UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
**School of Computer Science and Applied Mathematics**

# Numerical Methods
# Lab 3

**April 2021**

Some notes:

The bisection method is popular because it is robust. It always works, as long as there is a root in the interval. It is however, a very slow method, compared to the rest of the methods. In addition to being slow, it cannot find roots of multiplicity 2. The multiplicity of a root c is the power to which the factor $x-c$ is raised. Therefore, for functions like $f(x) = (x-c)^2$, the method fails since there are no such interval $[a\ b]$ for which $f(a)f(b) < 0$.

Also, the **continuity of the function** over the entire interval is an important hypothesis for the bisection method to work. This is already stated in Theorem 1 in your note. For example, the function $f(x) = \frac{1}{x}$ satisfies $f(-1) = -1$ and $f(1) = 1$. But, there is no root to the equation $\frac{1}{x} = 0$. Also, Theorem 1 in your note does not say that there is a unique root in the interval $[a\ b]$. There may be many roots, or even, possible infinitely many roots. The Theorem only guarantees that there will be at least a root in the interval.

## Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem.
- Please use the matlab template I sent to you for your mail file.

## Questions 1

(a) Use the methods for finding roots of equations, namely the bisection method, regular falsi method and the Newton's fixed point method (use starting point $x_0$ of your choice), which you have coded in questions 1, 2 and 3 of Lab 3 to find the root, accurate to within $10^{-5}$, of the nonlinear equations

(I) $f(x) = e^x + 2^{-x} + 2\cos x - 6, \quad 1 \le x \le 2.$

(II) $f(x) = 1 - \frac{2}{x^2-2x+2}, \quad -1 \le x \le 1.$

For questions (aI) and (aII), use matlab to present your answers in the form

| Method | root | Iteration count |
|---|---|---|
| Bisection | | |
| False position | | |
| Newton | | |

(III) For only Question (I), do plot the error at each iteration, given by $\frac{|c_n - c_{n-1}|}{|c_n|}$ against the number of iterations, for the Bisection and Falsi position method. On same graph, do plot the error $\frac{|x_n - x_{n-1}|}{|x_n|}$ against the number of iterations for the Newton method.

An observation:

Starting with $x_0 = 1$, it seems that the Newton's method is not as fast as it is praised to be, taking about 8 iterations to converge within the specified tolerance, right? First of all, we cannot really compare since Newton's method is not an interval based method. It is only safe to make comparison between the Bisection method and the Regular Falsi method.

Can you see that the Regular falsi method has outperformed the Bisection method. This will always be true for any function. This ability of the Regular falsi method is inherent in its formation, in that it uses linear interpolation which allows it to approach the root from one side ONLY, unlike its bisection method counterpart, which is all over the show.

In any case, to attempt a make a slight comparison amongst the three methods, it is fair to compute the first root $c_0$ for the bisection and the Regular falsi method. And then choose $x_0$ near the values. For the bisection method, $c_0 = 1.5$ and for the Regular falsi, $c_0 = 1.6783$.

Try plotting the function $f(x)$ in Question (I), choosing $x_0 = 1.5$. Then, you will be able to witness the full might of Newtons method.

(b) Using any root finding method, find **all** the point of intersection of the function $f(x) = \tan x$ and $g(x) = x$ on the interval $0 \leq x \leq 10$.

Use matlab to format your answer as below

Using the ________________ method, the roots are root1, root2, root3, root4, ....

## Questions 2

(a) Use Newton's method for finding roots of nonlinear system of equations, which you have coded in Question 4 to find the root near the point (1,1,1), to within $10^{-5}$ of the nonlinear system

$$\vec{f}(\vec{x}) = \begin{pmatrix} \sin x + y^2 + \ln z - 7 \\ 3x + 2^y - z^3 + 1 \\ x + y + z - 5 \end{pmatrix}, \qquad \text{where} \quad \vec{x} = (x, y, z)$$

Please use the format below when answering the question.

```
%% Question 2a
```

The root is ——— .

(b) Use Newton's method to find the point of intersection of the following pair of plane curves:

$$x^3 - 3xy^2 = 1, \qquad 3x^2y - y^3 = 0, \quad \text{near the point} \quad [-0.6, 0.6].$$

Please use the format below when answering the question.

```
%% Question 2b
```

The root is ——— .

(c) Use Newton's method to find the two intersection points of the parabola $y = x^2 - x$ and the ellipse $x^2/16 + y^2 = 1$.

Plot both the parabola and the ellipse on the same plot, showing the intersection points. Be sure to show the whole ellipse so you can be sure there are exactly two intersection points. Please include this plot when you send me your work

Please use the format below when answering the question.

```
%% Question 2c
```

The roots are root1, root2.