UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
School of Computer Science and Applied Mathematics

Numerical Methods
Questions for Lab 1

18 March 2021

Instructions

- Your code should be able to communicate the appropriate message, in the case of a computational problem.

The due date for this submission will be 20/02/2020 at 12 noon.

Questions 1 (17 Marks)

(a) Using your code in exercise 1 of Lab 1, solve the linear system $\mathbf{A}\vec{x} = \vec{b}$, where

(i) $$A = \begin{pmatrix} 2 & 1 & -1 & 2 \\ 4 & 5 & -3 & 6 \\ -2 & 5 & -2 & 6 \\ 4 & 11 & -4 & 8 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 5 \\ 9 \\ 4 \\ 2 \end{pmatrix}$$

(ii) $A = H(1:n, [n:-1:1])$ and $b = -H(1:n, n+1)$

for which H is the Hilbert matrix of size $n+1$. The entries of H are defined by

$$h_{ij} = \int_0^1 x^{(i-1)} x^{(j-1)}\, dx = \frac{1}{i+j-1}.$$

Choose $n = 3$.

(iii) $A = H$ and $b = (b_1, b_2, b_3, \ldots, b_n)$,

where H with entries h_{ij} defined as in (ii) is the Hilbert matrix of size $n = 5$ and

$$b_i = \sum_{j=1}^{n} h_{ij}.$$

(b) Modify your code in Exercise 1 of Lab 1 to perform a variation of Gaussian elimination method which returns a matrix in the form

$$A = \begin{pmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Note: Please do no just swap rows one with three.

Hence solve the linear system Ax = b, where

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & -4 & 2 \\ -2 & -1 & 5 \end{pmatrix}, \quad \text{and} \quad b = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

(c) Using your code in exercise 2 of Lab 1, solve the equation AX = B, where

$$A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -3 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{pmatrix}, \quad \text{and} \quad B = \begin{pmatrix} -8 & -10 & -100 \\ -20 & -20 & -250 \\ -2 & -2 & -25 \\ 4 & 8 & 80 \end{pmatrix}$$

Questions 2 (15 Marks)

(a) Using your function in Exercise 3, LU factorize the following matrices

(i) $$A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -3 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{pmatrix}. \qquad (\dagger)$$

(ii) $A = \rho$, where ρ is the pascal matrix of size 5.

Remember to output the permutation matrix P.

(b) Use your function in Exercise 4 to solve the linear system $Ax = b$, where A is defined in equation ($\dagger$) of question (i) and b is each of the column vector in matrix B in Question 1. Store the solutions x obtained as column vectors in matrix **X**. Present the answer in matrix form **X**.

Questions 3 (4 Marks)

BONUS MARK

LU factorization with partial pivoting is carried out without access to the right hand side vector of a linear system. We have to keep track of the row interchanges carried out during the factorization process, so that we can apply the same row interchange to the right hand side vector. We do this by applying all the row interchanges carried out during the LU - factorization to the identity matrix, as we did in an example in class. This gives rise to the permutation matrix P.

By definition, permutation matrices are matrices obtained by interchanging rows in the identity matrix, and it has precisely in each row and column, an entry 1.

Storing the permutation matrix P, just because we would like to keep track of the row interchanges we are carrying out, in the above task is a bit clumsy, and maybe unnecessary. Perhaps, performing PA = LU factorization does not require storage of an $n \times n$ matrix P.

Can you describe a more compact way to represent the information coming from row interchanges?