Lectu	re	12 - Fundamental Theorem of Algebra
Solving	co	instant coefficient linear homogeneous (see lecture 11):
J	<b>'</b>	root solution
		( e <sup>rt</sup>
		tib edtospt, edt sinpt  ort fort fort fort
•	W) i	k ert, tert, t <sup>2</sup> ert,, t <sup>(k-1)</sup> ert
e	χ.	y'' - 4y' + 5y = 0 $y'(0) = 1$ $y'(0) = 0$
		$(^2-4r+5=0$
		$r=2\pm \tilde{\iota}$ $\alpha=2$ $\beta=1$
		$y_1 = e^{2t}\cos t \qquad y_2 = e^{2t}\sin t$
		$y = Ae^{2t}cost + Bc^{2t}sint$ $y(0) = A-1+B\cdot 0 = 1$ $A = 1$
		$y' = 2Ae^{2t}cost - Ae^{2t}sint + 2Be^{2t}sint + Be^{2t}cost$
		y'(0) = 2A + B = 0
		g(0) = 2A + 0 = 0 g(0) = 2A = -2
		$y = e^{2t} cost - 2e^{2t} sint$
Fundo	rmen	tal Theorem of Algebra:
		omial P(r) of degree n has n roots counted by multiplicity
		r, has multiplicity k, (r-r,)k is a factor of P(r)
		$P(v) = (v-2)(r-5)^3(r+7)^4$
L.		degree 8
		r=2 (simple) multiplicity 1
		r=5 multiplicity 3
	_	r=-7 multiplicity 4
e	х.	y" + 3y" + 3y' + y = 0
		$r^3 + 3r^2 + 3r + r = 0$
		$(f+1)^3=0$ r=0 w/ multiplicity 3
		$y_1 = e^{-t}$ $y_2 = te^{-t}$ $y_3 = t^2 e^{-t}$

	ex.	y" -3y1+2y=0	y(0)=1 y	'(o) = o 'y'	··(o)=1						
		$\int_{0}^{3} -3 + 2 = 0$	U U								
		r=1 is root see prev lecture to	er factoring								
		$(r-1)^2(r+2)=0$									
		r=1 (multiplicity									
		y, = et yz = tc+	•								
		y = Act + Btet + C	Le-2t								
		do a bunch of	f algebra w/	initial o	auditions to	get !					
		y= 7 et - 1 tet + 3	2 -2t								
	ę X.	y(4) + 4y" + 8y" +	84 + 44 = 0	P(1) = (1	2+25+2)2						
		r= 1+i (multipli									
		y = Actcost + Betsi			,,,,,						
		y = Ac COST The S	incr cle cost t	/(2 3,4 0							
Opera	tors:										
	Acts	on function, ret		function	(derivatives,	integrals, m	ultiplying k	by a func	tion, et	·c.)	
	Acts			function	(derivatives,	integrals, m	ultiplying k	by a func	tion, et	·c.)	
	Acts ex.	on function, ret	t		(derivatives,	integrals, m	ultiplying b	a func	tion, et	·c.)	
	Acts ex.	on function, ret $L_1 = 2 \frac{d}{dt} \qquad L_2 = 1$ $L_1 \sin t = 2 \cos t$	t Lzsint=tsin	ŧ	(derivatives,	integrals, m	ultiplying k	py a func	tion, et	)	
	Acts ex.	on function, ret $L_1 = 2 \frac{d}{dt} \qquad L_2 = 1$	t $L_2 \sin t = t \sin t$ $L_2 e^{2t} = t e^{2t}$	ŧ	(derivatives,	integrals, m	ultiplying k	a func	tion, et	·c.)	
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	Acts ex.  Don't	on function, ret $L_1 = 2 \frac{d}{dt} \qquad L_2 = 1$ $L_1 \sin t = 2 \cos t$ $L_2 e^{2t} = 4 e^{2t}$ $L_1 t = 2$ $commute, order$ $ex.  L_1 = 2 \frac{d}{dt}$ $L_1 L_2 \sin t = L_1 (t \sin t)$	t $L_{2}sint = tsin$ $L_{2}e^{2t} = te^{2t}$ $L_{2}t = t^{2}$ $Usually matters$ $L_{2} = t$ $Ut = t$	t t	(derivatives,	integrals, m	ultiplying b	py a funci	tion, et	)	
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