



Numerical Methods Lab 2

01 April 2021

Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem.

Questions 1

Using your code for Exercise 1, 2 and 3 of Lab 2 (Iterative techniques for linear systems), solve the linear system where

(a)

$$A = \begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 5 \\ 9 \\ 4 \\ 2 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & -4 & 2 \\ -2 & -1 & 5 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

Use the zero vector as initial approximation and tolerance 10^{-6} .

Discuss in a line or two, why the results in one diverges and the result in the other converges.

Questions 2

The matrix $A = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix}$ arises in the finite difference discretization of the Poisson equation on a nine point grid.

- (a) Is matrix A diagonally dominant?
- (b) Find the spectral radius of the Jacobi and Gauss-Seidel iteration matrices.
- (c) Using the formula

$$\omega = \frac{2}{1 + \sqrt{1 - \rho_J^2}}, \quad (\star)$$

fix the optimal value of the relaxation parameter.

- (d) If $\rho_J = \max|\lambda|$ denotes the spectral radius of the Jacobi method, then the Gauss-Seidel iterative method has spectral radius $\rho_{GS} = \rho_J^2$, while the relaxation method with optimal relaxation parameter given in (\star) has spectral radius

$$\rho = \omega - 1. \quad (\Upsilon)$$

Verify that the spectral radius of the relaxation method, obtained from its iterative matrix agrees with that obtained in (Υ) .