CS 173 - Lecture 7

Collection of Sets

- Sets that contain other sets

ex. $\{\{0,1\},\{2,3\},\{4\}\}$

ex. $\mathbb{Z}_3 = \{[0],[1],[2]\} = \{\{0,3,6,\ldots\},\{1,4,7\ldots\},\{2,5,8\ldots\}\}$

- All $\mathbb{Z}_n$ are collections

Power Sets

- $\mathbb{P}(A)$ is a powerset of A iff. $\mathbb{P}(A)$ contains all subsets of A

ex. $A = \{1,2\}$ $\quad \emptyset \subseteq A \;\; \forall$ sets A $\quad |A| = 2$

$\mathbb{P}(A) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ $\quad |\mathbb{P}(A)| = 4$

ex. $B = \{1,2,3\}$ $\quad |B| = 3$

$\mathbb{P}(B) = \{\emptyset, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$ $\quad |\mathbb{P}(B)| = 8$

- If $|A| = n$, $|\mathbb{P}(A)| = 2^n$

Empty Set

$\emptyset \times A = \emptyset$

$\{\emptyset\} \times \{1,2\} = \{(\emptyset,1),(\emptyset,2)\}$

$|\mathbb{P}(\emptyset)| = 2^0 = 1$

$\mathbb{P}(\emptyset) = \{\emptyset\}$

Partitions

- Given set A, can partition into non-overlapping subsets which cover entire base set A

ex. $A = \{0,4,1,5,7\}$

$B = \{\{0,1\},\{4,5\},\{7\}\}$

$B' = \{\{7,4\},\{5,1,0\}\}$

Is B partion of A? T

Is B' partition of A? T

- Multiple ways to partition a set
- Rules:

1) No overlapping

$\forall\, b_1, b_2 \in B,\; b_1 \cap b_2 = \emptyset$

2) Covers entire base case

$b_1 \cup b_2 \cup b_3 \cdots b_n = A$

3) Empty set not included

$\phi \notin B$

- Non-partions

ex. $A = \{0, 1, 4, 5, 7\}$

1) $B = \{\{0\}, \{4, 5\}, \{7\}\}$

not partition bc doesn't cover 1

2) $B = \{\{0, 1\}, \{1, 4\}, \{5, 7\}\}$

not partition bc overlaps $(\{0, 1\} \cap \{1, 4\} \neq \emptyset)$

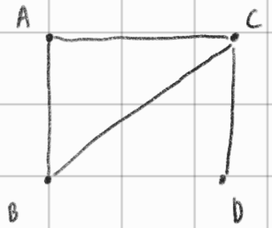
3) $B = \{\emptyset, \{0, 1\}, \{4, 5\}, \{7\}\}$

not partition bc $\emptyset$ included

Graphs Intro

- $Deg(n)$ = # edges connected to vertex n

ex.



$deg(A) = 2$ $deg(B) = 2$

$deg(C) = 3$ $deg(D) = 1$

$\overbrace{\{\underbrace{\{D\}}_{\substack{\text{all vals} \\ \text{w/ deg}=1}}, \underbrace{\{A, B\}}_{deg=2}, \underbrace{\{C\}}_{deg=3}\}}$ → partition of all vertecies of the graph $\{A, B, C, D\}$

Discussion Problems - 17.2abcd

17.2 Power Sets 2

Define the following sets:

$$
\begin{aligned}
A &= \{\{\text{Elm}\}, \{\text{Pine}\}\}\\
B &= \{\text{Elm}, \text{Oak}, \text{Maple}\}\\
C &= \{\text{Elm}, \text{Vine}, \text{Birch}, \text{Maple}\}\\
D &= \{\text{Tree}, \text{Disease}, \text{Street}\}
\end{aligned}
$$

List the elements of each of the following sets or calculate the cardinality (as indicated).

(a) $\{X \in \mathbb{P}(C) \ : \ |X| \text{ is even}\}$

(b) $A \cap \mathbb{P}(B \cap C)$

(c) $|\mathbb{P}(C \times D)|$

(d) $|\mathbb{P}(B \cap D)|$

a) $\{X \in \mathbb{P}(C) : |X| \text{ even}\}$

all x in power set of C such that cardinality of x is even

$\{\emptyset, \{\text{Elm}, \text{Vine}\}, \{\text{Elm}, \text{Birch}\}, \{\text{Elm}, \text{Maple}\}, \{\text{Vine}, \text{Birch}\}, \{\text{Vine}, \text{Maple}\}, \{\text{Birch}, \text{Maple}\}, \{\text{Elm}, \text{Vine}, \text{Birch}, \text{Maple}\}\}$

b) $A \cap \mathbb{P}(B \cap C)$

$B \cap C = \{\text{Elm}, \text{Maple}\}$

$\mathbb{P}(B \cap C) = \{\emptyset, \{\text{Elm}\}, \{\text{Maple}\}, \{\text{Elm}, \text{Maple}\}\}$

$A \cap \mathbb{P}(B \cap C) = \{\{\text{Elm}\}\}$

c) $|\mathbb{P}(C \times D)|$

$|(C \times D)| = 12$

$|\mathbb{P}(C \times D)| = 2^{12} = 4096$

d) $|\mathbb{P}(B \cap D)|$

$|B \cap D| = 0$

$|\mathbb{P}(B \cap D)| = 2^0 = 1$