Benjamin Schreyer Feb 3rd 2026

Also propose the invariant on simple graphs $g^i V_1 \cdot g^j V_1$ where $g = G - G^{\neg}$ the subtraction of the upper right graph formed from the adjacency matrix zeroing anything **under** the diagonal. Given two DAGs, apply topological sort so that they are both upper triangular, then apply the transformation where 0s in the upper triangle go to -1, except on the diagonal where it is left 0. Then apply the following algorithm to find a permutation between them. Let A and B be the modified adjacency matrices at that point. If they are isomorphic:

$$B = P^T A P \tag{1}$$

Multiply by the vector of all 1s V_1

$$B^0 V_1 = (P^T A P)^0 V_1 \tag{2}$$

This holds for higher powers of the matrices

$$B^k V_1 = (P^T A P)^k V_1 \tag{3}$$

Since $PV_1 = V_1$ we have found a system of equations on P. Now we need to show that $A^K V_1$ does not vanish before we have enough equations ($k = N - 1$ is needed). This because the sequence of vectors is linearly independent (a Krylov subspace) so as long as they dont vanish were good.

$$B^k V_1 = P^T A^k V_1 \tag{4}$$

Consider the sequence $u^{(k)}$ of the succesive powers A^k on V_1. By the upper triangular form entry $n - k$ and above of $u^{(k)}$ is zero. Consider the last nonzero entry at step k to be m. We show that at step $k + 1$ entry $m - 1$ is nonzero.

$$u_{m-1}^{(k+1)} = \sum_{j=1}^{n} A_{m-1,j} u_j^{(k)} \tag{5}$$

The sum only runs to m since entries above are 0, and the start of the sum is the row plus 1 since we have upper triangular form.

$$u_{m-1}^{(k+1)} = \sum_{j=m}^{m} A_{m-1,j} u_j^{(k)} \tag{6}$$

Then $u_{m-1}^{(k+1)} = A_{m-1,m} u_m^{(k)}$ which is nonzero by inductive hypothesis and structure of A.