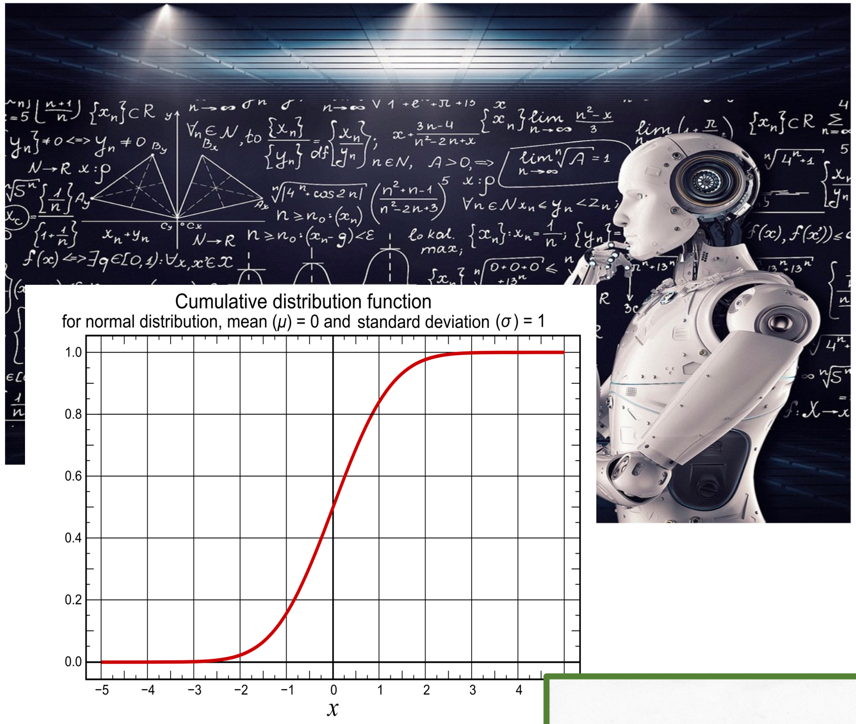
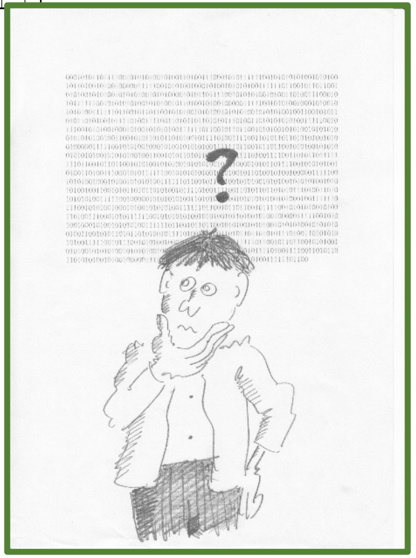
Inverse Transform Sampling





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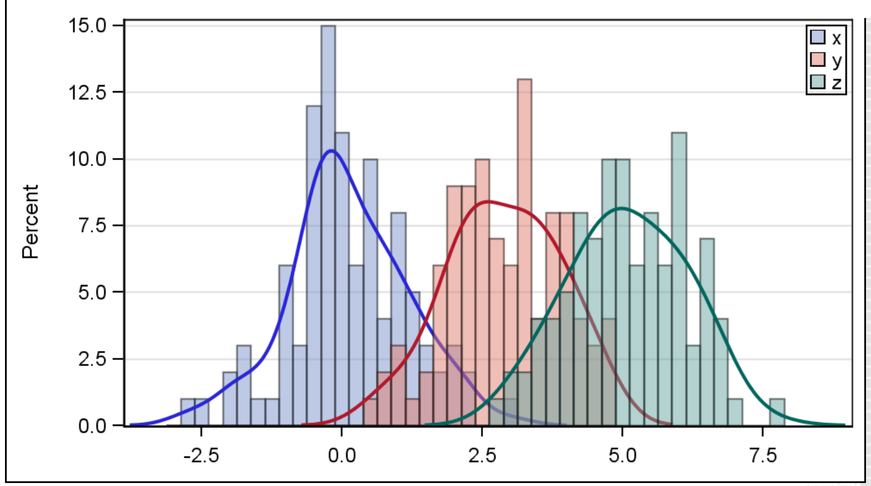

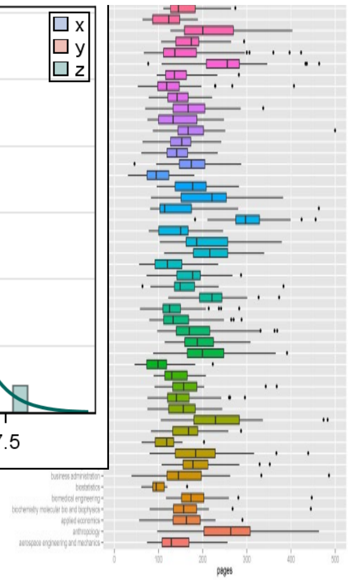

The distribution of $F(X)$

- Generate 1000 samples from a standard normal distribution: $x_1, \dots, x_{1000}$
- Trace each sampled value to its CDF: $c_i = \Phi(x_i)$

- How many c_i s do we expect to see in $[-5,-3]$?
- How many c_i s do we expect to see in $[0,1]$?
- How many c_i s do we expect to see in $[0,0.2]$?
- How many c_i s do we expect to see in $[0.4,0.6]$?

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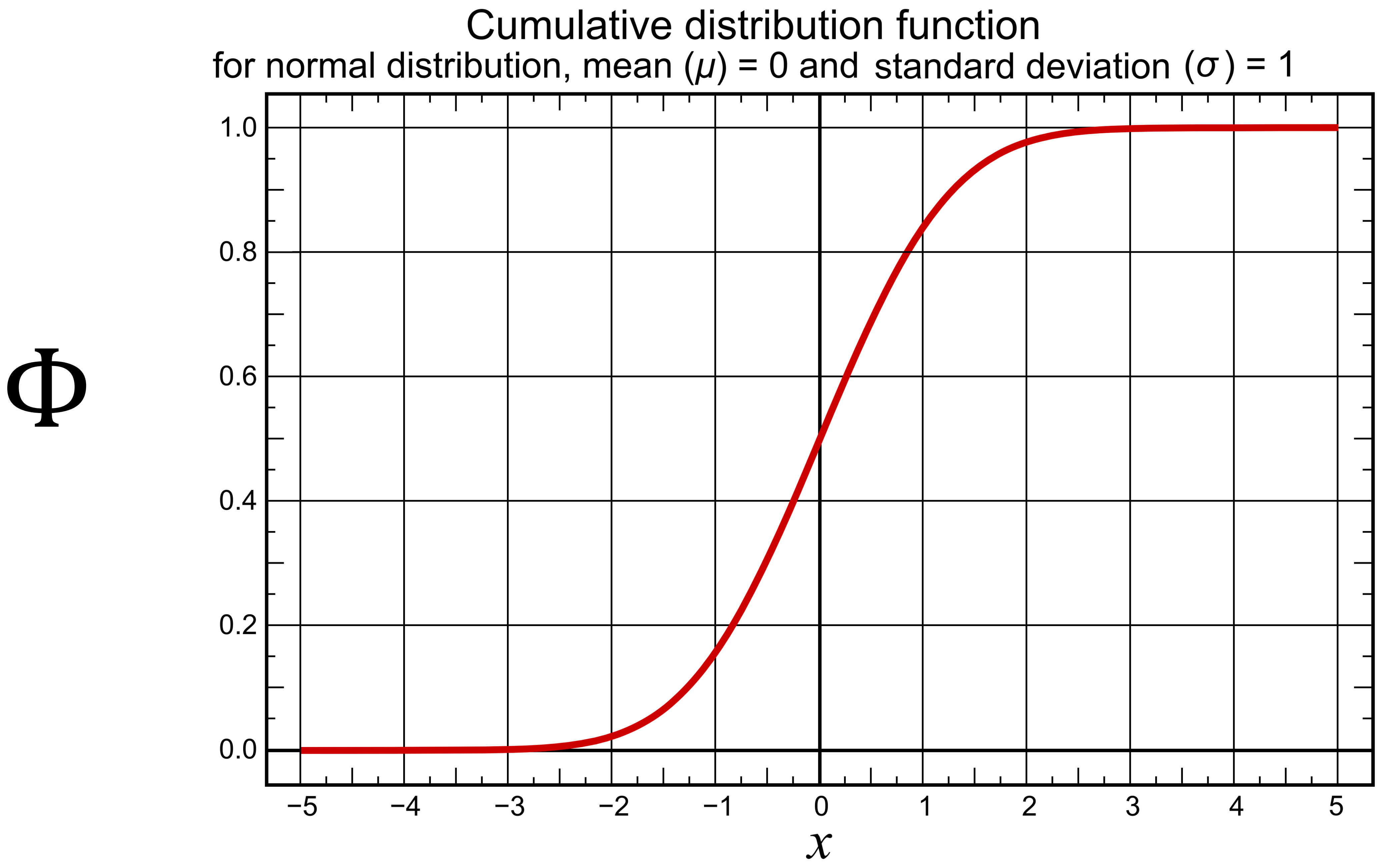
Cumulative distribution function
for normal distribution, mean (μ) = 0 and standard deviation (σ) = 1
1.0
0.8
0.6
0.4
0.2
0.0
−5
−4
−3
−2
−1
0
1
2
3
4
5
x
Φ

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- How many c_i s do we expect to see in $[0,1]$?
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- We can transform a sample from any distribution into a uniform sample, using the CDF of that distribution.
- By applying the inverse function of the CDF, we would be able to map the uniform distribution to a sample from any distribution we want

Inverse Mapping

- Formally, if X is a continuous random variable with cumulative distribution function F_X, then the random variable $U = F_X(X)$ has a uniform distribution on $[0,1]$.

- If U has a uniform distribution on $[0,1]$ and if X has a cumulative distribution F_X then the random variable $F_X^{-1}(U)$ has the same distribution as X.

$$P(a \le F(X) \le b)$$

$$P\left(\underset{u}{F^{-1}(a)} \le X \le \underset{v}{F^{-1}(b)}\right)$$

$$= F(\underset{v}{F^{-1}(b)}) - F(\underset{u}{F^{-1}(a)})$$

$$= b - a$$

$$\underline{P}\left(a \leq F^{-1}(U) \leq b\right)$$

$$\underline{P}\left(F(a) \leq U \leq F(b)\right)$$

$$= F(b) - F(a)$$

Practicalities

- Let X be a RV and let F_X be the CDF of X

- Generate a random number u from the standard uniform distribution U
- Compute $x = F_X^{-1}(u)$
- This procedure yields sampling from X