My Exam

- You have a certain amount of time to work on this exam.
- This exam is worth 50% of your final grade, so move with some alacrity.
- There are 3 questions, each of which is worth 30 points. You get 10 points for writing your name on all pages.

The grading scheme is as follows:

Question	Points	Score
0 (Name)	10	
1	30	
2	30	
3	30	

By signing below I affirm that I have read and intend to adhere to My University's code of Academic Integrity:

$\times$ _____

1. What is the airspeed velocity of an unladen swallow?

_____

Depends whether the swallow is African or European

2. Integrate $\sin x$ with respect to γ.

$$\int \sin x \, \mathrm{d}\gamma = \gamma \sin x + c$$

3. Solve the following integral:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, \mathrm{d}x.$$

Show your work.

First, let $I = \displaystyle\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, \mathrm{d}x$. Then,

$$\begin{aligned} I^2 &= \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, \mathrm{d}x \right)^2 \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\frac{1}{\sqrt{2\pi}} \right)^2 e^{-\frac{x^2}{2}} e^{-\frac{y^2}{2}} \, \mathrm{d}x \, \mathrm{d}y \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{x^2+y^2}{2}} \, \mathrm{d}x \, \mathrm{d}y \end{aligned}$$

Then, by polar conversion:

$$\begin{aligned} I^2 &= \int_0^{2\pi} \int_0^{\infty} \frac{1}{2\pi} e^{-\frac{r^2}{2}} \, r \, \mathrm{d}r \, \mathrm{d}\theta \\ &= 2\pi \int_0^{\infty} \frac{1}{2\pi} e^{-\frac{r^2}{2}} \, r \, \mathrm{d}r \\ &= \int_0^{\infty} e^{-\frac{r^2}{2}} \, r \, \mathrm{d}r \end{aligned}$$

Now, perform the u-substitution $u = \dfrac{r^2}{2}$. We get $\mathrm{d}u = \dfrac{\mathrm{d}}{\mathrm{d}r}\left(\dfrac{r^2}{2}\right) = r$. Then:

$$\begin{aligned} I^2 &= \frac{1}{2} \int_0^{\infty} e^{-u} \, \mathrm{d}u \\ &= \frac{1}{2} \left[-e^{-u} \right]_0^{\infty} \\ &= \frac{1}{2} \left[-e^{-\infty} - (-e^0) \right] \\ &= \frac{1}{2} \end{aligned}$$

Thus, $I = \sqrt{\dfrac{1}{2}} = \boxed{\dfrac{1}{\sqrt{2}}}$.