When you have a laser, everything looks like a diffraction pattern

Benjamin Schreyer

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The question is asked, how long does a suddenly ionized hydrogen atom take for its wave function to spread to the Bohr radius. We can answer this question quickly with axial optics.

Since clasically the proton has no momentum we use the free Schrödinger equation for a massive particle. Hopefully relativistic corrections dont matter, and hopefully a Gaussian models the initial wavefunction well.

$$-2i\frac{m}{\hbar}\frac{\partial}{\partial t}\psi = \Delta\psi \tag{1}$$

Now observe the paraxial dynamics of the EM field:

$$-2i\frac{k}{c}\frac{\partial}{\partial t}u = \Delta_\perp u \tag{2}$$

There is a direct correspondence between these equations by simply adding a transverse dimension to our optical propagation. Adding this third transverse axis is as easy as going from one dimensional diffraction patterns to two dimensional diffraction patterns. For axial optical propagation we have the following where $\hat{u}$ is the Fourier transform:

$$u(x, y, z, ct) = \hat{u}(\frac{2\pi}{\lambda ct}x, \frac{2\pi}{\lambda ct}y, \frac{2\pi}{\lambda ct}z) \tag{3}$$

Then a Gaussian with $\sigma^2(t=0)$ has width $\sigma^2(t) = \frac{\lambda^2c^2t^2}{4\pi^2\sigma^2(t=0)}$. So long as the Fraunhofer approximation holds $ct >> \frac{\sigma^2}{\lambda}$. Let us correspond variables to find that for the spreading proton the solution.

$$\sigma^2(t) = \frac{\hbar^2t^2}{4\pi^2m^2\sigma^2(t=0)} \tag{4}$$

The Fraunhofer condition is as follows.

$$t >> \frac{m}{\hbar}\sigma^2 \tag{5}$$

For the proton spreading to the Bohr radius it takes $10^{-19}s$ and the approximation holds by three orders of magnitude.