overperform asset B with a degree of confidence of 75%")
- Simplifies the input of macroeconomic scenarios, without specifying the density function of the returns implied by the views.

Step 1: CAPM equilibrium returns

Idea: determine first the "equilibrium" returns as a neutral starting point. These returns will be then tilted by the views. What are these equilibrium returns ? Simply the returns that clear the market according to market capitalization: "reverse optimization"

$$\Pi = \lambda \Sigma w_{mkt} \tag{1}$$

where Π is a $(N \times 1)$ vector of implied Excess Equilibrium Return Vector ;
λ is the risk aversion coefficient, can be calibrated or given by the data, generally comprised btw 2 and 6;
Σ is a $(N \times N)$ covariance matrix of excess returns ;
w_{mkt} is a $(N \times N)$ vector of market capitalization weights. Different methods to compute these equilibrium returns: true market capitalization, or composition of an index, etc.

Step 1: CAPM equilibrium returns

Example data: with 8 asset classes

Table : Example dataset stats

Asset	US Bonds	Int Bonds	Us LgGr	US LgVal	US SmGr	US SmVal	Int Dev	Intl Emg
M.cap	19,34%	26,13%	12,09%	12,09%	1,34%	1,34%	24,18%	3,49%
Π	0,08	0,67	6,42	4,09	7,44	3,71	4,81	6,61
$E(R)$	0,03	0,02	-0,06	-0,03	-0,07	-0,01	-0,07	-0,05

Covariance matrix Σ is computed from historical returns

λ set to 3.07

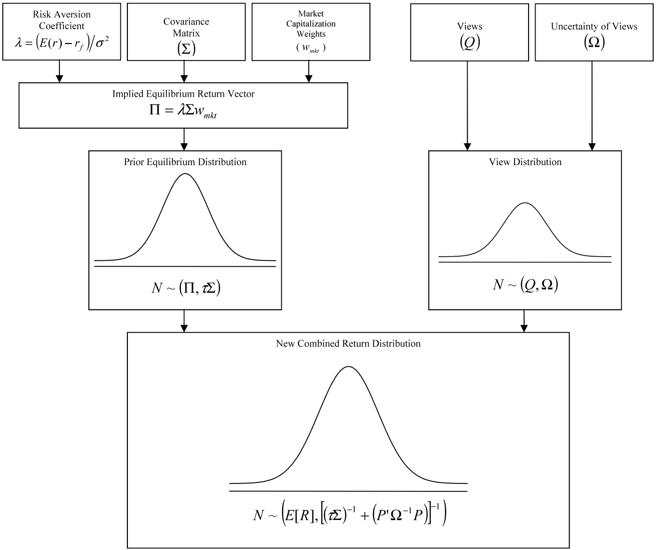
Step 2: Blending views

Idea: mixing the investor subjective view ("Prior") with the CAPM-implied equilibrium returns. Use works of Theil (1971, 1978) on mixed estimation. Views are incorporated in the E(R) vector through the formula:

$$E[R] = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \qquad (2)$$

$E[R]$ is the new (posterior) Combined Return Vector (N x 1 column vector);
τ is a scalar, difficult to interpret, something like a weight or a standard error applied to the covariance matrix, set by many practioners btw 0.01 and 0.05. Here $\tau = 0.025$
Σ is the covariance matrix of excess returns (N x N matrix);
P is a matrix that identifies the assets involved in the views (K x N matrix or 1 x N row vector in the special case of 1 view);
Ω is a diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix);
Π is the Implied Equilibrium Return Vector (N x 1 column vector);

Q is the View Vector (K x 1 column vector).


Risk Aversion Coefficient
$\lambda = (E(r) - r_f)/\sigma^2$
Covariance Matrix
(Σ)
Market Capitalization Weights
(w_{mkt})
Views
(Q)
Uncertainty of Views
(Ω)
Implied Equilibrium Return Vector
$\Pi = \lambda \Sigma w_{mkt}$
Prior Equilibrium Distribution
$N \sim (\Pi, \tau\Sigma)$
View Distribution
$N \sim (Q, \Omega)$
New Combined Return Distribution
$N \sim \left(E[R], \left[(\tau\Sigma)^{-1} + (P'\Omega^{-1}P)\right]^{-1}\right)$

Step 2: Blending views

"Clairvoyant" investor has 100 % confidence on his views
Rearranging eq(2), the posterior return vector simplifies to:

$$E[R_{100\%}] = \Pi + \tau\Sigma P'(P\tau\Sigma P')^{-1}(Q - P\Pi) \tag{3}$$

Views:

- View 1: Int Dev equities will have an abs. excess return of 5.25% with a confidence of 25%
- View 2: Int. bonds will outperform US bonds by 25bps with a confidence of 50%
- View 3: US large growth and US small growth will outperform US large values and US small values by 2% with a confidence of 65%

Step 3: Views and uncertainty, an alternative formulation

Contribution of the paper: "Incorporating user-specified confidence level".

Idea: starting point is the special case of 100% confidence in the views ie. Ω the diagonal matrix of uncertainty of the views is a matrix of zeros.

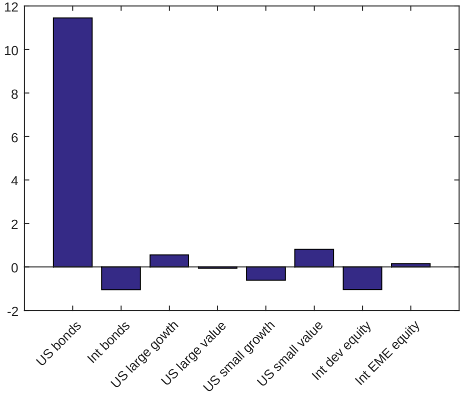
Views will tilt the market portfolio, this tilt will depend on a confidence level C_k

$$Tilt_k = [w_{100\%} - w_{mkt}] * C_k \tag{4}$$

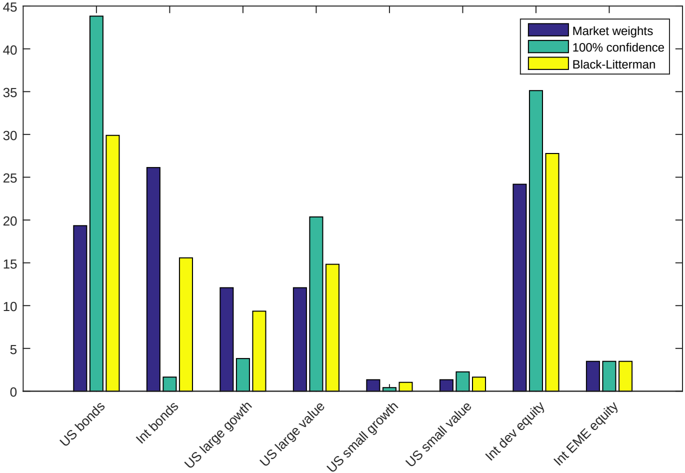
Idea: depending on your confidence level (eg. 65%) you will depart from the market portfolio toward the 100% confidence portfolio, but only in a 65% extent.

Standard MV weights

Figure : Standard unconstrained MV optimization: highly concentrated solution



Weights for different portfolio constructions



Risk return results

Table : Risk/Return by portfolio construction

	Hist MV	Mkt cap	100%	BL
Excess return %	0.0408	3.0041	3.4884	3.1007
Variance	0.0013	0.0098	0.0114	0.0101
Sharpe	0.0112	0.3037	0.3272	0.3085