* Some discussion problems are now included throughout the lecture

Graphs

$G = (V, E)$

vertecies ↑ edges

ex. $V = \{a, b, c\}$ $E = \{(a,b), (b,c)\}$

edges $a \to b$, $b \to c$ directed

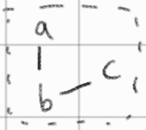
ex. $V = \{ \text{"} \}$ $E = \{ab, bc\}$

$a - b$ $b - c$
undirected

ex. $V = \{ \text{"} \}$ $E = \{\{a,b\}, \{b,c\}\}$

$a - b$ $b - c$
undirected

G:



- simple graph: no self edges, no multiple edges, at least one node/vertex
- degree: number of edges w/ vertex as endpoint

ex. (from above G) $\deg(a) = \deg(c) = 1$ $\deg(b) = 2$

- each edge adds +2 degree to entire graph
- sum of degrees of every node in any graph $G = 2|E| = \sum_{v \in V} \deg(v)$ (even).

Special Graphs

- complete graph (clique): K_n is a complete graph w/ n nodes/vertecies

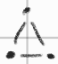
ex.

K_3

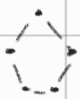
K_5

- cycle graph: C_n is a cycle graph w/ n nodes

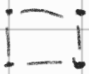
ex.



C_3



C_5



C_4

- wheel graph: W_n is a wheel graph w/ n+1 nodes

ex

W_3

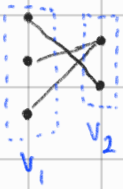
4 nodes

W_5

6 nodes

- **bipartite graph**: graph whose vertecies can be partitioned into sets V_1, V_2 where $V = V_1 \cup V_2$ and edges do not exist within V_1 or V_2

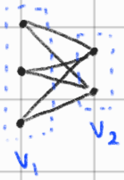
ex.



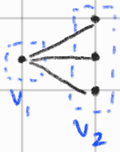
bipartite, not complete

- **complete bipartite**: $K_{m,n}$ is a bipartite graph w/ m+n nodes

ex.



$K_{3,2}$



$K_{1,3}$

Traversing a graph

- **Walk**: finite sequence of nodes from a to b described by nodes or edges
  - length = # edges traversed
  - closed if start node = end node
  - open if start node ≠ end node
- **Path**: walk where no node repeated

ex.

walks from a to d:

a, c, e, a, b, d or ac, ce, ea, ab, bd length 5 not path

a, b, d length 2 path

a, c, d length 2 path

- **cycle**: closed walk where no nodes used more than once other than start/end nodes
- **euler circuit**: closed walk which uses each edge in the graph exactly once (no restriction on nodes)
  - ↳ need all nodes w/ even degree
- **connected**: graph connected iff there is walk between every pair of nodes
- **distance**: length of shortest path between two nodes
- **diameter**: largest distance between two nodes

ex.

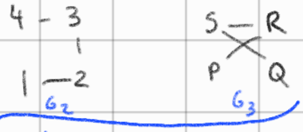
diameter = 3

Isomorphism

- Graphs are isomorphic if structure exactly same
- Suppose $G_1 = (V_1, E_1)$ and $G_2(V_2, E_2)$, G_1 and G_2 are isomorphic if we can construct an isomorphism from G_1 to G_2
- Isomorphism: bijection $f: V_1 \to V_2$ s.t edge (a,b) exists in G_1 iff edge $(f(a), f(b))$ exists in G_2

ex. a - b - c - d

G_1



G_2 G_3

iso morphic

ex. Isomorphism from G_1 to G_3?

definition of f: $f(a) = P$, $f(b) = R$, $f(c) = S$, $f(d) = Q$

$G_1 \xrightarrow{f} G_2$

ab $\to$ PR

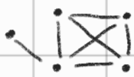
bc $\to$ RS

cd $\to$ SQ

↳ only edges in G_2? yes

- Not isomorphic?
  - diff # nodes
  - diff # edges
  - degree based structure difference (eg. G_1 has node w/ deg 5, G_2 doesn't)
  - contain different subgraphs
- Subgraphs: $G'(V', E')$ subgraph of $G = (V, E)$ iff $V' \subseteq V$, $E' \subseteq E$ and all endpts of E' are in V'

ex. G_1: G_2:



Both: 5 nodes
7 edges

not isomorphic: G_1 has K_4 subgraph, G_2 doesn't

or

G_1 has node w/ deg 1, G_2 doesn't

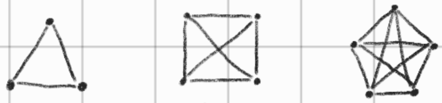
Discussion Problems - 8.4, 8.5, 9.1b

8.4 Graph Diameters

Recall that for a connected, simple graph G we define the **distance** between any two nodes v_i and v_j as the number of edges on the *shortest* path between them. Then the **diameter** G is the *maximum* distance between *any* pair of nodes in G.

Find the diameters of K_n, C_n, and W_n.

K_n:



K_3 diameter = 1 | K_4 diameter = 1 | K_5 diameter = 1

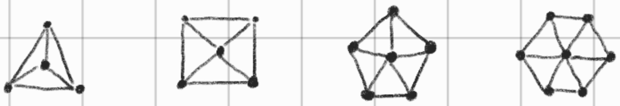
diameter of $K_n = 1$

C_n:

C_3 diameter = 1 | C_4 diameter = 2 | C_5 diameter = 2

diameter of $C_n = \begin{cases} \text{odd } n: & \frac{n-1}{2} \\ \text{even } n: & n/2 \end{cases}$

W_n:



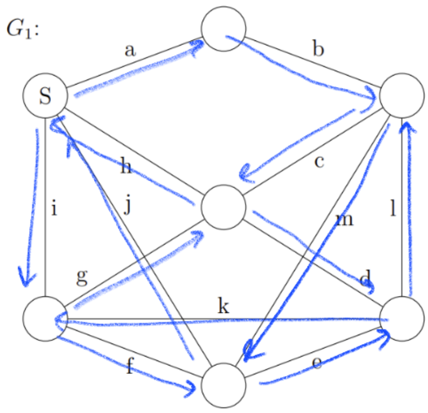
W_3 diameter = 1 | W_4 diameter = 2 | W_5 diameter = 2 | W_6 diameter = 2

diameter of W_n: $\begin{cases} n=3: & 1 \\ \text{otherwise}: & 2 \end{cases}$

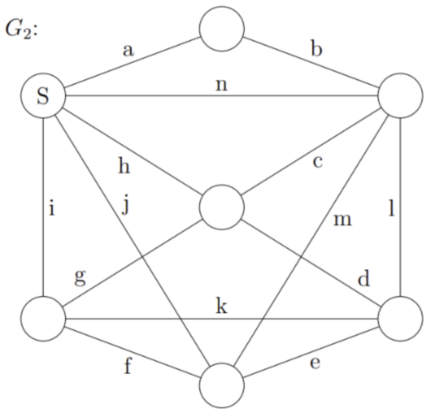
8.5 Euler circuits

Find an Euler circuit in each graph beginning at S, or explain why this isn't possible.

G_1:



G_2:



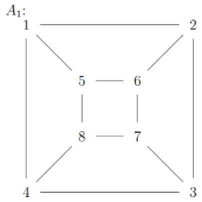
G_1: abcdlmekghifj

G_2: impossible bc at least one node w/ odd degree

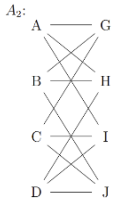
9.1 Isomorphic or not?

Give an isomorphism between the two graphs or briefly explain why this is not possible.

A_1:

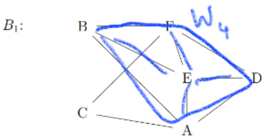


A_2:



b) impossible bc F and A are only nodes w/ deg 4 and aren't adjacent, but in B_2, 1 and 5 are only nodes w/ deg 4 but are adjacent

B_1:



B_2: