Lecture 7 - Bifurcations, Numerical Methods, and Euler Method

Autonomous Equations (from lecture 6)

$\frac{dy}{dt} = f(y)$

y^* fixed pt. if $f(y^*) = 0$

y^* stable if $f'(y^*) < 0$

y^* unstable if $f'(y^*) > 0$

Phase Line:

- Don't need a whole field cuz $f(y)$ not $f(y,t)$

ex. $\frac{dy}{dt} = y - y^2$

x = fixed point

Bifurcations

ex. Fishery model

$\frac{dy}{dt} = y - y^2$ logistic eqn, model for fish population

$\frac{dy}{dt} = y - y^2 - \underbrace{h}_{\text{rate fish are caught}}$ Allow fishing, constant number of fish caught per day

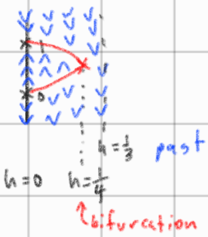
$h = 0$:

$\frac{dy}{dt} = y - y^2$ fixed pts $y = 0$ (unstable), $y = 1$ (stable)

$h \neq 0$:

$\frac{dy}{dt} = y - y^2 - h$

at fixed pts, $0 = y - y^2 - h$ $\quad y = \frac{1 + \sqrt{1-4h}}{2}$ (stable) $\quad y = \frac{1 - \sqrt{1-4h}}{2}$ (unstable)



$h = \frac{1}{3}$ past bifurcation pt $(h = \frac{1}{4})$, population collapses to zero

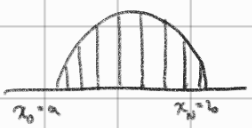
- Bifurcation - change in # or type of fixed pts

Numerical Methods

$\frac{dy}{dt} = f(y,t) \quad y(t_0) = y_0$

Euler Method

From calc II, remember riemann sums



$\int_a^b f(x)dx \approx \sum f(x_i)\Delta x$

$x_i = a + \frac{b-a}{N} i$

Given y_i at t_i, how do I find y_{i+1} at time $t_i + \Delta t = t_{i+1}$

$\frac{dy}{dt} = f(y,t) \quad \frac{dy_i}{dt} = f(y_i, t_i)$

By taylor's thmn:

$y(t_i + \Delta t) \approx y(t_i) + \frac{dy}{dt}(t_i)\Delta t + f(y_i, t_i)\Delta t$

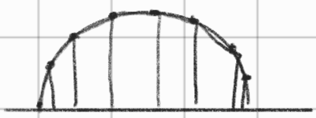
Euler Scheme (analogus to LH reimann sum)

$y_{i+1} = y_i + f(y_i, t_i)\underbrace{\Delta t}_{\text{step size}}$

$\frac{dy}{dt} = f(y,t) \quad y(t_0) = y_0 \quad y_1 = y(t_0 + \Delta t) = y_0 + f(y_0, t_0)\Delta t \quad y_2 = y(t_0 + 2\Delta t) = y_1 + f(y_1, t_1)\Delta t$

Improved Euler

Analogous to trapezoidal reimann



$k_1 = f(y_i, t_i)\Delta t$

$k_2 = f(y_i + k_1, t_i + \Delta t)\Delta t$

Rule

$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$

Quiz 8

$\frac{dy}{dt} = e^{ty} \quad y(0) = 1 \quad \Delta t = 0.1$

$y_1 = y_0 + f(y_0, t_0)\Delta t = 1 + e^{0 \times 1}(0.1) = 1.1$