Lecture 1 - Intro to Differential Equations

Differential equations

ex. $\frac{dY}{dT} = -Y$ $\qquad \frac{d^2Y}{dT} + \frac{dY}{dT} + 16Y = \cos T$ $\qquad \frac{d^3Y}{dT^3} + Y\frac{dY}{dT} - Y = 1$

In real life:

- Newton's law of cooling: $\frac{dT}{dt} = -k(T - T_0)$
- Newton's second law: $m\frac{d^2y}{dt^2} = F\left(\frac{dy}{dt}, y, t\right)$
- Hooke's Law: $m\frac{d^2y}{dt^2} = -kY$

Linear vs Non-Linear

Eqn. of line:

$$a_1 Y_1 + a_2 Y_2 + a_3 Y_3 + \dots + a_n Y_n = b$$

Linear diff. eqn.:

$$a_n(t)\frac{d^n y}{dt} + a_{n-1}(t)\frac{d^{n-1}y}{dt} + \dots + a_1(t)\frac{dy}{dt} + a_0(t)y = f(t)$$

anything not in this form is non-linear

ex. $\frac{dY}{dT} = -Y \rightarrow \frac{dY}{dT} + y = 0$ linear

$\frac{d^2Y}{dT^2} + \frac{dY}{dT} + 16Y = \cos t$ linear

$\frac{d^3Y}{dT^3} + \boxed{Y\frac{dY}{dT}} - Y = 1$ Non-linear

makes it non-linear

☆ Only dependance on $Y, Y', Y'', \dots Y^{(n)}$ matters

ex. $\frac{d^2y}{dt^2} + tY^2 = 0$ non-linear $\qquad \frac{d^2Y}{dt^2} + t^2Y = 0$ linear

Order of a Differential Equation

- Order = Order of highest derivative

ex. $\frac{dY}{dT} = -Y \rightarrow \frac{dY}{dT} + y = 0$ first order

$\frac{d^2Y}{dT^2} + \frac{dY}{dT} + 16Y = \cos t$ second order

$\frac{d^3Y}{dT^3} + Y\frac{dY}{dT} - Y = 1$ third order

- tells us what data we need to solve the problem