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ex. $\frac{d^3y}{dt^3} - 2\frac{d^2y}{dt^2} - 79\frac{dy}{dt} - 220y = 0$ P(r) = $r^3 - 2r^2 - 79r - 220$ roots are r, =-4, r <sub>2</sub> = 5, r <sub>3</sub> = 11 :. polynomial has 3 solutions:  y <sub>1</sub> = $e^{-4t}$ y <sub>2</sub> = $e^{-5t}$ y <sub>3</sub> = $e^{-1t}$ ex. Solve y <sup>11</sup> - 3y" + 2y = 0 y(6) = 1 y(0) = 0 y"(6) = 0  P(r) = $r^3 - 3r^2 + 2r = 0$ $r(r^2 - 3r + 2) = 0$ $r(r^2 - 3r + 2) = 0$ $r(r - 2)(r - 1) = 0$ $r = 0, 1, 2$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>2</sub> = $e^{1t} = e^{t}$ y <sub>3</sub> = $e^{2t}$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>2</sub> = $e^{1t} = e^{t}$ y <sub>3</sub> = $e^{2t}$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>2</sub> = $e^{1t} = e^{t}$ y <sub>3</sub> = $e^{2t}$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>2</sub> = $e^{1t} = e^{t}$ y <sub>3</sub> = $e^{2t}$ y <sub>2</sub> = $e^{0t} = 1$ y <sub>3</sub> = $e^{0t} = 1$ y <sub>4</sub> = $e^{0t} = 1$ y <sub>5</sub> = $e^{0t} = 1$ y <sub>7</sub> = $e^{0t} = 1$ y <sub>8</sub> = $e^{0t} = 1$ y <sub>9</sub> = $e^{0t} = 1$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>1</sub> = $e^{0t} = 1$ y <sub>2</sub> = $e^{0t} = 1$ y <sub>3</sub> = $e^{0t} = 1$ y <sub>4</sub> = $e^{0t} = 1$ y <sub>5</sub> = $e^{0t} = 1$ y <sub>7</sub> = $e^{0t} = 1$ y <sub>8</sub> = $e^{0t} = 1$ y <sub>9</sub> = $e^{0t} = 1$ y <sub>9</sub> = $e$	
$P(r) = r^{3} - 2r^{2} - 79r - 220$ $roots  are  r_{1} = -4,  r_{2} = -5,  r_{3} = 11  \therefore  polynomial  has  3  solutions:$ $y_{1} = e^{-4t}  y_{2} = e^{-5t}  y_{3} = e^{11t}$ $ex.  Solve  y''' - 3y'' + 2y' = 0  y(0) = 1  y'(0) = 0  y''(0) = 0$ $P(r) = r^{3} - 3r^{2} + 2r = 0$ $P(r$	
Foots are $r_1 = -4$ , $r_2 = -5$ , $r_3 = 11$ polynomial has 3 solutions. $y_1 = e^{-4t}$ $y_2 = e^{-5t}$ $y_3 = e^{1t}$ $y_4 = e^{-5t}$ $y_5 = e^{-5t}$ $y_7 = e^{-5t}$	
ex. Solve $y''' - 3y'' + 2y' = 0$ $y(0) = 1$ $y'(0) = 0$ $y'''(0) = 0$ $e(r) = r^3 - 3r^2 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r^3 + 2r = 0$ $e(r) = r^3 - 3r = 0$ $e(r)$	
ex. Solve $y''' - 3y'' + 2y' = 0$ $y(0) = 1$ $y'(0) = 0$ $y''(0) = 0$ $f(r) = r^3 - 3r^2 + 2r = 0$ $f(r^2 - 3r + 2) = 0$ $f(r - 2)(r - 1) = 0$ $f(r - 2)$	
$ \rho(r) = r^{3} - 3r^{2} + 2r = 0 $ $ r(r^{2} - 3r + 2) = 0 $ $ r(r - 2)(r - 1) = 0 $ $ r_{3} = e^{0t} = 1 $ $ y_{2} = e^{2t} = e^{t} $ $ y_{3} = e^{2t} $ $ y_{4} = e^{t} = 1 $ $ y_{2} = e^{2t} = e^{t} $ $ y_{3} = e^{2t} $ $ y_{4} = e^{t} = 1 $ $ y_{5} = e^{t} = 1 $ $ y_{7} = e^{t} = 1 $ $ y_{1} = e^{t} = 1 $ $ y_{2} = e^{t} = 1 $ $ y_{3} = e^{t} $ $ y_{1} = e^{t} = 1 $ $ y_{2} = e^{t} = 1 $ $ y_{3} = e^{t} $ $ y_{4} = e^{t} $ $ y_{5} = e^{t} $ $ y_{7} = e^{$	
$ \rho(r) = r^{3} - 3r^{2} + 2r = 0 $ $ r(r^{2} - 3r + 2) = 0 $ $ r(r - 2)(r - 1) = 0 $ $ r_{3} = e^{0t} = 1 $ $ y_{2} = e^{2t} = e^{t} $ $ y_{3} = e^{2t} $ $ y_{4} = e^{t} = 1 $ $ y_{2} = e^{2t} = e^{t} $ $ y_{3} = e^{2t} $ $ y_{4} = e^{t} = 1 $ $ y_{5} = e^{t} = 1 $ $ y_{7} = e^{t} = 1 $ $ y_{1} = e^{t} = 1 $ $ y_{2} = e^{t} = 1 $ $ y_{3} = e^{t} $ $ y_{1} = e^{t} = 1 $ $ y_{2} = e^{t} = 1 $ $ y_{3} = e^{t} $ $ y_{4} = e^{t} $ $ y_{5} = e^{t} $ $ y_{7} = e^{$	
$P(r) = r^{3} - 3r^{2} + 2r = 0$ $r(r^{2} - 3r + 2) = 0$ $r(r - 2)(r - 1) = 0 \qquad r = 0, 1, 2$ $y_{1} = e^{0t} = 1 \qquad y_{2} = e^{1t} = e^{t} \qquad y_{3} = e^{2t}$ $y_{2} = e^{1t} = e^{t} \qquad y_{3} = e^{2t}$ $y_{3} = e^{2t} = 1 \qquad y_{4} = e^{2t}$ $y_{1} = e^{2t} = 1 \qquad y_{2} = e^{2t} = 1 \qquad y_{3} = e^{2t}$ $y_{2} = e^{2t} = 1 \qquad y_{3} = e^{2t}$ $y_{3} = e^{2t} = 1 \qquad y_{4} = 1 \qquad y_{5} = 1 $	
$r(v^{2}-3r+2)=0$ $r(r-2)(r-1)=0  r=0,1,2$ $y_{1}=e^{0t}=1  y_{2}=e^{1t}=e^{t}  y_{3}=e^{2t}$ $y_{2}=e^{1t}=e^{t}  y_{3}=e^{2t}$ $y_{3}=e^{2t}$ $y_{4}=e^{2t}=1  y_{5}=e^{2t}=1$ $y_{5}=e^{2t}=1  y_{5}=e^{2t}=1$ $y_{5}=e^{2t}=1  y_{5}=e^{2t}=1$ $y_{5}=e^{2t}=1  y_{5}=e^{2t}=1$ $y_{5}=e^{2t}=1  y_{5}=e^{2t}=1$	
$r(r-2)(r-1) = 0 \qquad r = 0, 1, 2$ $y_1 = e^{0t} = 1 \qquad y_2 = e^{2t} = e^{t} \qquad y_3 = e^{2t}$ $general \qquad 50 \text{ whim:}$ $y = A + Be^{t} + Ce^{2t}$ $y(0) = A + Be^{t} + Ce^{2t} = 1 \qquad y'(0) = Be^{t} + 2Ce^{2t} = 0 \qquad y''(0) = Be^{t} + 4Ce^{2t} = 0$	
$y, = e^{0t} = 1$ $y_2 = e^{2t} = e^{t}$ $y_3 = e^{2t}$ general solution: $y = A + Be^t + Ce^{2t}$ $y(0) = A + Be^t + Ce^{2t} = 1$ $y'(0) = Be^t + 2Ce^{2t} = 0$ $y''(0) = Be^t + 4Ce^{2t} = 0$	
general solution: $y = A + Be^{t} + Ce^{2t}$ $y(0) = A + Be^{t} + Ce^{2t} = 1$ $y'(0) = Be^{t} + 2Ce^{2t} = 0$ $y''(0) = Be^{t} + 4Ce^{2t} = 0$	
general solution: $y = A + Be^{t} + Ce^{2t}$ $y(0) = A + Be^{t} + Ce^{2t} = 1$ $y'(0) = Be^{t} + 2Ce^{2t} = 0$ $y''(0) = Be^{t} + 4Ce^{2t} = 0$	
$y = A + Be^{t} + Ce^{2t}$ $y(0) = A + Be^{t} + Ce^{2t} = 1$ $y'(0) = Be^{t} + 2Ce^{2t} = 0$ $y''(0) = Be^{t} + 4Ce^{2t} = 0$	
$y(0) = A + Be^{t} + (e^{2t} = 1)$ $y'(0) = Be^{t} + 2Ce^{2t} = 0$ $y''(0) = Be^{t} + 4Ce^{2t} = 0$	
A + 8 + C = 1 B + 2C = 0 B + 4C = 0	
B = -2C 2C = 0	
A=1 B=0 C=0	
y=1 general solution	
Synthetic/Polynomial Division:	
ex. find three solutions to y"-3y'+2y=0	
$P(r) = r^3 - 3r + 2 = 0$ $= \int_{0}^{1} \int_{0}^$	
:- (r-1) divides P(r)	
$(r-1)$ $(r^3+0r^2-3r+2)$	
$\frac{(r-1)(r^2+0r^2-3r+2)}{-(r^3-r^2)} \qquad \qquad -9 \qquad (r-1)(r^2+r-2)=0$	
$(2-3i+2)$ $(-5)^2/(-5)=0$	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	e d
$\frac{-(-2r+2)}{0}$ $y_1 = e^{t}$ $y_2 = e^{-2t}$ $y_3 = te^{t}$ from double root $(r-1)^2$ , explain	