The problem

$$
\begin{aligned}
&\frac{dX_\epsilon(t,\frac{t}{\epsilon},X_\epsilon)}{dt} = a(t,\frac{t}{\epsilon},X_\epsilon)\\
&a(t,\frac{t}{\epsilon},X_\epsilon) = -X_\epsilon cos(2\pi\frac{t}{\epsilon})\\
&X_\epsilon(0) = x
\end{aligned}
\tag{1}
$$

X0 calculation

$$
\begin{aligned}
&\frac{\partial X_0}{\partial t} = \int_0^1 a(t,\sigma,X_0)d\sigma = 0\\
&X_0(0) = x => X_0(t) = x
\end{aligned}
\tag{2}
$$

Y1 calculation

$$
\begin{aligned}
\frac{\partial Y_1}{\partial t} &= -\{\int_0^1 \nabla_x a(t,\sigma,X_0)d\sigma\}Y_1\\
&+\{\int_0^1\int_0^\sigma \nabla_x a(t,\zeta,X_0)d\zeta - \int_0^1 \nabla_x a(t,\zeta,X_0)d\zeta\}(\int_0^1 a(t,\sigma,X_0)d\sigma)\\
&-\int_0^1\{\nabla_x a(t,\sigma,X_0)\}(\int_0^\sigma a(t,\zeta,X_0)d\zeta - \sigma\int_0^1 a(t,\zeta,X_0)d\zeta)d\sigma\\
&-(\int_0^1\int_0^\sigma \frac{\partial a(t,\zeta,X_0)}{\partial t}d\zeta d\sigma - \int_0^1 \frac{\partial a(t,\zeta,X_0)}{\partial t}d\zeta)
\end{aligned}
\tag{3}
$$

Intermediate calculations

$$
\begin{aligned}
&\frac{\partial a(t,\sigma,X_0)}{\partial t} = \frac{\partial a}{\partial t} + \frac{\partial a}{\partial X}\frac{\partial X}{\partial t} + \frac{\partial a}{\partial \sigma}\frac{\partial \sigma}{\partial t}\\
&=> \frac{\partial a(t,\sigma,X_0)}{\partial t} = \frac{2\pi X_0}{\epsilon}sin(2\pi\sigma) = \frac{2\pi\sigma X_0}{t}sin(2\pi\sigma)
\end{aligned}
\tag{4}
$$

$$\int_0^1 a(t,\sigma,X_0)d\sigma = 0 \tag{5}$$

$$\int_0^1 \nabla_x a(t,\sigma,X_0)d\sigma = 0 \tag{6}$$

$$\int_0^1\int_0^\sigma \nabla_x a(t,\zeta,X_0)d\zeta d\sigma = 0 \tag{7}$$

$$\int_0^1\{\nabla_x a(t,\sigma,X_0)\}(\int_0^\sigma a(t,\zeta,X_0)d\zeta)d\sigma = 0 \tag{8}$$

$$\int_0^1 \frac{\partial a(t, \zeta, X_0)}{\partial t} d\zeta = 0 \tag{9}$$

$$-\int_0^1 \int_0^\sigma \frac{\partial a(t, \zeta, X_0)}{\partial t} d\zeta d\sigma = 0 \tag{10}$$

we get

$$\frac{\partial Y_1}{\partial t} = 0 \tag{11}$$

$$\begin{aligned} Y_1(t) = C; Y_1(s, X, s) = 0 => C = 0 \\ Y_1(t) = 0 \end{aligned} \tag{12}$$

X1 calculation

$$X_1(t, \tau) = Y_1(t) + \int_0^\tau a(t, \sigma, X_0) d\sigma - \tau \int_0^1 a(t, \sigma, X_0) d\sigma \tag{13}$$

$$X_1(t, \tau) = -\frac{sin(2\pi\tau)}{2\pi} \tag{14}$$

Final Result

$$\begin{aligned} X_\epsilon(t) = X_0(t) + \epsilon X_1(t, \frac{t}{\epsilon}) + ... \\ X_\epsilon(t) = x - \epsilon \frac{sin(2\pi \frac{t}{\epsilon})}{2\pi} \end{aligned} \tag{15}$$