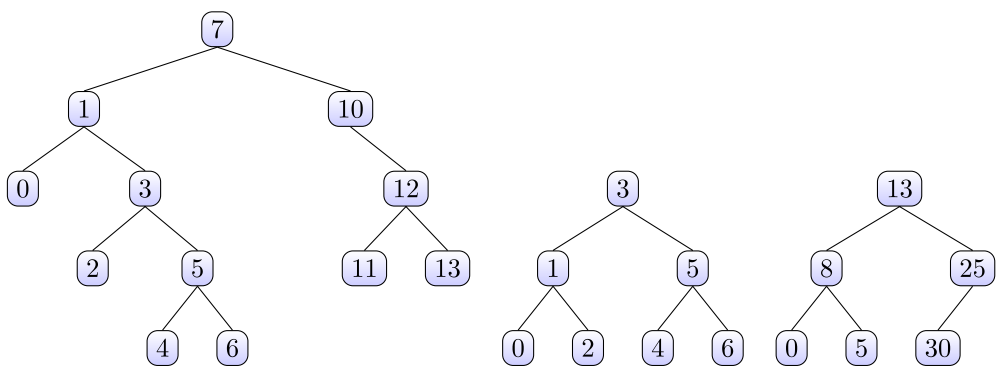
CS 284 A/B: Exercise Booklet - Trees
Fall 2016

1 Binary Tree Expressions

Exercise 1



(a) (b) (c)

1. Indicate which are full, perfect and/or complete.
2. Indicate their height.
3. List the ancestors of the node with label 11 in tree (a).
4. Indicate, for each, how many levels it has.
5. Indicate which are Binary Search Trees.
6. Every binary tree of height h has $2^h - 1$ nodes. True or false?

Exercise 2

Write the Tree Expression for each of the three trees of the previous exercise.

Exercise 3

Prove, by induction on the height h, that a perfect binary tree has $2^h - 1$ nodes.

Exercise 4

Given a perfect binary tree of height h, derive an expression $f(n)$ for the height as a function of the number of nodes n.

Exercise 5

Prove that the number of leaves in a perfect tree of height h is 2^{h-1}.

Exercise 6

From the two previous exercises, derive the number of leaves l in a perfect binary tree as a function of the number of nodes n.

Exercise 7

Prove that the number of nodes in a full binary tree of height h is between 2^{h-1} and $2^h - 1$.

2 Binary Trees

Exercise 8

Implement the following operations:

- `public int height()`
- `public int no_of_nodes()`
- `public boolean is_leaf()`
- `public int no_of_leaves()`
- `public boolean is_full()`
- `public boolean is_balanced()`. A binary tree is said to be *balanced* if both of its subtrees are balanced and the height of its left subtree differs from the height of its right subtree by at most 1.
- `public boolean is_perfect()`

Exercise 9

Implement the following operations:

- `public BinaryTree<E> copy()`
- `public BinaryTree<E> mirror()`
- `public void prune(int level)`. For example, `prune(0)` should yield the empty tree. Also, `prune(h)`, for `h` the height of the tree, should not modify the tree.

Exercise 10

A path in a binary tree is a sequence of 0s (left) and 1s (right) that leads to a node in the tree. Implement the following operations:

- `public ArrayList<ArrayList<Integer>> paths()`. Eg. for the empty tree it should return `[[]]`. For a tree such as (preorder):

```
7
  1
    2
      null
      null
    3
      null
      null
  null
```

it should produce [[0, 0, 0], [0, 0, 1], [0, 1, 0], [0, 1, 1], [1]].

- public ArrayList<ArrayList<Integer>> paths2(). Similar to previous exercise only that paths should stop at the leaves. Eg. for the example above it should return it should produce [[0, 0], [0, 1]].

3 Binary Search Trees

Exercise 11

Draw what a binary search tree would look like if the following numbers were inserted: 50 20 75 98 80 31 150 39 23 11 77.

Exercise 12

Define an operation public ArrayList<E> ancestors(E item) that returns the list of all the ancestors of item. For example, in the tree (a) of Exercise 1, given 5, it should return [7,1,3]. If the item is not in the tree, then an exception NoSuchElementException should be thrown.

Exercise 13

Define an operation public ArrayList<E> bfs() that returns the list of all the items in the tree, by levels. For example, in the tree (b) of Exercise 1, it should return [3,1,5,0,2,4,6].

Exercise 14

Define an operation public ArrayList<E> bfs2() that returns the list of all the items in the tree, by levels, where each level is a list itself. For example, in the tree (b) of Exercise 1, it should return [[3],[1,5],[0,2,4,6]].

Exercise 15

Consider the assertion: “Adding an element to a BST always increments its height”. True or false? Justify your answer

Exercise 16

What’s wrong with the following code for adding an element to a BST?

```java
public boolean add(E item) {
    root = add(root, item);
    return addReturn;
}

private Node<E> add(Node<E> localRoot, E item) {
   if (localRoot == null) {
       // item is not in the tree, insert it.
       addReturn = true;
       return new Node<E>(item);
   } else if (item.compareTo(localRoot.data) == 0) {
       // item is equal to localRoot.data
       addReturn = false;
       return localRoot;
   } else if (item.compareTo(localRoot.data) < 0) {
       // item is less than localRoot.data
       add(localRoot.left, item);
       return localRoot;
   } else {
       // item is greater than localRoot.data
```

```
      add(localRoot.right, item);
      return localRoot;
    }
}
```

Exercise 17

Define a method in the class `BinaryTree<E>` that returns a list with all the subtrees located at a given depth. Here are the two methods you should add to the above mentioned class, one of which you have to complete.

```
private ArrayList<BinaryTree<E>> projectLevel(Node<E> localRoot, int l) {
  // Complete here
}

public ArrayList<BinaryTree<E>> projectLevel(int l) {
  return projectLevel(this.root,l);
}
```

For example, if we use the binary tree from the file `Fig_6_12.txt` then the output of

```
FileReader fin = new FileReader("Fig_6_12.txt");
Scanner src = new Scanner(fin);
BinaryTree<String> tree = BinaryTree.readBinaryTree(src);
System.out.println(tree);
System.out.println(tree.projectLevel(0));
System.out.println(tree.projectLevel(1));
System.out.println(tree.projectLevel(2));
System.out.println(tree.projectLevel(3));
System.out.println(tree.projectLevel(4));
```

would be

```
3
  7
    null
    null
  8
    6
      null
      0
        null
        null
    null

[3
  8
    null
    6
      0
        null
        null
      null
  7
    null
    null
]
[8
  null
  6
    0
```

```
      null
      null
    null
, 7
  null
  null
]
[6
  0
    null
    null
  null
]
[0
  null
  null
]
[]
```

Exercise 18

Use the previous exercise and the height of a tree to implement the following operation, to be included in `BinaryTree<E>`, that returns a list with all the subtrees of a binary tree:

```
public ArrayList<BinaryTree<E>> st() {
  // Complete here
}
```

Sample output for `System.out.println(tree.st());` from the tree of the previous exercise would be:

```
[3
  8
    null
    6
      0
        null
        null
      null
  7
    null
    null
, 8
  null
  6
    0
      null
      null
    null
, 7
  null
  null
, 6
  0
    null
    null
  null
, 0
  null
  null
]
```

Exercise 19

Suppose we have int values between 1 and 1000 in a BST and search for 363. Which of the following cannot be the sequence of keys examined.

1. 2 252 401 398 330 363
2. 399 387 219 266 382 381 278 363
3. 3 923 220 911 244 898 258 362 363
4. 4 924 278 347 621 299 392 358 363
5. 5 925 202 910 245 363

Exercise 20

Draw the BST resulting from removing the following elements from the tree below:

1. kept
2. cow

References

- `http://www.cs.washington.edu/143/`
- `http://www.cs.princeton.edu/courses/archive/fall13/cos126/precepts/BSTex.pd`