PHYS3990 HW2

October 4, 2024

1. 2d Fourier Transform Separation Show that the two-dimensional Fourier transform (which we often take on the transverse spatial coordinates x, y in diffraction calculations) of a separable function $f(x, y)$ is the product of one-dimensional Fourier transforms of the separate functions that compose $f(x, y)$, $X(x), Y(y)$.

Given:

$$f(x,y) = X(x)Y(y) \tag{1}$$

$$FT_{x,y}\{f(x,y)\}(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy f(x,y)e^{-ik_x x - ik_y y} \tag{2}$$

Show:

$$FT_{x,y}\{f(x,y)\}(k_x,k_y) = FT_{x,y}\{X(x)Y(y)\}(k_x,k_y) = \left(FT_x\{X(x)\}(k_y)\right)\left(FT_y\{Y(y)\}(k_y)\right) \tag{3}$$

2. Composing 1d diffractions Consider the single Fourier transform Fresnel diffraction in one dimension, for x, and y transverse axes, with z (L, propagation distance) dependence removed.

$$A'(y') \propto e^{\frac{i\pi}{\lambda L}y'^2} FT_y\{A(y)e^{\frac{i\pi}{\lambda L}y^2}\}(\frac{y'}{\lambda L}) \tag{4}$$

$$B'(x') \propto e^{\frac{i\pi}{\lambda L}x'^2} FT_x\{B(x)e^{\frac{i\pi}{\lambda L}x^2}\}(\frac{x'}{\lambda L}) \tag{5}$$

Multiply the above two forms and show $f'(x', y') = A'(y')B'(x')$ can be expressed with a 2d Fourier transform, specifically $FT_{x,y}\{e^{\frac{i\pi}{\lambda L}(x^2+y^2)}f(x,y)\}$ where $f(x, y) = A(y)B(x)$. Equation 3 will be very useful to answer this question.