

We want to determine if some function implemented on a quantum computer is satisfiable ie $f(x) = 1$ for x from $\{0,1\}^n$. To do this, run quantum period finding on the function $f(x)$, if its period is greater than 1, then surely it is satisfiable. Period finding can be done in polynomial time $O(\text{poly}(n))$ quite fast. There are only four functions without output in $\{0,1\}$ with period one: $f(x) = \{(0,1),(1,0), (2,1),(3,0)\dots\}, \{(0,0),(1,0), (2,0),(3,0)\dots\}, \{(0,1),(1,1), (2,1),(3,1)\dots\}, \{(0,0),(1,1), (2,0),(3,1)\dots\}$. Once we know the period, we can know if our function is $f(x) = 0$ for all x , by simply testing a handful of values such as $f(0), f(1)$. The time to run period finding and do this small test is still polynomial in n . If there is worry about the period being too large, if say only 1 value of x gives $f(x) = 1$, then the period is 2^n , if this is an issue append an ancilla as the most significant bit to the input, but don't give it to the oracle for $f(x)$, just send the ancilla into oblivion. Then the period for such a contrived f will be half the total domain length. This seems to be exactly like the class NP, since quantum period finding is probabilistic but fast.