2-Way Bounding

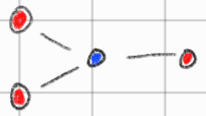
ex. prove $x=5$

if direct proof is difficult, can prove $x \le 5$ and $x \ge 5 \rightarrow x=5$

Graph Coloring

- Assigns color to each node in G s.t no two adjacent nodes have same color
- Chromatic number of G is min num of colors to color G validly

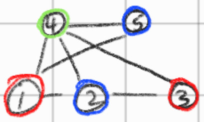
ex.



chromatic number = 2
(not a proof)

ex. given G_1, find chromatic num and prove correct

G_1:



Proof: Chromatic number is 3 (state answer)

Chromatic number is no more than 3. (upper bound)

Color nodes 1 and 2 red, 2 and 5 blue, 4 green. I know I can color w/ three colors. Can color in 2 colors?

Chromatic number is no less than 3. (lower bound)

Our graph has a K_3 subgraph (1-4-5), which we know needs three colors, so our graph needs at least 3 colors.

Set Equality

- Can show $A=B$ by showing $A \subseteq B$ and $B \subseteq A$ (see lecture 6 for subset proofs)

ex. Let $A = \{15p + 9q \mid p, q \in \mathbb{Z}\}$, $B = \{3k \mid k \in \mathbb{Z}\}$, show $A=B$

Proof: ① First, we'll show $A \subseteq B$. Let $x \in \mathbb{Z}$ be an element of A. Then, $x = 15p + 9q$ where $p, q \in \mathbb{Z}$. Then, $x = 3(5p+3q)$. Since $p, q \in \mathbb{Z}$, $5p+3q$ is also $\mathbb{Z}$, let $k = 5p+3q \in \mathbb{Z}$. Then, $x = 3k$, $k \in \mathbb{Z}$ so $x \in B$. Therefore, $A \subseteq B$. ② Now we'll show $B \subseteq A$. Let $y \in \mathbb{Z}$ where $y \in B$. Then, $y = 3k$, $k \in \mathbb{Z}$. Rewrite $3k$ as $y = 3k = -15k + 18k$. Then, $y = 15(-k) + 9(2k)$. Since $k \in \mathbb{Z}$, $-k \in \mathbb{Z}$, call it a, $2k \in \mathbb{Z}$, call it b. Then, $y = 15a + 9b$, $a, b \in \mathbb{Z}$, so $y \in A$, and thus $B \subseteq A$. ③ Since $B \subseteq A$ and $A \subseteq B$, $A = B$

Discussion Problems - 10.1b, 10.2cd

10.1 Set Equality Proofs

Prove that the following pairs of sets are equal. Or, if you are short on time, outline the proof. That is, write the main structure of the proof, and also apply the definitions of the two sets A and B, but leave out the algebra detail required to connect one definition to the other.

(a) $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 256\}$ and $B = \{(16\cos t, 16\sin t) \mid t \in \mathbb{R}\}$

(b) $X = \{10x + 15y : x, y \in \mathbb{Z}\}$ and $Y = \{n \in \mathbb{Z} : n = 5k \text{ for some } k \in \mathbb{Z}\}$

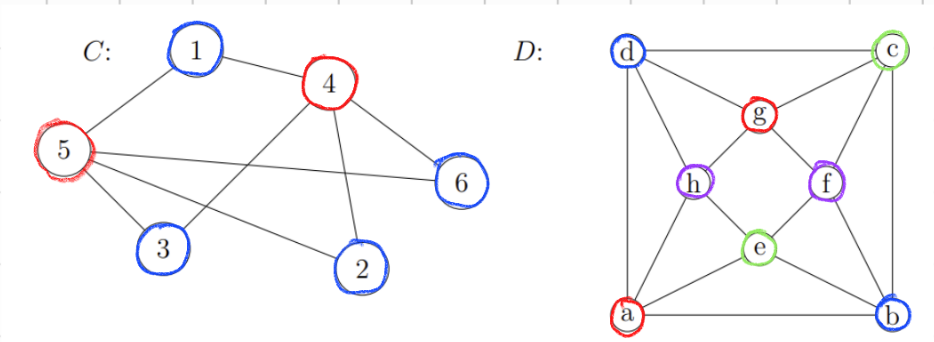
b) First, we'll prove $X \subseteq Y$. Let $a \in X$. By defn. of X, $a = 10x+15y$ where $x, y \in \mathbb{Z}$. Then, $a = 5(3x+5y)$. Let $w = 3x+5y$, $w \in \mathbb{Z}$ because $x, y \in \mathbb{Z}$. Now we have $a = 5w$, $w \in \mathbb{Z}$, therefore $a \in Y$, thus $X \subseteq Y$. Now, we'll prove $Y \subseteq X$. Let $b \in Y$. By defn. of Y, $b = 5k$ where $k \in \mathbb{Z}$. $b = 5k = 10(-k) + 15k$, and since $k \in \mathbb{Z}$, let $-k = d \in \mathbb{Z}$ and $k = h \in \mathbb{Z}$. Then, $b = 10d + 15h$, $b, h \in \mathbb{Z}$, $\therefore$ $b \in X$ and thus $Y \subseteq X$. Since $Y \subseteq X$ and $X \subseteq Y$, $Y = X$.

10.2 Chromatic Number

Recall that the justification that a particular chromatic number is valid requires bounding the number from above *and* below. Therefore you must give an *explicit* coloring to produce an upper bound *and* produce a valid argument that no smaller number of colors will work to produce a lower bound.

The argument justifying the lower bound often involves finding a copy of K_n (where n is the chromatic number you are attempting to validate) as a subgraph. Sometimes, however, you have to work through the space of possible $n-1$ colorings by hand and show that none of them work.

Find and justify the chromatic numbers for each of the following graphs.



c) Proof: Chromatic # is 2 ...

d) Proof: Chromatic # is 4 ...