

LP modeling tricks

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Overview

- ▶ Encoding various types of constraints ($=, \leq, \geq$)
- ▶ Other relationships (absolute value, ranges, maximum, minimum)
- ▶ Negative values with nonnegative decision variables
- ▶ Limitations of modeling with LPs

Encoding $=, \leq, \geq$

Below in the LP version column, we assume the LP is the opposite direction for \leq and \geq .

Constraint	LP version
$x \leq c$	$-x \geq -c$
$x \geq c$	$-x \leq -c$
$x = c$	$x \leq c; -x \leq -c$

Other relationships: absolute value

Say we have the below constraint:

$$|x| \leq t$$

We can turn this into the following constraints:

$$x \geq -t; \quad x \leq t$$

We can't do $|x| \geq t$ though, because then the space is no longer convex:

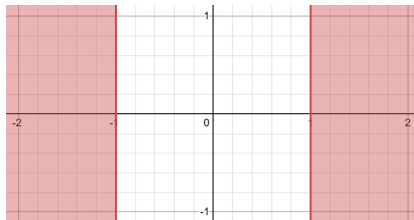


Figure 1: Feasible region for $|x| \geq 1$

Other relationships: range

For $a \leq x \leq b$, split the constraint into two:

$$x \geq a; x \leq b$$

Convert \leq to \geq or vice-versa if needed to get to canonical form.

Other relationships: max and min

For $\max(x_1, x_2, \dots, x_n)$, define some variable z as follows:

$$z \geq x_1; z \geq x_2; \dots; z \geq x_n$$

For $\min(x_1, x_2, \dots, x_n)$, define some variable z as follows:

$$z \leq x_1; z \leq x_2; \dots; z \leq x_n$$

Convert \leq to \geq or vice-versa if needed to get to canonical form.

Negative values with nonnegative decision variables

If we have some variable x that we want to be able to take on values that are both positive and negative, but we are restricted by canonical form to only use nonnegative decision variables (so $x \geq 0$):

- ▶ Define new decision variables x^+ and x^-
- ▶ Replace all instances of x with $x^+ - x^-$
- ▶ To determine the value of x given by a solution, use
$$x = x^+ - x^-$$

Limitations of modeling with LPs

The main limitation LPs have is that their feasible regions must be **convex**, as described earlier.

This means that we cannot model discrete decisions like this:

- ▶ If $x \geq 1$, then $y = 3$, otherwise $y = 0$
- ▶ Either $x = 0$ or $x = 1$

Essentially, anything that splits the feasible region into two parts is not permitted.

We will see with ILP (integer linear programming) that permitting a non-convex feasible region allows us to model significantly more complicated relationships.