Lab 07: The RLC Circuit

PHY310 Lab Journal

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1 Purpose

This lab aimed to measure the self-inductance of the Helmholtz coils using an RLC circuit.

2 Experimental Apparatus

Materials given were a Helmholtz coil, variable resistor, variable capacitor, voltmeter, function generator, multimeter, and various cables. My general setup is illustrated in Figure 1.

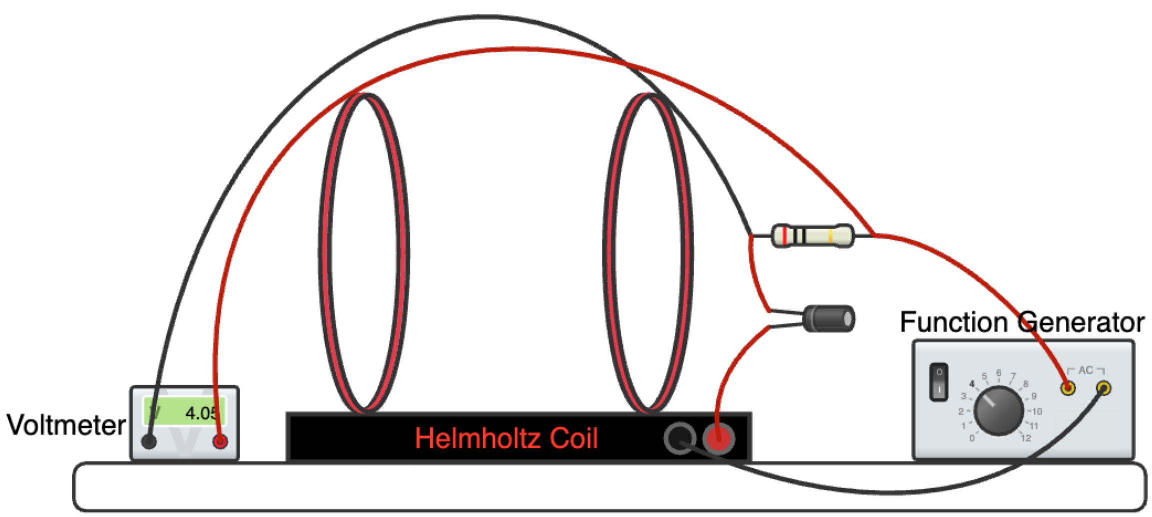

Figure 1: A diagram of my experimental setup

3 Procedure

To begin, we connected the Helmholtz coil in series with a 50 *Ohm* resistor and a 4 μF capacitor at either end of a function generator. We measured the actual attributes of the capacitor, resistor, and coil using multimeters.

We were told to assume for theoretical purposes a self-inductance of 0.02 Henries, so we computed the theoretical resonant frequency of the circuit with that value, as well as what we measured for the resistance and capacitance of the circuit.

Then we placed the voltmeter across the resistor and used the function generator at 4 Volts with a sinusoidal wave to sweep around our theoretical value to discover the actual self-inductance.

4 Results

We start with the series RLC circuit driven by an AC voltage source $V(t) = V_0 \sin(\omega t)$, where $\omega = 2\pi f$ is the angular frequency. The equation we use to model this (the current $I(t)$ through the circuit) is remarkably similar to our derivation for a driven damped harmonic oscillator!

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_{\text{source}}(t) \tag{1}$$

Here, Q is the charge, and $I(t) = \frac{dQ}{dt}$ is the current. We use an Ansatz for $I(t)$ of the form

$$I(t) = I_0 \sin(\omega t - \phi) \tag{2}$$

where ϕ is the phase shift between the current and the source voltage. As we saw in the lecture, the amplitude of the current I_0 can be found from the amplitude of the driving voltage V_0 and the properties of an RLC circuit:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{3}$$

Since we want RMS voltage, we remember from painful homework integrals that the RMS value of a sinusoidal function $A\sin(\omega t)$ is $\frac{A}{\sqrt{2}}$, so the RMS voltage across the resistor is:

$$V_{\text{Resistor RMS}} = \frac{I_{\text{peak}}R}{\sqrt{2}} \tag{4}$$

Therefore

$$V_{\text{Resistor RMS}} = \frac{V_0 R}{\sqrt{2}\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \tag{5}$$

Now we have an expression for our RMS voltage across the resistor as a function of frequency ω, resistance R, capacitance C, and the inductance L which we want to know.

See Table 1 of $V_{\text{Resistor RMS}}$ with respect to voltage. Errors have been propagated.

Frequency (Hz)	RMS Voltage (V)	Voltage Amplitude (V)
420.00	2.7310 ± 0.0273	3.8622 ± 0.0386
430.00	2.8046 ± 0.0280	3.9664 ± 0.0397
440.00	2.8792 ± 0.0288	4.0719 ± 0.0407
450.00	2.9462 ± 0.0295	4.1665 ± 0.0417
460.00	3.0069 ± 0.0301	4.2524 ± 0.0425
470.00	3.0611 ± 0.0306	4.3291 ± 0.0433
480.00	3.1075 ± 0.0311	4.3947 ± 0.0439
490.00	3.1476 ± 0.0315	4.4514 ± 0.0445
500.00	3.1779 ± 0.0318	4.4942 ± 0.0449
510.00	3.1986 ± 0.0320	4.5236 ± 0.0452
520.00	3.2109 ± 0.0321	4.5410 ± 0.0454
530.00	3.2161 ± 0.0322	4.5483 ± 0.0455
540.00	3.2064 ± 0.0321	4.5345 ± 0.0453
550.00	3.2034 ± 0.0320	4.5304 ± 0.0453
560.00	3.1833 ± 0.0318	4.5018 ± 0.0450
570.00	3.1548 ± 0.0315	4.4615 ± 0.0446
580.00	3.1213 ± 0.0312	4.4142 ± 0.0441
590.00	3.0947 ± 0.0309	4.3765 ± 0.0438
600.00	3.0516 ± 0.0305	4.3156 ± 0.0432
610.00	3.0156 ± 0.0302	4.2647 ± 0.0426

Table 1: RMS Voltages and Amplitudes at Frequencies

To compute a theoretical value to base our experimental results around, I used Wolfram Alpha to take the derivative of $\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R\omega}{L}\right)^2}$ and set it equal to zero. We used our capacitance and resistance, and a theoretical value of 0.02 Henries for self-inductance, and found the theoretical resonant frequency of that expression to be 468 Hz.

To find the actual self-inductance, I used Equation 5 to fit our data, shown in Figure 2. Scipy does all the heavy lifting for us, and we're left with the conclusion that the real resonant frequency was **530 Hz**, and $L = \mathbf{0.0209 \pm 0.0054}$ **Henries**.

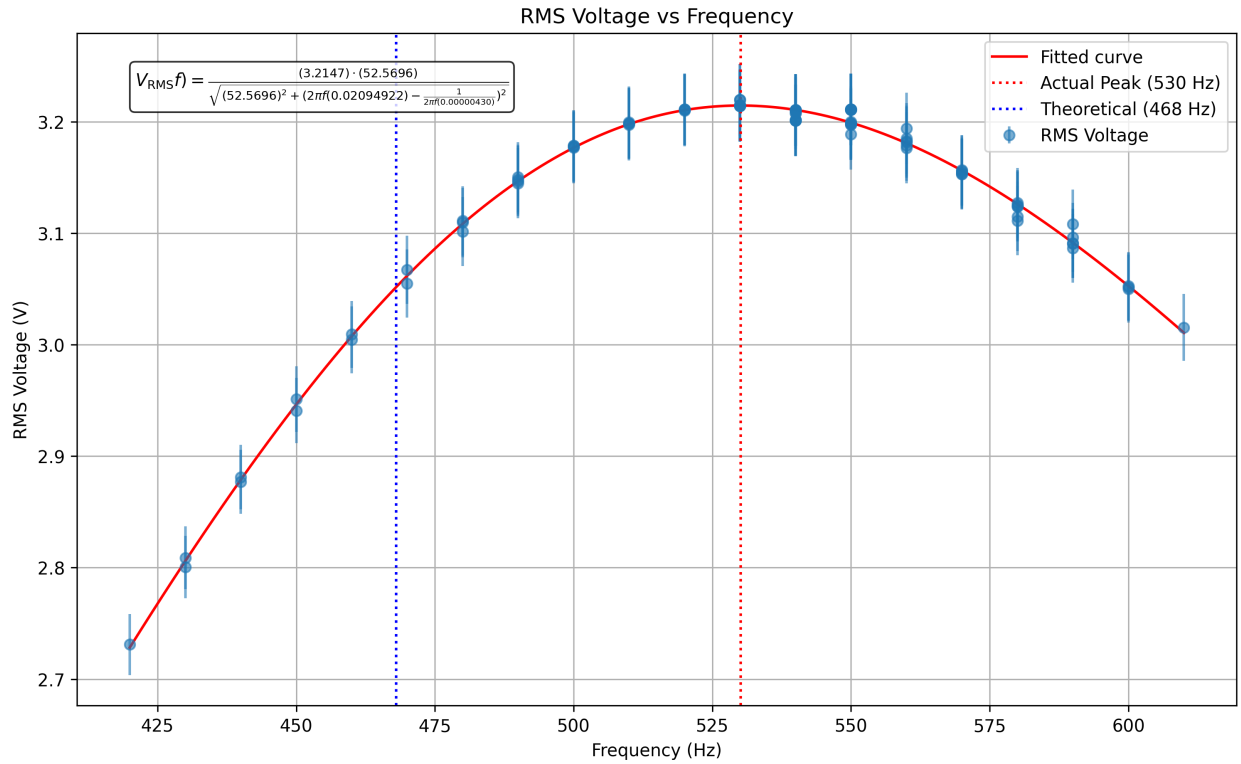

Figure 2: Voltage plotted against amplitude

5 Conclusions

Comparing this result to the results from Lab 6, my inductance there was measured to be **0.0393 ± 0.0004 Henries** where here it was **0.0209 ± 0.005 Henries**. This makes me think that something in one of the labs was off by a factor of two. Not bad, all things considered. Our samples showed a clear resonant frequency, which lead to a good, not highly error-prone computation for the self-inductance.